Adaptive Estimation and Compensation of the Time Delay in a Periodic Non-uniform Sampling Scheme

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***Abstract*—High sampling rate Analog-to-Digital Converters (ADCs) can be obtained by time-interleaving low rate (and thus low cost) ADCs into so-called Time-Interleaved ADCs (TI-ADCs). Nevertheless increasing the sampling frequency involves an increasing sensibility of the system to desynchronization between the different ADCs that leads to time-skew errors, impacting the system with non linear distortions. The estimation and compensation of these errors are considered as one of the main challenge to deal with in TI-ADCs. Some methods have been previously proposed, mainly in the field of circuits and systems, to estimate the time-skew error but they mainly involve hardware correction and they lack of flexibility, using an inflexible uniform sampling reference. In this paper, we propose to model the output of L interleaved and desynchronized ADCs with a sampling scheme called Periodic Non-uniform Sampling of order L (PNSL). This scheme has been initially proposed as an alternative to uniform sampling for aliasing cancellation, particularly in the case of bandpass signals. We use its properties here to develop a flexible on-line digital estimation and compensation method of the time delays between the desynchronized channels. The estimated delay is exploited in the PNSL reconstruction formula leading to an accurate reconstruction without hardware correction and without any need to adapt the sampling operation. Our method can be used in a simple Built-In Self-Test (BIST) strategy with the use of learning sequences and our model appears more flexible and less electronically expensive, following the principles of "Dirty Radio Frequency" paradigm: designing imperfect analog circuits with subsequently digital corrections of these imperfections.**

I. Introduction

In many areas such as satellite communications, the transmitted signals are characterized by very high frequencies, sometimes beyond the GHz order. In this case, the conversion of an analog signal into a digital one, performed by Analog-to-Digital Converters (ADCs), becomes a challenging operation related to the frequency sampling it requires [1]. In particular applications, because of economical and technological constraints, alternative solutions have to be considered. The most popular one rely on Time Interleaved ADCs (TI-ADCs) [2], [3]. TI-ADCs are composed of several ADC operating at the same sampling frequency f_s which are time interleaved in order to reach a higher global sampling frequency by sharing the sampling operation between the different components, at the prize of a strong synchronization between them.

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Indeed, TI-ADCs are particularly sensitive to desynchronization between the elementary ADCs. The different sampling channels have to be perfectly synchronized to perform the expected uniform sampling operation and limit the effect of time-skew errors as investigated in [4], [5]. Some methods have already been investigated for the desynchronization correction [6]–[11]. Most of these methods propose to further re-synchronize the ADCs for reconstruction error reduction. This implies a lack of flexibility through hardware corrections of the sampling devices and generally an increase in the system complexity and power consumption [5].

The method proposed in this paper directly operates on the digital signal and does not require any hardware correction. The delay between the ADCs due to desynchronization is first estimated on-line using an adaptive method and then taken into account by non-uniform sampling formulas that perform the signal reconstruction. We developped a simple method suitable for Built-In Self-Test (BIST) [12] strategies using learning sequences to calibrate the channels. These sequences can be injected at the input of our system by an auto-calibration method. We follow the principles of "Dirty Radio Frequency" as introduced in [13], adapting to the desynchronization instead of cancelling it. Nevertheless our method relies on an appropriate model of this desynchronization.

In the case of a TI-ADC composed of L ADCs, we propose to model the system using Periodic Non-uniform Sampling of order L (PNSL), a well-known sampling scheme. In this paper, the derivations are performed for two channels using the so-called PNS2 scheme, previously investigated for its anti-aliasing properties [14]–[16] or its efficiency for bandpass signals [17], [18].. The results can be easily extended to the case of L ADCs.For example a TI-ADC with L ADCs denoted $\text{ADC}_{0,\dots,L-1}$ can be calibrated by choosing ADC_0 as a reference line and then estimating successively the delay between each $\text{ADC}_{1,\dots,L-1}$ and ADC_0 in a PNS2 simple scheme and without lack of generality.

The paper is organized as follows. Section 2 presents the signal and sampling models. Section 3 details the proposed method. The performance analysis is conducted in section 4. Section 5 contains the concluding remarks.

II. Signal model and sampling scheme

A. Signal model and folded spectrum definition

For generality and possible application to telecommunications, the signal is modeled as a stationary random process $\mathbf{X} = \{X(t),\ t \in \mathbb{R}\}$ with zero mean, finite variance and power spectral density $s_X(f)$ defined by:

$$\mathrm{E}[X(t)X^*(t-\tau)] = \int_{-\infty}^{\infty} e^{2i\pi f\tau} s_X(f)\, df = K_X(\tau) \quad (1)$$

where $\mathrm{E}[\cdot]$ stands for the mathematical expectation and the superscript $*$ for the complex conjugate. Let define the folded spectrum of $\mathbf{X}$ as follows. Consider the sampling sequence $\mathbf{X}_\lambda = \{X(n+\lambda), n \in \mathbb{Z}\}$ for a normalized sampling rate and $\lambda \in [0, 1[$. $\mathbf{X}_\lambda$ is a zero mean stationary random sequence with spectral density $S_X^\lambda(f)$ defined for $f \in (-\frac{1}{2}, \frac{1}{2})$ by:

$$S_X^\lambda(f) = \sum_{n\in\mathbb{Z}} s_X(f+n)e^{2i\pi n\lambda}. \quad (2)$$

$S_X^\lambda(f)$ is called the generalized folded spectrum of $s_X(f)$, the spectral density of $\mathbf{X}$. Note that for $\lambda = 0$, we have a simple expression of the folded spectrum:

$$S_X^0(f) = \sum_{n\in\mathbb{Z}} s_X(f+n) \quad (3)$$

B. The PNS2 sampling scheme

The PNSL is a sampling scheme composed of L sequences $\mathbf{X}_{a_l}$, $l = 1, 2, ..., L$. When the sum in (2) contains at most L non-zero terms for each frequency f, an errorless reconstruction of $X(t)$ can be performed under light conditions on delay parameters a_l. The case of PNS2 is of particular interest for real bandpass signals whose band is composed of two symmetric intervals of unit length. In the following, we will focus on PNS2 for potential application to telecommunication bandpass signals and the signal power spectrum $s_X(f)$ support is then included in the k^{th} Nyquist band Δ_k defined by:

$$\Delta_k = \left(-(k+\frac{1}{2}), -(k-\frac{1}{2})\right) \cup \left(k-\frac{1}{2}, k+\frac{1}{2}\right) \quad (4)$$

In the case of PNS2, the sampling times are distributed according to two time interleaved periodic sequences $\{n, n \in \mathbb{Z}\}$ and $\{n + a, n \in \mathbb{Z}\}$ with $a \in [0, 1[$. The resulting mean sampling rate equals 2 and thus fits the signal effective bandwidth. This sampling condition was developed by Landau [19] and stipulates that a non-baseband signal with an effective bandwidth of B can be sampled at a rate of $2B$ and an errorless reconstruction can be performed.

Signal samples are then composed of the two sequences $\mathbf{X}_\lambda = \{X(n+\lambda), n \in \mathbb{Z}\}$ with $\lambda = \{0, a\}$. The parameter a controls the delay between the two interleaved uniform sequences. Under the additional condition that $2ka \notin \mathbb{Z}$, the exact reconstruction from an infinite number of samples is derived using the following formula [20]:

$$\begin{cases} X(t) = \dfrac{A_0(t)\sin[2\pi k(a-t)] + A_a(t)\sin[2\pi kt]}{\sin[2\pi ka]}, \\ \text{with: } A_\lambda(t) = \sum\limits_{n\in\mathbb{Z}} \dfrac{\sin[\pi(t-n-\lambda)]}{\pi(t-n-\lambda)} X(n+\lambda). \end{cases} \quad (5)$$

Note that in previous works, the authors have also proposed errorless reconstruction formulas (in terms of mean-squared error), whose convergence rate can be increased by the introduction of appropriate filters [21], [22] and which can perform joint reconstruction and interference cancelation or direct analytical signal reconstruction [23], [24].

C. Problem formulation

PNS2 reconstruction formulas (in our case Eq. 5) have been derived under the hypothesis that the time delay a is *a priori* known. However, in practical applications, this parameter may vary across time because of changes in the physical (mainly thermal) constraints imposed to the TI-ADC. The online estimation of this parameter is thus of particular interest to maintain reconstruction accuracy while avoiding expensive calibration as developed in [9], [10].Moreover, a direct use of this estimate in the reconstruction formulas prevent from the also expensive hardware delay corrections.

The influence of appropriate synchronization and then the impact of time-skew errors has been investigated in [4], [5], [25]. In [25] the authors showed the challenge of using high frequencies in the transmission of bandpass signals because of the impact on synchronization:

$$\Delta_f = \pi B \left(\left\lfloor \frac{2f_c}{B} \right\rfloor + 1 \right) \Delta_t \quad (6)$$

Indeed the sampling accuracy (the time error in other words, denoted as Δ_t) becomes increasingly important when the center frequency f_c of the signal increases for given bandwidth B and reconstruction precision Δ_f. For example, considering a signal around $f_c = 230$ MHz with a bandwidth $B = 50$ MHz and a reconstruction precision $\Delta_f = 1\%$, we must have a time error $\Delta_t \leq 6$ps, pushing the sampling time to be known very accurately using reliable methods.

Next section presents an adaptive strategy for the estimation of a from the observation of the sequences $\mathbf{X}_0$ and $\mathbf{X}_a$. The algorithm involves a learning sequence to estimate a in an auto-calibration purpose.

III. Proposed method: adaptive estimation of the PNS2 time delay

A. Method description

This section presents the strategy for the estimation of the time delay between the two channels of a PNS2 sampling scheme. This adaptive strategy is based on the use of a learning sequence, a signal with *a priori* known spectrum which could be transmitted when the signal of interest shuts down or before a transmission to calibrate the system. This simple methods can be embedded into a Self-Test strategy that does not require further complexity, only few components that are able to create and inject the learning sequence at the input of our system.

Moreover, although the PNS2 sampling scheme deals with bandpass signals as detailed below, the learning sequence is set as a baseband signal, allowing to take advantage of the sampling rate used in our reconstruction system. Indeed, we keep a normalized sampling frequency in each channel of the

PNS2 scheme, using the same sequences detailed below. Then, two main approaches can be considered. Our method gives a perfect estimation of a without iteration under the condition that the learning spectrum is known or can be estimated. For that purpose, simulations have been made with the example of a cosine wave. Then, another approach has been studied using a bandlimited white noise. By approximating the spectrum in the calculation, a recursive algorithm helps to estimate a with a very few iterations and simple operations. The simulation results will be detailed in the next section.

B. Principle

First, let consider a linear invariant filter of complex gain $\mu_a(f)$. We note $\mu_a[\mathbf{X}_0]$ the stationary sequence representing the filtered version of $\mathbf{X}_0$ by $\mu_a(f)$ and defined by the correspondence:

$$\mu_a(f) = \sum_k \alpha_k e^{-2i\pi k f} \iff \mu_a[\mathbf{X}_0](n) = \sum_k \alpha_k X(n-k) \tag{7}$$

We then define the complex gain of $\mu_a(f)$ using generalized folded spectrum (2):

$$\mu_a(f) = \frac{S_X^a(f)}{S_X^0(f)} e^{2i\pi f a} \ , \ f \in (-\frac{1}{2}, \frac{1}{2}) \tag{8}$$

and we set by periodicity $\mu_a(f) = \mu_a(f+n), n \in \mathbb{Z}$.

Now let us form the random sequence $\mathbf{D} = \{D_n, n \in \mathbb{Z}\}$:

$$D_n = X(n+a) - \mu_a[\mathbf{X}_0](n) \tag{9}$$

It is easy to show that $\mathbf{D}$ is orthogonal to $\mathbf{X}_0$, resulting in the following equality:

$$\begin{array}{c} \mathrm{E}[D_n X^*(m)] = 0 \ , \ \forall (n,m) \in \mathbb{Z} \\ \mathrm{E}[(X(n+a) - \mu_a[\mathbf{X}_0](n)) X^*(m)] = 0 \end{array} \tag{10}$$

This serie of equations represents the behaviour when a is known in our system. Now, let introduce the time-skew error that happen in the system by changing the parameter delay from a to an unknown $b \in [0,1[$ such as $b \neq a$. The parameter value a will only remain when we refer to the sampling sequence $\mathbf{X}_a$ which represents the samples taken according to a. Other occurences of a in the equations above will now be replace by b.

We introduce the following criterion:

$$\sigma_b^2 = \mathrm{E}\left[|X(n+a) - \mu_b[\mathbf{X}_0](n)|^2\right] \tag{11}$$

which can also be expressed as followed using (1):

$$\sigma_b^2 = \int_{-\infty}^{\infty} \left| e^{2i\pi f a} - \mu_b(f) \right|^2 s_X(f)\, df. \tag{12}$$

It is interesting to note that this criterion is minimum for $b = a$. As explained in section III-A the learning remains in baseband and then we have $s_X(f) = 0$ for $f \notin (-\frac{1}{2}, \frac{1}{2})$ leading to $\mu_b(f) = e^{2i\pi f b}$. In this case, the equation (12) reduces to:

$$\sigma_b^2 = \int_{-\frac{1}{2}}^{\frac{1}{2}} \left| e^{2i\pi f(b-a)} - 1 \right|^2 s_X(f)\, df. \tag{13}$$

In parallel, as $\mu_b(f) = e^{2i\pi f b}$ the filter reduces to a simple delay filter leading to:

$$\mu_b[\mathbf{X}_0](n) = X(n+b) \tag{14}$$

and then:

$$\sigma_b^2 = \mathrm{E}\left[|X(n+a) - X(n+b)|^2\right] \tag{15}$$

From now on, σ_b^2 can be estimated using (15) and the classical Shannon sampling formula for $X(n+b)$ derivation:

$$\mu_b[\mathbf{X}_0](n) = X(n+b) = \sum_k \frac{\sin[\pi(b-k)]}{\pi(b-k)} X(n+k) \tag{16}$$

It is well-known that the series involved in Shannon formula suffers from a poor convergence rate. This rate can be improved in the case of oversampling, following previous results [22]. For example, if $s_X(f) = 0 \ , \ f \notin (-\frac{1}{2}+\varepsilon, \frac{1}{2}-\varepsilon)$ and $0 < \varepsilon < \frac{1}{2}$ then the reconstruction can be performed with higher convergence rate (in $\frac{1}{k^2}$ rather than $\frac{1}{k}$) as follows:

$$\begin{array}{l} \mu_b[\mathbf{X}_0](n) = X(n+b) = \sum_k a_k X(n+k) \\ a_k = \frac{2}{(b-k)^2} \sin\left[\frac{\varepsilon}{2}(b-k)\right] \sin\left[(\pi - \frac{\varepsilon}{2})(b-k)\right]. \end{array} \tag{17}$$

Putting (15) into (13) gives an estimation method for a, under the assumption that $s_X(f)$ is *a priori* known for a given learning sequence. We detail now two examples of learning sequences that leads to simple estimation methods.

C. Examples

As explained previously, σ_b^2 can be derived according to Eq. (13) for a given learning sequence with known power spectral density $s_X(f)$. In the following, we provide simple closed form expressions of the criterion as functions of parameter a for two particular learning sequences.

1) Cosine wave: For a learning sequence defined as a cosine wave at the frequency f_0, we have the following spectrum: $s_X(f) = \frac{1}{2}(\delta(f-f_0) + \delta(f+f_0)) \ , \ -\frac{1}{2} < f_0 < \frac{1}{2}$. Then, from Eq. (13):

$$\sigma_b^2 = 4\sin^2\left[\pi f_0 (b-a)\right] \tag{18}$$

Because b is given as a parameter and σ_b^2 is estimated through (15) given an estimation that we denote $\widehat{\sigma}_b^2$, Eq. (18) leads to a straightforward estimation of parameter a as follows:

$$\hat{a} = b - \frac{1}{2\pi f_0} \arccos\left[1 - \frac{\widehat{\sigma}_b^2}{2}\right]. \tag{19}$$

2) Bandlimited white noise: This case is a bit different. Indeed, whereas the exact spectrum can be estimated and computed directly through the use of Eq. (13), it appears very simple to consider the hypothesis that $s_X(f) = 1$ on $(-\frac{1}{2}+\varepsilon, \frac{1}{2}-\varepsilon) \ , \ 0 < \varepsilon < \frac{1}{2}$, which is true in mean, and $s_X(f) = 0$ elsewhere. Under this hypothesis, the criterion can be written as:

$$\begin{array}{rcl} \sigma_b^2 & \approx & \int_{-\frac{1}{2}+\varepsilon}^{\frac{1}{2}-\varepsilon} \left| e^{2i\pi f(b-a)} - 1 \right|^2 df \\ & \approx & 2(1-2\varepsilon)\left(1 - \mathrm{sinc}[\pi(b-a)(1-2\varepsilon)]\right) \end{array} \tag{20}$$

with the convention $\text{sinc}(x) = \frac{\sin x}{x}$. Using (15) it leads to:

$$\text{sinc}[\pi(b-a)(1-2\varepsilon)] = 1 - \frac{\widehat{\sigma}_b^2}{2(1-2\varepsilon)} \quad (21)$$

that can be resolved using a calculation table of the cardinal sine function $f(x) = \text{sinc}[\pi x(1-2\varepsilon)]$.

An approximation error will be introduced by considering this hypothesis. Nevertheless, a simple recursive algorithm that iterates the estimation helps to estimate a. As shown by Fig. 2, a few number of iterations are sufficient to estimate a with a satisfying precision. Then with a reasonable number of iterations, an estimation error below 10^{-5} is possible whatever the initial error on a.

Algorithm 1: Iterative Algorithm

Initialization:
 Set $\hat{a}_0 = b$.
 Choose a threshold K.

Step n:
 Set $b_n = \hat{a}_{n-1}$.
 Estimate $\widehat{\sigma}_{b_n}^2$ using (15).
 Use (21) and the calculation table to estimate $b_n - \hat{a}_n$.

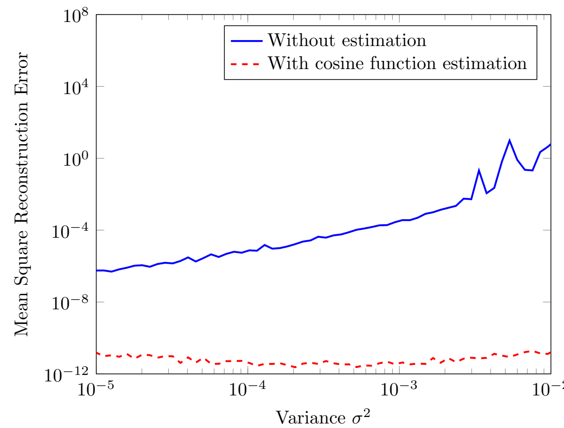
Until $|\hat{a}_n - \hat{a}_{n-1}| < K$

IV. Performance analysis

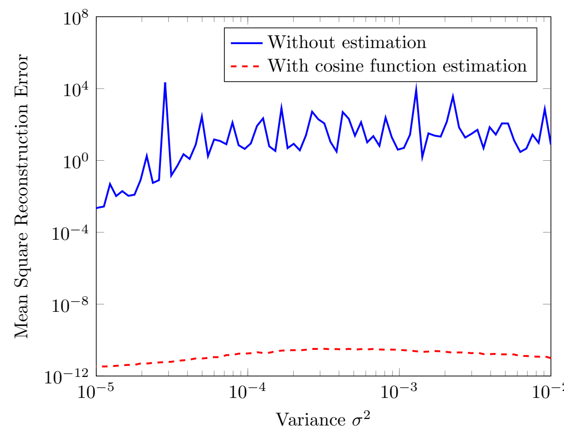
The simulation results are displayed on Fig. 1 and 2. In each case, reconstruction of the signal is performed using sliding windows of length N_{samp}. This parameter will be of interest in the case of the white noise iteration method.

First, Fig. 1 shows the performance of the cosinus estimation, helping to correct the reconstruction errors whatever the initial value of b. This estimation is done using $N_{\text{samp}} = 10$ in a one-tap algorithm without iteration, leading to a very low reconstruction error. It is important to note that the same conditions were used for the blue and red curves, the only difference being the use of cosine estimation on the red curve.

Concerning Fig. 2, the simulations were made choosing the Nyquist band number as $k = 3$ with $\varepsilon = \frac{5}{100}$ (5% of the bandwidth). The performance is shown through the plot of the 2 first iterations for each point and then the final estimation result. It is important to note that the dashed line represents the identity line, to which all the points belong initially. Fig. 2 then shows the performance of the estimation of a, at the first iteration, at the second iteration and then the final result, estimated after a certain number of iterations. In the case of $N_{\text{samp}} = 100$ in Fig. 2b, this final result estimated after an average of 20 iterations shows a precision below 10^{-5} for almost all the points. Fig. 2a has been tested using the same parameters, except that the reconstruction is now performed with a sliding window of only $N_{\text{samp}} = 10$ samples of the learning sequence. Even with this very low number of data, the estimation is performed quite well and leads to an estimation with a precision of almost 10^{-6} for all the points. The number of iterations however increases a bit but it must be related



(a) Bandpass signal with $k = 1$



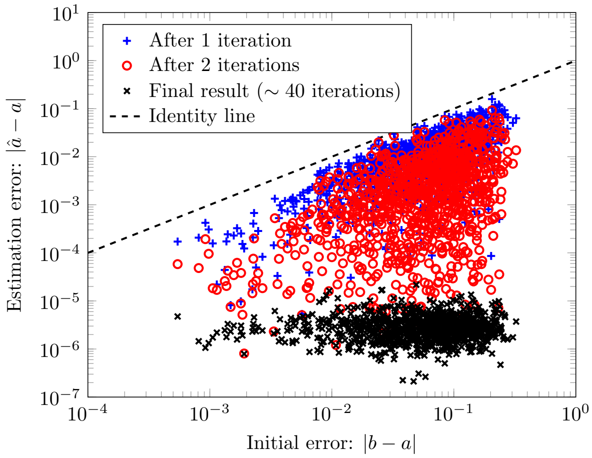
(b) Bandpass signal with $k = 7$

Fig. 1: Influence of the estimation on the signal reconstruction. (a), (b): Mean square reconstruction error as a function of variance with cosine estimation (red dashed) comparing to the reconstruction without estimation (blue solid)

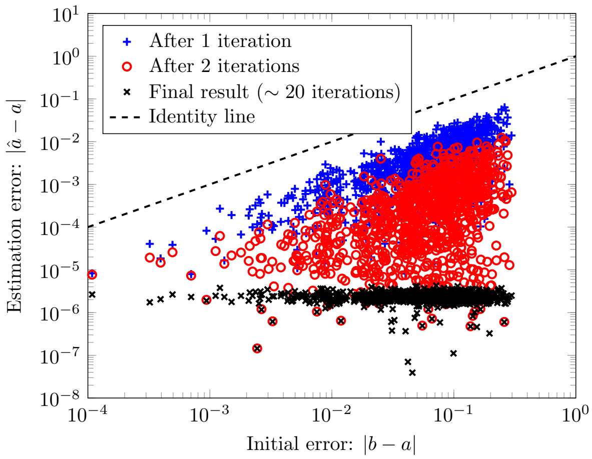
to the low value of $N_{\text{samp}} = 10$, leading to the use of $40 * 10 = 400$ samples to converge which is less than the case of $N_{\text{samp}} = 100$ for example ($20 * 100 = 2000$ samples).

V. Conclusion and future works

This article presents a method for the estimation and compensation of unknown delay between the channels in a PNS2 sampling scheme, used to model desynchronization errors in TI-ADCs. The extended case of PNSL that models a L channels TI-ADC can easily be retrieved by sucessively applying our method to each channel along with a fixed reference one, leading to a PNS2 scheme. The method developed here is very simple, easily computable (it requires only a few samples to be effective) and relies on the use of a learning sequence with known spectrum sent before the signal of interest or when it shuts down, by a simple signal injection as an input of the system. It differs from previous methods mainly based on hardware corrections of the sampling devices in order to cancel the desynchronization and return to the unflexible expected uniform grid. We propose here an extended and flexible framework for TI-ADC that adapts itself to imperfect



(a) $N_{\text{samp}} = 10$



(b) $N_{\text{samp}} = 100$

Fig. 2: Error estimation after algorithm running as a function of initial error (a) For $N_{\text{samp}} = 10$ signal samples (b) For $N_{\text{samp}} = 100$ signal samples

sampling devices. However, the learning sequence used must be known, exactly - as in the case of the cosine wave - or at least in mean as in the case of the bandlimited white noise. This could be a drawback of the proposed method if a learning sequence cannot be sent and/or spectrally known. In that purpose an extension of the method can be thought using existing sequences currently used in telecommunications such as Pseudo-Noise Sequences or Communication Pilots. By exploiting the known properties of these sequences (the seed initialization for PN Sequences or the symbols sent for pilots), a calibration method could be derived, allowing to overcome the need of an appropriate learning sequence creation device. Moreover, a blind calibration algorithm is a targeted upgrade of the method, allowing to estimate the delay using directly the samples taken from the signal of interest in bandpass. This will be part of a future work on the method developed here.

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