Discontinuity at origin in Volterra and band-pass limited models

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Abstract — Discontinuities at origin have been used to better approximate measured curves in recent papers but generally not explicitly and their physical validity has not always been demonstrated. In this communication, we show that these discontinuities can be explained by physically acceptable discontinuities in the real physical device. We propose simple criteria to accept or reject these discontinuities, in either passive or active devices, depending on the order of the discontinuity. In addition, we show that models having such discontinuities behave differently from classical models. In particular, these discontinuities explain non-integer dB/dB slopes of harmonic power and intermodulation power as a function of input power. Recent and older measurements of intermodulation products in passive devices, telephony base-station and RF transistors show such a behavior so that supposed lack of measurement cannot be used as a reason to reject discontinuities as non-physical.

Index Terms — behavioral model, Volterra model, band-pass limited model, Cann model, Rapp model, discontinuity, harmonics, intermodulation products.

I. Introduction

Classical models for non-linear electronic devices are generally based on analytic functions or their Taylor series development and particularly on polynomials for memory-less models or Volterra development for memory models. This implies that the model is continuous and infinitely derivable at origin.

However, discontinuities at origin have been used in non-linear models for better approximation of some measured characteristics of devices. The user may not be aware of the discontinuity of the model, and generally there is no discussion on the physical validity or the effect of the discontinuity.

Some authors have rejected these models as non-physical because they result in behavior that cannot be explained by the classical theory, e.g. non-integer slopes (in dB/dB) for harmonics and intermodulation (IM) products output power versus signal input power in small signal conditions.

In this communication, we show that such non-classical behavior has been measured and reported by many authors in measurements of intermodulation products in passive devices, telephony base-station and RF transistors so that it must have a physical explanation.

We show that some discontinuities at origin can be explained by physically acceptable discontinuities at origin in the real physical device. We propose simple criteria to accept or reject these discontinuities, in either passive or active devices, depending on the order of the discontinuity.

We compare the simulated results of these models with measurements and show that, in addition to better approximation of device characteristics, these discontinuities are essential to explain measurements that cannot be explained by classical theory.

II. Overview of Classical Theory

Classical theory [1] for non-linear memoryless devices is based on a polynomial expression of an input to output characteristic, e.g. the instantaneous voltage characteristic.

$$v_{out} = f(v_{in}) = \sum_{i=0}^{n} \alpha_i v_{in}^i \qquad (1)$$

When the input signal is a pure sine, the output signal contains DC and harmonics components in addition to the fundamental signal (at the same frequency as the input).

$$v_{in} = a.\cos(\omega t + \varphi) = a.\cos(\theta) \qquad (2)$$

$$v_{out} = f(a.\cos(\theta)) = \sum_{i=0}^{n} \alpha_i \,[a.\cos(\theta)]^i \qquad (3)$$

$$v_{out} = f(a.\cos(\theta)) = \frac{1}{2} f_0(a) + + \sum_{m=1}^{\infty} f_m(a).\cos(m\theta) \qquad (4)$$

Function $f_m(a)$ in (4) is the order m Chebyshev transform of function f [2]. It is computed as:

$$f_m(a) = \frac{1}{\pi} \int_{-\pi}^{+\pi} f[a.cos(\theta)] \; cos(m\theta)\, d\theta \qquad (5)$$

For the polynomial given in (1), the result is given by equation (6) where degree *i* must have the same parity as *m*:

$$f_m(a) = \sum_{i=0}^{n} 2\alpha_i \left(\frac{a}{2}\right)^i \frac{i!}{\left(\frac{i+m}{2}\right)! \left(\frac{i-m}{2}\right)!} \qquad (6)$$

For $i < m$, the factorial of $\left(\frac{i-m}{2}\right)$ is infinite and the coefficient is 0 for the term of degree i. These transforms are also polynomials. As can be seen, the fundamental output $f_1(a)$ is a polynomial with only odd degree terms:

$$f_1(\mathrm{a}) = \sum_{i=0}^{(n-1)/2} \beta_{2i+1} a^{2i+1} \qquad (7)$$

It is a low-pass equivalent (LPE) of the physical device in a small bandwidth around the fundamental carrier bandwidth.

Real coefficients in the model can be replaced by complex coefficients to take into account the AM/PM characteristic curve at fundamental centre frequency. Mathematical results are formally the same and will not be repeated [3].

This LPE can be used again in a Chebyshev transform to compute the intermodulation products created with a 2-carrier input signal [4]. Only odd order intermodulation products exist. In small signal conditions the power of an order $2i$+1 IM is a power function of input power with an exponent higher or equal to its order. It is equal to its order if the coefficient of the corresponding degree in the polynomial is not 0. This results generally in a small signal dB/dB slope equal to the order in a graph of IM output power versus input power.

III. CONTRADICTING MEASUREMENTS

The conclusion in previous paragraph is true only for functions that can be developed in a Taylor series at origin.

Measurements of intermodulation products in antennas, filters, coaxial connectors and other passive devices have been reported with even-integer or non-integer slopes in dB/dB [5, 6]. These slopes are fairly constant on a wide range of input power (up to 30 dB in [5]). Such results can be approximated in the classical theory only by using polynomials of high degree, e.g. 49 or more complicated functions [7]. These approximations are far from perfect in the measurement range; they diverge rapidly outside this range and generally do not permit to predict correctly higher order IM products.

Measurements on class C to A transistor amplifiers' IM products have been reported with dB/dB slopes between 2.2 to 2.8 in small signal conditions [8]. In this case, a model of the transistor (equivalent to BSIM3 model) has been used in a harmonic balance simulator with results in good agreement with measurements whereas computation with classical theory gives 3 dB/dB slopes. This behavior has been traced to the presence of a second degree term in the models [9]. This term has been discarded from BSIM3 as non-physical in [10].

IV. INTRODUCTION OF DISCONTINUITY AT ORIGIN

We will now demonstrate that all these measurement results can be explained by discontinuity at origin in the non-linearity.

In addition these non-linear and non-continuous models can be simple and their effect in small signal conditions can be computed as easily as in the polynomial case [11].

We replace the polynomial model in (1) by:

$$f(v_{in}) = sign(v_{in}) \sum_{i=0}^{k} \alpha_i |v_{in}|^i + \sum_{i=0}^{l} \beta_i |v_{in}|^i \quad (8)$$

This function contains two parts, one odd and one even. The odd part will produce only odd harmonics and odd IM products (particularly, the fundamental signal). The even part will produce only even harmonics and even products (particularly, the DC component and second harmonic).

In addition to the classical terms in (1), equation (8) contains terms with even degree of the modulus of input signal in the odd part and terms with odd degree in the even part.

Equation (6) must be modified by replacing factorials with Gamma function: $n! = \Gamma(n+1)$.

For odd m, we use the odd part of (8), we have:

$$f_m(a) = sign(a) \sum_{i=0}^{k} 2\alpha_i \left|\frac{a}{2}\right|^i \frac{\Gamma(i+1)}{\Gamma\left(\frac{i+m}{2}+1\right)\Gamma\left(\frac{i-m}{2}+1\right)} \quad (9)$$

For odd order m and even degree i, $\Gamma\left(\frac{i-m}{2}+1\right)$ in (9) is no longer infinite for $i < m$ and the corresponding coefficient is not 0 as it was in (6). The main result is that IM products of all odd orders will be produced from a single term of low even degree multiplied by $sign(v_{in})$.

In addition, in small signal conditions, all these IM products will have the same dB/dB slope equal to this even degree.

If more than one term exists, the slope will be equal to the lower degree in small signal conditions and to the higher degree in large signal conditions. In the intermediate range, variation of slope value will depend on the relative sign of both terms' coefficients, as in the classical theory [7].

V. GENERALIZATION OF MODEL WITH DISCONTINUITY AT ORIGIN

In equation (8), the parity of the function is no longer linked to the parity of the degree in each term. In addition, the power function of degree i is always computed on a positive or 0 term. Mathematically, we no longer need the degree to be an integer and we can use the following more general model:

$$f(v_{in}) = sign(v_{in}) \sum_{i=0}^{k} \alpha_i |v_{in}|^{p_i} + \sum_{i=0}^{l} \beta_i |v_{in}|^{q_i} \quad (10)$$

Degrees p_i and q_i are fractional or real numbers. They will result in dB/dB slopes having fractional or real values that fit correctly passive devices IM products measurements on a large input power range with a very small number of terms.

To take into account real degrees, equation (9) must be further modified. For odd m, we have:

$$f_m(a) = sign(a) \sum_{i=0}^{k} 2\alpha_i \left|\frac{a}{2}\right|^{p_i} \frac{\Gamma(p_i+1)}{\Gamma\left(\frac{p_i+m}{2}+1\right)\Gamma\left(\frac{p_i-m}{2}+1\right)} \quad (11)$$

The power function of modulus with a real degree (either multiplied by sign function or not) is an invariant of the Chebyshev transform, like for integer degree. This will allow us to compute easily the non-linear response of a device.

The same equation will be used to compute 2-carrier IM products in a second Chebyshev transform.

VI. PHYSICALLY ACCEPTABLE DISCONTINUITIES

For mathematical convergence, the real degree must be higher than -1. However, this is not sufficient for physical validity. We must at least verify that there is no creation of energy in the device.

Particularly, a passive device cannot have an infinite derivative at origin as this would give an output power larger than input power. For passive devices, the real degree must be at minimum 1.

An active device may have an infinitive derivative (gain) at origin only if the resulting output power is finite and is lower than the power that can be provided by its power supply. For active devices, the real degree must be at minimum 0.

This is valid for both odd and even parts of the model. Degree 0 model is a constant for the even part and a perfect limiter, $sign(v_{in})$, for the odd part. It needs a power supply.

We see that the function itself is continuous but its first derivative (for an active device) or its second derivative (for a passive device) may not be continuous. Higher derivatives will then be Dirac delta functions or other distributions.

VII. Behavior of simple models

A. Power functions

We present in Fig. 1 the values of the multiplicative term in equation (11) for odd functions (and odd harmonics or products). The real degree is given in abscissa.

For odd integer values, we find the classical results: all orders higher than the degree vanish.

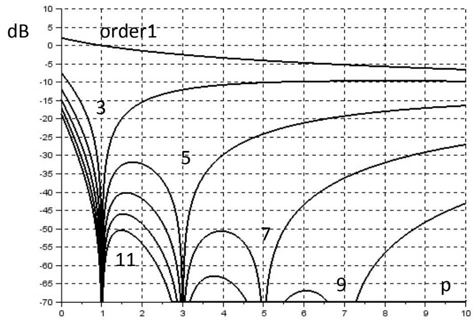

Fig. 1. Relative level in dB of odd order Chebyshev Transforms from 1 to 11 versus real degree.

For a single term of degree p, the ratio of two IM products will depend only on the degree and on the order of both IM products and not on the input power. A comparison of passive IM products measurements of order 3, 5 7 and 9 given in [12] and simulated results is shown in the following figure.

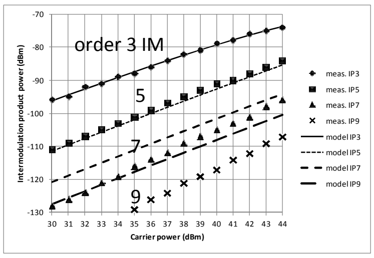

Fig. 2. Measured (dots, [12]) and simulated (lines, this work) levels of IM products of orders 3, 5, 7 and 9 versus input power.

As can be seen, measured slopes are far from classical values and would be difficult to simulate with an analytic model, it would need much more terms.

Figure 3 shows one of the amplifiers measured in [8]. It has been simulated with one term of degree 2.6 and one term of degree 3; giving the small signal IM slope of 2.6 dB/dB. Additional terms would be needed to better approximate the curves at higher input power.

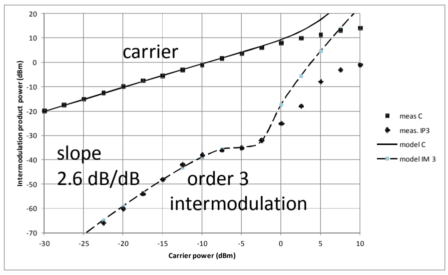

Fig. 3. Measured (dots, [8]) and simulated (lines, this work) levels of carrier and third order IM product versus input power.

B. Application to Volterra model

We can easily apply equation (8) with integer degrees to the case of polar Volterra model presented in [13, 14]. This model was proposed as an extension of the classical Volterra model that is suited to represent a physical device in a limited bandwidth as a low pass equivalent. The sign function is not defined in the complex plane but can be replaced by the ratio $\frac{v_{in}}{|v_{in}|}$ that is the exponential of the imaginary phase of the input signal complex envelope.

If we look only at the memory-less part of the model in [13], only one phase term remains and can be combined with a modulus of the input envelope to obtain the following simplified equation where the output signal envelope $y(n)$ results essentially from the product of the input signal envelope $x(n)$ by a gain that is a function of input envelope modulus at the same time:

$$\tilde{y}(n) = \tilde{h}_{0,0} + \tilde{x}(n) \cdot \sum_{p=0}^{P} \tilde{h}_p \cdot |\tilde{x}(n)|^{p-1} \qquad (12)$$

There is no discontinuity at origin if the gain is a function of the square of the modulus only, that is if: $\tilde{h}_p \neq 0$ only for odd integer values of p. The model has a discontinuity at origin in all other cases: real or odd $(p-1)$ degrees.

The minimum degree of $p = 0$ is acceptable for an active device: it gives an infinite gain at origin but a bounded output. It could be the model of a Schmidt trigger or an ideal clipper. For a passive device, we must have $p \geq 1$ to guarantee that the output energy will not be higher than the input energy. In addition the constant output $\tilde{h}_{0,0}$ must be 0.

Real degrees can certainly be used for the modulus terms. The number of variables in a kernel must obviously be an integer but there is no reason for the degree of each modulus term to be 1. It could be an arbitrary value or a fixed value e.g. 1/10 giving access to all fractional degrees $p/10$.

The phase terms must follow the rule given in [13] for an output around fundamental frequency: the sum of coefficients multiplying the phases must be 1. The sum of positive coefficients (phases of the complex envelope) and the sum of negative coefficients (phases of the conjugate of the envelope) must differ by 1 but can be arbitrary otherwise. They could be linked to the degrees of the modulus or be independent.

C. Application to Cann and Rapp SSPA models

Rapp proposed a model [15] for solid state power amplifiers (SSPA) that is based on a modified Saleh model [16]. When applied to a complex envelope, the model is:

$$\tilde{y} = \frac{\tilde{x}}{\sqrt[2p]{1+(|\tilde{x}|^2)^p}} \qquad (13)$$

This model is analytic provided that the parameter p is an integer. However, only terms with degrees multiples of $2p$ are present in the series development of the gain at origin. All intermodulation products of order up to $2p+1$ have slopes of $2p+1$ dB/dB. If the parameter is not an integer, all intermodulation products have small signal slopes of $2p+1$ dB/dB. For $p = 1/2$, the model is: $\tilde{y} = \frac{\tilde{x}}{1+|\tilde{x}|}$ (14)

The small signal gain in (14) is 1 and the first derivative is continuous at origin but the second derivative is not. The IM slope is 2 dB/dB, which may not be the expected behavior.

The AM/AM curve of Cann model [17] is defined as:

$$y = \frac{1}{\sqrt[s]{1+1/x^s}} \quad \text{or} \quad y = \frac{\text{signe(x)}}{\sqrt[s]{1+1/|x|^s}} \qquad (15)$$

When applied to a real signal or a complex envelope, it must be modified as: $\tilde{y} = \frac{\tilde{x}/|x|}{\sqrt[s]{1+1/|\tilde{x}|^s}} = \frac{\tilde{x}}{\sqrt[s]{1+|\tilde{x}|^s}}$ (16)

It is then equivalent to Rapp model with $s = 2p$. This model is analytic only for even integer values of parameter s. For other values, it will gives intermodulation products of all orders with slope equal to $1+s = 1+2p$ dB/dB.

The next figure gives examples with $s = 2p = 3$ giving small signal slopes of 4 dB/dB and $s = 2p = 4$ giving slopes of 5 dB/dB for orders 3 and 5 and 9 dB/dB for orders 7 and 9:

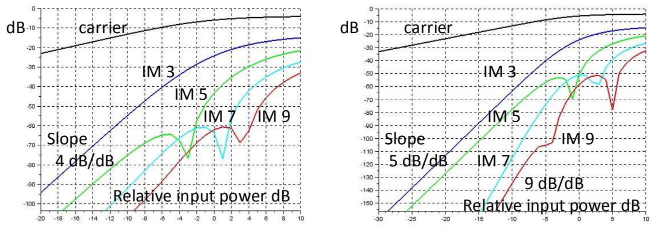

Fig. 4. Levels of carriers and intermodulation products of orders 3, 5, 7 and 9 versus input level for Rapp models 2p=3 and 2p=4.

These models cannot be put aside by considering them as not physical, they are. However, they may not represent correctly measurements and typical behavior of SSPAs.

VIII. Conclusion

For correct representation of non-linear devices, classical theory must be modified to include discontinuity at origin in the models. This allows for better approximation of devices' characteristics and explains some measurement results that seemed to be non-physical in view of classical theory.

Criteria for physical validity have been proposed.

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