Analysis and Improvement of GNSS Navigation Message Demodulation Performance in Urban Environments

PhD Dissertation Defense
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Introduction

**GNSS: Global Navigation Satellite System**

is a satellite-based system that allows a user to determine its position and velocity anywhere at any time, and to synchronize its clock with the ultra precise GPS time.

To compute a position:

- At least 4 emitting satellites are needed
- For each emitting satellite:
  - Satellite-receiver pseudo-range estimation
  - Navigation message demodulation:
    - **Key information:**
      - Satellite position (= Ephemeris)
      - Satellite Clock error corrections

  = **CED** (Clock error corrections & Ephemeris Data)
Introduction - Problematic

- The majority of new GNSS applications take place in urban environments.
- In these obstructed environments, the transmitted signal is impacted by obstacles.
- This impact induces fading and multipath on the resulting received signal.
- As a consequence, it can be difficult for the receiver to be able to process the received signal.
- Therefore, the performance in urban environments is degraded with respect to an AWGN channel.
- It is thus necessary to assess, and if needed to improve, the GNSS signals performance in an urban channel.

*Figure 2: Urban Environment*
The GNSS signals performance in urban environments can be improved in investigating:

- The satellite-receiver pseudo-range estimation process
- The navigation message demodulation process
  - only this aspect has been investigated during the PhD thesis

Final PhD thesis objective:

To make the GNSS navigation message more robust to the distortions introduced by the urban environment, optimizing:

- Channel coding
- Navigation message and signal structures
Introduction - Logic

- Development of a software simulator in C language, modeling the GNSS signal emission/reception chain in urban environments

- Development of an innovative method specially adapted to provide the GNSS signals demodulation performance in urban environments

- Provision of the GPS L1C signal demodulation performance in an urban environment for narrowband and wideband propagation channel models

- Demodulation performance improvement at the receiver level:
  - Development of an advanced method to adapt the decoding process

- Demodulation performance improvement at the signal level:
  - Design of a new GNSS signal, with an optimized channel code
Outline

1) Simulator Presentation
2) Demodulation Performance Analysis in Urban Environments
3) Demodulation Performance Improvement by Decoding Optimization
4) Demodulation Performance Improvement by Designing a New Signal
5) Conclusion
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1) Simulator Presentation
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1) Simulator Presentation - Model

**Objective:** To simulate the GNSS communication chain, with simulations as less time-consuming as possible, in keeping a real behavior.

- **Emitted GNSS signals:**
  \[ s_e(t) = A_{\text{data}} C_{\text{data}}(t) D(t) + A_{\text{pilot}} C_{\text{pilot}}(t) \]
  - \( A_{\text{data}} \) and \( A_{\text{pilot}} \) are respectively the data and pilot emitted amplitudes,
  - \( C_{\text{data}}(t) \) and \( C_{\text{pilot}}(t) \) correspond to the spreading codes,
  - \( D(t) \) is the data stream, protected by a channel code = the navigation message

- **Received signal:**
  \[ r_e(t) = \int_{-\infty}^{+\infty} h_e(t; \tau)s_e(t - \tau) d\tau + n(t) \]
  - \( h_e(t; \tau) \) is the equivalent low-pass channel impulse response,
  - \( n(t) \) is the equivalent low-pass AWGN

- **GNSS receiver processing:**

  ![Figure 3: Real GNSS receiver block diagram](image)

In the simulator, the signal is directly modeled at the correlator output level.
1) Simulator Presentation - Simulator

SiGMeP

Simulated GNSS Navigation Messages
- GPS L1 C/A
- GPS L2C
- GPS L1C
- Galileo E1 OS

Propagation Channel Models Impact Generation
- AWGN
- Narrowband (Prieto)
- Wideband (DLR)

\[ D(t) \]

Correlator Output Generation
- Received signal modeled at the correlator output level (through partial correlations)

\[ h_e(t; \tau) \]

GNSS Receiver Processing
- Carrier phase tracking by PLL or ideal phase estimation
- Demodulation and decoding

Demodulation Performance Computation
- BER
- WER
- CEDER

\[ CEDER \]

\[ I_p(i) \]

Figure 4: Simulator structure
1) Simulator Presentation - Navigation Message

GPS L1C Navigation message $D(t)$:

- **Subframe 1**
  - Data (9 bits)
  - TOI

- **Subframe 2**
  - Data (576 bits)
  - Clock and Ephemeris

- **Subframe 3**
  - Data (250 bits)
  - Variable Data

**Forward Error Correction**

- **Channel encoder:**
  - BCH (51,8)

- **Outer channel encoder:**
  - CRC-24Q

- **Inner channel encoder:**
  - Rate $\frac{1}{2}$ LDPC

**Interleaving**

- **Data’’** (548 coded bits)
  - Variable Data + redundant bits

- **Data’’** (1200 coded bits)
  - Clock and Ephemeris + redundant bits

- **Data’’** (52 coded bits)
  - TOI + redundant bits

*Figure 5: GPS L1C frame channel coding*

1 data symbol = 10 ms
Data rate = 100 sps
$\Rightarrow$ 1 message = 18 s
1) Simulator Presentation - Propagation Channel

- The Land Mobile Satellite (LMS) channel model in urban environment for the demodulation point of view is targeted, there are 2 candidates:
  - **A narrowband model**: designed by Perez-Fontan/Prieto
    - The delay of the direct signal and the delays of the echoes are assumed to be equal
  - **A wideband model**: designed by DLR
    - The time delay of each multipath echo is individually modeled

- Both reference models:
  - The Perez-Fontan model was referenced in the COST (European Cooperation in the field Of Scientific and Technical Research) in 2002
  - The DLR model is the reference wideband model for the ITU (International Telecommunication Union) since 2009
1) Simulator Presentation - Propagation Channel

Perez-Fontan/Prieto Model

- **Narrowband**
  \[ h_e(t; \tau) = c(t)\delta(t - \tau_{direct}(t)) \]

\[ r_{enb}(t) = c(t)[A_{data} C_{data}(t - \tau_{direct}(t)) D(t - \tau_{direct}(t)) + A_{pilot} C_{pilot}(t - \tau_{direct}(t))] \]

With: \[ c(t) = a_{channel}(t)e^{j\varphi_{channel}(t)} \]

- The amplitude of the received signal complex envelope is statistically determined: follows a Loo distribution with parameters \((M_A, \Sigma_A, MP)\)

\[ \text{Loo} \sim c(t) = a_{direct}(t)e^{j\varphi_{direct}(t)} + a_{multipath}(t)e^{j\varphi_{multipath}(t)} \]

Log-Normal\((M_A, \Sigma_A)\) Rayleigh\((MP)\) Uniform\((0, 2\pi)\)

- The Loo parameters \((M_A, \Sigma_A, MP)\) are not fixed, they follow a distribution law which parameters depend on the environmental conditions
1) Simulator Presentation - Propagation Channel

Perez-Fontan/Prieto Model

Channel states

- The received signal is classified into 2 states, according to the shadowing/blocking level of the direct signal component:
  - “Good” for direct signal to moderate shadowing
  - “Bad” for moderate to deep shadowing
- The Loo parameters \((M_A, \Sigma_A, MP)\) depend on the state
- Consecutive states: from bad to good, with a state duration variable which follows a log-normal distribution

Figure 6: Prieto received amplitude and phase

<table>
<thead>
<tr>
<th>User Speed</th>
<th>50 km/h</th>
</tr>
</thead>
<tbody>
<tr>
<td>Band of the measurements</td>
<td>S-band</td>
</tr>
<tr>
<td>Satellite Elevation Angle</td>
<td>40°</td>
</tr>
</tbody>
</table>
1) Simulator Presentation - Propagation Channel

**DLR Model**

- **Wideband**
  \[ h_e(t, \tau) = c_{direct}(t)\delta(\tau - \tau_{direct}(t)) + \sum_{l=1}^{L} c_l(t)\delta(\tau - \tau_l(t)) \]
  \[ r_{eb}(t) = c_{direct}(t)\left[A_{data} C_{data}(t - \tau_{direct}(t))D(t - \tau_{direct}(t)) + A_{pilot} C_{pilot}(t - \tau_{direct}(t))\right] \]
  \[ + \sum_{l=1}^{L} c_l(t)\left[A_{data} C_{data}(t - \tau_l(t))D(t - \tau_l(t)) + A_{pilot} C_{pilot}(t - \tau_l(t))\right] \]

  - \( c_{direct}(t) \) is the channel impact on the direct signal component,
  - \( L \) is the number of echoes,
  - \( c_l(t) = a_{channel,l}(t)e^{i\phi_{channel,l}(t)} \) is the channel impact on the \( l \)th echo,
  - \( \tau_l(t) \) is the propagation time of the \( l \)th echo.

- The model is based on an artificial scene with potential obstacles: buildings, trees, lampposts, reflectors

- Obstacles are statistically generated, but the resulting impact on the received signal is mainly deterministic, based on ray tracing and geometric techniques

*Figure 7: DLR model artificial scene*
1) Simulator Presentation - Propagation Channel

Comparison between both models:

<table>
<thead>
<tr>
<th>Multipath modeling</th>
<th>Perez-Fontan/Prieto</th>
<th>DLR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model type</td>
<td>Statistical</td>
<td>Hybrid: statistical/deterministic</td>
</tr>
<tr>
<td>Measurement campaigns date</td>
<td>1990</td>
<td>2002</td>
</tr>
<tr>
<td>Calculation burden</td>
<td>Simple to implement</td>
<td>Heavy and time-consuming</td>
</tr>
</tbody>
</table>

The Perez-Fontan/Prieto model is less time-consuming, but the DLR model is supposed to be more representative of reality thanks to its wideband characteristic.

Both models has thus been used, to investigate the narrowband/wideband modeling effect on demodulation performance.

Figure 8: Characteristics of two LMS channel models examples
1) Simulator Presentation - Correlator Output Model

- Classical correlator output model:
  \[ I_{p_{nb}}(i) = \frac{A}{2} d_i R[\varepsilon_{\tau_i}] a_{\text{channel}}(i) \cos(\varphi_{\text{channel}}(i) - \varphi_{\text{rep}}(i)) + n_I(i) \]

<table>
<thead>
<tr>
<th>Integration Time ( T_I )</th>
<th>Usually: ( T_I = ) spreading code sequence duration multiple</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples:</td>
<td>( T_I = 20 \text{ ms} ) (GPS L1C/A), ( 10 \text{ ms} ) (GPS L1C), ( 4 \text{ ms} ) (Galileo E1 OS)</td>
</tr>
</tbody>
</table>

Under these assumptions:
1) The phase error between the received signal and the local replica is constant over \( T_I \),
2) The propagation channel amplitude \( a_{\text{channel}}(t) \) is constant over \( T_I \).

- But in urban environments, these assumptions are not validated

A \( T_I \) duration where the assumptions are validated is researched

- A new correlator output model is proposed, based on partial correlations:

The \( T_I \) duration is thus divided into \( N \) smaller intervals lasting \( T_{I_{part}} \) seconds, where we assume that:

1) \( R[\varepsilon_{\tau_i}] \) can be divided into \( N \) equal portions corresponding to partial correlations,
2) The phase error between the received signal and the local replica is constant over \( T_{I_{part}} \),
3) The propagation channel amplitude \( a_{\text{channel}}(t) \) is constant over \( T_{I_{part}} \)
1) Simulator Presentation - Correlator Output Model

Partial correlation duration analysis:

- To investigate the optimal choice of the partial correlation duration $T_{I\text{part}}$
- Several values of $T_{I\text{part}}$ have been tested with the simulator SiGMeP, following two principles:
  - The $T_{I\text{part}}$ value has to be as long as possible in order to reduce the number of partial correlations to be generated,
  - The data error rate computed at the receiver output must be as faithful as possible to reality
- To determine the maximum acceptable $T_{I\text{part}}$ duration, a “reference case” supposed to represent reality with $T_{I\text{part}} = 0.01\, ms$ has been simulated with SiGMeP and compared with the “tested cases” with $T_{I\text{part}} > 0.01\, ms$
  - BER degradation computation

<table>
<thead>
<tr>
<th>Reference Partial Integration Time $T_{I\text{part}}^{\text{ref}}$</th>
<th>$T_{I\text{part}}^{\text{ref}} = 0.01, ms$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tested Partial Integration Time $T_{I\text{part}}^{\text{tested}}$</td>
<td>$T_{I\text{part}}^{\text{tested}} = 0.05, ms, 0.1, ms, 0.5, ms, 1, ms$</td>
</tr>
</tbody>
</table>

$\Rightarrow$ Simulations show that: $T_{I\text{part}}^{\text{MAX}} = 0.1\, ms$
Outline

1) Simulator Presentation

2) Demodulation Performance Analysis in Urban Environments

3) Demodulation Performance Improvement by Decoding Optimization

4) Demodulation Performance Improvement by Designing a New Signal

5) Conclusion
2) Demodulation Performance Analysis in Urban Environments

- Historically, GNSS signals have been firstly designed for open environments:
  - Demodulation performance studied in the AWGN channel model

- However, new GNSS applications are emerging in urban environments:
  - Necessary to assess the GNSS signals demodulation performance in an urban channel model

- The urban channel is very different from the AWGN channel:
  - Necessary to adapt the methodology of representing the GNSS signals demodulation performance in urban environments
2) Demodulation Performance Analysis in Urban Environments

Classical Method Limitation n°1:
The received C/N₀ is not constant in urban environments

Objective:
- Find a C/N₀ which is constant for a long time for any urban user
- Find a C/N₀ which is representative from an operational point of view

Proposition:
To use the theoretical \( C_{\text{pre-urban}}/N_0 \)
with \( C_{\text{pre-urban}} \) = received direct signal power without channel attenuation

Figure 9: GNSS signals demodulation performance

Figure 10: Difference in the C/N₀
2) Demodulation Performance Analysis in Urban Environments

Classical Method Limitation n°2:
Only punctual instead of continuous message demodulations are required because in GNSS the same information set is repeated for a given time interval (example: CED information set).

Objective:
Combine the next characteristics:
- GNSS requires punctual demodulation
- Urban environments have dynamic signal reception conditions

Proposition:
- To provide the demodulation performance for favorable reception conditions together with statistical information about its occurrence

Figure 11: Urban channel impact on the received signal
2) Demodulation Performance Analysis in Urban Environments

Which statistical occurrence values associated with demodulation performance are considered acceptable?

- Determined by the operational requirements
- **Example:** To determine if a GPS L1C receiver can calculate a continuous valid position during 4 consecutive hours, with a probability greater than 95%

How to link operational requirements with statistical occurrence and associated demodulation performance?

- In determining **low level requirements** = at least 1 demodulated information set by 1 satellite during a continuous duration

**Figure 12:** New method block diagram
2) Demodulation Performance Analysis in Urban Environments - Example

<table>
<thead>
<tr>
<th>Strategy n°1 Operational Requirement Example</th>
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<tbody>
<tr>
<td>To determine if a GPS L1C receiver can calculate a continuous valid position during 4 consecutive hours, with a probability greater than 95%, and with a CED error rate equal to 10⁻².</td>
</tr>
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</table>

**Step 0**: Interpreting this ‘high level’ operational requirement through a ‘low level’.

\[ P_{final - 4h} = 95\% = P_{1sat - 4h}^4 = \left( P_{1sat - 1h}^2 \right)^4 \Rightarrow P_{1sat - 1h} = \left( P_{final - 4h} \right)^{1/8} = 0.9936 \]

*Figure 13: CED emission and validity periods diagram for GPS L1C*
2) Demodulation Performance Analysis in Urban Environments - Example

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**Step 0**: Interpreting this ‘high level’ operational requirement through a ‘low level’.

**Determining:**

\[ P_{0_{fav-1h\ max}} = \text{the required probability that no ‘favorable state message’ has been received during the duration of interest } 1h, \text{ from } 1 \text{ satellite,} \]

According to the low level requirement $P_{1_{sat-1h}}$ defined before.

\[ P_{0_{fav-1h}} \leq 1 - P_{1_{sat-1h}} \Rightarrow P_{0_{fav-1h\ max}} = 0.0064 \]
2) Demodulation Performance Analysis in Urban Environments - Example

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**Step 1:** Finding the criterion to separate the ‘unfavorable state messages’ from the ‘favorable state messages’ which provides the best demodulation performance.

**Prieto model:**
The received signal is classified into 2 states, ‘Good’ and ‘Bad’ according to the channel impact level

$\Rightarrow$ ‘Favorable state message’ = message entirely received in ‘Good’ Prieto state

**DLR model:**

$\Rightarrow$ ‘Favorable state message’ = message for which its estimated received $C/N_0$ is above a threshold
2) Demodulation Performance Analysis in Urban Environments - Example

Strategy n°1 Operational Requirement Example
To determine if a GPS L1C receiver can calculate a continuous valid position during 4 consecutive hours, with a probability greater than 95%, and with a CED error rate equal to $10^{-2}$.

**Step 1**

**Prieto model:**

- $P_{0fav-1h} < P_{0fav-1h max} = 0.0064$
- $\Rightarrow$ validation of $P_{final-4h} > 95$

**DLR model:**

- $P_{0fav-1h} < P_{0fav-1h max}$
- $\Rightarrow$ validation of $P_{final-4h} > 95$

**Simulation Conditions**

<table>
<thead>
<tr>
<th>Environment</th>
<th>Urban</th>
</tr>
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<tr>
<td>Database Band</td>
<td>S</td>
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*Figure 14: ‘Favorable state messages’ histogram, for GPS L1C*
2) Demodulation Performance Analysis in Urban Environments - Example

Step 2: Calculating the CED Error Rate during ‘Favorable States’

- For the PLL tracking case, the CEDER presents a floor, due to PLL losses of lock during unfavorable conditions.
- It seems never possible to demodulate with $CEDER = 10^{-2}$ with the classical method.
- The most relevant information is hidden: the possibility of punctually obtaining much better demodulation performance in favorable reception conditions = new method (green lines).

Figure 15: GPS L1C GOOD state CED demodulation performance and total CED demodulation performance.
2) Demodulation Performance Analysis in Urban Environments - Example

Calculating the CED Error Rate during ‘Favorable States’

Step 2

- For the PLL tracking case, the CEDER presents a floor, due to PLL losses of lock during unfavorable conditions.
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Figure 16: GPS L1C GOOD state CED demodulation performance and total CED demodulation performance.
2) Demodulation Performance Analysis in Urban Environments - Example

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### Step 3: Operational requirements validation or non-validation

\[
P_{\text{final} - 4h} = P_{1\text{sat} - 1h,\text{abs}}^8 = \left( P_{1\text{fav}} \cdot P_1 + P_{2\text{fav}} \cdot P_2 + \cdots + P_{200\text{fav}} \cdot P_{200} \right)^8
\]

\[
P_{\text{final} - 4h} = \left( P_{1\text{fav}} \cdot \left( 1 - CEDER_{C/N_0} \right) + P_{2\text{fav}} \cdot \left( 1 - CEDER_{C/N_0}^2 \right) + \cdots + P_{200\text{fav}} \cdot \left( 1 - CEDER_{C/N_0}^{200} \right) \right)^8
\]

- **Prieto model:**
  - For \( C_{\text{pre-urban}}/N_0 > 26 \text{ dBHz} \)
  - \( P_{\text{final} - 4h} \approx 95.3\% \)

- **DLR model:**
  - For \( C_{\text{pre-urban}}/N_0 > 25.5 \text{ dBHz} \)
  - \( P_{\text{final} - 4h} \approx 99.8\% \)

\( \Rightarrow \) the required \( P_{\text{final} - 4h} \) equal to 95% being thus validated
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3) Demodulation Performance Improvement by Decoding Optimization

**MAP decoding criterion:** If \( \frac{p(b_x = +1|y)}{p(b_x = -1|y)} > 1 \) or \( \text{LLR} = \log \left( \frac{p(b_x = +1|y)}{p(b_x = -1|y)} \right) > 0 \) \( \Rightarrow \hat{x} = +1 \)

**Figure 17:** GNSS emission/reception chain block diagram

**Objective:** Since in classical receivers the LLR is derived assuming an AWGN channel whereas urban environments are targeted here, we want to derive the LLR for any urban user.
3) Demodulation Performance Improvement by Decoding Optimization

Detection function $\Rightarrow$ $\text{LLR} = \log\left(\frac{p(y/x=+1)}{p(y/x=-1)}\right)$

In GNSS, the received symbol $y$ to be decoded is the data component correlator output $I_P$ modeled through partial correlations for a narrowband channel by:

$$y(i) = I_P(i) = \sqrt{P}x(i) \left[ \sum_{n=1}^{N} a_{\text{channel}}(n + (i - 1)N) \cos(\varepsilon_{\theta}(n + (i - 1)N)) \right] + n_1(i)$$

Normalization by $\sqrt{P}$:

$$y(i) = x(i) \left[ \sum_{n=1}^{N} a_{\text{channel}}(n + (i - 1)N) \cos(\varepsilon_{\theta}(n + (i - 1)N)) \right] + n'_1(i)$$
3) Demodulation Performance Improvement by Decoding Optimization

Usually, the expression of the soft channel decoder input in GNSS receivers is obtained assuming an AWGN propagation channel.

Assumptions:

- The propagation channel is considered as an **AWGN channel**
- The noise power $\sigma^2_{n_I'}$ is known

$$y(i) = I'_P(i) = x(i) \sum_{n=1}^{N} a_{\text{channel}}(n + (i - 1)N) \cos(\varepsilon_\theta(n + (i - 1)N)) + n_I'(i)$$

The decoder input is thus: $LLR_{AWGN}(i) = \log \left( \frac{p(x=1/y)}{p(x=-1/y)} \right) = \frac{2y(i)}{\sigma^2_{n_I'}}$

**Objective:**

To derive the detection function LLR assuming an urban channel, considering 2 cases:

1) **Perfect** propagation channel impact knowledge is assumed
2) **No** propagation channel impact knowledge is assumed
3) Demodulation Performance Improvement by Decoding Optimization

**Case 1:** However, urban environments are targeted. Thus, the detection function must consider an urban channel. Firstly, perfect Channel State Information is assumed = ideal case \(\Rightarrow\) best achievable performance

**Assumptions:**
- The propagation channel is considered as an urban channel
- The channel impact on \(I'_p(i)\) is perfectly known
- The noise power \(\sigma^2_{n_i'}\) is known

\[
y(i) = I'_p(i) = x(i) \left[ \sum_{n=1}^{N} a_{\text{channel}}(n + (i - 1)N) \cos(\varepsilon_\theta (n + (i - 1)N)) \right] + n'_i(i)
\]

The decoder input \(LLR_{\text{Perfect CSI}}\) is thus:

\[
LLR_{\text{Perfect CSI}}(i) = \frac{2 \left[ \sum_{n=1}^{N} a_{\text{channel}}(n + (i - 1)N) \cos(\varepsilon_\theta (n + (i - 1)N)) \right] y(i)}{\sigma^2_{n_i'}}
\]
3) Demodulation Performance Improvement by Decoding Optimization

Case 1 Simulations Results

**Simulation Conditions**

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Signals</strong></td>
<td>GPS L1C</td>
</tr>
<tr>
<td><strong>Channel Model</strong></td>
<td>Perez-Fontan/Prieto</td>
</tr>
<tr>
<td><strong>Environment</strong></td>
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<td><strong>Database Band</strong></td>
<td>S</td>
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<tr>
<td><strong>Satellite Elevation Angle</strong></td>
<td>40°</td>
</tr>
<tr>
<td><strong>Phase Estimation</strong></td>
<td>Ideal/PLL</td>
</tr>
</tbody>
</table>

- **For the PLL tracking case**, the CEDER floor obtained using $LLR_{AWGN}$ due to PLL losses of lock totally disappears with the use of $LLR_{Perfect\ CSI}$
- **In the ideal phase estimation case**, decoding gain of 3 dB
- **It is really promising since it represents the best achievable demodulation performance**

*Figure 18: GPS L1C demodulation performance obtained with $LLR_{AWGN}$ and $LLR_{Perfect\ CSI}$ in the Prieto channel model*
3) Demodulation Performance Improvement by Decoding Optimization

Case 2: However, urban environments are targeted. Thus, the detection function must consider an urban channel. Secondly, no Channel State Information is assumed = real cases

Assumptions:
- The propagation channel is considered as an urban channel
- The channel impact on $I'_p(i)$ is unknown

$$y(i) = I'_p(i) = x(i) \left[ \sum_{n=1}^{N} a_{\text{channel}}(n + (i - 1)N) \cos(\epsilon_\theta (n + (i - 1)N)) \right] + n'_1(i)$$

Inspired by the $LLR_{Perfect \, CSI}$ expression, the detection function is modeled by a linear function of the observations $y$ [Yazdani, 2009]:

$$LLR_{No \, CSI}(i) = \alpha y(i)$$
3) Demodulation Performance Improvement by Decoding Optimization

Case 3:

\[ LLR_{No\,CSI}(i) = \alpha y(i) \]

\( \alpha \) is determined maximizing the mutual information \( I(LLR; X) \) [Yazdani, 2009]:

\[ \alpha_{MCLA} = \arg \max_{\alpha} I(LLR; X) \]

With \( I(LLR; X) \) estimated with no learning sequence (\( x \) unknown) and without any statistical knowledge (LLR’s pdf unknown).

<table>
<thead>
<tr>
<th>Over a sliding windows of several symbols ( y ), ( I(LLR; X) ) is estimated for each tested ( \alpha )</th>
</tr>
</thead>
<tbody>
<tr>
<td>The ( \alpha ) value which provides the maximum estimated ( I(LLR; X) ) is chosen</td>
</tr>
<tr>
<td>For the corresponding symbol, the LLR is computed with this ( \alpha ) value</td>
</tr>
</tbody>
</table>
3) Demodulation Performance Improvement by Decoding Optimization

Case 2 Simulations Results

<table>
<thead>
<tr>
<th>Simulation Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Signals</strong></td>
</tr>
<tr>
<td><strong>Channel Model</strong></td>
</tr>
<tr>
<td><strong>Environment</strong></td>
</tr>
<tr>
<td><strong>Database Band</strong></td>
</tr>
<tr>
<td><strong>Satellite Elevation Angle</strong></td>
</tr>
<tr>
<td><strong>Phase Estimation</strong></td>
</tr>
</tbody>
</table>

- For ideal phase estimation, we approach the ideal case using $LLR_{no \ CSI}$
- For the PLL tracking case, the CEDER floor obtained using $LLR_{AWGN}$ still remains with the use of $LLR_{no \ CSI}$ but at a lower level
- The optimization method could be more investigated, especially for an important PLL estimated phase error (by channel estimation or taking into account a phase error model or the PLL losses of lock detector output)

*Figure 19: GPS L1C demodulation performance obtained with $LLR_{AWGN}$ and $LLR_{Perfect \ CSI}$ in the Prieto channel model*
Outline

1) Simulator Presentation
2) Demodulation Performance Analysis in Urban Environments
3) Demodulation Performance Improvement by Decoding Optimization
4) Demodulation Performance Improvement by Designing a New Signal
5) Conclusion
4) Demodulation Performance Improvement by Designing a New Signal

Objective
New GNSS signal design with improved demodulation performance in urban environments

Research axis
- LDPC channel code
  - Modern channel code able to approach the channel capacity
- CSK modulation
  - Possibility to non-coherently demodulate the navigation message

Proposition
LDPC code optimization for a GNSS CSK-based signal.
4) Demodulation Performance Improvement by Designing a New Signal - LDPC Channel Coding

**LDPC = Low-Density Parity-Check**

- Able to provide performance which approaches the best possible performance: the channel capacity
- Linear block codes:

```
Encoder
```

\[
\begin{align*}
\mathbf{u} \quad & \rightarrow \quad \text{(k bits)} \\
\mathbf{c} \quad & \rightarrow \quad \text{(n bits)} \\
\mathbf{c} & = \mathbf{uG} \\
\text{or } \mathbf{cH}^T & = 0 \\
\text{with } \mathbf{H} \text{ obtained by } \mathbf{GH}^T & = 0
\end{align*}
\]

- Defined by the null space of the parity-check matrix \( \mathbf{H} \)

\[
\mathbf{C} = \{ \mathbf{c} \in \mathbb{GF}(2)^{n \times n} / \mathbf{cH}^T = 0 \} \text{ that has a low density of 1s}
\]

**Channel code rate:** \( R = \frac{k}{n} \)

**Generator matrix:** \( \mathbf{G} \)

**Parity check matrix:** \( \mathbf{H} \)
4) Demodulation Performance Improvement by Designing a New Signal - Tanner Graph

A Tanner graph, based on the parity-check matrix $H$:

- Completely represents a LDPC encoder
- Is made by nodes, separated into two types:
  - The Variables Nodes (VN), representing the **coded bits**
  - The Check Nodes (CN), representing the **parity check equations**
- Is drawn as follows: the Check Nodes $CN_i$ are connected to the Variable Nodes $VN_j$ whenever element $h_{ij}$ in the parity check matrix $H$ is equal to 1:

$$
H = \begin{pmatrix}
V_{N1} & V_{N2} & V_{N3} & V_{N4} & V_{N5} & V_{N6} & V_{N7} & V_{N8} \\
1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\
1 & 1 & 0 & 1 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \\
1 & 1 & 0 & 0 & 1 & 1 & 1 & 0
\end{pmatrix}
$$

*Figure 20: Tanner graph example*

- Node degree = number of edges connected to a node, $d_{v_{\text{max}}}$ and $d_{c_{\text{max}}} =$ maximum degrees
4) Demodulation Performance Improvement by Designing a New Signal - CSK Modulation

CSK = Code Shift Keying

- M-ary orthogonal modulation
- Each waveform corresponds to the same spreading sequence, but circularly shifted:

<table>
<thead>
<tr>
<th>CSK symbol 1</th>
<th>PRN chip 1</th>
<th>PRN chip 2</th>
<th>PRN chip 3</th>
<th>PRN chip …</th>
<th>PRN chip 10 229</th>
<th>PRN chip 10 230</th>
</tr>
</thead>
<tbody>
<tr>
<td>00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CSK symbol 2</td>
<td>PRN chip 10 230</td>
<td>PRN chip 1</td>
<td>PRN chip 2</td>
<td>PRN chip …</td>
<td>PRN chip 10 228</td>
<td>PRN chip 10 229</td>
</tr>
<tr>
<td>01</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CSK symbol 3</td>
<td>PRN chip 10 229</td>
<td>PRN chip 10 230</td>
<td>PRN chip 1</td>
<td>PRN chip 2</td>
<td>PRN chip …</td>
<td>PRN chip 10 228</td>
</tr>
<tr>
<td>10</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CSK symbol 4</td>
<td>PRN chip 10 228</td>
<td>PRN chip 10 229</td>
<td>PRN chip 10 230</td>
<td>PRN chip 1</td>
<td>PRN chip …</td>
<td>PRN chip 10 227</td>
</tr>
<tr>
<td>11</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Figure 21: CSK waveforms example*

- Possibility to non-coherently demodulate the navigation message
- Data rate increase
4) Demodulation Performance Improvement by Designing a New Signal - $LLR_{CSK}$

LLR derivation for a GNSS CSK-modulated signal in an AWGN channel:

$$LLR_{q-CSK} = \log \left( \frac{p(b_{xq} = 1/y)}{p(b_{xq} = 0/y)} \right)$$

$$LLR_{q-CSK} = \log \left( \frac{\sum_{\text{All CSK symbols which } b_{xq}=1} \left[ \frac{1}{e^{\sigma_b^2}} \sum_{i=1}^{N} (y_{chip_i} x_{chip_i}) \Pi_{j\neq q} p(b_{xj}) \right]}{\sum_{\text{All CSK symbols which } b_{xq}=0} \left[ \frac{1}{e^{\sigma_b^2}} \sum_{i=1}^{N} (y_{chip_i} x_{chip_i}) \Pi_{j\neq q} p(b_{xj}) \right]} \right) + \log \left( \frac{p(b_{xq} = 1)}{p(b_{xq} = 0)} \right)$$

$LLR_{E_q}$

**Extrinsic:**
Information exclusively brought by the other bits

$LLR_{A_q}$

**A priori:**
Input information
4) Demodulation Performance Improvement by Designing a New Signal - Iterative Decoding

The decoding process can be made through two different methods:

Classical Decoding

Iterative Decoding

Figure 22: CSK demodulator and LDPC decoder combination, for the classical decoding method

Figure 23: CSK demodulator and LDPC decoder combination, for the iterative decoding method
4) Demodulation Performance Improvement by Designing a New Signal - EXIT Chart

To determine if iterative decoding provides better performance than non-iterative decoding in the case of a CSK-modulated GNSS signal:

⇒ the **EXtrinsic-Information-Transfer (EXIT) chart** is used

- Developed in the late 1990s
- Consists in plotting the output metric of interest, the extrinsic mutual information $I_E$, as a function of the input metric of interest, the a priori mutual information $I_A$

- $I_E = I(LLR_E; b_x)$ is the mutual information between the emitted coded bit $b_x$ and the **extrinsic** LLRs and $I_A = I(LLR_A; b_x)$ is the mutual information between the emitted coded bit $b_x$ and the **a priori** LLRs

- If $I_E$ increases with $I_A$ (= if bringing more a priori information to the demodulator involves a higher extrinsic information quantity at its output), it means that iterative decoding will improve the performance
4) Demodulation Performance Improvement by Designing a New Signal - EXIT Chart Plots

- bringing more a priori information to the demodulator involves a higher extrinsic information quantity at its output, meaning that iterative decoding will improve the performance.

- Optimization of the LDPC code for a CSK-modulated signal in an AWGN channel, considering iterative decoding.

**Figure 24**: CSK EXIT charts for different numbers of bits per CSK symbols
4) Demodulation Performance Improvement by Designing a New Signal - EXIT Chart Areas

The area under the CSK demodulator EXIT curve can be linked to the capacity:

\[ A_{CSK} = \int_0^1 T_{CSK}(i) di \approx R_0 \]

Where:

- \( A_{CSK} \) is the area under the EXIT chart of the CSK demodulator,
- \( T_{CSK} \) is the EXIT chart function associated with the CSK demodulator,
- \( R_0 \) is the maximum achievable channel code rate:
  
  \[ \text{maximum code rate} = 0.25 \Rightarrow 4 \text{ coded bits for 1 information bit} \]

\[ \text{maximum code rate} = 0.01 \Rightarrow 100 \text{ coded bits to encode 1 information bit} \]

**Figure 25:** Comparison between the maximum achievable code rate \( R_0 \) with iterative and non-iterative decoding.
4) Demodulation Performance Improvement by Designing a New Signal - LDPC Optimization

Derivation of the **asymptotic analysis** for the LDPC code optimization under iterative decoding *[Ten Brink, 2004] [Pouliiat, 2010]*:

- Method based on the demodulator EXIT chart function and on the updating equations of the exchanged LLR messages between the demodulator and the decoder

- Consists in solving a **linear programming optimization problem**, the cost function being to maximize the channel code rate

- **Leading to the optimized degree distributions** for the LDPC channel code parameters (the edges profile)
4) Demodulation Performance Improvement by Designing a New Signal - LDPC Codes Generation

According to the asymptotic analysis, **generation of finite length H matrices:**

- 2 bits and 6 bits per CSK symbol
- 600 information bits with channel code rate $\frac{1}{2}$
  
  since the GPS L1C signal is used as a benchmark to test the demodulation performance improvement
- Different maximum VN degrees: $d_{v_{max}} = 10, 15$
- Number of degree 2 VNs: limited or no limited

**Finite length H matrices generation with the PEG algorithm**

PEG algorithm: well-known to construct LDPC codes at finite block lengths with very good performance

**Comparison between:**

**Signal 0** (600 info bits, code $\frac{1}{2}$): GPS L1C LDPC code + BOC modulation + non-iterative decoding

**Signal 1** (600 info bits, code $\frac{1}{2}$): GPS L1C LDPC code + CSK modulation + non-iterative decoding

**Signal 2** (600 info bits, code $\frac{1}{2}$): GPS L1C LDPC code + CSK modulation + iterative decoding

**Signal 3** (600 info bits, code $\frac{1}{2}$): Optimized LDPC code 1 + CSK modulation + iterative decoding

**Signal 4** (600 info bits, code $\frac{1}{2}$): Optimized LDPC code 2 + CSK modulation + iterative decoding
4) Demodulation Performance Improvement by Designing a New Signal - Optimized LDPC Results

Approximately 0.5 dB of improvement compared with the current GPS L1C LDPC code, both CSK-modulated and iteratively decoded.

\[ d_{v_{\text{max}}} = 15 \] provides worse performance than \( d_{v_{\text{max}}} = 10 \), because of the short message length (600 bits).

Figure 26: Finite length results: BER according to Eb/N0 for 2 bits per CSK symbol
4) Demodulation Performance Improvement by Designing a New Signal - Optimized LDPC Results

6 bits/CSK symbol

1.2 dB of improvement compared with the current GPS L1C LDPC code but with CSK and iterative decoding

Figure 27: Finite length results: BER according to Eb/N0 for 6 bits per CSK symbol
Outline

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Conclusion

- Development of a software simulator, able to model the entire GNSS signal emission/reception chain in urban environments

- Development of an innovative method specially adapted to provide the GNSS signals demodulation performance in urban environments, able to evaluate the fulfillment of operational needs

- Detection function computation adaptation to any kind of user reception environment, without any CSI knowledge, improving the demodulation performance compared with the classical detection function use, only in modifying a part of the receiver process
  - The demodulation performance is improved by 3 dB in the ideal phase estimation
  - If a PLL phase tracking is considered, the floor due to PLL losses of locks drops

- Optimization of a LDPC code profile for a CSK-modulated signal and iterative decoding in an AWGN propagation channel model, thanks to the EXIT charts analysis and asymptotic optimization method
  - The best performing designed code outperforms significantly the GPS L1C LDPC code under iterative decoding by about 1.2 dB at BER= 10^{-4} for 6 bits per CSK symbol
Future Work

- It would be interesting to exhibit all the current GNSS signals demodulation performance in urban environments with the new method and to vary the user speed, the emitting satellites elevation angle and the azimuth angle.
- Demodulation performance has been investigated through the CED error rate, but Time To First Fix (TTFF) is a figure of merit which remains to be studied.
- Future work on the detection function optimization would consists in improving the innovative method for PLL tracking.
- The new designed LDPC-coded and CSK-modulated signal dedicated to iterative decoding needs to be compared with the current GPS L1C signal, and tested in urban environments.
- Future investigations will consider the extension to the non-binary case, since our work leads us to cycles codes.
Questions

Publications:


