Table of Content

CPM Modulation
- System Model
- Notable CPM schemes
- Interests
- Trellis representation

Decomposition and Detection of CPM
- Rimoldi’s Decomposition
- PAM Decomposition

Thesis Contribution
- Thesis Charles-Ugo
- Thesis Romain
The complex envelop of the transmitted signal for CPM systems in baseband can be described as follows:

\[ s(t) = \sqrt{\frac{E_s}{T}} \cdot e^{j\phi(t,\alpha)} \]

The information-carrying phase is:

\[ \phi(t, \alpha) = 2\pi h \sum_{i=0}^{N-1} \alpha_i q(t - iT) \]

\( \alpha_i \in \{ \pm 1, ..., \pm (M - 1) \} \) the information symbols, \( E_s \) is the symbol energy, \( T \) is the symbol period, \( h \) the modulation index (\( h = \frac{P}{Q} \), with \( P \) and \( Q \) are relatively prime).
Phase Response

- Keeps the phase of the CPM signal continuous.
- Satisfies the following equation:

\[
q(t) = \begin{cases} 
0, & t \leq 0 \\
\int_0^t g(u)du, & 0 < t \leq LT \\
\frac{1}{2}, & t > LT 
\end{cases}
\]

Where \( g(t) \) is the pulse response. It defines the shape of the trajectory.

The spectral efficiency is highly dependent on this parameter.
Figure: Phase $g(t)$ and pulse $q(t)$ response of some CPM
CPM parameters

- **L** is the CPM memory.
  - support length of the pulse response
  - the number of past symbols required to determine the signal waveform
- \( L = 1 \) total response CPM, \( L > 1 \) partial response.
- Greater \( L \) leads to less out-of-band energy (smaller side lobes)

![Figure: Influence of parameter L for a RC h=0.5](image_url)
CPM parameters

- **h** is the modulation index
  - Usually rational number $< 1$
  - small $h$ leads to narrow occupied bandwidth

**Figure**: Influence of parameter $h$ for a REC $L=2$
Notable CPM schemes and some application

- CPFSK: Telemetry
- SOQPSK: UHF SatCom (MIL-STD-188-181A)
- GMSK: Global System for Mobile Communication (GSM), Automatic Identification System (AIS)
- Mixed RC/REC: Satellite Communication (DVB-RCS2)
- MSK, SFSK: considered for deep space communication
- Generalized MSK: Bluetooth data transmission
Interests

- Constant envelop waveform.
  - The transmitted power is constant.

**Figure:** Binary 3RC $h=2/3$ in a three dimensional plan

- Phase continuity.
  - High spectral efficiency.
- Memory.
  - Fit to turbo decoding.
How to detect the emitted sequence?

- Maximum Likelihood (ML) Detection

\[ \hat{s} = \arg\max \int r(t) \hat{s}^*(t) dt \]

- Need a trellis representation to perform a Viterbi algorithm

- Decomposition of the phase, at \( t \in [kT; (k + 1)T] \)

\[
\phi(t, \alpha) = h\pi \sum_{i=0}^{k-L} \alpha_i + 2h\pi \sum_{i=k-L+1}^{k} \alpha_i q(t - iT) \\
\hat{=} \phi_k
\]

The signal can be modelled only from

\( \sigma_k = \{ \phi_k, \alpha_{k-L+1}, \ldots, \alpha_{k-1} \} \) which forms a state of our trellis and \( \alpha_k \) the current symbol.
Example of Trellis representation

- MSK scheme (M=2, L=1, h=1/2 and REC pulse shape)
- state \( \sigma_k = \{ \phi_k \} \)
- \( \phi_k = \pi h \sum_{i=0}^{k-1} \alpha_i \) takes 4 values modulo \( 2\pi \) \( \{0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}\} \)

- Time-variant trellis (only 2 of the 4 states are accessible in each symbol period)
Time invariant phase trellis for CPM can be obtained by defining the tilted phase $\psi$

$$\psi(t, \alpha) = \phi(t, \alpha) + \frac{\pi h(M - 1)t}{T}$$

The modified data sequence is introduced and defined as:

$$u_i = \alpha_i + \frac{(M - 1)}{2}$$

$u_i \in \{0, 1, ..., M - 1\}$ is called the tilted symbol and $\psi$ the tilted phase.
Figure: (a) Tilted-phase tree of MSK (b) Physical tilted-phase trellis of MSK
BCJR Algorithm with a maximum a posteriori (MAP) criteria
- Turbo Demodulation
- BER minimisation

State is defined as follows: $\delta_k = \{u_{k-1}, \ldots, u_{k-L+1}, \phi_k\}$

Transition $\{\delta_k \rightarrow \delta_{k+1}\}$ is done such that
- $\phi_{k+1} = \phi_k + 2\pi hu_{k-L+1}$.
- Symbol $u_k$ emitted

Complexity $Q \cdot M^{L-1}$

Figure: State Diagram of the usual BCJR
PAM Decomposition and complexity reduction

- Developed by Laurent for binary CPMs, extended to M-ary CPMs by Mengali and Morelli
- Idea: $\text{CPM} = \text{sum of modulated PAM}$

$$s(t) = \sum_{k=0}^{K-1} a_{k,n} g_k(t - nT)$$

$\{a_{k,n}\}$ can be expressed in closed form from $\{\alpha_n\}$ and $h$
$\{g_k\}$ can be obtained in closed form from $q(t)$ and $h$
- Most signal power in the first $M-1$ components (known as principal components)
  - can be used to design the detection
  - for $k \in [0; M-2]$, $\{a_{k,n}\}$ can be expressed only from $a_{0,n-1}$ and $\alpha_n$
  - only $Q$ states in the detection!
  - only $M-1$ matched filters!
PAM Decomposition: Example

- 2REC, $h = 1/4$ and $M = 4$
Thesis Charles-Ugo

- Thesis co-funded by CNES and CNRS
- Academic supervisors: M.-L. Boucheret, C. Pouliiat and N. Thomas
- Application: Launchers Telemetry system (Ariane, Vega and Soyuz)
Context
- Low data transmission rate
- 'Effets Flammes'
- Channel undergoes an unknown phase rotation $\theta$

Modulation: CPFSK
- Memory $L=1$
- $g(t)$ is a rectangular phase response.

Key points
- Deal with the phase shifting.
- Channel characterisation.
- Increase the Rate.
Thesis Romain

- Thesis co-funded by CNES and TAS
- Academic supervisors: M.-L. Boucheret, C. Pouliiat and N. Thomas
- Application: Unmanned Air Vehicle (UAV)
Equalization and Synchronization for CPM

- **System Model**

- **Key points**

- More information (research context, publication, teaching...) are available [here](#)
Thanks for your attention!

Pierre Laurent, Exact and approximate construction of digital phase modulations by superposition of amplitude modulated pulses (AMP), *IEEE Trans. on Communications*, vol. 34, no. 2, 1986.


References II

- Tarik Benaddi, Sparse Graph-Based Coding Schemes for Continuous Phase Modulations, *PhD Dissertation*, INP-Toulouse, 2015