Cardiac Motion Estimation by Using Convolutional Sparse Coding

PhD(c) Nelson Diaz[1]
Ph.D Adrian Basarab [3]
Ph.D Jean-Yves Tourneret [3]
Advisor: Ph.D Henry Arguello [2]

[1]Department of Electrical and Computer Engineering,
[2]Department of Computer Science
Universidad Industrial de Santander, Bucaramanga, Colombia.
[3]University of Toulouse, Toulouse, Francia.

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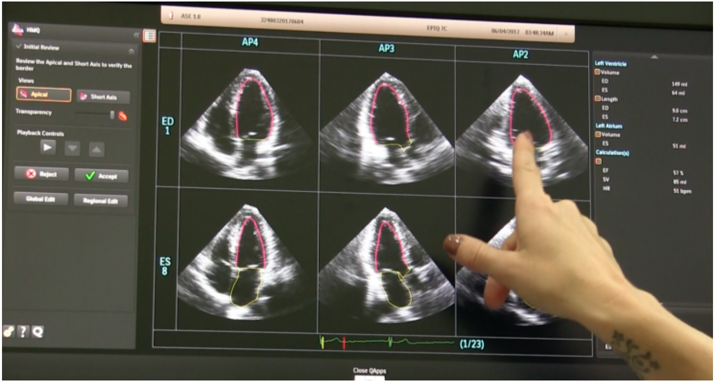
Outline

Motivation

- Optimal treatment of cardiac disease requires early detection of abnormalities and accurrate monitoring tools.



Introduction to convolutional sparse coding (CDL)

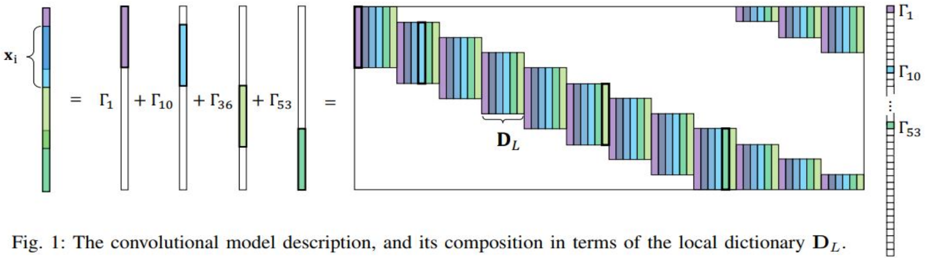


Fig. 1: The convolutional model description, and its composition in terms of the local dictionary $\mathbf{D}_L$.

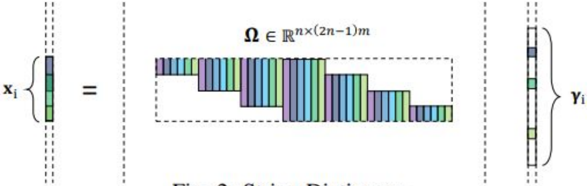
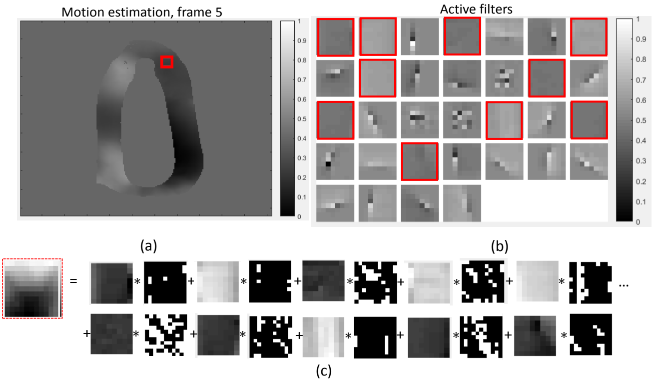


Fig. 2: Stripe Dictionary

Example of CDL in Ultrasound Imaging



$\mathbf{s}_k$ is modeled as a convolution between the coefficient maps $\mathbf{x}_m$ and a set of M filters $\mathbf{d}_m$.

$$\mathbf{s}_k \approx \sum_{m=1}^{M} \mathbf{d}_m * \mathbf{x}_m \tag{1}$$

Dictionary filters and coefficient maps

A dictionary is estimated off-line by using a set of training cardiac motions denoted as $\hat{\mathbf{s}}$.

$$\underset{\mathbf{d}_m,\mathbf{x}_{k,m}}{\operatorname{argmin}} \frac{1}{2}\sum_k \left\| \sum_m \mathbf{x}_{k,m} * \mathbf{d}_m - \mathbf{s}_k \right\|_2^2 + \lambda \sum_m \sum_k \|\mathbf{x}_{k,m}\|_1 \quad (2)$$

$$\text{s.t.} \quad \|\mathbf{d}_m\| = 1 \ \forall m = 1, ..., M$$

The coefficient maps $\mathbf{x}_m$ are computed from cardiac motions $\mathbf{s}_k$

$$\underset{x_m}{\operatorname{argmin}} \frac{1}{2} \left\| \sum_m \mathbf{x}_m * \mathbf{d}_m - \mathbf{s}_k \right\|_2^2 + \lambda \sum_m \|\mathbf{x}_m\|_1 . \quad (3)$$

Eq (2) and (3) can be solved using alternating direction method of multipliers (ADMM).

Motion estimation

- A pair of successive frames $(\mathbf{r}_k, \mathbf{r}_{k+1}) \in \mathbb{R}^{J \times N}$ acquired at time instants k and $k+1$
- $(\mathbf{s}_x, \mathbf{s}_y) \in \mathbb{R}^{J \times N}$ where $\mathbf{s}_x$ and $\mathbf{s}_y$ are the motions along the x and y axes.
- Since the motion estimation problem is considered independently the displacement vector is equal to $\mathbf{s} = \mathbf{s}_x$ or $\mathbf{s} = \mathbf{s}_y$.
- The motion estimation field is formulated as the minimization of a cost function with energy $E_{\text{data}}(\mathbf{s})$ penalized by spatial and sparse regularizations, i.e.,

$$\underset{\mathbf{x},\mathbf{s}}{\operatorname{argmin}} \left\{ E_{\text{data}}(\mathbf{s}) + \lambda_d E_{\text{sparse}}(\mathbf{s}, \mathbf{x}) + \lambda_s E_{\text{spatial}}(\mathbf{s}) \right\} \tag{4}$$

Data fidelity

The ML estimator is classically computed in the negative log-domain

$$\underset{\mathbf{s}}{\operatorname{argmin}} -\ln\left[p(\mathbf{r}_{k+1})|\mathbf{r}_k(n)\right]. \tag{5}$$

Straightforward computations exploiting the Rayleigh statistics of ultrasound imaging detailed in [1] lead to the following data fidelity term

$$E_{\mathrm{data}}(\mathbf{s}) = -2d(\mathbf{s}) + 2\log[e^{2d(\mathbf{s})} + 1] + C \tag{6}$$

where

$$d(\mathbf{s}) = \frac{1}{b}\sum_{n=1}^{N}[\mathbf{r}_{k+1}(n + \mathbf{s}(n)) - \mathbf{r}_k(n)],$$

n indicates the pixel index, $\mathbf{s} = [s(1), \ldots, s(N)]^T$ is the vectorized motion, and $\mathbf{r}_k = [r_k(1), \ldots, r_k(N)]^T$ is the vectorized ultrasound image in frame k, and $C = -\log(2\sigma^4/b)$ is a known constant

Regularization term

The spatial regularization term promotes the smoothness of the motion estimation field and is defined as

$$E_{\mathrm{spatial}}(\mathbf{s}) = \|\nabla \mathbf{s}\|_2^2 \tag{7}$$

The proposed sparse regularization determines the motion $\mathbf{s}_k$ that is best represented as a convolution between M filters $\mathbf{d}_m$ and the coefficient maps $\mathbf{x}_m$, i.e,

$$E_{\mathrm{sparse}}(\mathbf{s}) = \left\| \mathbf{s}_k - \sum_{m=1}^{M} \mathbf{x}_m * \mathbf{d}_m \right\|_2^2 . \tag{8}$$

Motion estimation algorithm

Input: $\mathbf{r}_{b,1}, \mathbf{r}_{b,2}, \lambda_s, \lambda_d, K, J, \lambda, \rho,$
$\widetilde{\mathbf{s}}_0 =$ LADdist motions, $\widehat{\mathbf{s}}_0 =$ LADprox motions

Output: $\mathbf{s}$

function $\mathrm{MEFCDL}(\mathbf{r}_{b,1}, \mathbf{r}_{b,2}, \lambda_s, \lambda_d, K, J, \lambda, \rho, \widetilde{\mathbf{s}}_0, \hat{\mathbf{s}}_0)$
    $\mathbf{d}_m \leftarrow$ Computes the dictionary by solving (2)
    $\mathbf{x}_m \leftarrow$ Computes the coefficient maps by solving (3)
    for $k \leftarrow 1, K$ **do**
        for $j \leftarrow 1, J$ **do**
            $\mathrm{argmin}_{\mathbf{s}} \{ E_{\mathrm{data}}(\mathbf{r}_{b,1}, \mathbf{r}_{b,2}, \mathbf{s}_{j-1}) + \lambda_s \|\nabla \mathbf{s}_{j-1}\|_2^2 + \lambda_d(k) \|\mathbf{s}_{j-1} - \sum_m \mathbf{d}_m * \mathbf{x}_m\|_2^2 \}$
            s.t. $\|\mathbf{d}_m\| = 1 \ \forall m$ ▷ Motion estimation
    return $\mathbf{s}$ ▷ (Estimated motion field)

Experimentation setup

Table 1: Parameters for each step of algorithm 7, dictionary learning, sparse coding, and cardiac motion estimation.

Step	**Parameters**	**Values**
Dictionary learning	Database	LADdist
	Filter size	16×16
	Filters number	$M = 48$
	Sparsity term	$\lambda = 0.001$
	Number of iteration	500
Sparse coding	Database	LADprox
	Number of iteration	500
Cardiac motion estimation	Regularization parameter	$\lambda_s = 0.75$
	Sparsity term (Systole)	$\lambda_d = \{1 \times 10^{-6} \times 10^{-3}\}$
	Sparsity term (Diastole)	$\lambda_d = \{1 \times 10^{-9} \times 10^{-2}\}$

Parameters λ and ρ

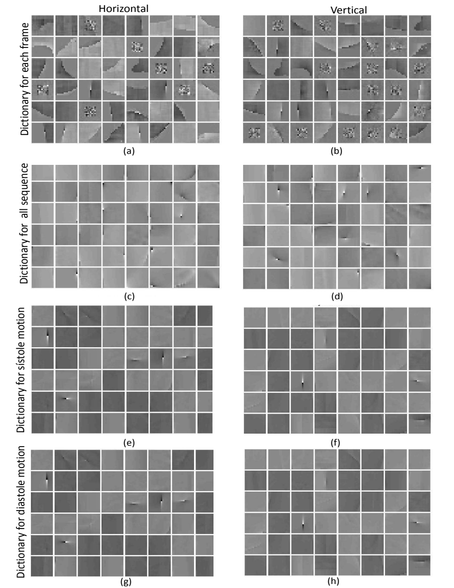
	$\rho_0 = 25$	$\rho_1 = 50$	$\rho_2 = 100$	$\rho_3 = 150$
$\lambda_0 = 0.05$	28.9992	29.2101	28.6905	**29.3802**
$\lambda_1 = 0.1$	24.9587	24.9932	25.2910	25.2395
$\lambda_2 = 0.2$	20.9748	21.1248	21.2963	21.3028
$\lambda_3 = 0.3$	18.7247	18.8688	18.8703	18.8667
$\lambda_4 = 0.5$	16.1208	16.1271	15.9824	16.0115

Table 2: Image quality (PSNR) with 100 iteration of (CBPDN) for different choices of λ and ρ by using the motion horizontal ground-truth.

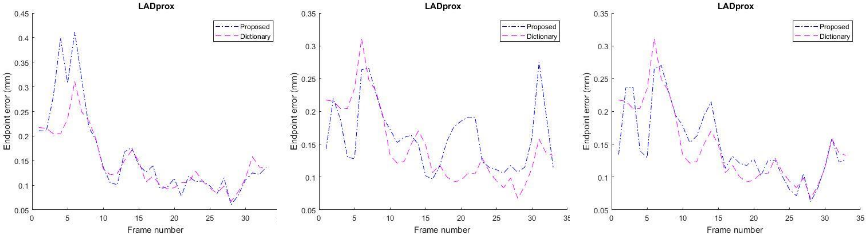
	$\rho_0 = 25$	$\rho_1 = 50$	$\rho_2 = 100$	$\rho_3 = 150$
$\lambda_0 = 0.05$	26.9361	27.2072	27.2159	**27.3542**
$\lambda_1 = 0.1$	22.8906	22.8575	22.9179	23.1302
$\lambda_2 = 0.2$	19.2978	19.2049	19.3376	19.3352
$\lambda_3 = 0.3$	17.3294	17.4144	17.4043	17.3599
$\lambda_4 = 0.5$	15.1404	15.2372	15.2145	15.1526

Table 3: Image quality (PSNR) with 100 iteration of (CBPDN) for different choices of λ and ρ by using the motion horizontal ground-truth.

3 dictionaries, 3 scenarios



Mean Endpoint Error



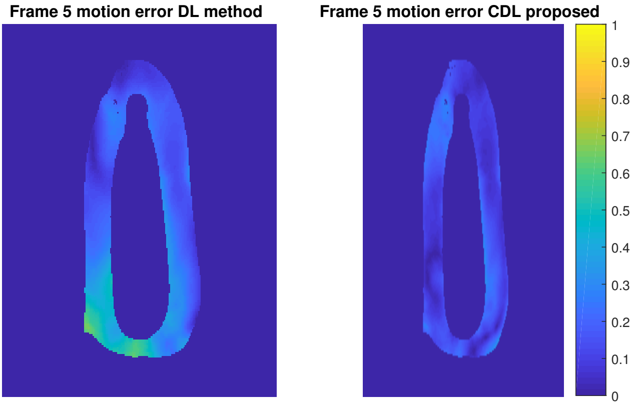
(a) (b) (c)

This error is defined for the nth pixel as

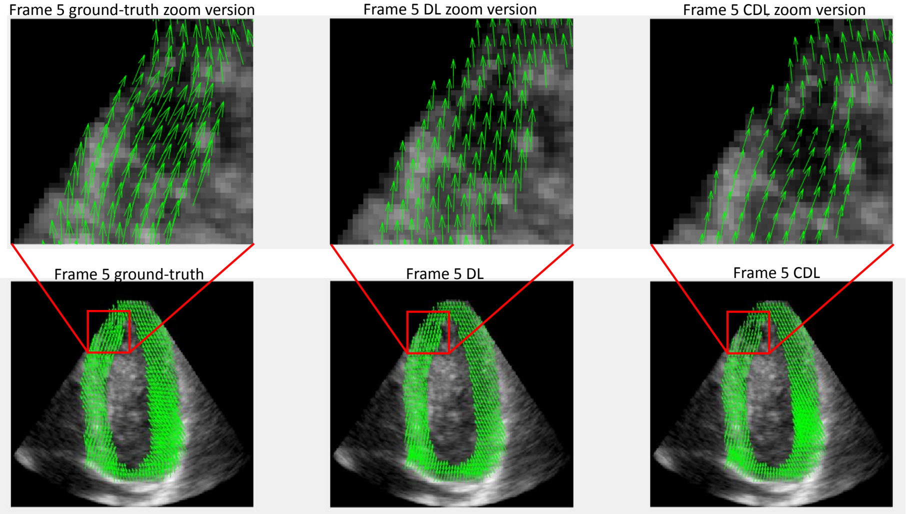
$$e_n = \sqrt{[\mathbf{s}_x(n) - \hat{\mathbf{s}}_\mathbf{x}(n)]^2 + [\mathbf{s}_y(n) - \hat{\mathbf{s}}_y(n)]^2}, \tag{9}$$

where $\mathbf{s}_x(n)$, $\mathbf{s}_y(n)$, $\hat{\mathbf{s}}_x(n)$, $\hat{\mathbf{s}}_y(n)$ are the true and estimated (horizontal and vertical) motions at pixel n.

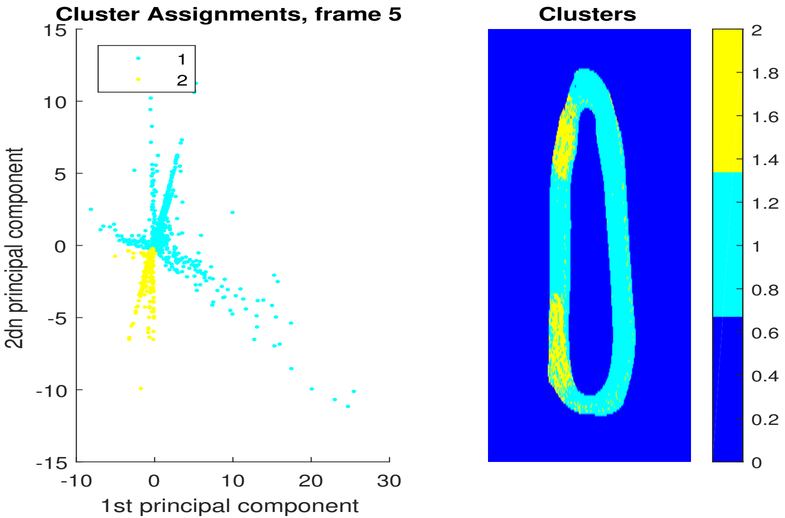
Error maps estimation



Estimation motion comparison

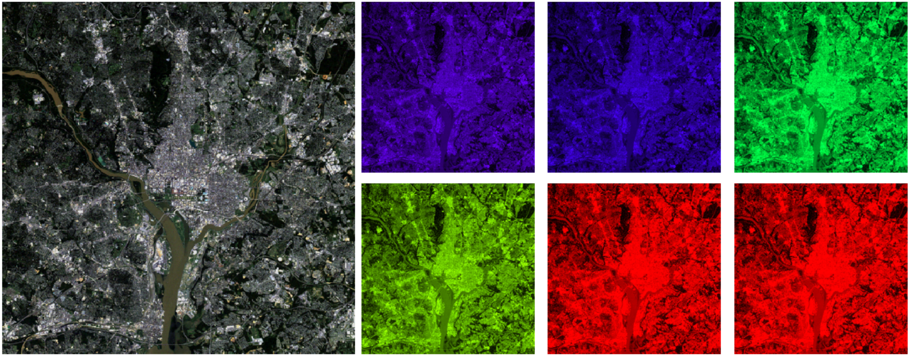


Clustering the PC of **x** (kmeans with 2 classes)



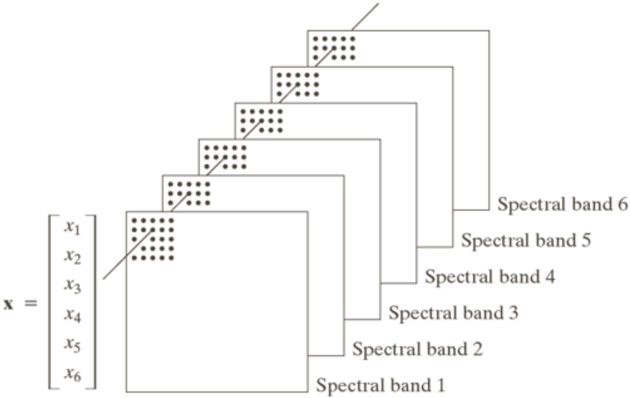
PCA for multispectral imaging 1

- **Implement the principal component transform to compress and reconstruct multispectral images.**



PCA for multispectral imaging 2

- **Understand the applications of satellital images in agriculture, geology and surveillance** (you will need to understand which is the structure multispectral images)



Understand the mathematics that PCA involves.

Compute the mean vector $\mathbf{m}_x = \frac{1}{K}\sum_{k=1}^{K}\mathbf{x}_k$ and the covariance matrix $\mathbf{C}_x = \frac{1}{K}\sum_{k=1}^{K}\mathbf{x}_k\mathbf{x}_k^T - \mathbf{m}_k\mathbf{m}_k^T$ Let $\mathbf{A}$ a matrix whose rows are formed from the eigenvectors of $\mathbf{C}_\mathbf{x}$ ordered in descending order,

$$\mathbf{y} = \mathbf{A}(\mathbf{x} - \mathbf{m}_x) \tag{10}$$

Because the rows of $\mathbf{A}$ are orthonormal vectors, it follows that $\mathbf{A}^{-1} = \mathbf{A}^T$, and any vector $\mathbf{x}$ can be recovered from its corresponding $\mathbf{y}$ by using the expression

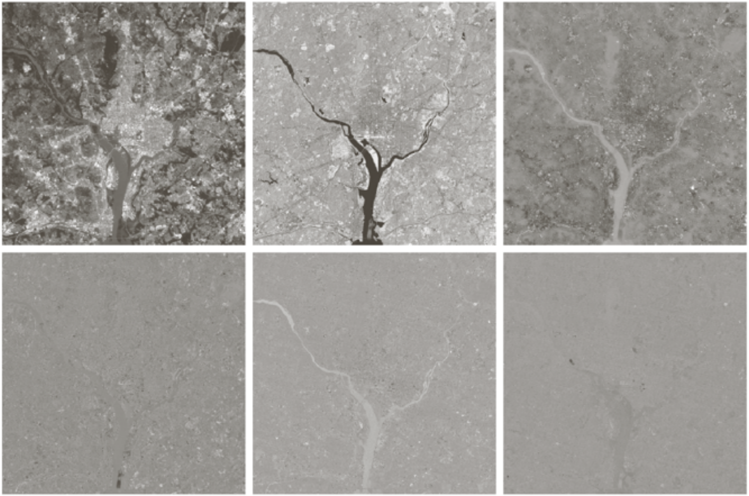
$$\mathbf{x} = \mathbf{A}^T\mathbf{y} + \mathbf{m}_x \tag{11}$$

Using just k eigenvectors

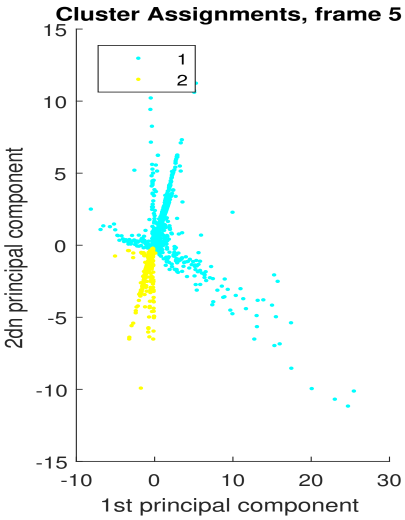
$$\hat{\mathbf{x}} = \mathbf{A}_k^T\mathbf{y} + \mathbf{m}_x \tag{12}$$

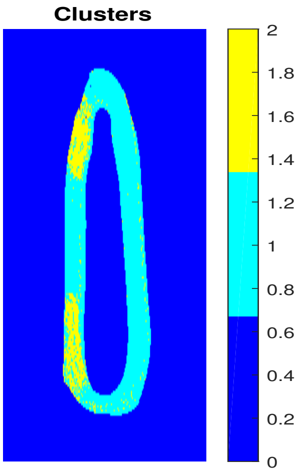
Principal components

- **Six principal component images, obtained using equation (5).**

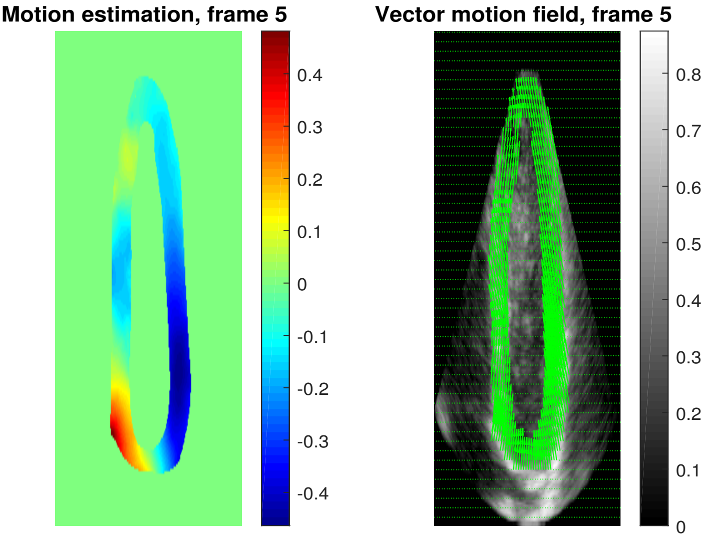


Clustering the PC of **x** (kmeans with 2 classes)

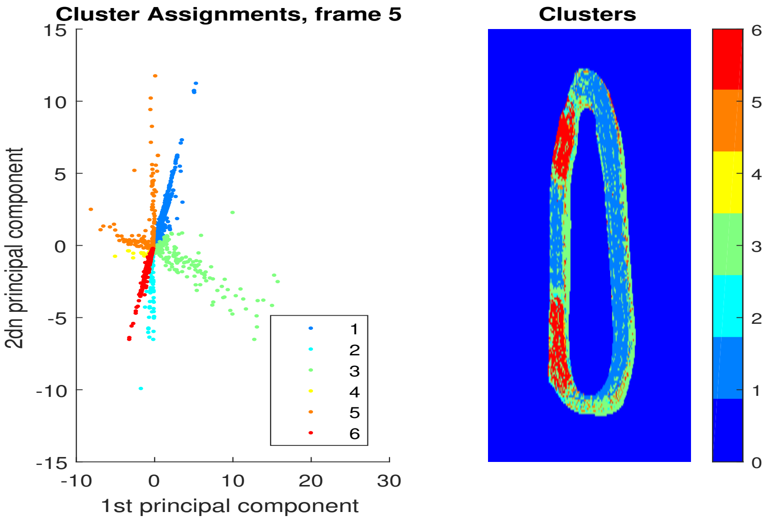




High-correlation between **x** and **s**



Clustering the PC of **x** (kmeans with 6 classes)

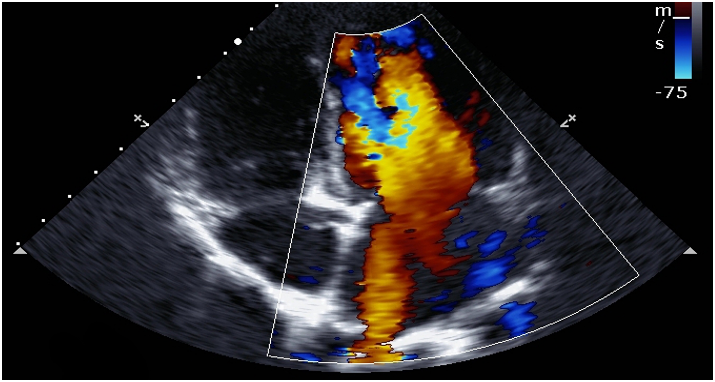


Products

- It was submitted the paper entitled: Cardiac motion estimation by using convolutional sparse coding.

Conclusions and future work

- A new method for cardiac motion estimation in ultrasound imaging was presented.
- In the future will be studied the sparse decomposition for anomaly detection and cardiac tissue classification.



Questions?

Thanks for your attention!