New Models for Passive Non Linearities Generating Intermodulation Products with Non-Integer Slopes

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***Abstract*—Many measurement results on passive intermodulation products exhibit slopes of third order intermodulation product level as a function of input level different from the classical 3 dB/dB slope. Even-integer and real values between 1 and 3 are commonly reported for telephony base station towers antennas and filters. No classical model has been able to approximate these measurements up to now. We propose a non-analytic model that explains this behaviour and may serve as theoretical basis to find a physical model.**

Index Terms* — *PIM, intermodulation product, slope, non linearity, behavioural model

I. Introduction

Passive intermodulation products have always been a problem on satellite receive and transmit antennas. Passive equipment such as antennas, when transmitting high power signals, generate intermodulation products. When the difference between transmit and receive frequencies is not large enough with respect to frequency bandwidth, intermodulation products with significant power may fall into the receive bandwidth. They may interfere with the low power signals received by the same antenna. This may lead to the loss of some functions in the satellite. Design, manufacturing and test of materials, antennas and filters take this problem into account but this may lead to higher cost, sometimes even preventing the dual use of an antenna for receive and transmit.

In recent years, the same problem appeared on telephony base stations where transmitted power, number of frequency bands, number of antennas and frequency bandwidth have increased. Many causes have been identified such as non-linear resistance, nickel plating, corrosion and loose connections. A case of intermodulation products generated by a rusty fence around the antenna tower has been reported.

Problems may also occur on aircrafts for the same reasons.

II. Analytic Models Theory

From the first articles on passive intermodulation products [1] to recently [2-5], many authors have reported even-integer and non-integer slopes, between 1 and 3 dB/dB, for passive intermodulation products level versus input signal level

This seems to contradict the classical behaviour of active intermodulation products e.g. a third order product level increases with the input level along a 3 dB/dB slope, a fifth order product along a 5 dB/dB slope and so on.

We remark first that:

- There is no physical or mathematical reason for passive non-linear components to behave differently from active non-linear components if energy is conserved.
- It is not always true that active intermodulation products increase with a small signal slope equal to their order, even in a classical model of active components.

What is true in classical theory [6-8] is that, in small signal conditions, a non linearity modeled by a polynomial transfer function (or by some function that can be replaced by its Mac Lauren series development e.g. its Taylor series development in a given domain around 0) will generate even harmonics and intermodulation products with even integer dB/dB slopes (2, 4, 6 …) and odd harmonics and intermodulation products with odd integer dB/dB slopes (1, 3, 5 …).

These slopes come, in fact, from the lowest degree that is of the same parity and higher than or equal to the order of the product. The third harmonic and the third order intermodulation product generated by a transfer function modelled by a polynomial with a single term of fifth degree would have a small signal slope of 5 dB/dB and not 3 dB/dB.

The Saleh model is a classical example [9] that follows the "slope equals order" theory in small signal conditions. This model uses a fraction of two polynomials and all the coefficients in its Mac Lauren series development are alternates +2 and -2 for odd degree terms and all 0 for even degree terms.

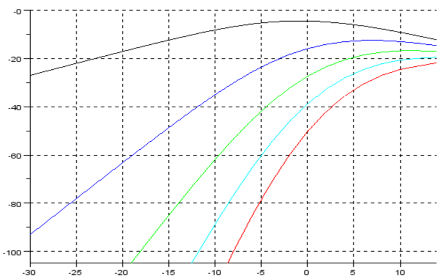

Fig. 1. Level of carriers and intermodulation products of orders 3, 5, 7 and 9 at the output of Saleh model

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As can be seen in fig. 1, in small signal conditions, slopes are equal to the lowest odd degree lower or equal to the order, which, in this case, is in effect equal to the order. These slopes decrease at higher input power because of saturation. In other models they may go through nulls and have high positive or negative slopes in a small range of input power.

III. NON-ANALYTIC MODEL THEORY

A non linearity couldn't be modelled with a polynomial or an analytic function if it generates, in small signal conditions, harmonics or products with:

- non-integer (fractional or real) slopes;
- even integer slopes for odd order products;
- odd integer slopes for even order products;

except in a limited input power range.

We propose to replace the classical polynomial by the following non-linear function that is not smooth, and so is non-analytic, at the origin:

$$f(u) = P(|u|) + sign(u) \cdot Q(|u|) \quad (1)$$

P and Q are arbitrary functions of the modulus of input variable and $sign(u)$ is the sign function.

- $P(|u|)$ is the even part of function *f* and it will generate only even harmonics and even intermodulation products.
- $sign(u).Q(|u|)$ is the odd part of function *f* and it will generate only odd harmonics and odd intermodulation products.

We compute levels of harmonics at the output of this non-linear function by using the Chebyshev transform as in the classical theory [6-8].

When *P* or *Q* is a power function of $|u|$ such as $|u|^p$, where the exponent *p* is an arbitrary real number, and when the input signal is $u = a.\cos(\omega t + \varphi) = a.\cos(\theta)$, the output signal is composed of harmonics of the input signal, especially a DC component (harmonic 0) and a fundamental component (harmonic 1):

$$f(u) = f[a\cos(\theta)] = \frac{1}{2} f_0(a) + \sum_{m=1}^{\infty} f_m(a) \cdot \cos(m\theta) \quad (2)$$

The amplitude of order *m* harmonic is $f_m(a)$ which is given by the Chebyshev transform of function *f*:

$$f_m(a) = \frac{1}{\pi} \int_{-\pi}^{+\pi} f[a\cos(\theta)] \cos(m\theta) d\theta \quad (3)$$

- For *m* even and from the even part of f, $P(|u|) = |u|^p$:

$$f_m(a) = 2.\left(\frac{|a|}{2}\right)^p \frac{\Gamma(p+1)}{\Gamma\left(\frac{p+m}{2}+1\right).\Gamma\left(\frac{p-m}{2}+1\right)} \quad (4)$$

- For *m* odd and from the odd part of *f*, $sign(u).Q(|u|) = sign(u) \cdot |u|^p$:

$$f_m(a) = 2.sign(a).\left(\frac{|a|}{2}\right)^p \frac{\Gamma(p+1)}{\Gamma\left(\frac{p+m}{2}+1\right).\Gamma\left(\frac{p-m}{2}+1\right)} \quad (5)$$

As can be seen, power functions of the modulus of input variable and these same functions multiplied by the sign of the input variable are both invariants of Chebyshev transform as the polynomials are in the classical theory [7].

The result of the Chebyshev transform is formally equivalent to that for integer powers where factorials have been replaced by Γ functions as the argument are no longer integer.

This will permit us to compute the AM/AM function, which is the amplitude of fundamental (or first harmonic) output amplitude given by $m = 1$: , $f_1(a)$. It comes only from the odd part of function f. This function is of type: $f_1(a) = sign(a).R(|a|)$ where *R* is a power function such as $|a|^p$ with the same exponent as the initial function *Q*. From this AM/AM function, we compute again Chebyshev transforms that give us the carriers and intermodulation products for a two-carrier input signal having an amplitude (or envelope): $a = A.\cos(\Omega t + \Phi) = A.\cos(\Theta)$, [8]. The level of odd order IM products has the same form as (5):

$$F_m(A) = 2.sign(A).\left(\frac{|A|}{2}\right)^p \frac{\Gamma(p+1)}{\Gamma\left(\frac{p+m}{2}+1\right).\Gamma\left(\frac{p-m}{2}+1\right)} \quad (6)$$

Carrier amplitude will be given by $m = 1$ while third order products will be given by $m = 3$ in this equation.

As can be seen in (6), intermodulation products of all orders (and so the third order) will depend on p^{th} power of *A* and so will have a slope of *p* dB/dB versus input power.

An equivalent equation can be derived from (4) for even order IM products (around even order harmonics):

$$F_m(A) = 2.\left(\frac{|A|}{2}\right)^p \frac{\Gamma(p+1)}{\Gamma\left(\frac{p+m}{2}+1\right).\Gamma\left(\frac{p-m}{2}+1\right)} \quad (7)$$

Levels of harmonics or IM products are given in figure 2.

If there is only one power term in function *Q* (and so in *R*) the ratio between two given products will be constant when input power varies. These ratios can be easily computed from equations (6) and (7).

Arbitrary fractional or real slopes can be obtained, provided that the exponent *p* is not lower than 1, for passive devices, to guarantee that output power is never higher than input power.

These functions are continuous but their second or higher order derivatives may not be continuous at the origin. They are not smooth and so are not analytic.

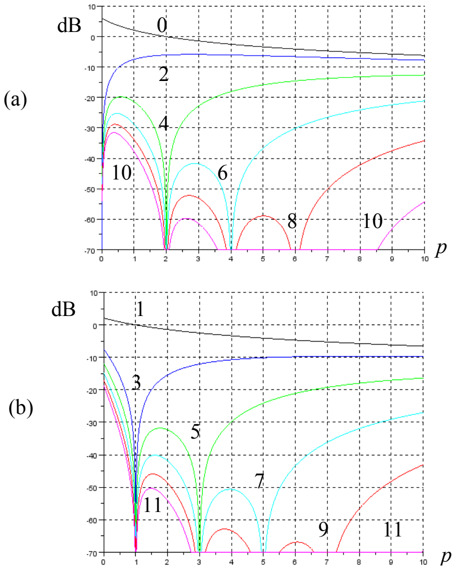

Fig. 2 Harmonics (or IM products) level as a function of real exponent p in power function of modulus of input for:

(a) even order (0 to 10) transforms of even functions
(b) odd order (1 to 11) transforms of odd functions

Measurements on micro strip lines and coplanar lines published in [2] support this result, as shown in next chapter.

If the exponent is an odd integer $p = 2n + 1$, we are in the special case studied by the classical theory: only odd order harmonics or products of order lower or equal to p exist.

The linear term ($p = 1$) present in the odd part of all passive transmission devices will generate only the undistorted signal. All terms with a different exponent will generate harmonics or IM products and will modify the signal.

If there are two or more terms with different exponents, the small signal slope will be equal to the lowest one while the large signal slope will be equal to the higher one. The slope may saturate in the case of an infinite series of terms. This behaviour is the same as in the classical theory [3].

IV. COMPARISON WITH PUBLISHED MEASUREMENTS

In figures 3 and 4, we reproduce measurement results obtained on a micro strip line around 900 MHz and published in figure 6 of article [2]. Slopes of intermodulation products have been estimated by the authors to:

1.6 dB/dB for IM3,
1.9 dB/dB for IM5,
2.3 dB/dB for IM7
2.5 dB/dB for IM9.

Ratios between successive products are nearly constant: IM3/IM5≈ 13 dB; IM5/IM7≈ 15 dB; IM7/IM9≈ 12dB.

A. *One term model*

We approximate first these results with a simple non-analytic model using only one term with an exponent of 1.6 in addition to the linear term. So all IM products vary with a constant slope of 1.6 dB/dB versus input power.

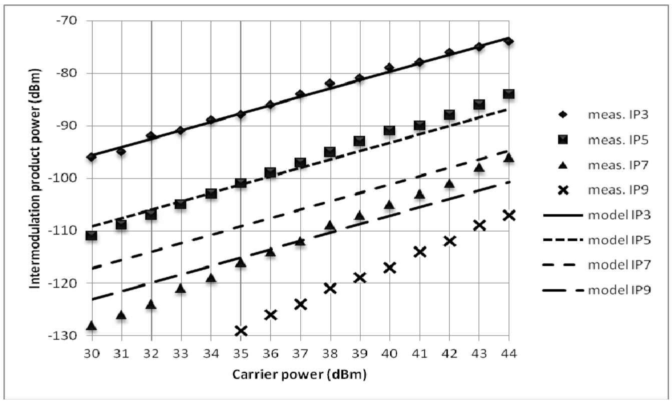

Fig. 3. Comparison of measured output power of intermodulation products of order 3 to 9 (dots, from [2]) and results from our non-analytic model (solid and dashed lines).

As can be seen, this model, which has been adjusted only to obtain the correct slope and value for the third order IM product in the 14 dB input power range, gives an average error of less than 0.5 dB on third order IM product, a correct fifth order value and slope and pessimistic seventh and ninth order IM products.

Higher slope values have given better approximation of higher order products but worse approximations of third order product.

B. *Two terms model*

We approximate the same results with a non-analytic model using two terms in addition to the linear term. Slopes are 2 and 2.5 dB/dB. Both real coefficients have been adjusted to obtain the lowest possible average error on third order intermodulation product only, in the 14 dB input power range.

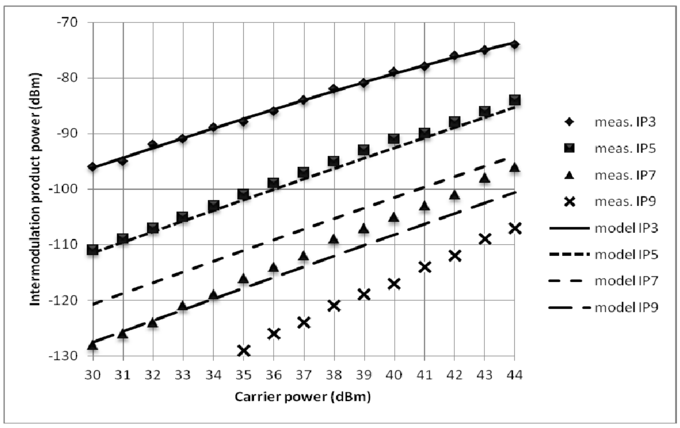

Fig. 3. Comparison of measured output power of intermodulation products of order 3 to 9 (dots, from [2]) and results from our non-analytic model (solid and dashed lines).

With this model, IM3 error is less than 0.4 dB, IM5 error around 1 dB, IM7 error less than 5 dB and IM9 error less than 10 dB.

The non-analytic 2-term model is quite correct for third and fifth intermodulation products and pessimistic but acceptable for seventh and ninth intermodulation products.

C. Polynomial model

A simple polynomial model would have a slope of 3 dB/dB for the third order instead of 1.6 and the minimum error would be ± 10dB in the 14 dB range.

To obtain fifth order intermodulation products 13 dB below the third order in the classical theory, we would have to use a fifth degree term with a high enough coefficient.

This would give us a strong variation of third order product with a notch if the coefficients have opposite signs and then a 5dB/dB slope.

An analytic model using a Saleh model or a hyperbolic tangent function as in figure 5, would present a saturation of the products and decreasing slopes as power increases.

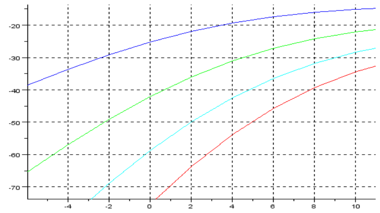

Fig. 5. Hyperbolic tangent model, orders 3 to 9 products

A slope of 1.6 dB/dB can be observed but on a very limited input power range. Values and slopes for other orders are quite different from measured data.

V. CONCLUSION

We proposed new behavioural models for passive non linearity. They are based on functions that are not smooth, and so not analytic, at the origin. These functions are continuous but their second or higher order derivatives may not be. This is acceptable as they respect the energy conservation principle.

These models result in even-integer and non-integer small signal slopes in dB/dB for odd harmonics and odd intermodulation products.

Such arbitrary slopes have been frequently found in measurement results and were up to now unexplained.

In addition, simpler models, with a smaller number of coefficients, give a quite good agreement with measured values of third and fifth order products. They can be used for more precise and more efficient behavioural simulations.

A good approximation in a behavioural model is not sufficient as a formal demonstration of the physical effects that happens in the device.

However, simple non-analytic at the origin behavioural models explain not only third order intermodulation product behaviour for which they have been adjusted but also fifth and higher order intermodulation products behaviour.

This may orient research toward new, non-analytic, physical models explaining this behaviour or new investigations on already tested non-analytic physical models. In these new investigations, models should not be replaced by polynomial or analytical approximations, as this would destroy the non-classical small signal behaviour.

VI. AKNOWLEDGMENT

We would like to thank all authors that published, since 1976, measurement data not obeying the classical theory; some of them have been cited in references.

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