Higher Dynamic Measurement of Antenna Passive Intermodulation Products, using Ray Optics

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***Abstract*—Passive intermodulation products may occur when two or more carriers are transmitted through a passive device such as a filter, a transmission line or an antenna or if they are reflected from one object. These products are generally due to distributed non-linearity along the transmission path or the reflector of the antenna. We show that ray optics can be used to determine easily the directions in which these products are in phase and reinforced. This is particularly significant for multi-horn fed reflector antennas, for multiple antennas systems and for higher dynamic in measurement of passive intermodulation.**

***Index Terms*—antenna, measurement, non-linearity, passive intermodulation, ray optics, space, telecommunications, telephony base station.**

I. Introduction

Passive intermodulation products are mostly detrimental in a transmit and receive system when two or more carriers are transmitted with high power and when a combination of some transmitted carrier frequencies falls inside one of the reception bands of the system.

This was found in 1972 on a telecommunications spacecraft on which a deployable antenna was used for transmission and reception. It also happens now in aircrafts and in mobile telephony base stations. It happens more and more frequently as systems use many different bands for transmission and reception, some of them on the same antenna or on antennas that can be coupled directly or through reflection on an object in their field of view. It may become a critical problem when upgrading some base stations for 5G.

We propose a determination of the main directions along which intermodulation products generated by a great number of identical micro non-linearity, such as those on a reflector or any non-linear object, add in phase. We use ray optics for a simple geometric construction of these directions as for reflection and refraction [1].

We present the application of ray optics to intermodulation products generated by a non-linear plane as is done in [2] and [3] for the second harmonic and the product at sum frequency.

Then we apply this method to spherical waves and to other shapes such as a parabolic reflector with multi-horn feed.

We show how this can be used to perform spatial filtering to separate intermodulation products from carriers when measuring intermodulation products on an antenna and thus improve the dynamic range of the measurement.

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II. Intermodulation Products Computation

A. Products frequencies

When two carriers at frequencies f_1 and f_2 are amplified by the same non-linear amplifier or transmitted through the same non-linear passive device, harmonics and products are generated.

The frequencies of the harmonics and products are given by the following formula:

$$f_{m,n} = \left| m f_1 + n f_2 \right| \tag{1}$$

Coefficients m and n are positive, null or negative integers. We define the harmonic number as $H = |m+n|$ and the product order as $O = |m| + |n|$.

Equation (1) can be extended to any number of carriers:

$$f_{m_1, m_2, \cdots, m_p} = \left| m_1 f_1 + m_2 f_2 + \cdots + m_p f_p \right| \tag{2}$$

In this paper, we will limit the presentation to 1 or 2 carriers, to harmonics 0, 1 and 2 and to orders 0 to 3.

Equation (1) comes from the fact that the product of 2 cosines can be expressed as the sum of 2 cosines of the sum and the difference of phases:

$$\cos(\phi_1).\cos(\phi_2) = \left\{ \cos(\phi_1 + \phi_2) + \cos(\phi_1 - \phi_2) \right\} / 2 \tag{3}$$

Carrier phases are given by: $\phi_i = \omega_i t + \varphi_i = 2\pi f_i t + \varphi_i$ and products phases by: $\phi_{m,n} = m\,\omega_1\,t + m\,\varphi_1 + n\,\omega_2\,t + n\,\varphi_2$

Equation (3) gives second order products at harmonics 0 (difference frequency) and 2 (sum frequency). It can be generalized for higher degrees. By multiplying again by a cosine for degree 3, order 3rd order products at harmonics 1 and 3 are generated. It can be extended also to any number of cosines.

We get the well-known simple geometric construction for the frequencies of intermodulation products around the carriers. Frequency differences are proportional to the difference between both carriers' frequencies.

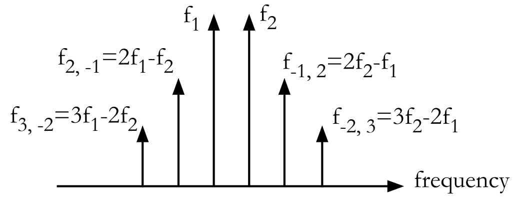

Fig. 1. Frequencies of carriers and intermodulation products of order 3 and 5

B. Products wave-vectors

Equation (3) is generally used when multiplying guided signals. It can be used also for plane waves but the carrier phase must be modified to take into account each carrier wave-vector $\vec{k}$ and the position $\vec{r}$ where the non-linearity occurs:

$$\phi_i = \omega_i t - \vec{k}_i . \vec{r} + \varphi_i = \omega_i t - k_x x - k_y y - k_z z + \varphi_i \tag{4}$$

with the condition that: $k_i = \left|\vec{k}_i\right| = 2\pi / \lambda_i = 2\pi f_i / c = \omega_i / c$.

From (4) and (3) we obtain the phases of the products generated at a given point as:

$$\phi_{m,n} = m\,(\omega_1 t + \varphi_1 - \vec{k}_1 . \vec{r}) + n\,(\omega_2 t + \varphi_2 - \vec{k}_2 . \vec{r}) \tag{5}$$

When all carriers are transmitted from the same antenna (or the same feed in a multi-horn antenna) incident waves wave-vectors are aligned. The wave-vector of the product is then given by:

$$\vec{k}_{m,n} = m\,\vec{k}_1 + n\,\vec{k}_2 \tag{6}$$

This solution is not correct in the general case where carriers are transmitted from different sources such as multiple antennas or multiple feeds of an antenna because (6) would not respect the condition between frequencies and wave-vectors. If two vectors are not aligned then the modulus of the sum is strictly lower than the sum of the moduli of the summed vectors. The solution for the products wave-vectors must obey the condition: $k_{m,n} = \left|\vec{k}_{m,n}\right| = (m\omega_1 + n\omega_2)/c$.

III. NON-LINEAR PLANE REFLECTOR

Only the projections (noted as $k_{//}$) of the wave-vectors of incident waves on the non-linear reflector must be added with the same formula (1) or (2) as the incident frequencies in the intermodulation product frequency. This is shown in [1] for diffracted waves and in [2, 3] for second order harmonics and second order products. We generalize these formulas for higher order products, for spherical waves and for other reflector shapes. We will have in the case of two waves:

$$f_{m,n} = \left|m f_1 + n f_2\right| \qquad \vec{k}_{m,n\,//} = m\vec{k}_{1\,//} + n\,\vec{k}_{2\,//}$$

and

$$k_{m,n} = 2\pi f_{m,n} / c \tag{7}$$

We consider a reflection plane that supports non-linear defaults that are much smaller than the wavelength. We consider that the non-linear plane is homogeneous, that is the distribution of non-linearity is constant.
The projections of the wave-vectors of third order intermodulation products are obtained with the same formulas as their frequencies:

$$\begin{aligned} f_{2,-1} &= 2f_1 - f_2 & \qquad \vec{k}_{2,-1\,//} &= 2\vec{k}_{1\,//} - \vec{k}_{2\,//} \\ f_{-1,2} &= 2f_2 - f_1 & \qquad \vec{k}_{-1,2\,//} &= 2\vec{k}_{2\,//} - \vec{k}_{1\,//} \end{aligned} \tag{8}$$

A. Non-linear reflection of two plane waves

Figure 2 gives the geometric construction of the 4 reflected rays for fundamental intermodulation products wave-vectors in the particular case of third order.
In cases where the computed projection is longer than the corresponding radius, the product will not propagate.
This is comparable to the Brewster angle in refraction.

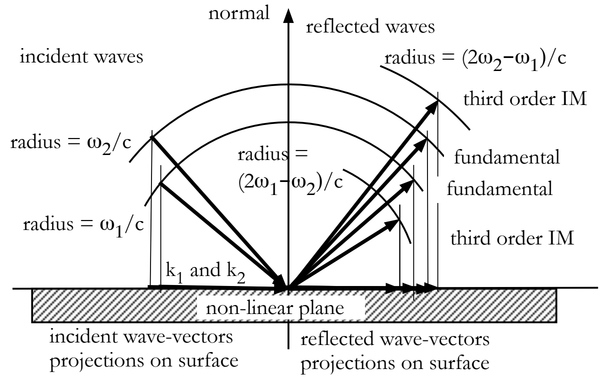

Fig. 2. Fundamental and third order intermodulation products wave-vectors reflected by a non-linear plane

B. Non-linear reflection of waves coming from two sources

For spherical waves coming from two point sources (the phase centers of source feeds), we draw the same geometric construction at multiple points along a reflector made of a limited area of a non-linear plane. In Fig. 3, we draw the rays for the center and the edges of this reflector in the plane orthogonal to the reflector and containing the point sources. We use geogebra; a geometry software [4]. We see that the third order intermodulation products are reflected in directions that appear to come from two virtual sources that are on each side of the real sources. Their exact position depends on the real sources positions and on the signal frequency transmitted by each real source. Real sources are D and E. Rays coming from D are blue; rays coming from E are green. Segments DM and EN represent the wave-vectors along incident rays. Intermodulation products are red and purple. Virtual sources are J_1 and K_1. For better clarity, the figure has been drawn for the case of transmission; transmitted rays are on the side of the non-linear plane opposite to the real or virtual sources. Virtual rays are dashed and intersect at virtual sources.

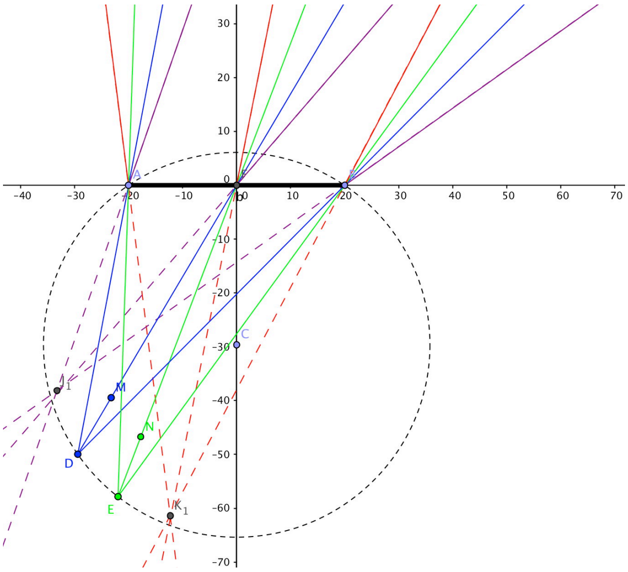

Fig. 3. Fundamental and third order intermodulation products wave-vectors; two real sources and a non-linear plane. Virtual rays are dashed.

In some cases, one or the other virtual source may disappear, as the corresponding rays no longer propagate (same as Brewster angle). In intermediate cases, only some part of the plane reflects rays from a given virtual source.

IV. NON-LINEAR PARABOLIC REFLECTOR

We draw the same figure for an offset fed parabolic reflector; which will be the case for multi-source parabolic antennas.

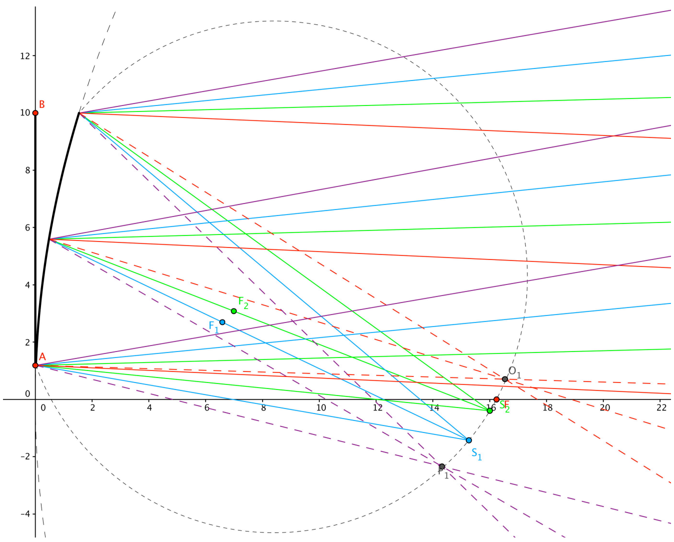

Fig. 4. Fundamental and third order intermodulation products wave-vectors; two real sources and a non-linear parabolic reflector. Virtual rays are dashed.

The two real sources are placed near the focus. In the case of Fig. 4, they are on a circle defined by the focus and the edges of the parabolic reflector so that they see the reflector under the same angle. As can be seen on Fig. 4, intermodulation products come approximately from two virtual point sources on the same circle and are focalized by the parabolic reflector in parallel rays with directions different from the ones of the fundamental signals.

V. SHAPED REFLECTORS

Shaped reflectors or reflector objects of any shape would give the same type of construction. On each point of the non-linear reflector we will draw the projections of the wave-vectors on the plane tangent to the surface of the object and apply the same formula for projected wave-vectors as for frequencies in the product. Then we draw the wave-vector having this projection and the modulus corresponding to the intermodulation product frequency.

VI. HIGH DYNAMIC PIM MEASUREMENTS

When measuring a parabolic reflector or a small piece of plane reflector material, the directions that each intermodulation product will follow can be computed first. We can thus find places where the PIM product power will be higher or maximum and the carrier power much lower than its maximum. Either by moving the receiving antenna, or by rotating the non-linear transmit reflector, we can measure separately the power of the carriers and the power of the PIM products, each at, or near, its maximum. In these conditions the dynamic range of the measurement may be improved and the needed equipment is simplified. Generally, PIM products are measured at 120 to 170 dB under the carrier power. A frequency filter or diplexer with very high rejection is needed. Spatial filtering may allow to use less stringent specification for this rejection filter.
CNES filed a patent on this measurement method.
A satellite manufacturer is measuring space reflectors in this way to assess the practical gain in measurement dynamic. More work was done to compute complete diagrams of the intermodulation products from fields on the reflector.

VII. CONCLUSION

This paper shows that intermodulation products generated by incident waves coming from different directions or different point sources are transmitted in directions that are different from the directions of the reflected waves.Ray optics has been used to obtain these directions. At any point, projections of wave-vectors on the plane tangent to the non-linear reflector add, as vectors, in the same manner as frequencies in the intermodulation product. Parabolic reflectors may focalize these intermodulation products. Measurement dynamic may be improved by this spatial filtering.

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