Spectrometer Calibration using Sparse Inverse Problems for Atmospheric Sounding

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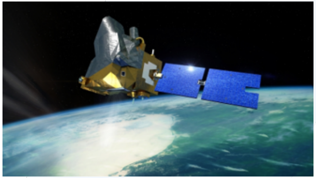
by PhD candidate Jihanne El Haouari



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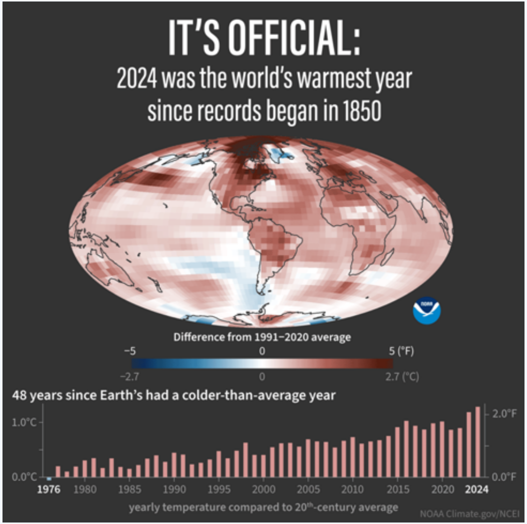
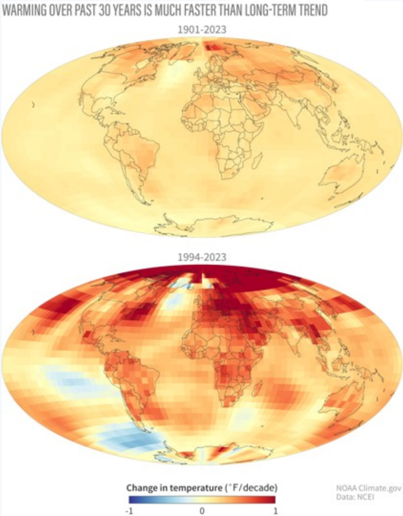
November 25, 2025

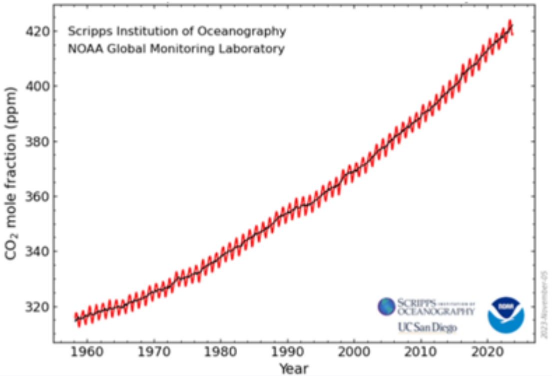
Climate change challenges



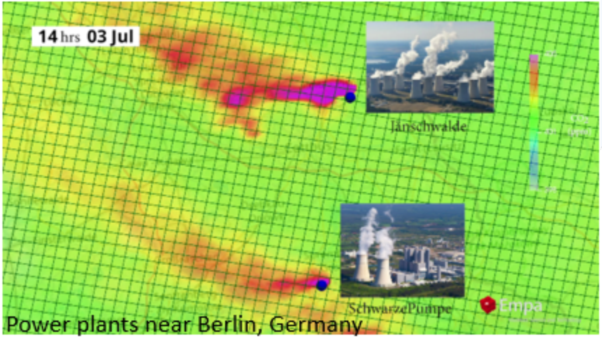


Climate change challenges

Atmospheric CO_2 at Mauna Loa Observatory



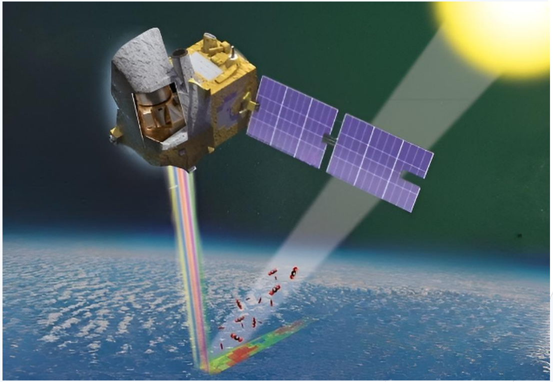
Simulated CO_2 at 2x2 km^2 grid



Climate change challenges

Climate change challenges

Monitoring Atmospheric CO_2 Concentrations via Remote Sensing



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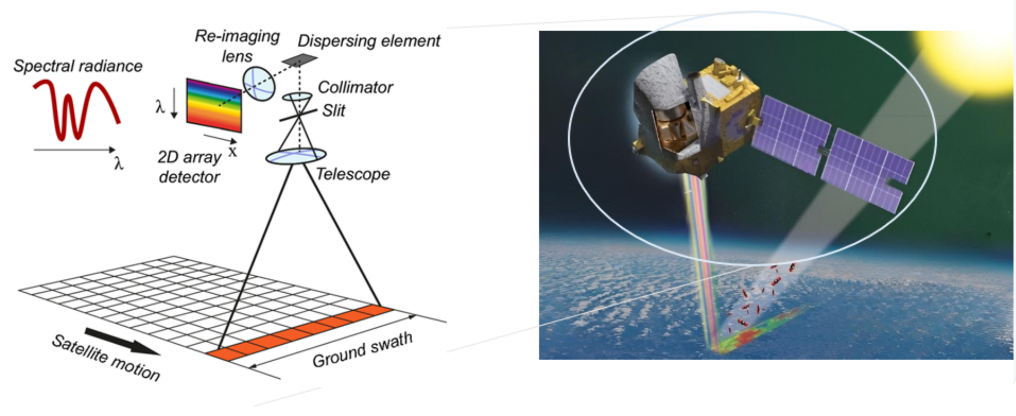
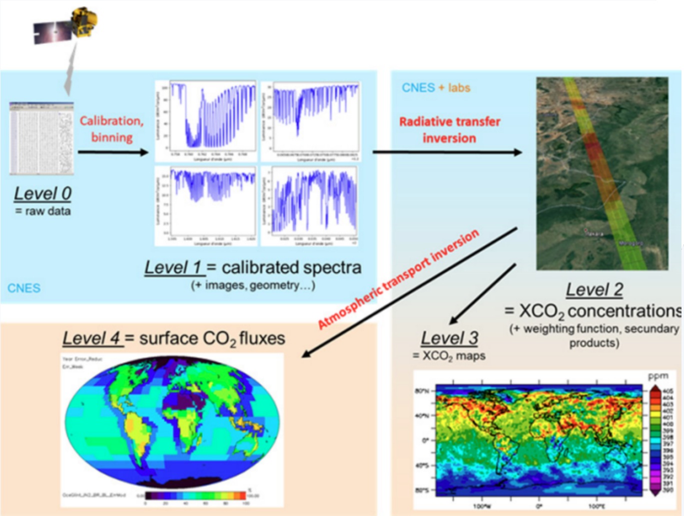
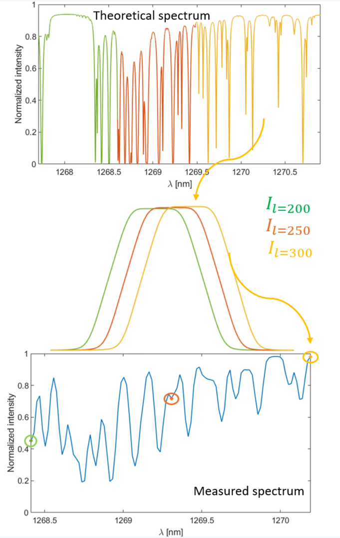


Spectrometer Calibration for Accurate CO_2 Concentration Measurements



Estimation of Instrument Spectral Response Functions (ISRFs)



An ill-posed problem

For each pixel $\ell = 1, \ldots, N_\lambda$

- 1 wavelength λ_ℓ
- 1 observation $y_\ell \in \mathbb{R}^+$
- 1 ISRF $\boldsymbol{I}_\ell \in \mathbb{R}^{N+1}$
- 1 nonlinear function f_ℓ
- 1 reference spectrum: $\boldsymbol{r}_\ell = \pi(\boldsymbol{r}, \lambda_\ell)$

$$\underbrace{y_\ell}_{\text{known}} = \underbrace{f_\ell}_{\text{unknown}} \left(\left\langle \underbrace{g}_{\text{unknown}} \left(\underbrace{\boldsymbol{r}_\ell}_{\text{known}} \right), \underbrace{\boldsymbol{I}_\ell}_{\text{unknown}} \right\rangle \right) + \epsilon_l$$

Contributions of this Thesis

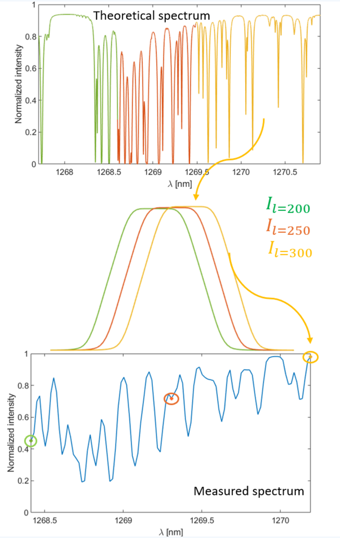
An ill-posed problem

For each pixel $\ell = 1, \ldots, N_\lambda$ $\quad \underbrace{y_\ell}_{\text{known}} = \underbrace{f_\ell}_{\text{unknown}}(\langle \underbrace{g}_{\text{unknown}}(\underbrace{\boldsymbol{r}_\ell}_{\text{known}}), \underbrace{\boldsymbol{I}_\ell}_{\text{unknown}} \rangle) + \epsilon_l$

- **Linear inverse problem:** $y_\ell = \langle \boldsymbol{r}_\ell, \boldsymbol{I}_\ell \rangle + \epsilon_l$
  - New model: Sparse representation of ISRFs
- **Sparse nonlinear inverse problem:** $y_\ell = f_\ell(\langle \boldsymbol{r}_\ell, \boldsymbol{I}_\ell \rangle) + \epsilon_l$
  - Separable nonlinearities [Golub1973]
  - General model of nonlinearities
  - Dictionary preconditioning
- **Sparse nonlinear inverse problem with shifts:** $y_\ell = f_\ell(\langle g(\boldsymbol{r}_\ell), \boldsymbol{I}_\ell \rangle) + \epsilon_l$
  - Model of shifts

Outline

ISRF Estimation as an Inverse Problem



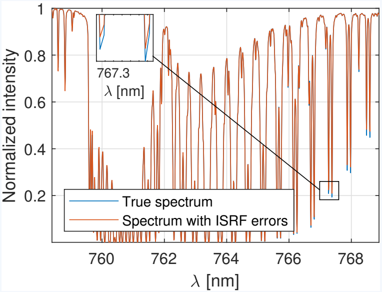
An ill-posed problem

For each pixel $\ell = 1, \ldots, N_\lambda$

- 1 wavelength λ_ℓ
- 1 observation $y_\ell \in \mathbb{R}^+$
- 1 ISRF $\boldsymbol{I}_\ell \in \mathbb{R}^{N+1}$
- nonlinearities: functions f_ℓ and g
- 1 reference spectrum: $\boldsymbol{r}_\ell = \pi(\boldsymbol{r}, \lambda_\ell)$

$$\underbrace{y_\ell}_{\text{known}} = \underbrace{f_\ell}_{\text{unknown}} (\langle \underbrace{g}_{\text{unknown}} (\underbrace{\boldsymbol{r}_\ell}_{\text{known}}), \underbrace{\boldsymbol{I}_\ell}_{\text{unknown}} \rangle) + \epsilon_l$$

ISRF Estimation as an Inverse Problem



An ill-posed problem

For each pixel $\ell = 1, \ldots, N_\lambda$

- 1 wavelength λ_ℓ
- 1 observation $s_\ell \in \mathbb{R}^+$
- 1 ISRF $\boldsymbol{I}_\ell \in \mathbb{R}^{N+1}$
- 1 reference spectrum: $\boldsymbol{r}_\ell = \pi(\boldsymbol{r}, \lambda_\ell)$

$$\underbrace{s_\ell}_{\text{known}} = \langle \underbrace{\boldsymbol{r}_\ell}_{\text{known}} , \underbrace{\boldsymbol{I}_\ell}_{\text{unknown}} \rangle + \epsilon_l$$

ISRF Estimation as an Inverse Problem

Assumption: ISRFs $\boldsymbol{I}_\ell$ vary little from ℓ to $\ell+1$

Sliding window $\mathcal{W}(\lambda_\ell)$ of $L+1$ measurements around λ_ℓ

- $\boldsymbol{R}_\ell = \Pi\left(\boldsymbol{r}, \mathcal{W}(\lambda_\ell)\right) = [\boldsymbol{r}_{\ell-\frac{L}{2}}, \ldots, \boldsymbol{r}_{\ell+\frac{L}{2}}] \in \mathbb{R}^{(L+1)\times(N+1)}$
- $\boldsymbol{s}_\ell = [s_{\ell-\frac{L}{2}}, \ldots, s_{\ell+\frac{L}{2}}] \in \mathbb{R}^{L+1}$

Observation model (matrix form)

$$\boldsymbol{s}_\ell = \boldsymbol{R}_\ell \boldsymbol{I}_\ell$$

Assumptions about ISRF $\boldsymbol{I}_\ell$: positivity

Parametric models

Gaussian model [1]

$$\boldsymbol{I}_{\ell,\beta_{\mathrm{G}}}(x) = A_{\mathrm{G}} \ \exp\left(-\frac{(\lambda_\ell - x - \mu_{\mathrm{G}})^2}{2\sigma_{\mathrm{G}}^2}\right), \quad x \in \boldsymbol{\Delta}$$

$\boldsymbol{\Delta}$ wavelength grid, $\beta_{\mathrm{G}} = (A_{\mathrm{G}}, \mu_{\mathrm{G}}, \sigma_{\mathrm{G}}^2)^T$ - parameters

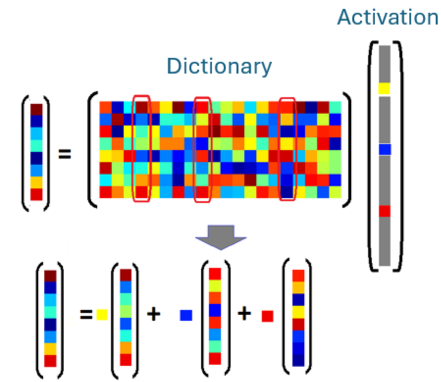
Super-Gaussian model [1]

$$\boldsymbol{I}_{\ell,\beta_{\mathrm{SG}}}(x) = A_{\mathrm{SG}} \ \exp\left(-\left|\frac{\lambda_\ell - x - \mu_{\mathrm{SG}}}{w_{\mathrm{SG}}}\right|^{k_{\mathrm{SG}}}\right), \quad x \in \boldsymbol{\Delta}$$

$\beta_{\mathrm{SG}} = (A_{\mathrm{SG}}, \mu_{\mathrm{SG}}, w_{\mathrm{SG}}, k_{\mathrm{SG}})^T$- parameters

[1] Beirle S. et al. (2017) – Parameterizing the instrumental spectral response function and its changes by a super-Gaussian and its derivatives, *Atmos. Meas. Tech.*

Proposed non-parametric model



Sparse representation of ISRFs (SPIRIT)

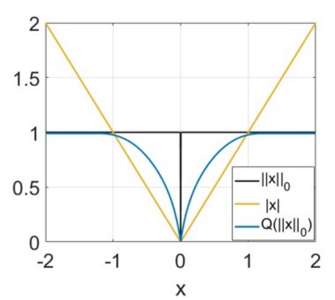
- Dictionary $\boldsymbol{\Phi} \in \mathbb{R}^{(N+1)\times N_D}$ (SVD of representative ISRFs)

$$\boldsymbol{I}_\ell \approx \boldsymbol{I}_\ell^K = \boldsymbol{\Phi}\boldsymbol{\alpha}_\ell = \sum_{k=1}^{K} \alpha_k \boldsymbol{\Phi}_{\gamma_k}$$

$$\boldsymbol{s}_\ell \approx \boldsymbol{R}_\ell \boldsymbol{I}_\ell^K = \boldsymbol{\Psi}_\ell \boldsymbol{\alpha}_\ell$$

where $\boldsymbol{\Psi}_\ell = \boldsymbol{R}_\ell \boldsymbol{\Phi} \in \mathbb{R}^{(L+1)\times N_D}$

ISRF Estimation as a sparse inverse problem



Optimization problem with ℓ_0 norm [2]

$$\arg\min_{\boldsymbol{\alpha}_\ell} \|\boldsymbol{s}_\ell - \boldsymbol{\Psi}_\ell \boldsymbol{\alpha}_\ell\|_2^2$$

$$s.t. \|\boldsymbol{\alpha}_\ell\|_0 \leq K$$

Relaxation via ℓ_1 regularization (LASSO)

$$\arg\min_{\boldsymbol{\alpha}_\ell} \|\boldsymbol{s}_\ell - \boldsymbol{\Psi}_\ell \boldsymbol{\alpha}_\ell\|_2^2 + \mu\|\boldsymbol{\alpha}_\ell\|_1$$

Relaxation via quadratic envelope regularization [3]

$$\arg\min_{\boldsymbol{\alpha}_\ell} \|\boldsymbol{s}_\ell - \boldsymbol{\Psi}_\ell \boldsymbol{\alpha}_\ell\|_2^2 + Q_\gamma(\mu\|\boldsymbol{\alpha}_\ell\|_0)(\boldsymbol{\alpha}_\ell),\ \gamma > 0$$

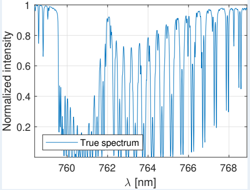
[2] Mallat, S., Zhang, Z. (1993) - Matching pursuits with time-frequency dictionaries. *IEEE Trans. Signal Process.*

[3] Carlsson M. (2019) – On convex envelopes and regularization of non-convex functionals, *J. Optim. Theory Appl.*

Experimental Setup and Evaluation Metrics

Data description

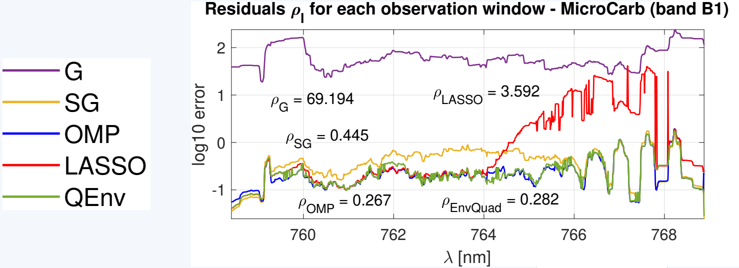
- Reference spectra: 4A/OP radiative transfer software
- Simulated ISRFs for the MicroCarb spectrometer
- Measured spectra: $s(\lambda_l) = (r * I_l)(\lambda_l) = \int_{-\infty}^{\infty} r(\lambda_l - u) I_l(u) du$



Metrics

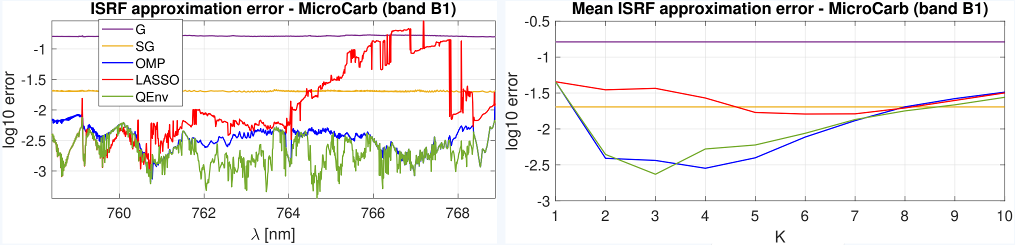
- ISRF approximation errors: $E_\ell = \sum_{n=-N/2}^{N/2} |I_\ell(n\Delta) - \hat{I}_\ell(n\Delta)|$
- Residual errors: $\rho = \frac{1}{N_\lambda} \sum_{\ell=1}^{N_\lambda} ||\boldsymbol{s}_\ell - \boldsymbol{r}_\ell \hat{\boldsymbol{I}}_\ell||_2^2$

Numerical results



$K = 3$

Numerical results



ISRF estimation error ($K = 3$) and mean ISRF estimation error

- Poor performance with parametric models (0.1s)
- Limited performance of the LASSO algorithm (0.01s)
- Best performance achieved with quadratic envelopes (1s)
- OMP: best trade-off between computation time and estimation performance (0.001s)

Sparse nonlinear inverse problem

Forward model

$$\underbrace{y_\ell}_{\text{known}} = \underbrace{f_\ell}_{\text{unknown}} (\langle \underbrace{g}_{\text{unknown}} (\underbrace{\boldsymbol{r}_\ell}_{\text{known}}), \underbrace{\boldsymbol{l}_\ell}_{\text{unknown}} \rangle) + \epsilon_l \quad \ell = 1, ..., N_\lambda$$

$$\boldsymbol{l}_\ell = \boldsymbol{\Phi}\boldsymbol{\alpha}_\ell, \ \boldsymbol{\alpha}_\ell \text{ sparse}$$

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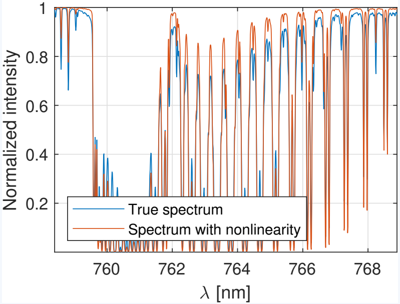

- <u>Parameterized nonlinearities:</u> function f_θ
  - Spectrometer calibration
  - Hyperspectral imaging
  - Epidemiology

Description of nonlinearities

Parameterized nonlinearities

- f_θ bijective
- Separable nonlinearity[4]: $f_\theta(z) = \sum_{p=0}^{P} g_p(z)\theta_p \ \forall z \in \mathbb{R}^+$
  - Polynomials[5]
- General model of nonlinearities [6,7]
  - $f_\theta = \theta_1 x^{\theta_2}$
  - $f_\theta = \tanh(\theta_1 x) + \theta_2 x$

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[4] Golub,G. H., Pereyra V. (2003) - Separable nonlinear least squares: the variable projection method and its applications, *Inverse Probl.*

[5] Altmann Y. et al. (2012), Supervised nonlinear spectral unmixing using a postnonlinear mixing model for hyperspectral imagery, *IEEE Trans. Image Process.*

[6] Taghvaei A. et al. (2020), Fractional SIR epidemiological models, *Sci. Rep.*

[7] Babaie-Zadeh M. et al. (2001), Blind separating convolutive post non-linear mixtures, *Proc. 3rd ICA Workshop.*

Iterative optimization method

Extended Invariance Principle (EXIP) with sparsity

- Problem formulation with sparsity: $\arg\min_{\boldsymbol{\theta},\boldsymbol{\alpha}_\ell} \|\boldsymbol{y}_\ell - f_{\boldsymbol{\theta}} \circ (\boldsymbol{\Psi}_\ell \boldsymbol{\alpha}_\ell)\|_2^2 + \mu\|\boldsymbol{\alpha}_\ell\|_0$
- Resolution method: An iterative approach
  - Step 1: Estimation of $\boldsymbol{\theta}$: $\arg\min_{\boldsymbol{\theta}} \|\boldsymbol{y}_\ell - f_{\boldsymbol{\theta}} \circ (\boldsymbol{\Psi}_\ell \boldsymbol{\alpha}_\ell)\|_2^2$
  - Step 2: EXtended Invariance Principle (EXIP)[8] for estimating $\boldsymbol{\alpha}_\ell$: $\beta_\ell = g(\boldsymbol{\alpha}_\ell) = \boldsymbol{\Psi}_\ell \boldsymbol{\alpha}_\ell$

$$\arg\min_{\boldsymbol{\alpha}_\ell} \left[\hat{\beta}_\ell - g(\boldsymbol{\alpha}_\ell)\right]^T R''_{\beta_\ell}(\hat{\beta}_\ell) \left[\hat{\beta}_\ell - g(\boldsymbol{\alpha}_\ell)\right] + \mu\|\boldsymbol{\alpha}_\ell\|_0$$

where $R_{\beta_\ell}(\beta_\ell) = \| \boldsymbol{y}_\ell - f_{\boldsymbol{\theta}} \circ \beta_\ell\|_2^2$ and $R''_{\beta_\ell}(\hat{\beta}_\ell) = \nabla^2 R_{\beta_\ell}(\hat{\beta}_\ell)$

[8] Stoica P. & Soderstr T. (1989) - On reparametrization of loss functions used in estimation and the invariance principle, *Signal Process.*

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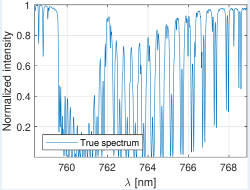

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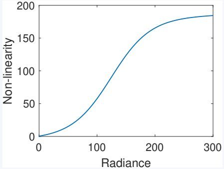
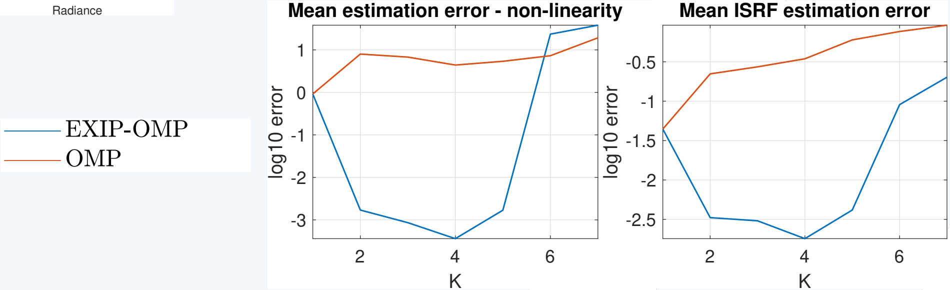
- Residual errors: $\rho = \frac{1}{N_\lambda} \sum_{\ell=1}^{N_\lambda} ||\boldsymbol{s}_\ell - \boldsymbol{r}_\ell \hat{\boldsymbol{I}}_\ell||_2^2$
- Estimation errors for the parameters of the nonlinearities:
  $r_{\text{NL}} = \frac{1}{Q} \sum_{i=1}^{Q} \left(f_{\theta}(x_i) - f_{\hat{\theta}}(x_i)\right)^2, x_i \in \mathcal{D}$

Numerical results - ISRF estimation

- Nonlinearity: $f_{\theta}(z) = \frac{\theta_1}{1+\exp(-\theta_2 z)} + \theta_2 z$



- Measured spectrum divided into 14 channels of $Q = 70$ wavelengths SNR=55dB
- 1 ISRF to estimate per wavelength



Competitive performance with EXIP / OMP

Numerical results - ISRF estimation

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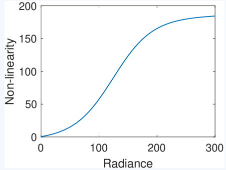

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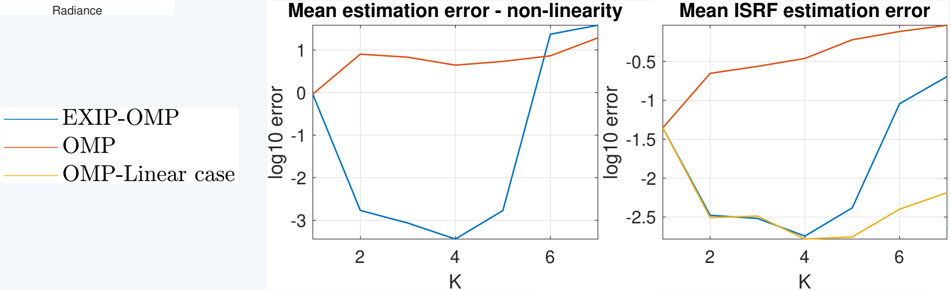


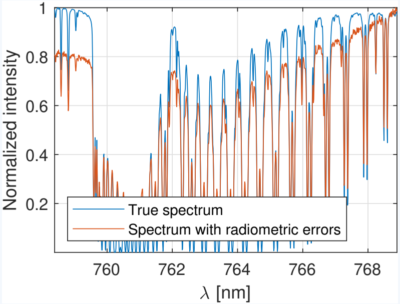
Competitive performance with EXIP / OMP

Radiometric calibration using multiple reference spectra

MicroCarb spectrometer

$$\underbrace{y_\ell}_{\text{known}} = \underbrace{f_{\boldsymbol{\theta}_\ell}}_{\text{unknown}} (\langle \underbrace{\boldsymbol{r}_\ell}_{\text{known}} , \underbrace{\boldsymbol{l}_\ell}_{\text{unknown}} \rangle) + \epsilon_l \quad \ell = 1, \ldots, N_\lambda$$

$\boldsymbol{\theta}_\ell$ - parameter vector of pixel ℓ



Radiometric errors: (functions $f_{\boldsymbol{\theta}_\ell}$)

- Linear gains
- Dark current
- Nonlinear gains

- 1 function $f_{\boldsymbol{\theta}_\ell}$ per pixel ℓ
- Q reference spectra $\boldsymbol{r}^{(q)}$

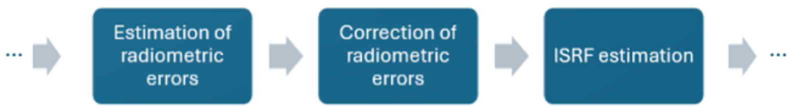
Radiometric Calibration Using Multiple Reference Spectra

Joint estimation of ISRF and radiometric errors (SPIRITUAL)

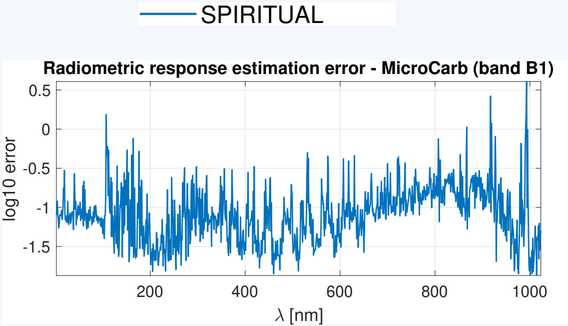
- Polynomial model:

$$y_{\ell,q} = f_{\boldsymbol{\theta}_\ell}\left(s_{\ell,q}\right) = \sum_{p=0}^{P} \theta_p^\ell \, \left(s_{\ell,q}\right)^p = \sum_{p=0}^{P} \theta_p^\ell \, \left(\left(\boldsymbol{r}_\ell^{(q)}\right)^T \boldsymbol{I}_\ell\right)^p, \; q = 1, .., Q$$

- Iterative estimation:
  - **Step 1:** Estimate nonlinearity parameters $\boldsymbol{\theta}_\ell$ using Q reference spectra
  - **Step 2:** Estimate ISRFs via sliding window with all reference spectra

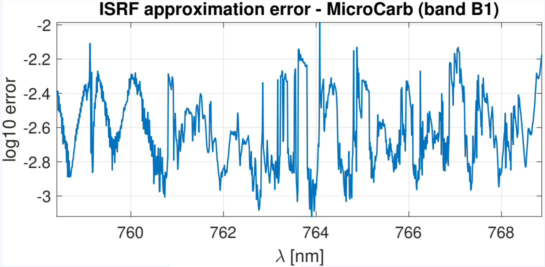


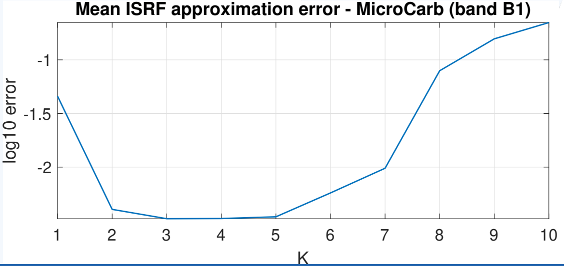
Numerical results - Radiometric calibration using multiple reference spectra



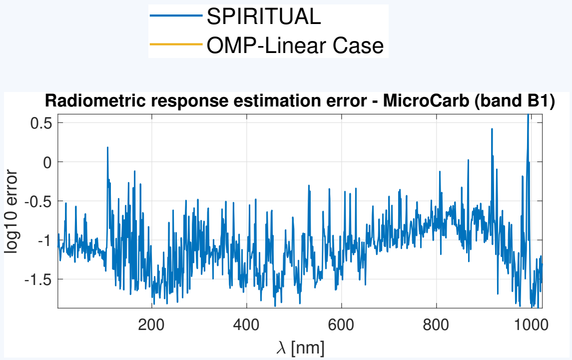
K=4, P=3, Q=11

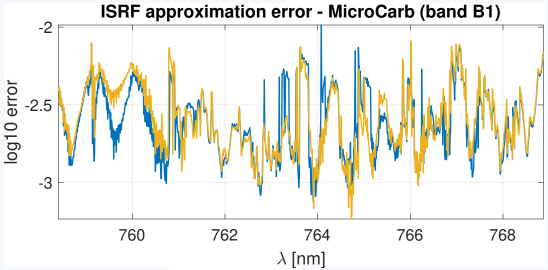
Competitive performance with SPIRITUAL

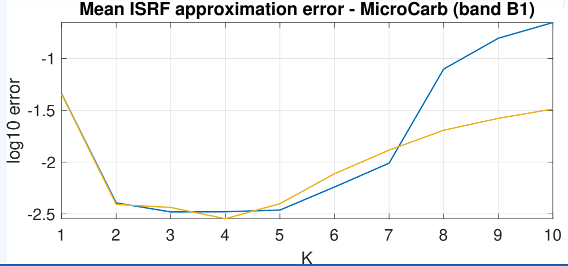




Numerical results - Radiometric calibration using multiple reference spectra







K=4, P=3, Q=11

Competitive performance with SPIRITUAL

Sparse nonlinear inverse problem with shifts

Forward model

$$\underbrace{y_\ell}_{\text{known}} = \underbrace{f_\ell}_{\text{unknown}} (\langle \underbrace{g}_{\text{unknown}} (\underbrace{\boldsymbol{r}_\ell}_{\text{known}}), \underbrace{\boldsymbol{I}_\ell}_{\text{unknown}} \rangle) + \epsilon_l \quad \ell = 1, \ldots, N_\lambda$$

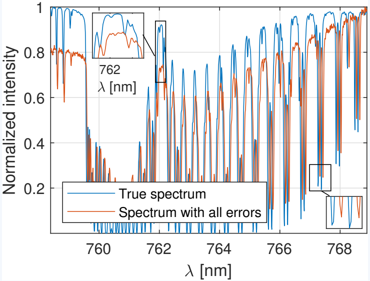
$\boldsymbol{I}_\ell = \boldsymbol{\Phi}\boldsymbol{\alpha}_\ell, \ \boldsymbol{\alpha}_\ell$ sparse

Sparse nonlinear inverse problem with shifts

Forward model

$$\underbrace{y_\ell}_{\text{known}} = \underbrace{f_\ell}_{\text{unknown}} (\langle \underbrace{g}_{\text{unknown}} (\underbrace{r_\ell}_{\text{known}}), \underbrace{l_\ell}_{\text{unknown}} \rangle) + \epsilon_l \quad \ell = 1, \ldots, N_\lambda$$

$$l_\ell = \Phi \alpha_\ell, \quad \alpha_\ell \text{ sparse}$$



Parameterized nonlinearities:

- f_θ
  - Spectrometer calibration
  - Hyperspectral imaging
- shifts g_c
  - Incorrect values of λ_ℓ
  - Doppler effect

Sparse nonlinear inverse problem with shifts

Joint estimation of sparse vectors, nonlinearity and shifts

- Shift: $\lambda'_\ell = \lambda_\ell + g_{\boldsymbol{c}}(\ell)$
- Joint estimation problem: $\arg\min_{\boldsymbol{\alpha},\boldsymbol{\theta},\boldsymbol{c}} \sum_{\ell=1}^{N_\lambda} \|\boldsymbol{y}_\ell - f_{\boldsymbol{\theta}} \circ (\boldsymbol{R}_\ell(\boldsymbol{c})\boldsymbol{\Phi}\boldsymbol{\alpha}_\ell)\|_2^2 + \mu \sum_{\ell=1}^{N_\lambda} \|\boldsymbol{\alpha}_\ell\|_0$
  $\boldsymbol{R}_\ell(\boldsymbol{c}) \triangleq \Pi\left(\boldsymbol{r}, \mathcal{W}\left(\lambda_\ell + g_{\boldsymbol{c}}(\ell)\right)\right)$

Sparse nonlinear inverse problem for MicroCarb

Problem in presence of radiometric errors and spectral shifts

- Q reference spectra $\boldsymbol{r}^{(q)}$
- 1 function $f_{\boldsymbol{\theta}_\ell}$ per pixel ℓ
- 1 function $g_{\boldsymbol{c}_q}$ per reference spectrum q
- Forward model assuming polynomial nonlinearities for pixel ℓ, reference spectrum q:

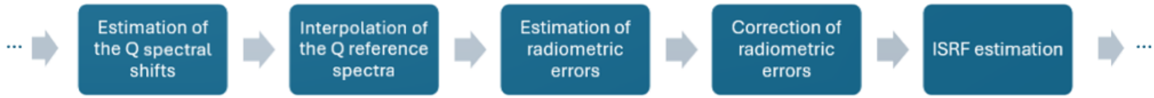
$$y_{\ell,q} = \sum_{p=0}^{P} \theta_p^\ell \left(\left(\boldsymbol{r}_\ell^{(q)}(\boldsymbol{c}_q)\right)^T \boldsymbol{l}_\ell \right)^p,$$

$$\boldsymbol{r}_\ell^{(q)}(\boldsymbol{c}_q) \triangleq \pi\left(\boldsymbol{r}^{(q)}, \lambda_\ell + \sum_{m=0}^{M} c_m^q \left(\frac{\ell}{\ell_{\max}}\right)^m\right)$$

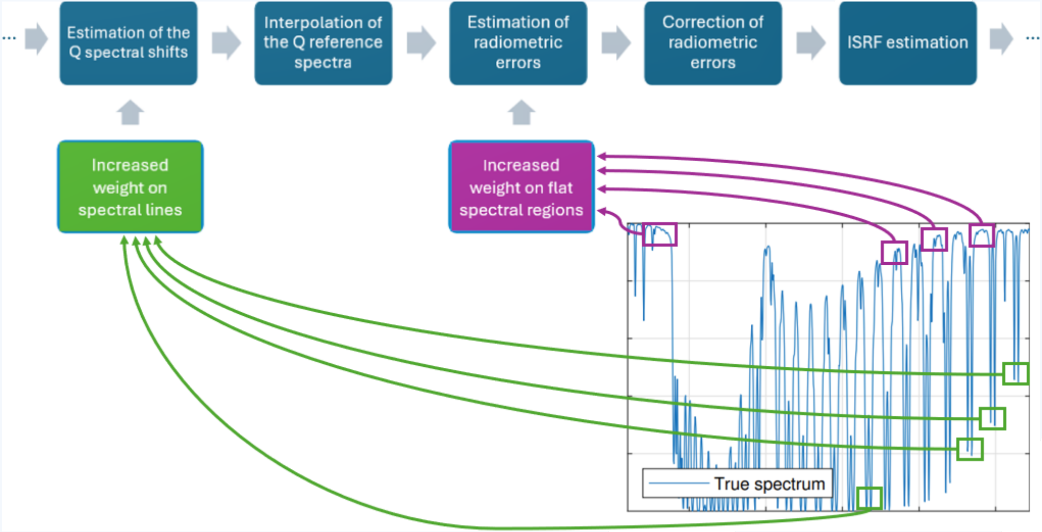
Sparse nonlinear inverse problem for MicroCarb

Iterative estimation in presence of radiometric errors and spectral shifts - ASPIRIT

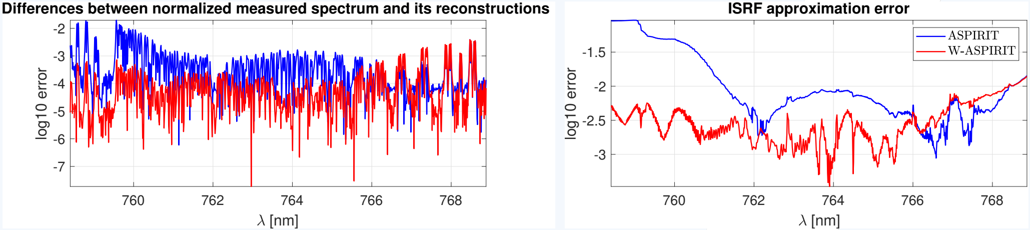
- Estimate spectral shift parameters $\boldsymbol{c}_q$ for the Q reference spectra
- Estimate radiometric parameters $\boldsymbol{\theta}_\ell$ for the N_λ pixels
- Estimate ISRFs via sliding window with all reference spectra



Radiometric and spectral calibration - Weighted ASPIRIT algorithm



Numerical results - In presence of radiometric errors and spectral shifts

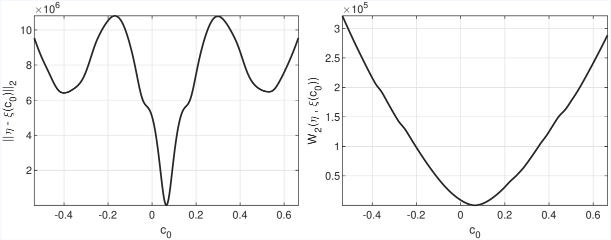


Measurement reconstruction error and ISRF estimation error versus λ for
$K = 4,\ Q = 11,\ P = 3$ and $M = 3$

Promising results with the proposed method
Best performance obtained when using weights

Optimal Transport with Wasserstein distance

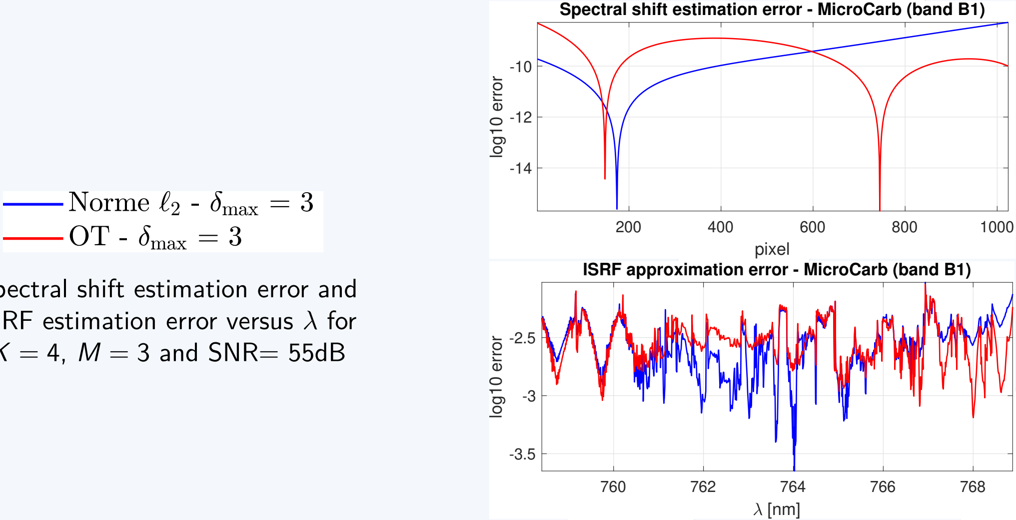
- Joint estimation problem: $\arg\min_{\boldsymbol{\alpha},\boldsymbol{\theta}_\ell,\boldsymbol{c}} \sum_{\ell=1}^{N_\lambda} \|\boldsymbol{y}_\ell - f_{\boldsymbol{\theta}_\ell} \circ (\boldsymbol{R}_\ell(\boldsymbol{c})\boldsymbol{\Phi}\boldsymbol{\alpha}_\ell)\|_2^2 + \mu \sum_{\ell=1}^{N_\lambda} \|\boldsymbol{\alpha}_\ell\|_0$
- Optimal Transport



$$\arg\min_{\boldsymbol{c}} \sum_{\ell=1}^{N_\lambda} W_2(\boldsymbol{y}_\ell, \boldsymbol{R}_\ell(\boldsymbol{c})\boldsymbol{I}_\ell)$$

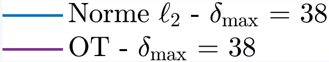
Distances between measurements $\boldsymbol{s}_\ell$ and model $\boldsymbol{R}_\ell(\boldsymbol{c})\boldsymbol{\Phi}\boldsymbol{\alpha}_\ell$ versus shift value

Numerical results - In presence of spectral shifts

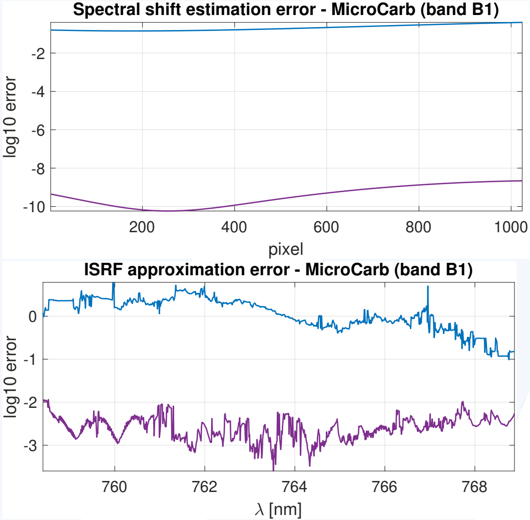


Spectral shift estimation error and ISRF estimation error versus λ for $K = 4$, $M = 3$ and SNR$=$ 55dB

Numerical results - In presence of spectral shifts



Spectral shift estimation error and ISRF estimation error versus λ for $K = 4$, $M = 3$ and SNR$= 55$dB



Conclusion

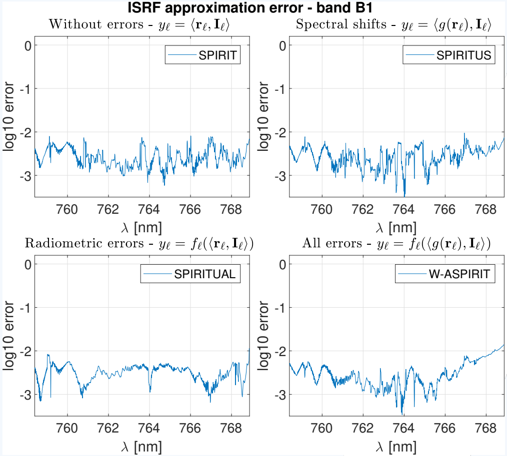
- Sparse inverse problem for spectrometer calibration
  - Best performance with OMP
- Nonlinear sparse inverse problem models with different nonlinearities
  - Competitive performance using EXIP and OMP
- Impact of Shifts
  - Importance of including weights

Conclusion

- Enhanced calibration strategies for atmospheric spectrometers (MicroCarb, CO2M)
- Sparse inverse problem models enabling high-accuracy recovery of ISRFs, spectral shifts, and radiometric nonlinearities
- Framework applicable beyond the targeted missions

Conclusion & Perspectives

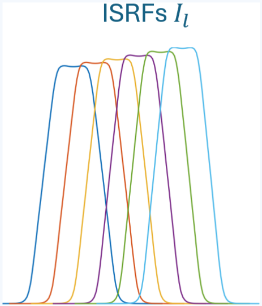
- Moving toward operational integration for space instruments

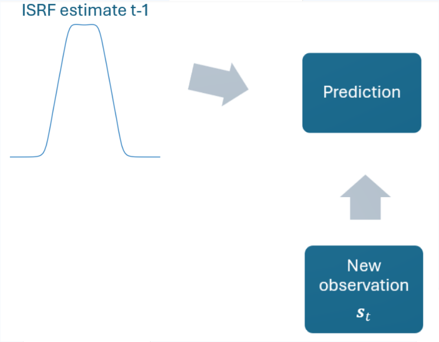


Perspectives

Alternative ISRF Estimation Models

- State-space models
- Possible use of Kalman filter

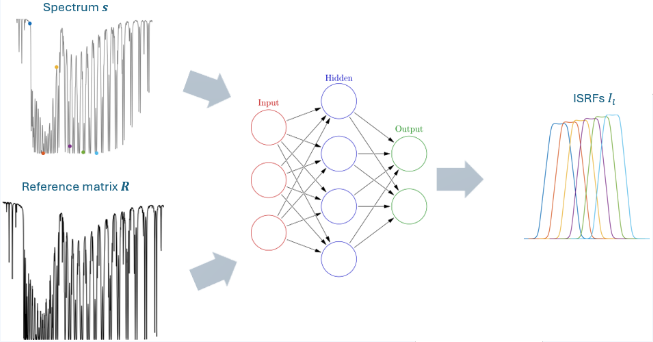




Perspectives

Potential use of Machine Learning

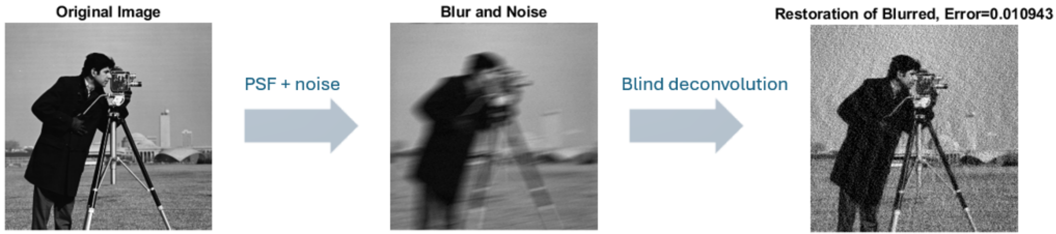
- Neural Network to estimate ISRF
- Learn nonlinear behaviors from data



Perspectives

Imperfect Knowledge of Reference Spectra

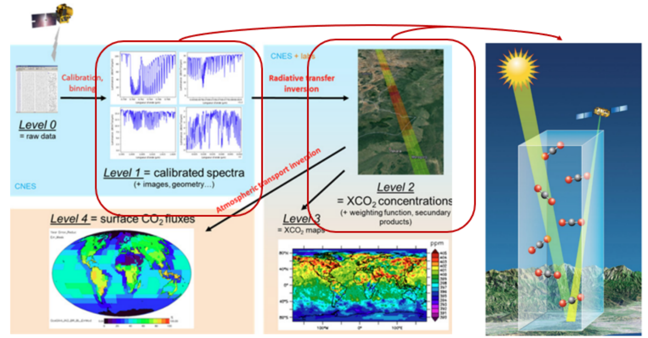
- ISRF estimation with reference spectrum uncertainties
- Blind deconvolution



Perspectives

Toward integration within CO_2 Retrieval Algorithms

- Use the proposed inversion methods to retrieve CO_2
- Perform joint calibration (level 1) and gas retrieval (level 2)



Publications

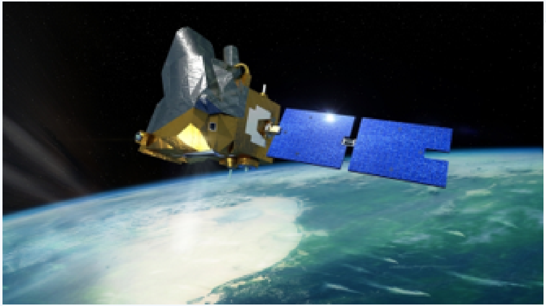
Journal papers

1. Jihanne El Haouari, Jean-Michel Gaucel, Christelle Pittet, Jean-Yves Tourneret and Herwig Wendt, In-Flight Estimation of Instrument Spectral Response Functions Using Sparse Representations, Atmospheric Measurement Techniques, 2024
2. Jihanne El Haouari, Jean-Michel Gaucel, Christelle Pittet, Jean-Yves Tourneret and Herwig Wendt, Solving parametric nonlinear inverse problems through sparse modeling, IEEE Transactions on Signal Processing, 2025, in preparation

Conference papers

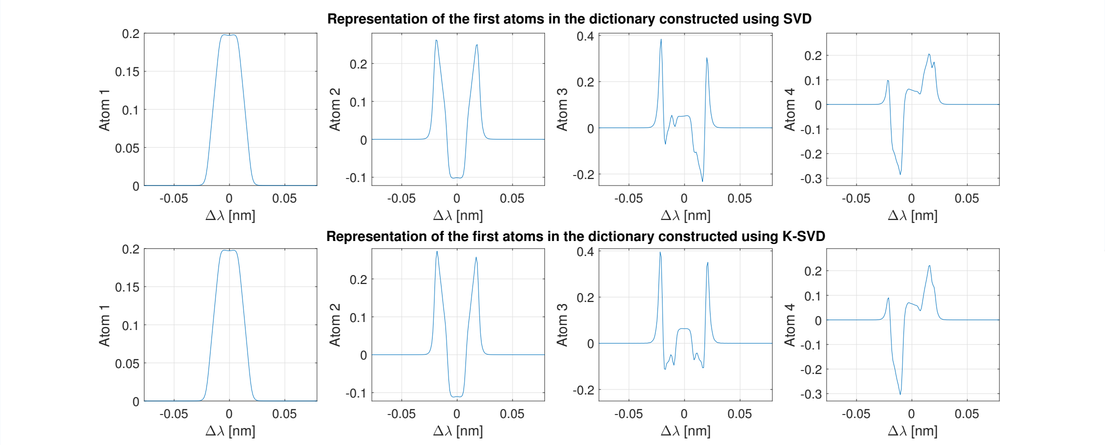
1. Jihanne El Haouari, Jean-Yves Tourneret, Herwig Wendt, Christelle Pittet, Jean-Michel Gaucel, Approximation et Estimation de Réponses Spectrales d'Instruments, GRETSI, Grenoble, France, August 28- September 1, 2023
2. Jihanne El Haouari, Jean-Michel Gaucel, Christelle Pittet, Jean-Yves Tourneret and Herwig Wendt, Estimation of Instrument Spectral Response Functions using Sparse Representation in a Dictionary, Proc. IEEE IGARSS, Athens, Greece, July 7-12, 2024
3. Jihanne El Haouari, Jean-Michel Gaucel, Christelle Pittet, Jean-Yves Tourneret and Herwig Wendt, Estimation of Instrument Spectral Response Functions in Presence of Radiometric Errors, Proc. EUSIPCO, Lyon, France, Aug. 26-30, 2024.
4. Jihanne El Haouari, Jean-Michel Gaucel, Christelle Pittet, Jean-Yves Tourneret and Herwig Wendt, On-ground and in-flight estimation of instrument spectral responses in the presence of measurement errors, ICSO2024, Antibes Juan-Les-Pins, Oct. 21-25, 2024
5. Jihanne El Haouari, Marcus Carlsson, Jean-Yves Tourneret, Herwig Wendt, Jean-Michel Gaucel and Christelle Pittet, Estimating Instrument Spectral Response Functions Using Sparse Representations and Quadratic Envelopes, ICASSP, Hyderabad, India, Apr. 06-11, 2025
6. Jihanne El Haouari, Filip Elvander, Jean-Michel Gaucel, Christelle Pittet, Jean-Yves Tourneret and Herwig Wendt, Joint ISRF and Spectral Shift Estimation for Spectrometer Calibration using Optimal Transport, Proc. EUSIPCO, Palermo, Italy, Sept. 8-12, 2025

Thank you for your attention



I look forward to your questions

Atoms of the dictionaries



Robustness to noise

Mean ISRF approximation error and mean residual as a function of SNR and method (G, SG, OMP, LASSO, SVD, K-SVD) for MicroCarb (band B1) — in blue: ISRF estimation errors under the mission criterion; in bold: best results

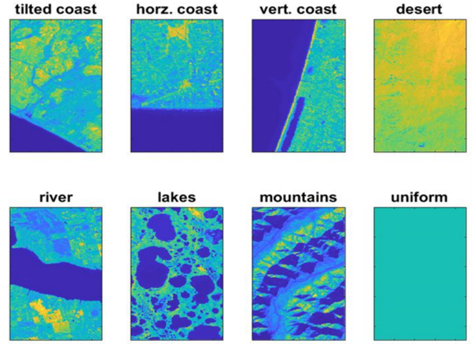
Instrument	/ SNR	ISRF approximation error (%) G	SG	OMP SVD	OMP K-SVD	LASSO SVD	LASSO K-SVD	Residual G	SG	OMP SVD	OMP K-SVD	LASSO SVD	LASSO K-SVD
	20 dB	16.28	**3.39**	4.58	4.37	14.38	14.23	185.5	116.4	**112.3**	112.4	112.7	112.9
MicroCarb	40 dB	16.27	2.04	**0.54**	0.56	2.05	2.37	70.2	1.56	**1.32**	**1.32**	1.53	1.70
bande B1	55 dB	16.27	2.03	**0.29**	0.33	1.33	1.66	69.21	0.43	**0.23**	0.24	0.38	0.61
	80 dB	16.27	2.03	**0.28**	0.32	1.27	1.68	69.2	0.39	**0.20**	**0.20**	0.34	0.60
	120 dB	8.03	7.79	**0.51**	0.55	0.83	0.73	45.07	37.32	**1.11**	1.13	1.14	1.13

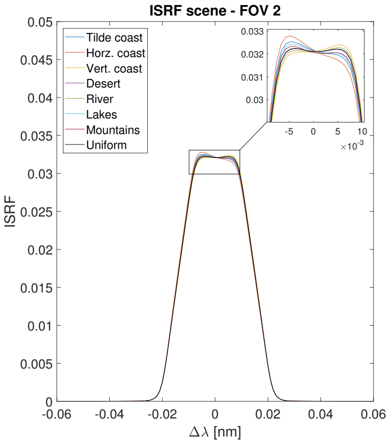
Robustness to noise in presence of radiometric errors

Mean ISRF approximation error for MicroCarb (band B1) — in blue: ISRF estimation errors under the mission criterion

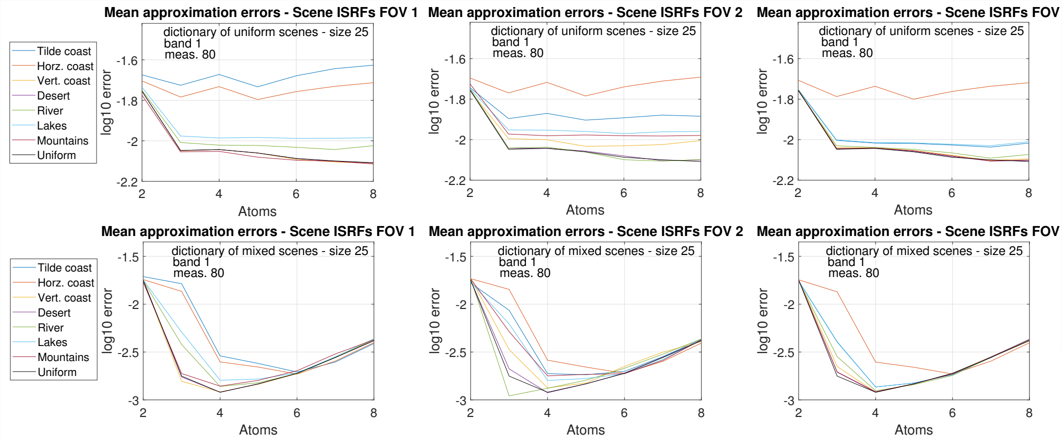
		ISRF approximation error (%)
Instrument	/ SNR	SPIRITUAL
MicroCarb bande B1	20 dB	1.01
	40 dB	0.29
	55 dB	0.27
	80 dB	0.27

ISRF sensitivity to the scene

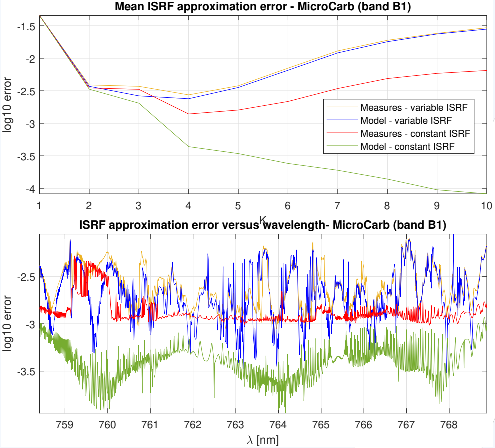




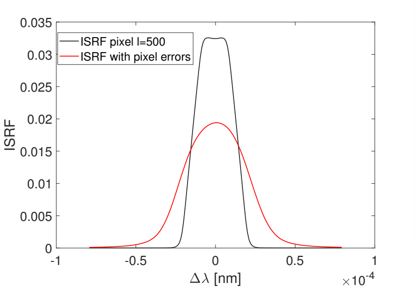
ISRF sensitivity to the scene

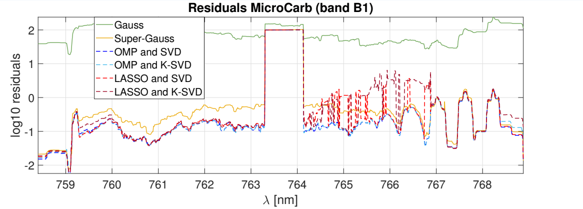


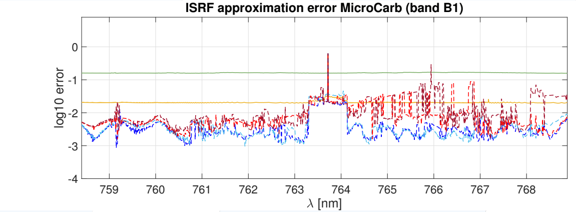
Interpolation-induced Mismatch



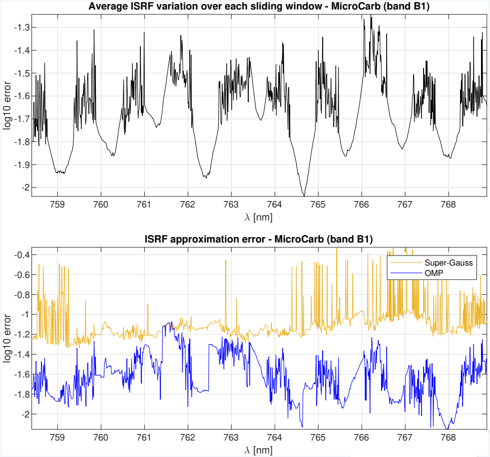
Uncertainties about one ISRF



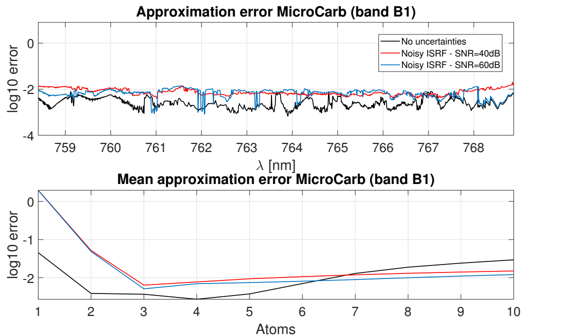


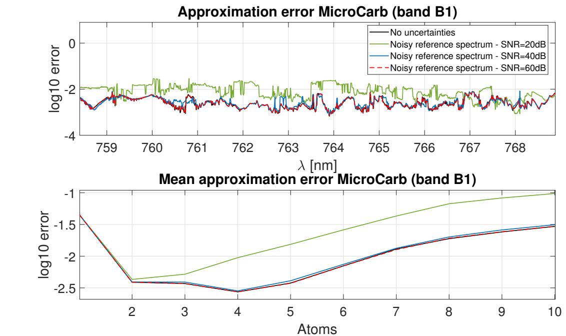


Uncertainties about the sliding window

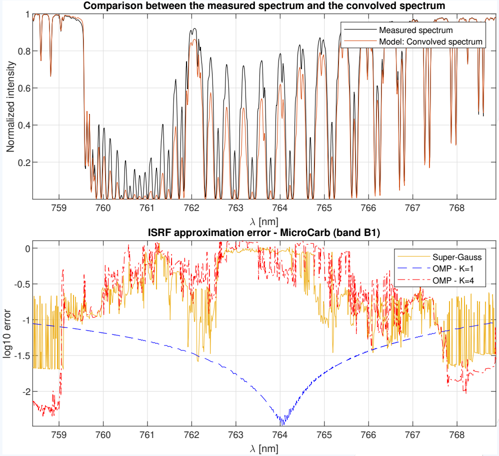


Uncertainties about the dictionary and the reference spectrum



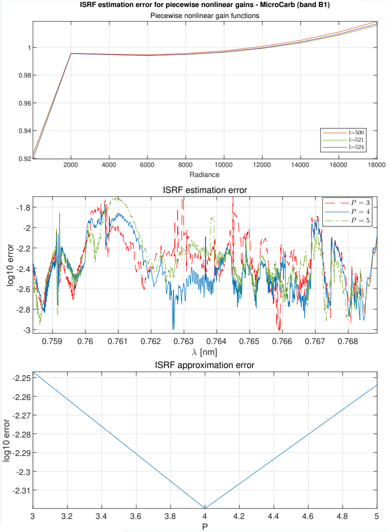


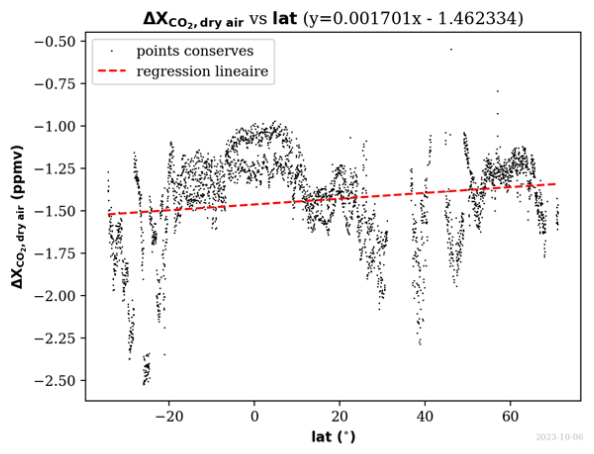
Error in the selection of the reference spectrum



Uncertainties about the nonlinearities

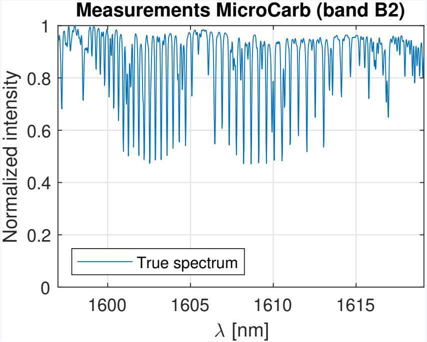
- $f_l(x) = Gnlin_l(a_l x + b_l)(a_l x + b_l)$ with
  - $Gnlin_l$ piecewise nonlinear gain
  - a_l multiplicative gain
  - b_l additive gain
- Sparsity $K = 4$

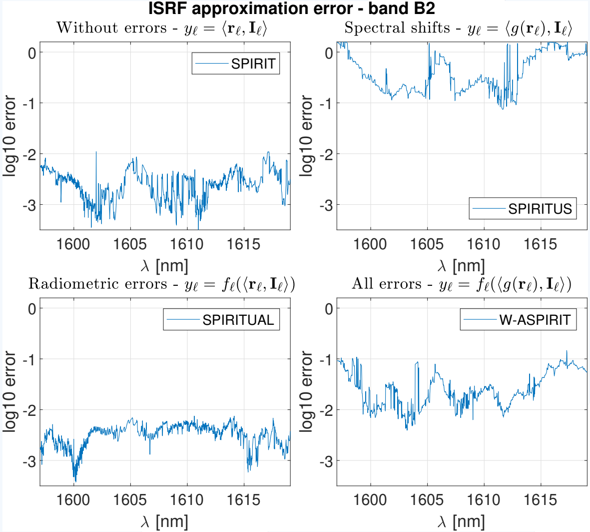




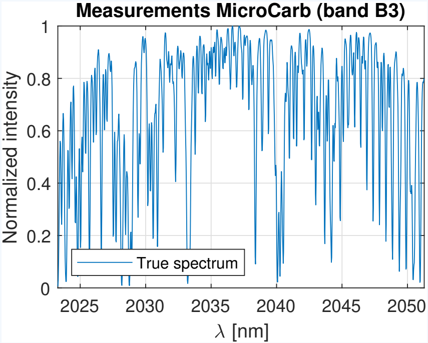
$\Delta X_{CO_2, dry\ air}$ vs lat (y=0.001701x - 1.462334)
points conserves
regression lineaire
−0.50
−0.75
−1.00
−1.25
−1.50
−1.75
−2.00
−2.25
−2.50
$\Delta X_{CO_2, dry\ air}$ (ppmv)
−20
0
20
40
60
lat (°)
2023-10-06

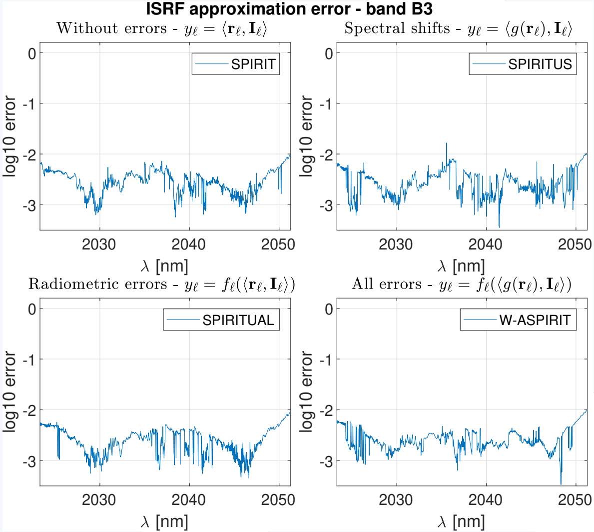
Results for all scenarios and all bands





Results for all scenarios and all bands





Results for all scenarios and all bands