Telecommunications for Space and Aeronautics

EVM algorithm proposed to IEEE standard P 1765

presentation to IRT, TAS, CNES

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Outline of presentation

- Introduction and context of work
- Classical definition of EVM
- Hypotheses for the proposed algorithm
- Transmitter and optimum receiver
- Definition of EVM used in the algorithm
- Algorithm description and time Vernier
- Errors and standard deviation
- Comparison of other algorithms
- Processing of DC offset
- Future work and conclusion

Introduction

EVM is defined in many communication standards

These definitions are generally procedures to result in a percentage

Always slightly different in different standards

Cookbook recipes and not mathematical definitions

Uncertainty on this measurement difficult to assess

Problem for evaluation of measurement equipment by users

Problem for calibration of measurement equipment

Context of work

Proposal by NIST (National Institute of Standards and Technology) to standardize EVM uncertainty and calibration

Followed by NPL(UK) and other metrology laboratories

Followed by measurement industry (Keysight, National Instruments, Tektronix, Rohde & Schwarz, Anritsu)

Strong participation of Aerospace Corp and TéSA with metrology laboratories

IEEE P1765 workgroup uncertainty in EVM

- Propose an EVM “golden algorithm”
- Propose test waveforms for calibration of algorithms
- Propose calibration methods for measurement equipment
- Propose best practice document

Participation of TéSA

I proposed to this IEEE group the simulation algorithm for EVM computation that I first developed in CNES for NPR

I improved the algorithm and the theory while working in the IEEE group

This algorithm is based on a mathematical definition that results in a closed form expression for EVM

Exactitude and some other parameters like bias and standard deviation can be computed and minimized

Comparison and assessment of other algorithms

It is as fast and as exact as possible because part of the work is already done, no optimization is necessary

It can be implemented in simulations and in test benches that are based on digitally generated and measured signals

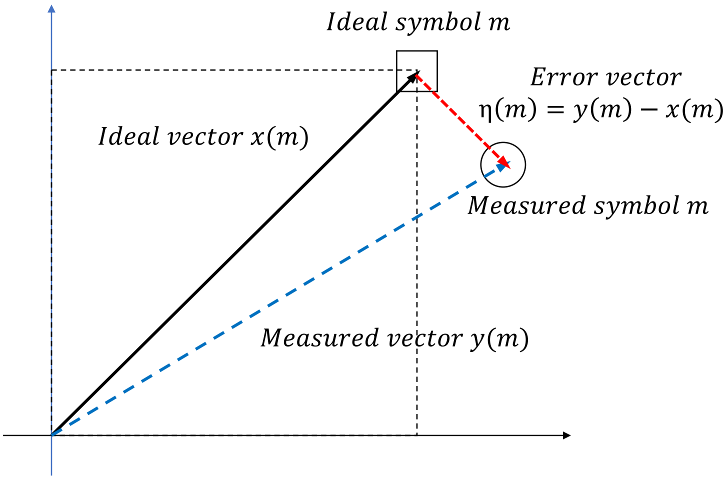
It can be applied to other metrics such as NPR and CCPR

Classical definition of EVM

EVM is classically defined by the following algorithm

1) The RF or IF frequency and symbol clock frequency are synchronized (e.g. by phase lock loops in a receiver)
2) The measured signal is filtered by a matched filter and sampled at optimum times at symbol clock frequency to generate the measured symbols
3) The measured symbols are normalized, with respect to the ideal symbols, by adding an offset (in some cases) and by multiplying by a complex gain (amplitude and phase)
4) This complex gain may vary in time along the sequence of symbols (in some cases, if the receiver will have this capability)
5) The signal may be equalized (in some cases, if the receiver will have this capability)
6) The signal may be corrected for IQ imbalance (in some cases, if the receiver will have this capability)
7) The EVM noise amplitude is the smallest possible RMS difference between normalized measured symbols and ideal symbols. This EVM noise is minimized by optimizing the complex gain, and if applicable, offset , equalization and complex gain variation
8) EVM is the ratio (expressed generally in %) of the EVM noise RMS amplitude and the RMS amplitude (or in some cases the maximum amplitude) of the ideal symbols

Ideal symbols, measured symbols and error vectors



In this figure, only one symbol of the constellation is shown

The error vector is the vector difference of measured and ideal vectors

The magnitude of this error vector is quadratically averaged on all symbols and normalized to the average power of ideal symbols:

$$EVM^2 = \frac{\sum_{m=1}^{N} |\eta(m)|^2}{\sum_{m=1}^{N} |x(m)|^2}$$

Data-aided and non data-aided EVM

- In data-aided EVM, the ideal symbols sequence (or the ideal signal) that has been transmitted is known by the receiver of the measured signal
- In non data-aided EVM, this sequence of symbols (or signal) is not known. The receiver knows only the ideal constellation of symbols to be received (modulation type and coding if necessary).
  - The receiver may first regenerate this sequence of symbols but may do so with some errors (as in the normal operation of a receiver). Then the operation is the same as in the data-aided case
  - The receiver may also include this regeneration of symbols (with some errors) in the EVM optimization process.
- Non data-aided EVM gives a lower (optimistic) value than data-aided EVM as symbol errors appear when the signal crosses a decision threshold and is nearer to another symbol. The erroneous decision on the symbol decreases the difference between the measured symbol and the chosen symbol. It is optimistic, particularly for high values of raw bit error rate.

Hypotheses for the algorithm

The algorithm is used in simulations to prove its correctness and in laboratory to calibrate measurement equipment

The algorithm uses the data-aided method for better accuracy. The ideal symbols sequence (or the ideal signal) is known by the receiver of the measured signal (either by transmission or by generation with a method identical to the one used in the transmitter)

The RF or IF frequency and clock frequency phase lock loops are supposed to be locked and stable with only a residual phase error and time error, no frequency error, no frequency drift. All simulations could be done with complex baseband IQ signals without RF or IF

No equalization is done (in present version)

No variation of amplitude or phase of gain versus time is used (in present version)

The algorithm determines the optimum complex gain, optimum time alignment and optimum sampling time of measured signal and generates measured symbols as the best optimum receiver would do

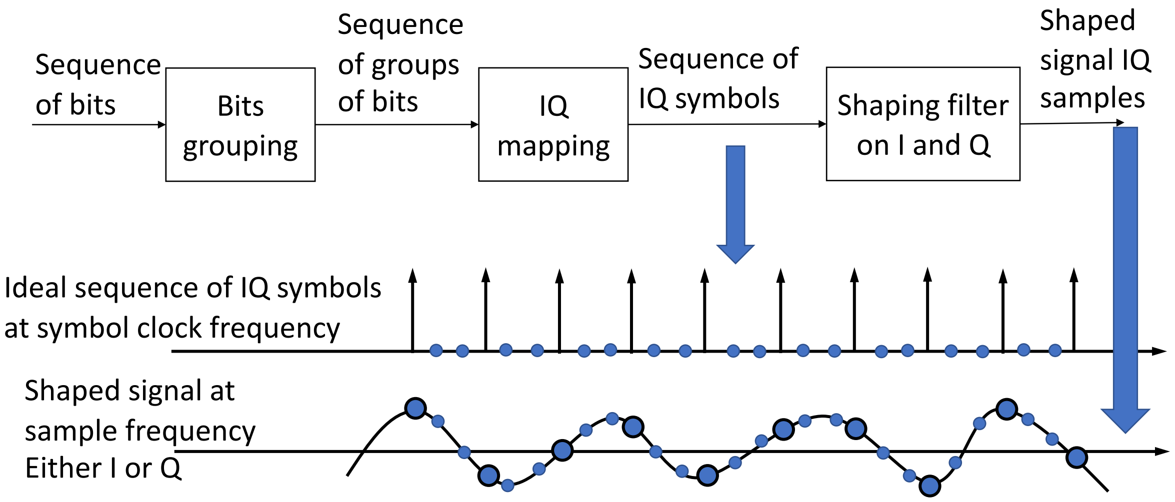
The algorithm compares measured and ideal symbols to compute EVM

The algorithm must be able to work with a low number of samples per symbol (2) for application to signals quantified near the Shannon limit by measurement equipment

Transmitter and channel

- The transmitter transforms a bit stream in a symbol stream by generating groups of M bits.
- M is the modulation order (e.g. 6 bits per symbol for 64 QAM modulation)
- The symbol stream is mapped to complex IQ symbols
- The signal is composed of a sequence of Dirac pulses at symbol frequency, each Dirac pulse having the IQ complex value of one symbol
- The signal is filtered by a shaping filter (e.g. a square-root raised cosine filter)
- The filtered signal modulates an IF or RF signal
- The channel is linear (complex gain, filter, delay, …) and stochastic (random) noise independent of the dignal is added

Transmitter schematics (signal generation)



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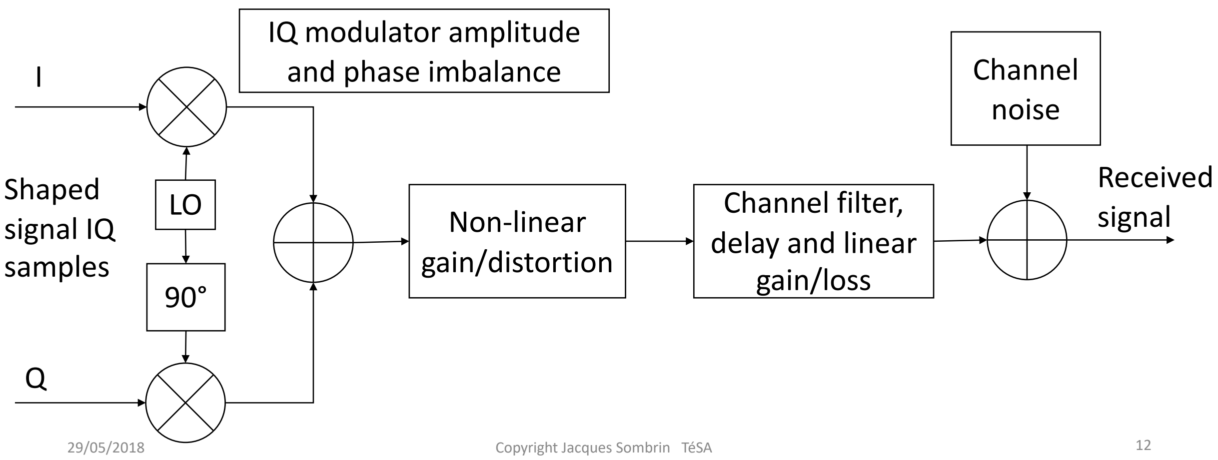
Transmitter and channel schematics

Main distortions imposed by the transmitter and channel



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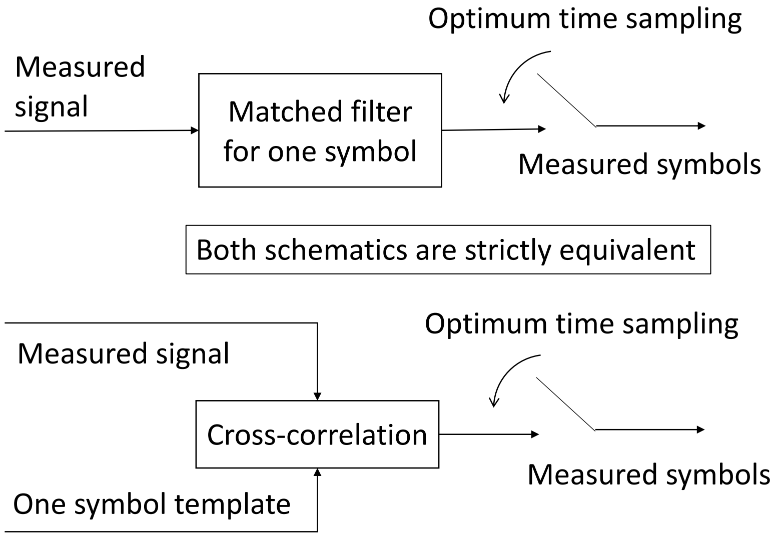
Optimum receiver

- The optimum receiver is the best possible receiver that can be used when the shape of ideal transmitted signal (or ideal symbols sequence) called the template is known and the channel is a linear channel with additive white stochastic noise
- These are exactly the conditions given by the EVM definition and the hypotheses the golden algorithm:
  - The EVM noise is defined as an addition to the ideal signal or symbols
  - The channel is linear, it is defined as a complex gain and delay and eventually a linear filter that can be equalized.
  - Gain and phase variations in time are not considered here.
- The optimum receiver performs the correlation of the received signal with the template (ideal signal or symbols) followed by the optimum sampling of the correlation result
- The correlation with the template is equivalent to the convolution with the matched filter time response (the conjugate of the time reversed template)
- The matched filter response (or template) has infinite length (at least one signal period)
- The matched filter is the optimal linear filter for maximizing the signal to noise ratio in the presence of additive stochastic noise
- This gives the lowest possible correct value for EVM

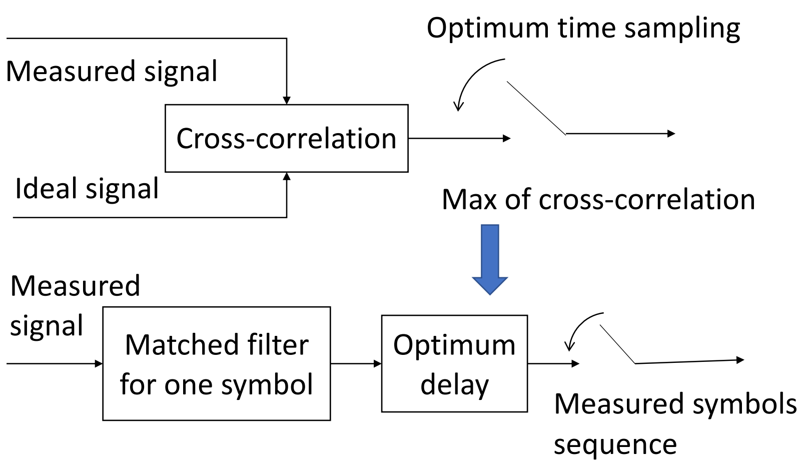
Optimum receiver for symbols (for reference)



- This is the classical optimum receiver for streams of unknown symbols
- Only the template (or shape) for one symbol is known
- If the measured signal is only one symbol, the optimum time sampling is at maximum of response (or cross-correlation)
- For a stream of symbols, optimum time samplings is at clock frequency (once per symbol)

Optimum receiver for a known ideal signal (1)

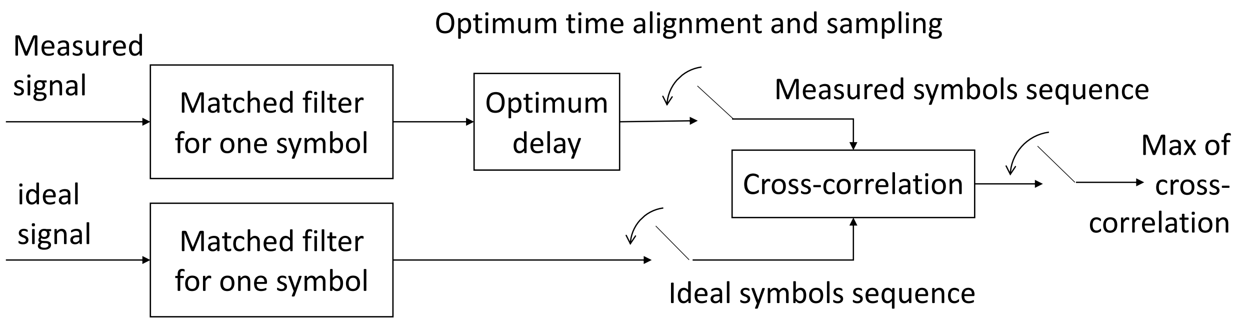
direct cross-correlation of signals



- The optimum receiver definition can be applied to the complete signal (at least one full period of this signal)
- The optimum time sampling is at the maximum of the cross-correlation of signals (once for each full signal period)
- Matched filters are not used to compute the cross-correlation but are necessary to extract symbols at clock frequency otherwise wide band noise on the measured signal is taken into account in the EVM

Optimum receiver for a known ideal signal (2)

cross-correlation of sequences of symbols



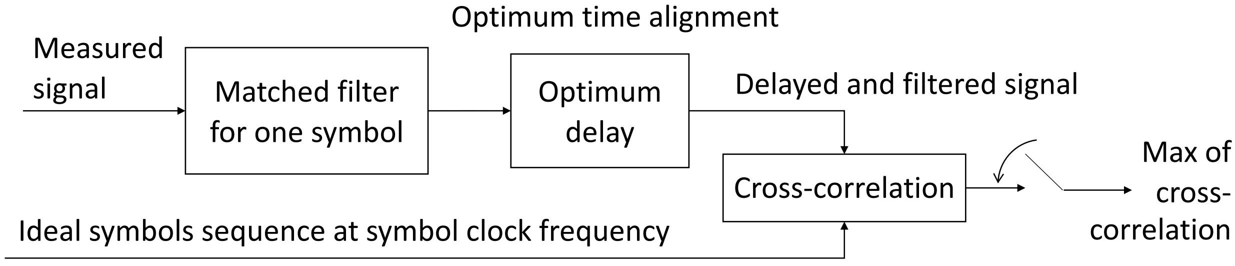
The optimum time alignment and sampling is the one that result in the maximum of the cross-correlation between symbols (once for each full signal period)

The optimum delay for the signal is generally negative because the channel introduces a real positive delay

The negative delay of the signal can be obtained through a positive delay of the ideal symbols or through a positive delay of the signal by one signal period plus the optimal negative delay

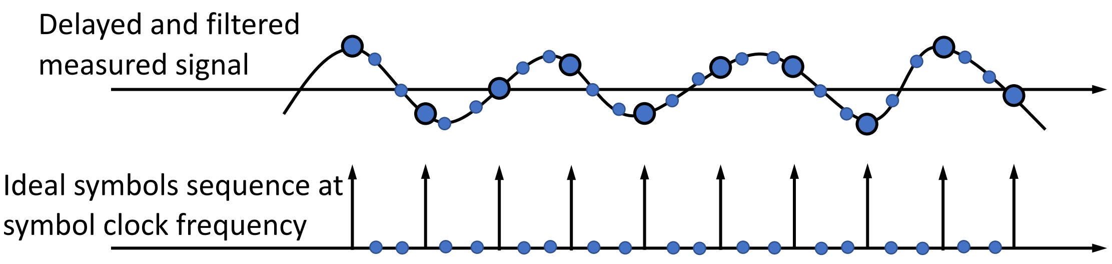
Optimum receiver for a known sequence of symbols

cross-correlation of measured signal with ideal symbols sequence



- If the ideal sequence of symbols is known, it can be used directly instead of obtaining it at the output of an optimum receiver
- The cross-correlation with ideal symbols sequence (stream of IQ Dirac pulses) realizes the sampling of the measured signal at one sample per symbol
- The optimum delay is defined as the one that gives the maximum of the cross-correlation between signal and symbols (once for each full signal period)

Cross-correlation of measured signal and ideal symbols



For a given delay (an integer multiple of sampling periods) of the filtered measured signal, the cross-correlation is the sum (along one signal period) of the products of ideal symbols and samples of the signal at the same time (one product for each symbol in the sequence or signal)

Definition of EVM used in the algorithm (1)

- The algorithm uses the cross-correlation between measured y and ideal x symbols after Nyquist filtering, optimum time alignment of signals and optimum sampling of the ideal and measured signals
- Instead of filtering the ideal signal and sampling it at optimum times, the ideal symbols can be used if they are already known (or after extraction from the ideal signal)
- Optimum time alignment and optimum sampling of the measured signal are defined as the ones giving the maximum modulus for the cross-correlation of measured symbols with ideal symbols
- The autocorrelation of ideal symbols at 0 delay (= the total or average power of ideal symbols sequence) can be used for normalization
- The normalized cross-correlation maximum is the equivalent complex gain of the transmission link
- No normalization of the signals is necessary
- EVM noise at receiver output is equal to the difference between measured symbols and ideal symbols multiplied by the equivalent complex gain

$$cc = E[y.x^*]$$

$$ac = E[x.x^*]$$

$$\gamma = \frac{\max cc}{ac}$$

$$\gamma = \frac{\max E[y, x^*]}{E[x, x^*]}$$

$$\eta = y - \gamma.x$$

29/05/2018 19

Definition of EVM used in the algorithm (2)

- This noise has zero correlation with the ideal symbols

$$E[\eta, x^*] = \mathrm{E}[(y - \gamma x), x^*] = \mathrm{E}[y, x^*] - \gamma.E[x, x^*] = 0$$

- The square of the EVM is the ratio of the average power of this noise to the average power of measured noiseless symbols (equal to the average power of ideal symbols multiplied by the equivalent complex gain)

$$EVM^2 = \frac{E[\eta, \eta^*]}{|\gamma|^2.E[x, x^*]} = \frac{\mathrm{E}[y, y^*] - \|\gamma\|^2.E[x, x^*]}{|\gamma|^2.E[x, x^*]} = \frac{\mathrm{E}[y, y^*]}{|\gamma|^2.E[x, x^*]} - 1$$

- The algorithm results in a closed form expression for EVM as:

$$EVM^2 = \frac{E[y,y^*] .E[x,x^*]}{|\max E[y.,]|^2} - 1 \qquad \mathrm{EV}M = \sqrt{\frac{E[y,y^*] .E[x,x^*]}{|\max E[y,x^*]|^2} - 1}$$

- This gives the wanted EVM only if the noise has been filtered with the matched filter (this is done in the schematics 2 and 3 where the matched filter is applied to the measured signal)
- Using the angle θ between ideal and measured signals in the complex N-space (see slides on geometric interpretation), we have: $\cos(\theta) = \frac{|\max E[y,x^*]|}{\sqrt{E[x.x^*]\,E[y.y^*]}}$ and $EVM = |\tan(\theta)| = \sqrt{\frac{1}{cos^2(\theta)} - 1}$

29/05/2018 20

Formulas for cross-correlations used in the algorithm

- Cross-correlation is an inner (or dot) product in the vector space of signals or symbols (or in the vector space of their spectra)
- The vector space may have a finite or an infinite number of dimensions
- For continuous signals: $E[y,x^*](\tau) = \int_{-\infty}^{+\infty} y(t+\tau)\, x^*(t)dt$
- For periodic continuous signal: $E[y,x^*](\tau) = \frac{1}{T}\int_{-T/2}^{+T/2} y(t+\tau)\, x^*(t)dt$
- For discrete signals: $E[y,x^*](\tau = n\,dt) = \sum_{m=-\infty}^{+\infty} y(m+n)\, x^*(m)$
- For periodic discrete signals: $E[y,x^*](\tau = n\,dt) = \frac{1}{N}\sum_{m=-N/2}^{N/2-1} y(m+n)\, x^*(m)$
- $\max E[y,x^*] = E[y,x^*](\tau_{max})$ with $|E[y,x^*](\tau_{max})| \geq |E[y,x^*](\tau)| \quad \forall\tau$
- Then: $|\max E[y,x^*]|^2 = \max|E[y,x^*]|^2$
- $E[x,x^*] = \|x\|^2 = E[x,x^*](0) = \max E[x,x^*]$ does not change if x is delayed
- But its value depends on the exact optimum sampling time of the signal in the symbol. This is a problem only for the measured signal as the ideal signal has not been delayed or symbols are known

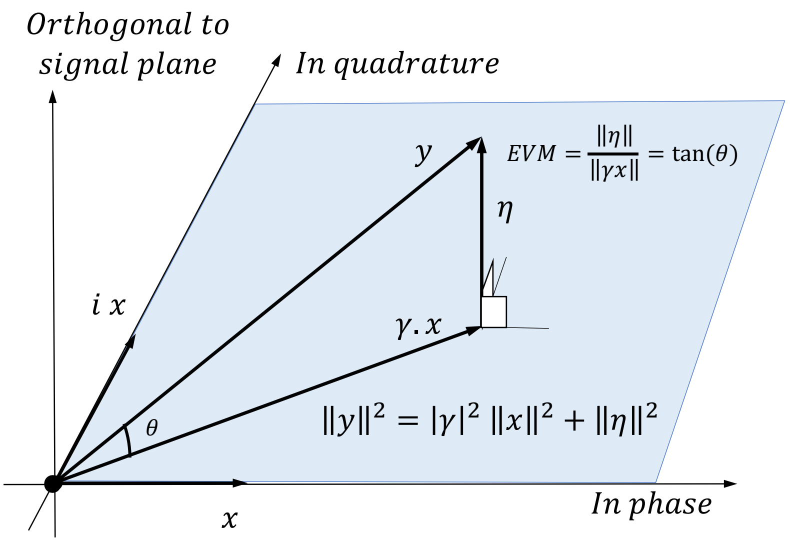
Geometric interpretation of EVM

- The signal γx is the orthogonal projection of the noisy symbols vector y on the complex coordinate proportional to x (i.e. the complex plane x, ix)
- The noise η is orthogonal to x (because their cross-correlation is 0) and has the smallest possible length
- The autocorrelation of vector x is the square of the Euclidian or L^2 norm of x and the total power of x is: $E[x,x^*] = \|x\|^2$
- The symbols and noise autocorrelations (powers) obey Pythagoras theorem:

$$E[y,y^*] = \|y\|^2 = |\gamma|^2 E[x,x^*] + E[\eta,\eta^*] = |\gamma|^2\,\|x\|^2 + \|\eta\|^2$$

- The measured symbols power is the sum of its ideal symbols component power and an uncorrelated noise power (having zero cross-correlation with ideal symbols)

Geometric presentation



Orthogonal projection of the measured symbols sequence y on the complex plane that contains ideal symbols sequence x and $i\ x$

Noise η is orthogonal to the complex plane of the symbols

For clarity, only one real dimension orthogonal to the symbols plane is shown

For a symbols sequence made of N complex symbols, this dimension would be $(N-1)$ complex

Practical EVM computation

- The optimum time alignment is based either on the maximum of the cross-correlation of symbols sequences (schematics 2 and 3) or on the maximum of the cross-correlation of signals (schematic 1)
- The value of the cross-correlation at the optimum time divided by the autocorrelation of the ideal symbols (or ideal signal) is the equivalent complex gain
- **The EVM can be computed:**
- Either from the root square sum of error vectors at optimum times (as the difference between measured symbols and ideal symbols multiplied by the equivalent complex gain)

$$EVM = \sqrt{\frac{E[\eta, \eta^*]}{|\gamma|^2 . E[x, x^*]}} = \sqrt{\frac{\mathrm{E}[(y - \gamma x), (y - \gamma x)^*]}{|\gamma|^2 . E[x, x^*]}}$$

- Or directly from the total power of ideal symbols, total power of measured symbols and the complex gain or the maximum of cross-correlation

$$EVM = \sqrt{\frac{\mathrm{E}[y, y^*] - |\gamma|^2 . E[x, x^*]}{|\gamma|^2 . E[x, x^*]}} = \sqrt{\frac{\mathrm{E}[y, y^*]}{|\gamma|^2 . E[x, x^*]} - 1} = \sqrt{\frac{\mathrm{E}[y, y^*] . E[x, x^*]}{|\max E[y, x^*]|^2} - 1}$$

- In the direct cross-correlation of signals schematics (like schematic 1), it is necessary to filter the measured signal (or at least the noise) before computing the measured symbols power or the filtered noise power
- The formal definition for NPR (noise power ratio) is exactly the same, using a different ideal signal and a noise filter that could be different from the SRC filter

Possible normalization of signals

- The cross-correlation algorithm does not need normalization of signals
- Some normalization can be applied for drawing constellations and for comparison with other algorithms
- The ideal sequence of symbols can be normalized to an average power of 1: $E[x, x^*] = \|x\|^2 = 1$
- The measured symbols sequence is normalized by division by the gain and its power is not 1 but:

$$E[y, y^*] = \|y\|^2 = E[x, x^*] + E[\eta, \eta^*]/|\gamma|^2 = 1 + \|\eta\|^2/\,|\gamma|^2$$

- After this normalization, the maximum of cross-correlation of measured and ideal symbols is equal to 1

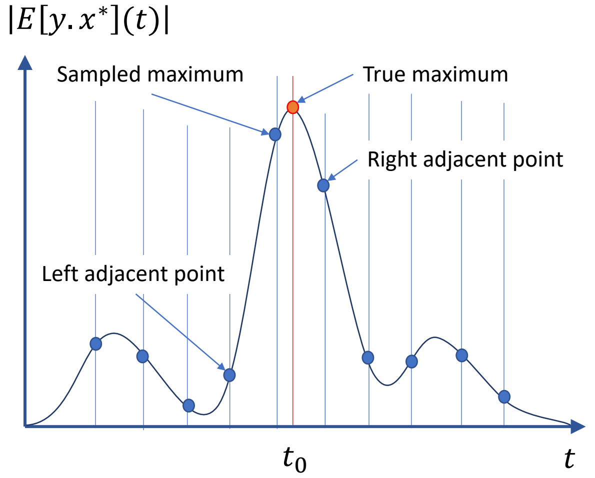
$$\max E[y, x^*] = E[x, x^*] = 1$$

- The normalized equivalent complex gain is now also equal to 1: $\gamma = \frac{\max E[y,x^*]}{E[x,x^*]} = 1$
- Error vectors are obtained as: $\eta = y - x$
- EVM is obtained as: $EVM^2 = \frac{\mathrm{E}[y,y^*]}{|\gamma|^2.E[x,x^*]} - 1 = \frac{E[y,y^*]\,.E[x,x^*]}{|\max E[y,x^*]|^2} - 1 = \mathrm{E}[y, y^*] - 1 = \|\eta\|^2$
- With this normalization, we have: $EVM = \|\eta\|$

Alignment algorithm description

- The alignment algorithm is based on the optimum receiver schematic for a known sequence of symbols
- The measured signal is filtered via a matched filter for one symbol template (e.g. an SRC filter)
- The cross-correlation between the filtered signal and the ideal symbols is computed for all possible delays at integer numbers of sample periods along the signal or a period of the signal
- The rough optimum delay (generally negative) is the one that gives the maximum sampled value for the modulus of the cross-correlation
- At this stage, the granularity is equal to the sampling period of the measured signal and is not fine enough for EVM accuracy (see slide on timing error)
- A time alignment Vernier must be used after that

Modulus of the cross-correlation curve



The true maximum of modulus of cross-correlation is not necessarily on a sample time

Even with 20 samples per symbol or more

Maximum timing error is

$\pm 1/2$ sample period

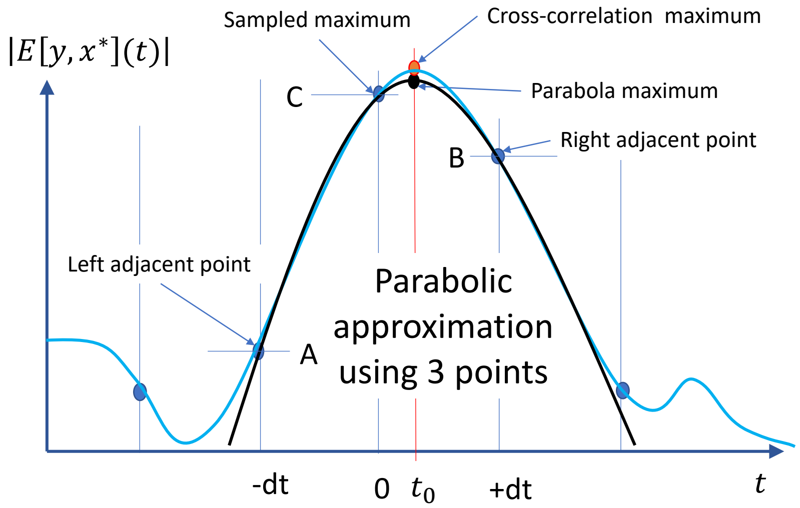
$= \pm 1/40$ sample period

$= \pm 0.025$ symbol period

Time alignment Vernier algorithm

- A time alignment Vernier must be used to obtain a delay granularity of less than 1/1000 of a symbol period and an EVM error less than 0.1%
- The maximum modulus point and the two adjacent points of the cross-correlation curve are used to approximate the modulus of cross-correlation curve by a parabola
- The optimum delay is computed as the time where the parabola is maximum (the derivative of the parabola is zero)
- It is a fraction of sample period (positive or negative) relative to the sampled maximum found before
- The maximum of the parabola is not used as the maximum of the cross-correlation
- The complex value of cross-correlation is computed at the optimum time (which is 0 after the cross-correlation has been delayed)
- This is simply the sum of all terms of the spectrum of the cross-correlation
- The measured and filtered signal is delayed by the optimum delay
- The measured symbols are obtained by sampling the delayed signal at time 0 and at all integer symbol periods after that

Time alignment Vernier relative to sampled maximum



$$y = ax^2 + bx + c$$
$$x = \frac{t}{dt}$$
$$A = a - b + c$$
$$B = a + b + c$$
$$C = c$$
$$2a = A + B - 2C$$
$$2b = B - A$$
$$\frac{t_0}{dt} = \frac{-b}{2a}$$
$$\frac{t_0}{dt} = \frac{(B - A)/2}{2C - A - B}$$

Fractional delay algorithm (warning)

- The time alignment Vernier needs to delay the cross-correlation and the measured signal by a fraction of the sampling period.
- This is obtained by multiplying the spectrum of the cross-correlation or signal by the complex exponential of a phase that is proportional to the frequency
- The corresponding total phase variation along the frequency vector is not a multiple of 2π so there is a phase discontinuity between the ends of the spectrum
- There is a spectrum discontinuity if there is any spectral power at the ends of the spectrum
- This will result in a Gibbs phenomenon on the time cross-correlation and a possible perturbation of the maximum position and its value
- In our case, the spectral power is null because the spectrum has been filtered by the shaping or matched filter and this filter has a null response at the ends of the spectrum for a sampling rate of 2 samples per symbol or more
- But this may be wrong if the time response of the filter has been truncated. In that case the frequency response of the filter is infinite and so it is not null at the ends of the spectrum
- The problem is worse if the FFT algorithm is not followed by a shift of 0 or center frequency to the center of the spectrum and if the phase slope is incorrectly applied. The phase discontinuity may happen at 0 frequency where there is always spectral power

Improved time alignment Vernier algorithm

- When the number of signal samples per symbol is low, (e.g. between 1 and 3) the parabola approximation using 3 points may not be a good enough fit because the values of the two adjacent points are nearly zero
- A delay of one-half sampling period is applied to the filtered measured signal and the cross-correlation is computed again for all possible delays at integer numbers of sample periods along the signal
- This can be done by delaying the cross-correlation itself by one-half sample period instead of computing it again
- This is identical to an interpolation that doubles the sampling rate of the signal
- The highest modulus of both sequences of cross-correlations is found
- The two adjacent points are taken on the other sequence
- The time Vernier algorithm is applied to these 3 points with $dt/2$ time intervals
- If necessary, the position of the maximum may be improved again by using a smaller value for a new time Vernier applied to the cross-correlation around the previously found maximum point

EVM error due to time alignment error

- For an ideal signal, we consider a parabolic approximation of the normalized cross-correlation main lobe and of the measured signal power around the optimum time as:

$$\frac{E[x,x^*].E[y,y^*](t)}{|E[y,x^*](t)|^2} = 1 + a(t/T)^2$$

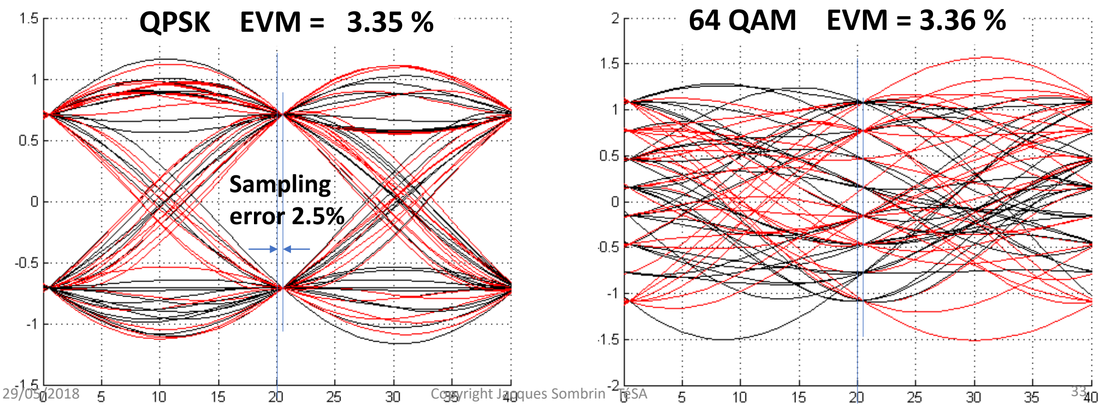
- An error of $t = \varepsilon T$ near the maximum results in an EVM of:

$$EVM(\varepsilon) = \sqrt{\frac{E[x,x^*].E[y,y^*](t)}{|E[y,x^*](t)|^2} - 1} = \sqrt{a\varepsilon^2} = \sqrt{a}|\varepsilon| \quad \text{instead of 0}$$

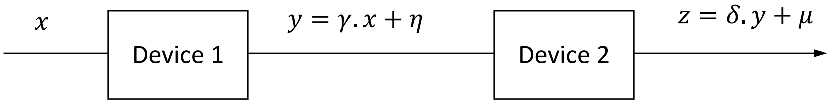
- The coefficient a depends slightly on the roll-off factor but not on the modulation or coding.
- It is around 1.759 for a roll-off of 0.35. Its square root is around 1.326
- A timing error of ± 2.5 % symbol period on an ideal signal (having zero EVM) would give a 3.316 % EVM error
- Simulations (next slide) agree with this value for EVM error: 3.35 % and 3.36 %
- This error must be added in root sum square (RSS) with other EVM errors

Alignment error of QPSK and 64QAM ideal signals

- Sampling rate = 20 samples per symbol
- Ideal signal has been delayed by one half sampling period
- Optimum sampling time is rounded to an integer number of sampling periods
- Residual sampling error = ± 0.5 sampling period = ± 2.5 % symbol period



EVM of cascaded devices (1)



- Symbols are x at input, y at device 1 output and z at device 2 output
- We have: $y = \gamma . x + \eta$ and $z = \delta\, y + \mu = \delta\, \gamma\, x + \delta\, \eta + \mu$
- With conditions of orthogonality: $\eta \perp x$ and $\mu \perp y = \gamma\, x + \eta$
- In general, μ is not orthogonal to either x or η . We project the EVM noise μ on plane $x\eta$:

$$\mu = \mu_x + \mu_\eta + \mu_0 = \mu_{x\eta} + \mu_0 \quad \text{with} \quad \mu_{x\eta} \ \ in\ x\eta\ plane \quad \text{and} \quad \mu_0 \perp x\eta\ plane$$

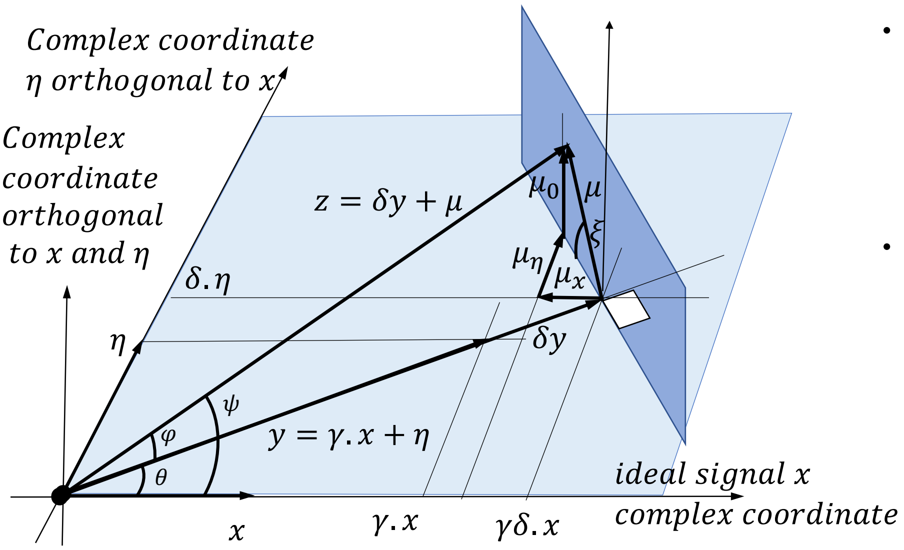
$$\|\mu_{x\eta}\| = \|\mu\| \cos\xi \qquad \|\mu_0\| = \|\mu\| \sin\xi$$

- The resulting signal z can be projected on 3 axes: $z = \delta\, \gamma\, x + \delta\eta + \mu_x + \mu_\eta + \mu_0 = (\delta\, \gamma\, x + \mu_x) + (\delta\, \eta + \mu_\eta) + \mu_0$

$$\mu_x \parallel x \qquad \|\mu_x\| = \|\mu_{x\eta}\| \frac{\|\eta\|}{\|y\|} = \|\mu_{x\eta}\| \sin\theta = \|\mu\| \cos\xi \sin\theta$$

$$\mu_\eta \parallel \eta \qquad \|\mu_\eta\| = \|\mu_{x\eta}\| \frac{\|\gamma\, x\|}{\|y\|} = \|\mu_{x\eta}\| \cos\theta = \|\mu\| \cos\xi \cos\theta$$

Geometric interpretation of cascaded EVM



- The EVM noise of the second device is in the plane orthogonal to its input signal y but not generally orthogonal to both x and η
- If it is uncorrelated with the signal and the noise in the first device, it is orthogonal to the plane of signal and first noise

$\mu_x = \mu_\eta = 0 \qquad \mu = \mu_0$

EVM of cascaded devices (2)

- EVM of device 1: $EVM_1 = \frac{\|\eta\|}{|\gamma|\ \|x\|} = \tan(\theta) \qquad \|\eta\| = \tan(\theta)|\gamma|\ \|x\| = EVM_1|\gamma|\ \|x\|$
- EVM of device 2: $EVM_2 = \frac{\|\mu\|}{|\delta|\ \|y\|} = \frac{\|\mu\|}{|\delta|\ \|\gamma\ x+\eta\|} = \tan(\varphi) \qquad \|\mu\| = \tan(\varphi)|\delta|\ \|y\| = EVM_2|\delta|\ \|y\|$
- EVM of cascaded devices: $EVM = \frac{\|\delta\ \eta+\mu_\eta+\mu_0\|}{\|\gamma\ \delta\ x+\mu_x\|} = \tan(\psi)$

$$\|y\| = \frac{|\gamma|\ \|x\|}{\cos\theta} \qquad \|y\|^2 = \frac{|\gamma|^2\|x\|^2}{\cos^2\theta} = |\gamma|^2\|x\|^2(1+\tan^2\theta) = |\gamma|^2\|x\|^2(1+EVM_1^2)$$

$$\cos\theta = \frac{1}{\sqrt{1+\tan^2\theta}} = \frac{1}{\sqrt{1+EVM_1^2}} \qquad \sin\theta = \frac{EVM_1}{\sqrt{1+EVM_1^2}}$$

$$\|\mu_x\| = \|\mu\|\cos\xi\sin\theta = EVM_2|\delta|\ \|y\|\cos\xi\sin\theta = EVM_2|\delta||\gamma|\|x\|\cos\xi\frac{\sin\theta}{\cos\theta}$$

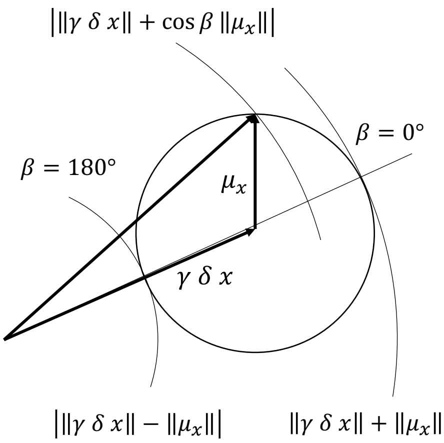
$$\|\mu_x\| = EVM_1\ EVM_2|\delta||\gamma|\|x\|\cos\xi$$

$$\|\mu_\eta\| = \|\mu\|\cos\xi\cos\theta = EVM_2|\delta|\ \|y\|\cos\xi\cos\theta = EVM_2|\delta||\gamma|\|x\|\cos\xi$$

$$\|\mu_0\| = \|\mu\|\sin\xi = EVM_2|\delta|\ \|y\|\sin\xi = EVM_2|\delta||\gamma|\|x\|\sin\xi\ /cos\theta$$

EVM of cascaded devices (3)

- Square of EVM of cascaded devices: $EVM^2 = \frac{\left\|\delta\,\eta + \mu_\eta\right\|^2 + \|\mu_0\|^2}{\|\gamma\,\delta\,x + \mu_x\|^2}$
- The term in the denominator is:
  $$\|\gamma\,\delta\,x + \mu_x\| = \|\gamma\,\delta\,x\| + \cos\beta\,\|\mu_x\| =$$
  $$\|\gamma\,\delta\,x\|\{1 + EVM_1 EVM_2 \cos\beta\cos\xi\}$$
- As we have generally $\|\gamma\,\delta\,x\| > \|\mu_x\|$, the magnitude is always positive
- Angle β is not known, it depends on the relative complex argument between components $\gamma\,\delta\,x$ and μ_x on the complex coordinate x
- In the case of $\left\|\delta\,\eta + \mu_\eta\right\|^2$ the result depends on the relative magnitude of the two terms, the cosine multiplier must be applied to the smaller term
  $$\left\|\delta\,\eta + \mu_\eta\right\| = \max\left(\|\delta\,\eta\|, \left\|\mu_\eta\right\|\right) + \cos\alpha\min\left(\|\delta\,\eta\|, \left\|\mu_\eta\right\|\right) =$$
  $$\|\gamma\,\delta\,x\|\{\max(EVM_1, EVM_2\cos\xi) + \cos\alpha\min(EVM_1, EVM_2\cos\xi)\}$$
- The cosines of angles α and β can have, independently, any value between -1 and +1



EVM of cascaded devices (4)

$$EVM^2 = \frac{\|\gamma\,\delta\,x\|^2\{\max(EVM_1, EVM_2\cos\xi) + \cos\alpha\min(EVM_1, EVM_2\cos\xi)\}^2 + \{EVM_2\|\gamma\,\delta\,x\|\sin\xi\,/cos\theta\}^2}{(\|\gamma\,\delta\,x\|\{1 + EVM_1 EVM_2\cos\beta\cos\xi\})^2}$$

$$EVM^2 = \frac{\{\max(EVM_1, EVM_2\cos\xi) + \cos\alpha\min(EVM_1, EVM_2\cos\xi)\}^2 + (1 + EVM_1^2)\{EVM_2\sin\xi\}^2}{\{1 + EVM_1 EVM_2\cos\beta\cos\xi\}^2}$$

$$EVM^2 = \frac{\{\max(EVM_1, EVM_2\cos\xi) + \cos\alpha\min(EVM_1, EVM_2\cos\xi)\}^2 + EVM_2^2(1 + EVM_1^2)(\sin\xi)^2}{(1 + \cos\beta\cos\xi\,EVM_1\,EVM_2)^2}$$

- Angles α and β depend on relative complex arguments between components on each complex coordinate such as between $\delta\,\eta$ and μ_η and between $\gamma\,\delta\,x$ and μ_x
- Uncorrelated case, $\xi = 90°$: $EVM^2 = EVM_1^2 + EVM_2^2 + EVM_1^2 EVM_2^2$
- Correlated case with $\xi = 0°$: $EVM^2 = \frac{(EVM_1 + \cos\alpha\,EVM_2)^2}{(1 + \cos\beta EVM_1 EVM_2)^2}$ or $EVM^2 = \frac{(EVM_2 + \cos\alpha\,EVM_1)^2}{(1 + \cos\beta EVM_1 EVM_2)^2}$
- In the correlated case,: $\min EVM = \frac{|EVM_1 - EVM_2|}{1 + EVM_1 EVM_2}$ and $\max EVM = \frac{EVM_1 + EVM_2}{1 - EVM_1 EVM_2}$
- These last expressions are the ones for the tangent of the sum or difference of 2 angles in the plane

Bias due to the limited number of symbols

- When generating a theoretically uncorrelated distortion, e.g. a random noise, it has coordinates along all axes in the complex N-space. (this may not be true for distortion correlated with the signal)
- It is not exactly orthogonal to the signal. One coordinate out of N of the noise is parallel to the signal
- The EVM algorithm will suppose this noise coordinate to be part of the signal and make an error both on signal power and on noise power
- For a normalized signal with N symbols, the measured noise power will be $\frac{N-1}{N}\sigma^2 = \left(1-\frac{1}{N}\right)\sigma^2$ instead of σ^2
- The measured signal amplitude will be between $1-\sigma/\sqrt{N}$ and $1+\sigma/\sqrt{N}$ instead of 1
- The measured EVM will be between $\frac{\sigma\sqrt{1-1/N}}{1+\sigma/\sqrt{N}}$ and $\frac{\sigma\sqrt{1-1/N}}{1-\sigma/\sqrt{N}}$ instead of the expected value σ
- For N=512 and $\sigma = 10\%$, the result is between 9.946 % and 10.034%
- For N=512 and $\sigma = 20\%$, the result is between 19.80 % and 20.16%
- For N=2048 and $\sigma = 20\%$, the result is between 19.91 % and 20.09%
- A higher number of symbols should be used, particularly for EVM higher than 10%

Standard deviation of the estimated EVM (1)

- Valid only for EVM referenced to average amplitude of signal (not to maximum)
- After normalization of x and complex gain γ, errors are: $\eta_m = y_m - x_m$
- We define the squares of their moduli: $X_m = |\eta_m|^2 = |y_m - x_m|^2$
- The average of variables X_m is the square of the EVM:

$$EVM^2 = \overline{X_m} = \frac{1}{N}\sum X_m \qquad EVM = \sqrt{\overline{X_m}} = \sqrt{\frac{1}{N}\sum X_m}$$

- The variance of variables X_m is:

$$Var(X_m) = \frac{1}{N-1}\sum(X_m - \overline{X_m})^2 = \frac{1}{N-1}\left\{\sum X_m^2 - N\,\overline{X_m}^2\right\}$$

- The standard deviation of variables X_m is: $\sigma_{X_m} = \sqrt{Var(X_m)}$
- The variance of the average $\overline{X_m}$ of variables X_m is: $Var(\overline{X_m}) = Var(X_m)/\text{N}$

Standard deviation of the estimated EVM (2)

- The standard deviation of the average $\overline{X_m}$ of variables X_m is:

$$\sigma_{\overline{X_m}} = \sigma_{EVM^2} = \sqrt{Var(\overline{X_m})} = \sqrt{Var(X_m)/N} = \sigma_{X_m}/\sqrt{N}$$

- The relative STD of the EVM is: $\frac{\sigma_{EVM}}{EVM} = \frac{1}{2}\frac{\sigma_{EVM^2}}{EVM^2} = \frac{1}{2}\frac{\sigma_{\overline{X_m}}}{\overline{X_m}} = \frac{\sigma_{X_m}}{2\sqrt{N}\,\overline{X_m}} = \frac{\sqrt{Var(X_m)}}{2\sqrt{N}\,EVM^2}$
- The standard deviation of EVM is:

$$\sigma_{EVM} = \frac{\sqrt{Var(X_m)}}{2\sqrt{N}\ EVM} = \frac{\sigma_{X_m}}{2\sqrt{N\,\overline{\overline{X_m}}}} = \frac{\sqrt{\sum X_m^2 - N\,\overline{X_m}^2}}{2\,\sqrt{N(N-1)\,\overline{\overline{X_m}}}}$$

- The standard deviation obtained with this formula seems to be OK for noise but it is pessimistic (different from 0) for deterministic distortions that depend on the amplitude or phase of the signal as it takes into account the difference between the magnitudes of errors for different points of the constellation

Standard deviation between estimated EVMs

- The same formula is applied to the average of EVMs measured on several trials or runs with the same parameters. The variable X_m is the square of the EVM obtained at the end of run m: $X_m = EVM(m)^2$
- N is the number of runs. The averaged EVM is obtained as the quadratic average:

$$EVM^2 = \overline{X_m} = \frac{1}{N}\sum X_m \qquad EVM = \sqrt{\overline{X_m}} = \sqrt{\frac{1}{N}\sum X_m}$$

- The standard deviation of the averaged EVM is:

$$\sigma_{EVM} = \frac{\sqrt{Var(X_m)}}{2\sqrt{N}\ EVM} = \frac{\sigma_{X_m}}{2\sqrt{N\,\overline{\overline{X_m}}}} = \frac{\sqrt{\sum X_m^2 - N\,\overline{X_m}^2}}{2\,\sqrt{N(N-1)\,\overline{\overline{X_m}}}}$$

- The standard deviation obtained seems to be OK for noise but it may be optimistic (it can be 0 or nearly 0) for pure distortions because all the $EVM(m)$ obtained are identical or nearly identical if the signal stays the same in each run or if the distortion is memoryless and the signal contains the same number of symbols of each type in the constellation

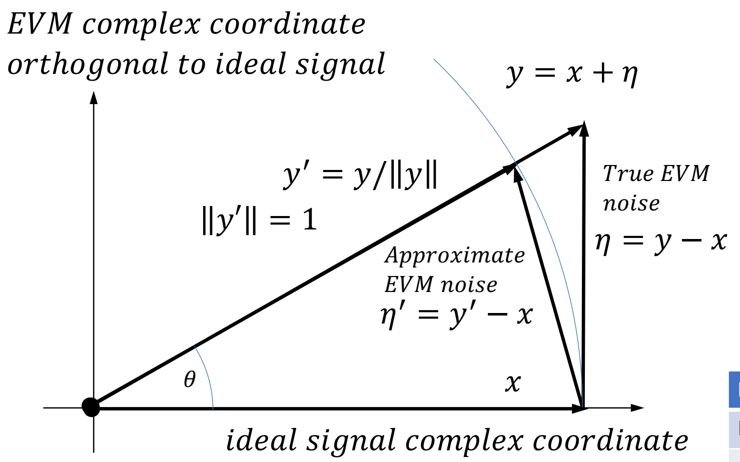
Bias in one approximate computation of EVM

- In some algorithms, the ideal and measured signals are both normalized to an average power of 1:

$$E[x, x^*] = \|x\|^2 = 1 \qquad E[y, y^*] = \|y\|^2 = 1$$

- The phase between ideal and measured signals is generally optimized by using a minimization of the EVM algorithm (but the cross-correlation phase could also be used)
- Error vectors are obtained as: $\eta' = y - x$
- EVM is obtained as: $EVM'^2 = \frac{E[\eta', \eta'^*]}{E[y, y^*]} = \frac{\|\eta'\|^2}{\|y\|^2} = \|\eta'\|^2 \qquad EVM' = \|\eta'\|$
- This computation is only approximate as the noise definition is not exact and the noise power is in fact divided by the sum of signal and noise power, see next slide
- This approximation is acceptable for low EVM but the use of this approximation as definition of EVM prevents any correct mathematical study of errors
- The result is always optimistic, it gives a lower value for EVM, like the non-data-aided algorithm
- Both biases add in the same direction

Geometric 2D comparison of EVM and approximation



The ideal signal and the cross-correlation have been normalized to 1 for easier comparison

$$\|x\| = 1 \qquad \gamma = 1 \qquad y' = y/\|y\|$$

$$\eta = y - x \qquad \eta' = y' - x$$

$$EVM = \|\eta\| = \tan(\theta)$$

$$EVM' = \|\eta'\| = 2.\sin(\theta/2)$$

$$EVM' = 2.\sin\left[\frac{\text{atan}(EVM)}{2}\right]$$

EVM	10 %	17.5%	20 %	30 %
EVM'	9.96 %	17.3%	19.7 %	29 %
error	-0.04%	-0.2%	-0.3%	-1%

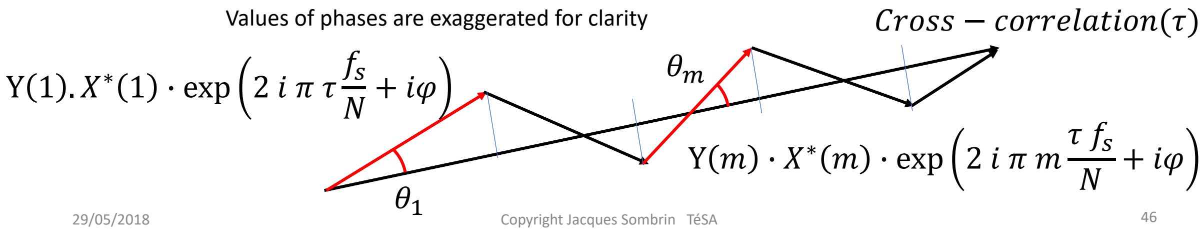
Time domain or frequency domain algorithm

- The algorithm may work in the time domain only or alternately in time and frequency domains
- Cross-correlation of two continuous infinite signals can be computed in the time domain or as the inverse Fourier transform of the product of Fourier transform of the signals
- The Fourier transform Is replaced by the Fourier series in the case of one or more periods of a continuous periodic signals
- The fast Fourier transform (FFT) can be used for one or more periods of a sampled periodic signals
- FFT can be applied to any finite sampled signal and is mathematically exact if the ideal and measured signal or symbols can be seen as one period of an infinite periodical signal or sequence of symbols
- This is obtained:
  - Either by generating directly one period of a periodic signal by using Fourier transforms to simulate all filters in the frequency domain
  - Or by circularizing a signal that is generated in the time domain. Time domain filters are applied to more than one period of the signal so that it is possible to extract from the signal a length of one period that is free of filter transients and has no discontinuity between end and beginning.
    In real time operation of a receiver or measurement equipment, a much longer signal must be used and processed (with windowing applied if necessary). The initial transient part of the signal is discarded and one full sequence-length of signal is processed for EVM measurement. This part is effectively circular and could also be processed using Fourier transforms.
- Matched filters are easy to simulate in the frequency domain
- Delays of signals or cross-correlations are easy to simulate by adding a linear phase slope (phase proportional to frequency) to the spectrum
- Cross-correlation at time 0 can be computed as the integral of the cross-correlation spectrum

Comparison of NIST and proposed algorithms (1)

- The algorithm used by NIST finds the delay τ that minimizes the RMS phase difference between the terms of the spectra of the measured signal (with added delay τ and phase φ) and ideal signal: $\theta_m = \text{atan}[Y(m)/X(m)] + 2\pi\, m\, \tau \frac{f_s}{N} + \varphi$
- $X(m)$ and $Y(m)$ are the m^{th} terms of the Fourier transforms of the ideal and measured signals $x(t)$ and $y(t)$
- We show here the sum of vectors having the same phases (the products of the measured spectrum and the conjugate of the ideal spectrum) with a delay τ and phase φ

Values of phases are exaggerated for clarity



Comparison of NIST and proposed algorithms (2)

- The following sum, for all frequency samples, of the terms we considered is in fact the cross-correlation of signals $y(t)$ and $x(t)$ for the delay τ :

$$cross - correlation(\tau) = \sum_{m=1}^{N} Y(m) \cdot X^{*}(m) \cdot \exp\left(2\, i\, \pi\, m \frac{\tau\, f_s}{N} + i\varphi\right)$$

- Assuming that we are near enough to the maximum of the cross-correlation, the phase differences between elements of the sum and the maximum of the cross-correlation are small
- As the amplitude of any element of the sum do not change when its phase is changed, the sum can be expressed as the sum of the projections of the elements on the cross-correlation for this value of delay.

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Comparison of NIST and proposed algorithms (3)

- It is equal to the sum of amplitudes of the products of spectra multiplied by the cosines of the phase differences with the cross-correlation
- As the phase differences are small, we can replace $\cos(\theta_m)$ by $1 - \theta_m^2$
- The projection of element m is:

$$|Y(m) \cdot X^{*}(m)| \cos(\theta_m) \approx |Y(m) \cdot X^{*}(m)|(1 - \theta_m^2)$$

- As the amplitudes of the elements in the sum do not vary, maximizing the sum of these projections is equivalent to minimizing the sum of: $|Y(m) \cdot X^{*}(m)|\ \theta_m^2$
- This is equivalent to finding τ by a least square fit of phases θ_m with a weighting equal to: $|Y(m) \cdot X^{*}(m)| = |Y(m) \cdot X(m)|$

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Weighting used in NIST algorithm

- An identical phase θ_m is obtained in NIST algorithm from the complex ratio: $Y(m)/X(m)$ instead of $Y(m) \cdot X^*(m)$
- The weighting used by NIST is slightly different from the one that would optimize the cross-correlation $|Y(m) \cdot X^*(m)| = |Y(m) \cdot X(m)|$
- It is either:

$$|X(m) \cdot X^*(m)| = |X(m)|^2$$

- Or

$$|Y(m) \cdot Y^*(m)| = |Y(m)|^2$$

- When the amplitudes of the terms of the ideal and measured spectra are not far from each other, the result on alignment must be near enough cross-correlation maximum.

Other algorithms

- Some algorithms simulate the operation of a real time receiver to determine the received symbols and the errors
- Typically, this is what must be done in a non-data aided measurement equipment
- In addition, if the ideal sequence of symbols is not known, the nearest ideal symbol must be determined for each received symbol as is done in a real receiver

- Other algorithms rely on a minimization of EVM by optimizing all parameters such as gain amplitude and phase, RF and clock frequency error, frequency, phase and amplitude drift as a function of time
- This result in a non-convex optimization problem with many local minima that can be difficult to solve efficiently

- For both theses types of algorithms, the resulting EVM is not obtained in closed form.
- It is difficult to compute the error on the EVM measurement but the algorithms could be calibrated by comparison to the proposed algorithm

Processing of the DC offset in standards

- ETSI technical report 290 “Digital Video Broadcasting (DVB); Measurement guidelines for DVB systems”, 1997, proposes to remove the effect of DC offset in the received symbols from computation of EVM
- Mashhour and Borjak, (2001) compute EVM for 8 PSK modulation following GSM standard (1999) by removing an optimized DC offset from measured symbols
- Jensen and Larsen (2013) refer to different standards (WLAN, LTE, UMTS, EDGE) that either remove or not the DC offset of the measured symbols in the EVM determination
- None of the authors or standards evoke the DC offset that may be present in the ideal transmitted sequence of symbols (even while the ideal full constellation is symmetrical and has no offset)

DC offset operation of receivers

- Most analog receivers would output a random value for the DC offset of the analog signal because of the operation of components and amplifiers
- The DC offset of the analog signal is generally removed by using coupling capacitors between stages of amplification and before the analog to digital converter
- The DC offset can also be removed from the symbols after optimum sampling and before decision on the symbol values
- Because of this, many receivers do not accept long streams of 0 or 1 because such a signal would be confused with a DC offset and be removed. The DC block is a high-pass filter. The inverse of its frequency cut-off limits the length of the accepted stream.
- In addition, the receiver clock must stay synchronized during this time.
- All-digital receivers (from the RF or IF) and simulations have less problems with processing the DC offset and keeping its correct value along all the processing

DC offset of pseudo-random sequences of symbols

Many sequences of symbols are used to test transmission links

- Random permutation (shuffled) sequences and de Bruijn sequences contain an equal number of each symbol in the constellation (equal occurrence). They have zero DC offset and IQ imbalance
- Maximum length shift register sequences (MLSR) have nearly equal occurrence of symbols. The 0 symbol appears one less time than the others. It is generally on the diagonal. They have nearly zero DC offset
- Random sequences and Galois fields sequences have varying numbers of symbols of each type in the constellation (either random or depending on the initialization). They have high DC offset

We propose to use sequences with low or zero DC offset

DC offset variants of the EVM computation (1)

Computation of the equivalent complex gain

We consider 2 main variants of the alignment algorithm:

1) We compute complete correlations (with the DC components) to find the optimum time alignment, optimum sampling of measured signal and equivalent complex gain
2) We compute reduced correlations (without the DC components) to find the optimum time alignment, optimum sampling of measured signal and equivalent complex gain

If the DC offset of the ideal sequence of symbols is 0, these 2 variants give the same result

The second variant may be useful when the DC offset of the measured sequence is quite different from the DC offset of the ideal sequence multiplied by the complex gain

DC offset variants of the EVM computation (2)

Computation of the EVM noise power and EVM ratio

The choice is linked to the previous one

1) Either we keep the DC offsets in the comparison of symbols.
   This will add a DC component to the EVM noise.
   It is applicable mainly in the first variant as the optimum complex gain will take into account the DC offsets and the total EVM noise with DC component is optimized
2) Or we neglect the DC offsets in the comparison of symbols.
   This seems to be the normal choice for the second variant

Future work

- Future receivers will be able to equalize the channel response and to correct IQ phase and amplitude imbalance (and IQ skew, different delay on transmitted I and Q signals)
- This will have to be included in the algorithm
- Equalization and IQ imbalance correction possible using cross-correlation and spectrum of ideal and measured signals
- Different optimum time alignment on I and Q possible also
- Computation of phase and amplitude EVM ?
- Frequency correction more difficult, long time signal and receiver algorithm needed
- Gain and frequency variation during measurement frame

Conclusion

- The proposed algorithm simulates the operation of an optimum receiver that would receive a known sequence of symbols after known modulation and unknown multiplicative channel, unknown delay and addition of stochastic noise
- The smallest EVM possible in these conditions is obtained
- A standard deviation for the computed EVM is given
- Other algorithms are compared with the proposed algorithm
- Two variants are proposed: EVM with or without DC offset
- Simulation of filters or delays and computation of cross-correlations can be done in the time domain or in the frequency domain