Continuous Phase Modulation

A short Introduction

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System Model

- The complex envelop of the transmitted signal for CPM systems in baseband can be described as follows:
  ∀ t,

$$s(t) = \sqrt{\frac{\mathcal{E}_s}{T}} \cdot e^{j\phi(t,\alpha)}$$

- The information-carrying phase is:

$$\phi(t,\alpha) = 2\pi h \sum_{i=0}^{N-1} \alpha_i q(t - iT)$$

$\alpha_i \in \{\pm 1, ..., \pm(M-1)\}$ the information symbols, $\mathcal{E}_s$ is the symbol energy, T is the symbol period, h the modulation index ($h = \frac{P}{Q}$, with P and Q are relatively prime).

Phase Response

- Keeps the phase of the CPM signal continuous.
- Satisfies the following equation:

$$q(t) = \begin{cases} 0, & t \leq 0 \\ \int_0^t g(u)du, & 0 < t \leq LT \\ \frac{1}{2}, & t > LT \end{cases}$$

Where g(t) is the pulse response. It defines the shape of the trajectory.

The spectral efficiency is highly dependent on this parameter.

Phase and Pulse Response examples

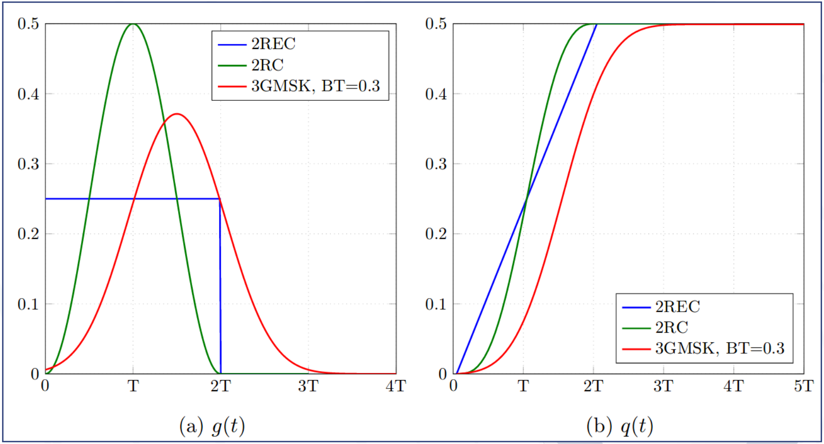


Figure: Phase g(t) and pulse q(t) response of some CPM

CPM parameters

- **L** is the CPM memory.
  - support length of the pulse response
  - the number of past symbols required to determine the signal waveform
  - $L = 1$ total response CPM, $L > 1$ partial response.
  - Greater L leads to less out-of-band energy (smaller side lobes)

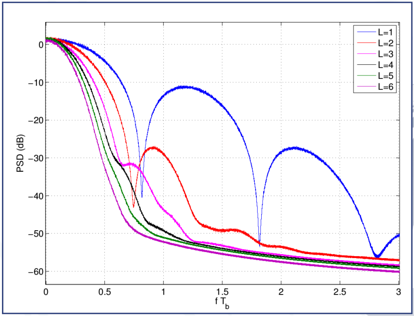


Figure: Influence of parameter L for a RC h=0.5

- **h** is the modulation index
  - Usually rational number < 1
  - small h leads to narrow occupied bandwidth

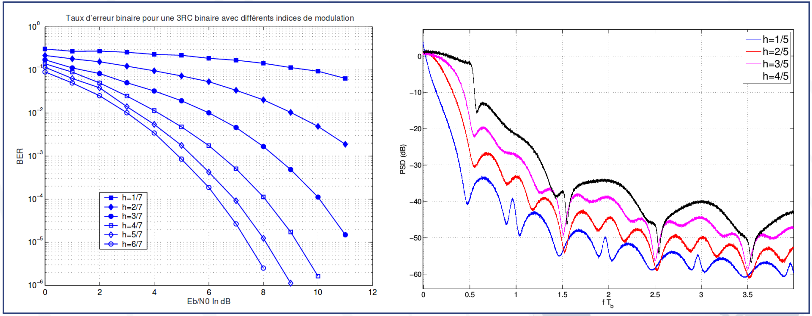


Figure: Influence of parameter h for a REC L=2

Notable CPM schemes and some application

- CPFSK: Telemetry
- SOQPSK: UHF SatCom (MIL-STD-188-181A)
- GMSK: Global System for Mobile Communication (GSM), Automatic Identification System (AIS)
- mixed RC/REC: Satellite Communication (DVB-RCS2)
- MSK, SFSK: considered for deep space communication
- Generalized MSK: Bluetooth data transmission

Interests

- Constant envelop waveform.
  - The transmitted power is constant.

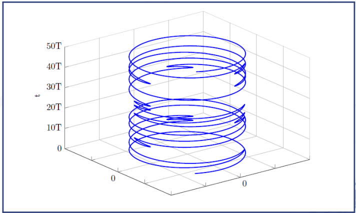


Figure: Binary 3RC h=2/3 in a three dimensional plan

- Phase continuity.
  - High spectral efficiency.
- Memory.
  - Fit to turbo decoding.

Trellis representation

- How to detect the emitted sequence?
  - Maximum Likelihood (ML) Detection

$$\widehat{s} = \text{argmax} \int r(t)\widetilde{s}^*(t)dt$$

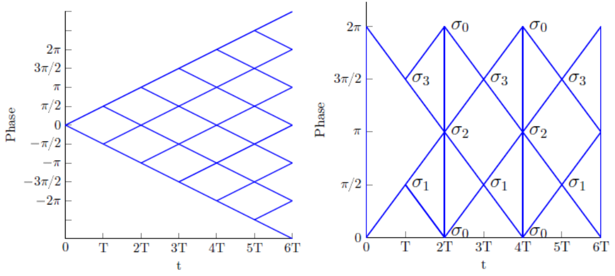
  - Need a trellis representation to perform a Viterbi algorithm
- Decomposition of the phase, at $t \in [kT; (k+1)T[$

$$\phi(t,\alpha) = \underbrace{h\pi \sum_{i=0}^{k-L} \alpha_i}_{\doteq \phi_k} + 2h\pi \sum_{i=k-L+1}^{k} \alpha_i q(t - iT)$$

The signal can be modelled only from $\sigma_k = \{\phi_k, \alpha_{k-L+1}, \ldots, \alpha_{k-1}\}$ which forms a state of our trellis and α_k the current symbol.

Example of Trellis representation

- MSK scheme (M=2, L=1, h=1/2 and REC pulse shape)
- state $\sigma_k = \{\phi_k\}$
- $\phi_k = \pi h \sum_{i=0}^{k-1} \alpha_i$ takes 4 values modulo 2π $\{0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}\}$



(a) Phase tree of the MSK (b) Modulo-2π phase tree of the MSK

- Time-variant trellis (only 2 of the 4 states are accesible in each symbol period)

Rimoldi's Decomposition

- Time invariant phase trellis for CPM can be obtained by defining the tilted phase ψ

$$\psi(t,\alpha) = \phi(t,\alpha) + \frac{\pi h(M-1)t}{T}$$

- The modified data sequence is introduced and defined as:

$$u_i = \frac{\alpha_i + (M-1)}{2}$$

$u_i \in \{0, 1, ..., M-1\}$ is called the tilted symbol and ψ the tilted phase.

Rimoldi's Time-invariant trellis

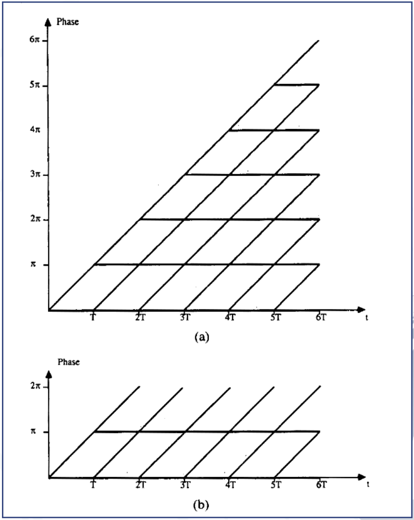


Figure: (a) Tilted-phase tree of MSK (b) Physical tilted-phase trellis of MSK

CPM Detection using Rimoldi's Representation

- BCJR Algorithm with a maximum a posteriori (MAP) criteria
  - Turbo Demodulation
  - BER minimisation
- State is defined as follows: $\delta_k = \{u_{k-1}, ..., u_{k-L+1}, \phi_k\}$
- Transition $\{\delta_k \rightarrow \delta_{k+1}\}$ is done such that
  - $\phi_{k+1} = \phi_k + 2\pi h u_{k-L+1}$.
  - Symbol u_k emitted
- Complexity $Q \cdot M^{L-1}$

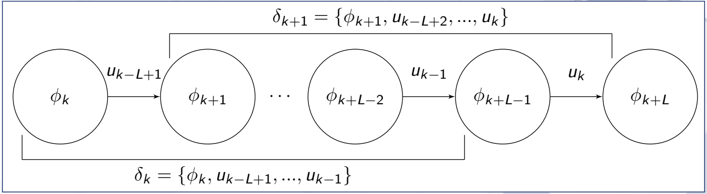


Figure: State Diagram of the usual BCJR

PAM Decomposition and complexity reduction

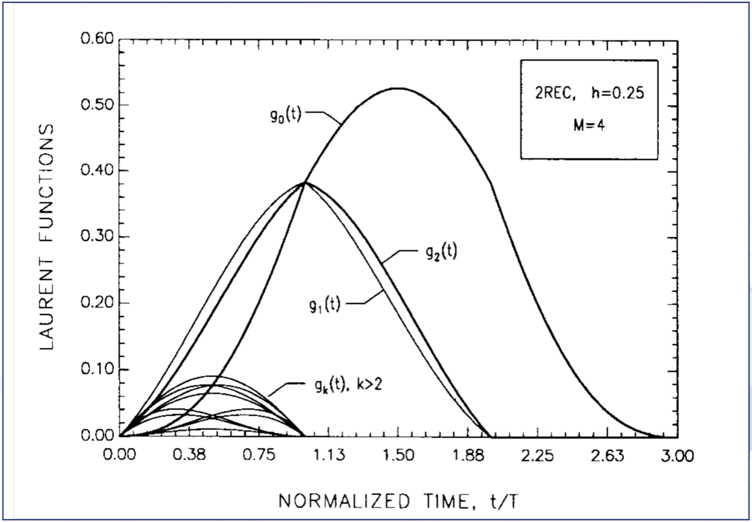
- Developed by Laurent for binary CPMs, extended to M-ary CPMs by Mengali and Morelli
- Idea: CPM = sum of modulated PAM

$$s(t) = \sum_{k=0}^{K-1} a_{k,n} g_k(t - nT)$$

  $\{a_{k,n}\}$ can be expressed in closed form from $\{\alpha_n\}$ and h
  $\{g_k\}$ can be obtained in closed form from $q(t)$ and h
- Most signal power in the first $M-1$ components (known as principal components)
  - can be used to design the detection
  - for $k \in [0; M-2]$, $\{a_{k,n}\}$ can be expressed only from $a_{0,n-1}$ and α_n
  - only Q states in the detection!
  - only $M-1$ matched filters!

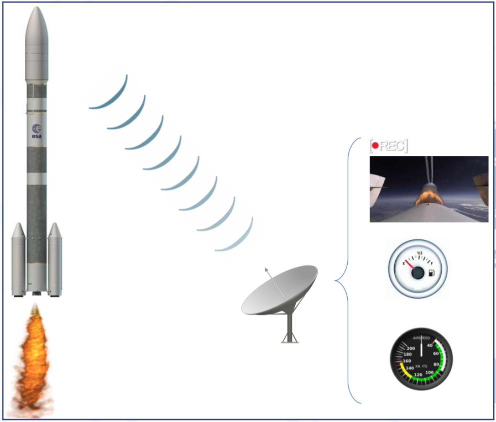
PAM Decomposition: Example

- 2REC, $h = 1/4$ and $M = 4$



Thesis Charles-Ugo

- Thesis co-funded by CNES and CNRS
- Academic supervisors: M.-L. Boucheret, C. Poulliat and N. Thomas
- Application: Launchers Telemetry system (Ariane, Vega and Soyuz)



CPM for telemetry launchers

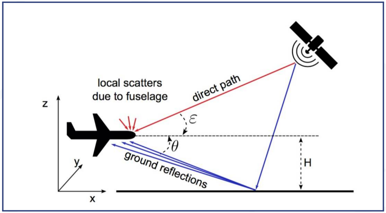
- Context
  - Low data transmission rate
  - 'Effets Flammes'
  - Channel undergoes an unknown phase rotation θ
- Modulation : CPFSK
  - Memory L=1.
  - g(t) is a rectangular phase response.
- Key points
  - Deal with the phase shifting.
  - Channel characterisation.
  - Increase the Rate.

Thesis Romain

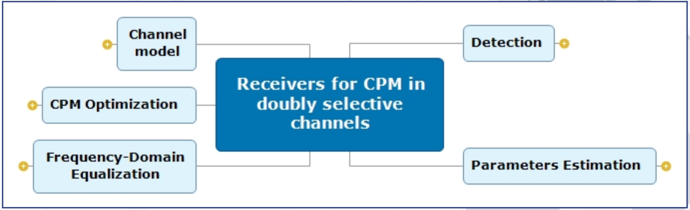
- Thesis co-funded by CNES and TAS
- Academic supervisors: M.-L. Boucheret, C. Poulliat and N. Thomas
- Application: Unmanned Air Vehicle (UAV)

Equalization and Synchronization for CPM

- System Model



- Key points



- More information (research context, publication, teaching...) are available here

Thanks for your attention!

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