Optimal Channel Coding Structures for Fast Acquisition Signals in Harsh Environment Conditions.

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BIOGRAPHY

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ABSTRACT
In this article, we provide the method to construct two error correcting structures for GNSS systems, which are capable to provide Maximum Distance Separable (MDS), full diversity and rate-compatible properties. Thanks to those properties, the GNSS receiver is capable to reduce the Time-To-First-Fix (TTFF) and to enhance the robustness of the data demodulation under low Carrier to Noise ratio environments, urban environments and pulsed jamming environments. The proposed error correcting structures are then simulated and compared with the GPS L1C subframe 2 error correcting scheme under the precedent transmission environments. Simulations show an outstanding improvement of the error correction capabilities (which reduce the TTFF in harsh environments) mainly caused by the rate-compatible and the full diversity properties. Moreover, thanks to the MDS property a high reduction of the TTFF under good environments is appreciated.

I-INTRODUCTION

In recent works [1] [2] [3] [4], the interest for reducing the Time-To-First-Fix (TTFF) on the GNSS system has motivated the design of new channel coding schemes, which decrease the time to retrieve the Clock and Ephemerides Data (CED), also called Time-To-Data (TTD). In [1] and [2], the coding schemes exploit both serial concatenation and the Maximum Distance Separable (MDS) property in order to retrieve the information data as fast as possible and to improve the demodulation threshold of the data. In [3], the coding schemes exploit both the MDS property and the full diversity in order to both reduce the complexity of the decoder and improve the error correcting performance in hash environments, while the information data is retrieved as fast as possible. Finally in [4], the coding scheme exploits not only the MDS property and the full diversity, but also the rate-compatible property [5] is added in the coding scheme design. Thanks to the rate-compatible property, an outstanding enhancement of the error correction capabilities and the demodulation threshold is achieved.

In this paper, we provide the methodology to construct channel coding structures able to provide MDS property, full diversity property and rate-compatible property. Thanks to those combined properties, the decoder is capable to **reduce the TTD and to provide enhanced error correction capabilities and lower demodulation threshold under low Carrier to Noise ratio (C/N_0) environments, urban environments and pulsed jamming environments**. Moreover, in order to ensure the robustness of the CED, error detecting techniques are combined with the channel coding structure, as it was already the case in [1] [2] [3] [4].

For this purpose, in order to construct the channel coding structures, we start by modeling the message structure (information bits and redundant bits) under the non-ergodic channel assumption [6] [7] (commonly presented as block fading channel and which can be seen as an extension of the already presented [2] erasure channel model). Accordingly, the information and the redundant bits from the channel coding encoder are divided into different data blocks and each block is weighted by a fading coefficient. Modeling the message structure under the erasure channel assumption [2], helps us to finely describe how the CED can be retrieved under lack of received data (labelled as erased data) and also, as a direct consequence, to describe the method to reduce the TTD.

Moreover, this model enables us to provide the requirements to obtain the three desired channel coding properties: the MDS, the full diversity and the rate-compatible properties. The MDS property allows to retrieve k data information units from any k free error data units (no matter if it is information or redundant data units). It must be noted that, in this case, the information units correspond to the block defined by the message structure design. The second full diversity property allows creating an error correction code structure to lower the resilience degradation under rough environments. The last rate-compatible property allows to improve the error correction capabilities and the demodulation threshold by combining different data units and tuning a lower final channel coding rate (providing lower rate involves better error correction capabilities since more data units can be used to decode the information data) at the decoder.

In this paper, the construction of two channel coding structures to provide MDS property, full diversity property and rate-compatible property is presented:

- The first construction is a rate-compatible root LDPC code of rate 1/3.
- The second construction is a rate-compatible root LDPC code of rate 1/4.

Both error channel coding structures are simulated and compared with the GPS L1C subframe 2 error correcting scheme under the AWGN channel assumption, the urban channel assumption and the pulsed jamming assumption.

The paper is organized as follow: Section II sets the necessary background for the remainder of the article by presenting the block fading channel and the desired coding properties to construct the rate-compatible root LDPC codes. Section III presents the methodology to construct the rate-compatible root LDPC code of rate 1/3 and the optimal message structure to transmit the information. Section IV presents the methodology to construct the rate-compatible root LDPC code of rate 1/4 and the optimal message structure to transmit the information. Their performances are presented and analyzed in Section V for three different channels: AWGN, urban environment and pulsed jamming environment. Conclusions are finally drawn in Section VI.

II-THEORETICAL BACKGROUND

One of the most changeling issues to design a fast acquisition GNSS signal is to provide the lowest Time-To-First-Fix (TTFF) as possible. This time is defined as the time needed by the receiver to calculate the first position and can be considered as a contribution of different times including the time to retrieve the CED data: Time To Data (TTD). In this paper, we construct channel coding structures in order to reduce the TTD, since this time represents the higher contribution to the TTFF. This new channel coding structure manages to reduce the TTD under high C/N_0 environments and enhance the error correction capabilities under degraded environments: low C/N_0 conditions, urban environments or pulsed jamming environments.

In order to achieve the precedent remarks, we propose the following requirements to construct the channel coding structure:

- Within the CED, a Cyclic Redundancy Check (CRC) is needed in order to provide information about the reliability of the message.
- Provide a channel coding scheme with MDS, full diversity and rate-compatible properties under **non-ergodic block fading channel** [6] (those schemes are presented in section IV).

Non-Ergodic Block Fading Channel

The non-ergodic block-fading channel [6] is a simplified channel model that characterizes slowly-varying fading channels (and can be very precise to model the urban and the pulsed jamming channels, since those channels present transmission moments where deep fades, generated by multipath or intentional interferences, harm the transmitted information). It can be viewed as an extension of the well-known block-erasure channel which considers that some parts of the codeword are completely erased due to a deep fade of the channel or, because of the lack of received data. Indeed the block-erasure channel corresponds to the specific case of the largest signal to noise ratio regime of the block fading channel, where some parts of the codeword are received with high Signal to Noise Ratio SNR (SNR $\rightarrow\infty$) and the other parts are received with low SNR (SNR $\rightarrow 0$). Under this context (non-ergodic channel), the transmitted codeword can be viewed as finite number N of independent realization (degree of freedom) of the channel.

We consider a block-fading channel with nc fading blocks, whose discrete-time prompt correlator output at time i is given by:

$$y_i = h_i x_i + z_i \quad i = 1 \dots N_f \qquad (1)$$

where N_f denotes the frame length, $x_i \in \{-1, +1\}$ is the i_{th} BPSK modulated symbol, $z_i \sim N(0, \sigma^2)$ are the i.i.d. Gaussian noise samples, $\sigma^2 = N_0/(4T_i)$ with T_i the correlator integration time, and h_i is a real fading coefficient that belongs to the set $\aleph = \{\alpha_1, \alpha_2, \dots, \alpha_{nc}\}$. Figure 1 illustrates a codeword under the block fading channel scenario.

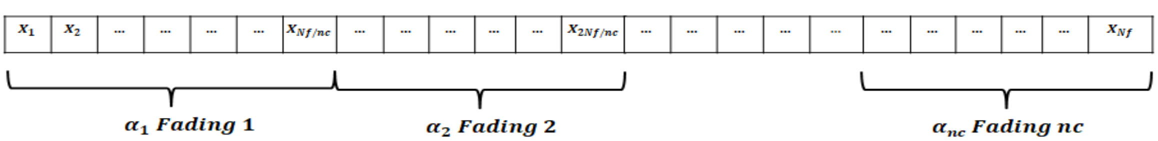

Figure 1 ***Block Fading Channel Model***

Similarly to any other non-ergodic channels, the block-fading channel has zero capacity in the strict Shannon sense since there is an irreducible probability that the decoder makes an error. In other words, it exits an irreducible probability where all channel are erased or all data blocks are not received and as a consequence the reliable communication is not possible. This probability is the *information outage probability* defined as:

$$P_{out} = Pr\{I_\aleph \leq R\} \qquad (2)$$

where $I_\aleph$ denotes the instantaneous mutual information between the input and output of the channel for a particular channel realization $\aleph$, and R is the transmission rate in bits per channel use. In such non ergodic channels, P_{out} is the best possible word error probability.

Thanks to the definition of the block fading channel, we model our specific channel assumption, where the set $\alpha_i \in \{0, 1\}$. In case of $\alpha_i = 0$ the data block i is erasure, otherwise the data block is modeled by an ergodic AWGN channel. Under those channel specifications, the channel coding schemes, presented in section IV, are designed for being as close as possible to the information outage probability limit.

Code Desired Properties
In order to design codes suited for the non-ergodic channels, two main properties must be accomplished:

- 1-MDS (Maximum Distance Separable property),
- 2-Full Diversity.

Consider a channel coding scheme, which provides codewords divided in *n* blocks. Consider that the information data is equivalent to *k* blocks with $k<n$. The *MDS* property allows to retrieve *k* data units of information from any *k* free error information units (no matters information or redundant data). In other words, thanks to this property we can reduce the time to retrieve CED under high C/N_0 environments, since only with *k* free error data units, the CED can be retrieved. By the other hand, several references exhibit the poor error correction performances over non-ergodic channels, which are not able to achieve a good *coding gain*. In order to achieve better error correction capabilities, the *full diversity* property is required.

*Definition 1***:** An error correcting code is said to have full diversity over block fading channel if the diversity order is equal to the number of fading blocks. The *diversity order* d determines the slope of the error-rate curve as a function of the SNR on a log-log scale for Rayleigh fading distribution.

$$d = -\lim_{\gamma \to +\infty} \frac{\log(P_{ew})}{\log(\gamma)} \quad (3)$$

where P_{ew} is the word error probability at the decoder output and γ is the average SNR per symbol (remark that the diversity order depends on the decoding algorithm). Since the error probability of any coding scheme is lower-bounded by the outage probability P_{out}, the diversity order is upper-bounded by the intrinsic diversity of the channel, which reflects the slope of the outage limit. When maximum diversity is achieved by a code, the coding gain yields a measure of SNR proximity to the outage limit. This optimal design yields the optimal code, which is given by the singleton bound:

$$d \leq 1 + \lfloor nc(1 - R) \rfloor \quad (4)$$

Codes achieving the singleton bound are termed *MDS*. *MDS* codes are outage-achieving over the (noiseless) block-erasure channel, but may not be close to the outage probability limit on noisy block-fading channels. As a matter of fact, *MDS* codes are necessary, but not sufficient to approach the outage probability of the channel and it is the *full diversity* the desired and sufficient condition in order to approach the outage probability. Codes with *full diversity* under the Belief Propagation (BP) decoding algorithm [8] (root LDPC codes) were proposed in [7].

In order to design codes with low demodulation threshold and to enhance the flexibility of the message structure, this last property must be accomplished:

- 3-Rate-compatibility.

An error correcting code of rate $1/n$ and divided in a block fading channel structure of n blocks is said to have rate-compatibility if with just k blocks fading data units, the information bits has a diversity order equal to k. Moreover, independently of the retrieved blocks, the same error correction capabilities are achieved. As example, if we have a rate compatible root LDPC code of rate $1/3$ and $k = 2$ block fading data units have been received, then the information bits can be retrieved with the same diversity order that a root LDPC code of rate $1/2$.

III-CONSTRUCTION OF THE RATE COMPATIBLE ROOT LDPC CODE OF RATE 1/3

The family of rate compatible root LDPC codes of rate *1/3* is an error correcting structure of rate *1/3*, which divided its information and redundant data in *3* block fading data unit. The family of codes is characterized by the following properties:

- The MDS property, which allows to retrieve the information data from any free error block fading data unit.
- The full diversity under the use of the BP algorithm [8], which allows the information bits to be decoded with a diversity order equal to the number of fading blocks.

- The rate-compatibility. Assuming a rate compatible root LDPC code of rate $1/3$ with $k = 2$ block fading data units, the information bits can be retrieved with the same diversity order that the information bits of a root LDPC code of rate $1/2$. Moreover, independently of the retrieved block fading data units, the same error correction capabilities are achieved.

In the following subsection, we are going to describe the methodology to construct the rate-compatible root LDPC codes for a rate of *1/3*. The information and the redundant data are divided in *3* block fading data units (equal to the coding rate). Each block fading data unit included *1/3* of the information data *1/3* of the redundant data. Since the coding rate is *1/3*, the redundant data is twice the information data. Let's present the rate-compatible root LDPC parity check matrix for a code rate 1/3:

$$H_\beta = \begin{pmatrix} I_{11} & 0 & 0 & H_{41} & H_{51} & H_{61} & 0 & 0 & 0 \\ I_{12} & 0 & 0 & 0 & 0 & 0 & H_{72} & H_{82} & H_{92} \\ H_{13} & H_{23} & H_{33} & I_{43} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I_{44} & 0 & 0 & H_{74} & H_{84} & H_{94} \\ H_{15} & H_{25} & H_{35} & 0 & 0 & 0 & I_{75} & 0 & 0 \\ 0 & 0 & 0 & H_{46} & H_{56} & H_{66} & I_{76} & 0 & 0 \end{pmatrix} \qquad (5)$$

where $I_{i,j}$ are identity matrices , $H_{i,j}$ are a combination of QC matrices [9] with different Hamming weights and subindexes i and j represent the column and row position inside the matrix. We remark that the 3 first columns represent the codeword data sent in the first block fading data unit (the first column represent the information data i_1 and columns 2-3 the redundant data), the columns 4-6 represent the codeword data sent in the second block fading data unit (the 4th column represent the information data i_2 and columns 5-6 the redundant data) and the columns 7-9 represent the codeword data sent in the third block fading data unit (the 7th column represents the information data i_3 and columns 8-9 the redundant data).

As it happened for the root LDPC codes, in order to compute the generator matrix, the submatrix corresponding to the redundant data must be full rank [9]. Let's re-structure the parity check matrix by inserting the information data in the first columns as it is illustrated in *(6).*

$$H_\beta = \begin{pmatrix} I_{11} & H_{41} & 0 & 0 & 0 & H_{51} & H_{61} & 0 & 0 \\ I_{12} & 0 & H_{72} & 0 & 0 & 0 & 0 & H_{82} & H_{92} \\ H_{13} & I_{43} & 0 & H_{23} & H_{33} & 0 & 0 & 0 & 0 \\ 0 & I_{44} & H_{74} & 0 & 0 & 0 & 0 & H_{84} & H_{94} \\ H_{15} & 0 & I_{75} & H_{25} & H_{35} & 0 & 0 & 0 & 0 \\ 0 & H_{46} & I_{76} & 0 & 0 & H_{56} & H_{66} & 0 & 0 \end{pmatrix} = (A \quad B) \qquad (6)$$

where A represents the 3 first columns of the matrix and B represents the reminder columns of the matrix (corresponding to the redundant data). In figure 2, it is represented the mapping of the information and redundant/parity bits by using the bipartite Tanner protograph representation that also shows how the different information and parity bits are connected to rootchecks ($1c_2$, $1c_3$, $2c_1$, $2c_3$, $3c_1$, $3c_2$).

Construction:
In order to construct the rate-compatible root LDPC codes of rate 1/3, the following constraints must be accomplished:

1st constraint-MDS property
When one block is retrieved under the erasure block channel the information data should be retrieved:

- If we retrieve the first block fading data unit, the data corresponding to the 1st, 2nd and 3rd columns of the parity check matrix in equation (5) is available. Considering that, from the 3rd and the 5th rows in equation (5) we generate the following submatrix:

$$H_{block1} = \begin{pmatrix} H_{13} & I_{43} & 0 & H_{23} & H_{33} \\ H_{15} & 0 & I_{75} & H_{25} & H_{35} \end{pmatrix} \qquad (7)$$

  The submatrix (7) can be used to retrieve the part of the codeword corresponding to the information data i_2 and i_3. Remark that the information is determined by using the rootchecks $2c_1$ or the rootchecks $3c_1$. Remark that both: a linear equation system or the BP algorithm can be used to retrieve the erase information data.

- If we retrieve the second block fading data unit, the data corresponding to the 4th, 5th and 6th columns of the parity check matrix in equation (5) is available. Considering that, from the 1st and the 6th rows in equation (5) we generate the following submatrix:

$$H_{block2} = \begin{pmatrix} I_{11} & H_{41} & 0 & H_{51} & H_{61} \\ 0 & H_{46} & I_{76} & H_{56} & H_{66} \end{pmatrix} \qquad (8)$$

  The submatrix (8) can be used in order to retrieve the part of the codeword corresponding to the information data i_1 and i_3. Remark that the information is determined by using the rootchecks $1c_2$ or the rootchecks $3c_2$. Remark that both: a linear equation system or the BP algorithm can be used to retrieve the erase information data.

- If we retrieve the third block fading data unit, the data corresponding to the 7th, 8th and 9th columns of the parity check matrix in equation (5) is available. Considering that, from the 2nd and the 4th rows in equation (5) we generate the following submatrix:

$$H_{block3} = \begin{pmatrix} I_{12} & 0 & H_{72} & H_{82} & H_{92} \\ 0 & I_{44} & H_{74} & H_{84} & H_{94} \end{pmatrix}. \qquad (9)$$

  The submatrix (9) can be used in order to retrieve the part of the codeword corresponding to the information data i_1 and i_2. Remark that the information is determined by using the rootchecks $1c_3$ or the rootchecks $2c_3$. Remark that both: a linear equation system or the BP algorithm can be used to retrieve the erase information data.

From equations *(7)*, *(8)* and *(9)*, it is shown that with just one of the block fading data unit, under the erasure channel, all the information data can be retrieved. Therefore the structure in equation (5) is MDS.

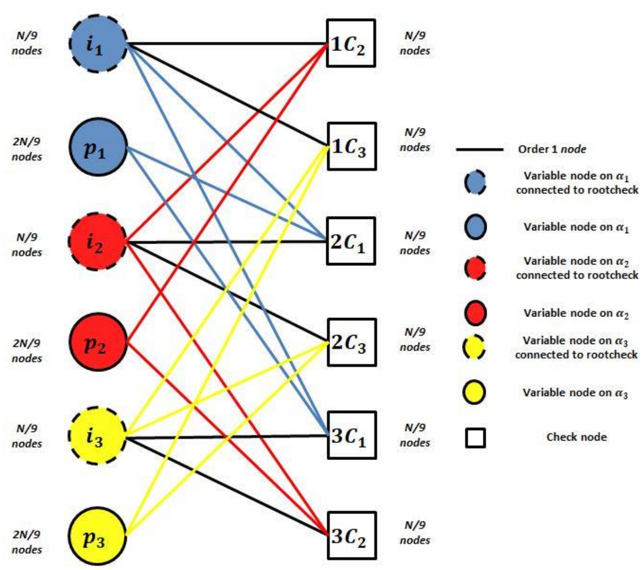

Figure 2 Tanner graph for a Root LDPC code of rate 1/3

2nd constraint-Full diversity

Let's study this property under block erasure channel:

The three fading coefficients α_1, α_2 and α_3 are independent and can belong to $\aleph \in \{0,1\}$. If we examine the tanner graph in figure 2, we can observe that the outage event occurs when $\alpha_1 = \alpha_2 = \alpha_3 = 0$ (all the block fading units are erased). Indeed, when:

- $\alpha_1 = 0$, $\alpha_2 = 1$ and $\alpha_3 = 1$, the information data i_1 are determined by using the rootchecks $1c_2$ or the rootchecks $1c_3$.
- $\alpha_1 = 1$, $\alpha_2 = 0$ and $\alpha_3 = 1$, the information data i_2 are determined by using the rootchecks $2c_1$ or the rootchecks $2c_3$.

- $\alpha_1 = 1$, $\alpha_2 = 1$ and $\alpha_3 = 0$, the information data i_3 are determined by using the rootchecks $3c_1$ or the rootchecks $3c_2$.

Let be ϵ the probability that α_i, $i = 1{,}2{,}3$, be equal to 0. The word error probability is equal to ϵ^3, which is precisely the outage probability of the channel [10]. Therefore, the matrix structure in equation (5) is outage achieving over the block erasure channel and as consequence full diversity. Remark that all the data information is retrieved through the same received block fading data units.

<u>3rd constraint-Rate-compatible</u>

If just one block is retrieved (first block fading data unit, second block fading data unit or third block fading data unit), under block erasure channel, the information data has the same diversity order (equal to 1):

- $\alpha_1 = 0$, $\alpha_2 = 0$ and $\alpha_3 = 1$, the information data i_1 and i_2 are determined by using the rootchecks $1c_3$ or the rootchecks $2c_3$.
- $\alpha_1 = 0$, $\alpha_2 = 1$ and $\alpha_3 = 0$, the information data i_1 and i_3 are determined by using the rootchecks $1c_2$ or the rootchecks $3c_2$.
- $\alpha_1 = 1$, $\alpha_2 = 0$ and $\alpha_3 = 0$, the information data i_3 and i_2 are determined by using the rootchecks $2c_1$ or the rootchecks $3c_1$.

If two blocks are retrieved (first and second block fading data units, first and third block fading data units or second and third block fading data units) under the block erasure channel, the information data has the same diversity order (equal to 2):

- Considering that 1st and the 2nd block fading data units are received, the data corresponding to the 1st-6th columns of the parity check matrix in equation (5) are available. Considering that, from the 1st, the 3th, 5th and the 6th rows in equation (10) we generate the following submatrix:

$$H_{block12} = \begin{pmatrix} I_{11} & H_{41} & 0 & 0 & 0 & H_{51} & H_{61} \\ H_{13} & I_{43} & 0 & H_{23} & H_{33} & 0 & 0 \\ H_{15} & 0 & I_{75} & H_{25} & H_{35} & 0 & 0 \\ 0 & H_{46} & I_{76} & 0 & 0 & H_{56} & H_{66} \end{pmatrix} \quad (10)$$

  This submatrix shows that the information data i_3 can be determined by using the rootchecks $3c_1$ or the rootchecks $3c_2$. Moreover, the information data i_1 is connected to the rootcheck $1c_2$ and the information data i_2 is connected to the rootcheck $2c_1$.

- Considering that 1st and the 3rd block fading data units are received, the data corresponding to the 1st-3rd and 7th-9th columns of the parity check matrix in equation (5) are available. Considering that, from the 1st-4th rows in equation (5) we generate the following submatrix:

$$H_{block13} = \begin{pmatrix} I_{12} & 0 & H_{72} & 0 & 0 & H_{82} & H_{92} \\ H_{13} & I_{43} & 0 & H_{23} & H_{33} & 0 & 0 \\ 0 & I_{44} & H_{74} & 0 & 0 & H_{84} & H_{94} \\ H_{15} & 0 & I_{75} & H_{25} & H_{35} & 0 & 0 \end{pmatrix} \quad (11)$$

  This submatrix shows that the information data i_2 can be determined by using the rootchecks $2c_1$ or the rootchecks $2c_3$. Moreover, the information data i_1 is connected to the rootcheck $1c_3$ and the information data i_3 is connected to the rootcheck $3c_1$.

- Considering that 2nd and the 3rd block fading data units are received, the data corresponding to the 4th-9th columns of the parity check matrix in equation (5) are available. Considering that, from the 1st, 2nd, 4th and 6th rows in equation (5) we generate the following submatrix:

$$H_{block23} = \begin{pmatrix} I_{12} & 0 & H_{72} & 0 & 0 & H_{82} & H_{92} \\ H_{13} & I_{43} & 0 & H_{23} & H_{33} & 0 & 0 \\ 0 & I_{44} & H_{74} & 0 & 0 & H_{84} & H_{94} \\ H_{15} & 0 & I_{75} & H_{25} & H_{35} & 0 & 0 \end{pmatrix} \quad (12)$$

This submatrix shows that the information data i_1 can be determined by using the rootchecks $1c_2$ or the rootchecks $1c_3$. Moreover, the information data i_2 is connected to the rootcheck $2c_3$ and the information data i_3 is connected to the rootcheck $3c_2$.

In order to provide a structure with the same error correction capabilities, independently of the received blocks:

- The demodulation threshold of the matrices in equation (7) (retrieved the first block fading data unit), (8) (retrieved the second block fading data unit) and (9) (retrieved the third block fading data unit) must be equal.
- The demodulation threshold of the matrices (10) (retrieved the first and the second block fading data units), (11) (retrieved the first and the third block fading data units) and (12) (retrieved the first and the third block fading data units) must be equal.

Considering the precedent, the structure in equation (5) must provide a symmetrical pattern for each of the block fading data unit. Now, we present the construction of rate compatible root LDPC code of rate 1/3. Considering the equations (7), (8) and (9), the submatrices $I_{i,j}$ are QC submatricesof Hamming weight of 1, the submatricesH_{13}, H_{15}, H_{41}, H_{46}, H_{72} and H_{74}, are QC submatrices of Hamming weight of 2. Finally, the subset of submatrices H_{23}, H_{33}, H_{25} and H_{35} of equation *(7)*, the subset of submatrices H_{51}, H_{61}, H_{56} and H_{66} of equation *(8)* and the subset of submatrices H_{82}, H_{92}, H_{84} and H_{94} of equation *(9)* generate the submatrices H_1, H_2 and H_3 of Hamming weight 3.

$$H_1 = \begin{pmatrix} H_{23} & H_{33} \\ H_{25} & H_{35} \end{pmatrix}, H_2 = \begin{pmatrix} H_{51} & H_{61} \\ H_{56} & H_{66} \end{pmatrix} \; and \; H_3 = \begin{pmatrix} H_{82} & H_{92} \\ H_{84} & H_{94} \end{pmatrix} \tag{13}$$

Remark from equation (13) that equation (5) can be designed as a multi-edge LDPC code [12].

Considering the precedent construction, we have computed the protograph EXIT chart algorithm [13] in order to verify that matrices in equations (7), (8) and (9) converge to the same demodulation threshold. Also, we have computed the same algorithm in order to verify that matrices in equations (10), (11) and (12) converge to the same demodulation threshold. Moreover, we also compute the EXIT chart algorithm [11] in order to verify that the average extrinsic information between the variable and check LDPC nodes converge equally.

Optimal Message Structure

The message is proposed to be structured as follows.

First, the message is split in two parts: the first part carries the CED (information data) and redundant data, the second part (equivalent to subframe 3 of the GPS L1C message [14]) carries additional data (called in figure 3 as Data 1, 2 and 3 and Redundant Data 1,2 and 3). The first part can carry three different messages, described as follows (considering as an example that the CED represent 600 bits):

- The 1st message sends the first and the second blocks (equivalent to the first block fading data unit and the second fading block data unit) and contains 2/3 of the CED and ephemerides data (400 bits) and 2/3 of the redundant data (800 bits). With this information, we are able to retrieve the CED with the same resilience as a Root-LDPC code of rate 1/2.
- The 2nd message sends the third and the first blocks (equivalent to the third block fading data unit and the second fading block data unit) and contains 2/3 of the CED and ephemerides data (400 bits) and 2/3 of the redundant data (800 bits). With this information we are capable to retrieve the CED with the same resilience as a Root-LDPC code of rate 1/2. Moreover, if we have already received the 1st message, we can enhance the resilience of the data since we have already received the precedent 200 bits of CED and the 400 bits of redundant data, equivalent to a channel coding scheme of 1/3.
- The 3rd message sends the second and the third blocks (equivalent to the second block fading data unit and the third fading block data unit) and contains 2/3 of the CED and ephemerides data (400 bits) and 2/3 of the redundant data (800 bits). With this information we are capable to retrieve the CED with the same resilience as a Root-LDPC code of rate 1/2. Moreover, if we have already received the second message, we can enhance the resilience of the data.

- Remark that since the proposed codes are MDS, under good channel conditions with just one block fading data unit we can retrieve the information and as a consequence we can reduce the TTD in high percentage of the cases.

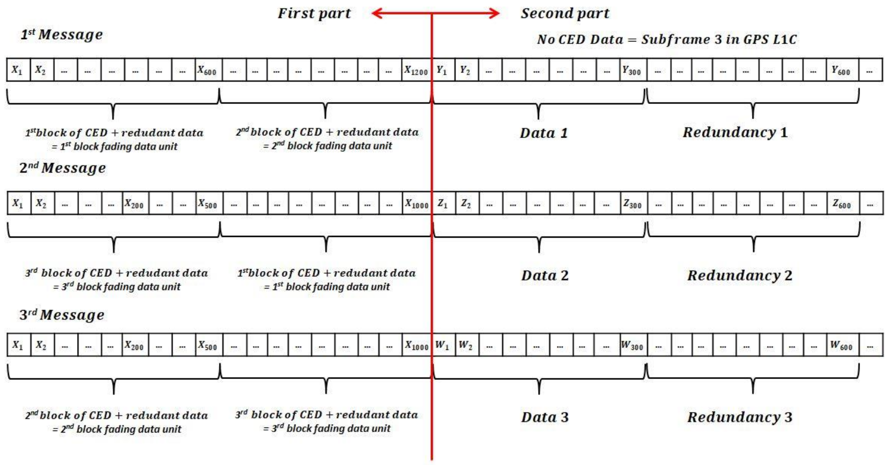

Figure 3 Navigation Message Structure Rate Compatible Root LDPC of Rate 1/3

IV-CONSTRUCTION OF THE RATE-COMPATIBLE ROOT LDPC CODE OF RATE ¼

We can extend the construction of the rate compatible Root LDPC codes of rate 1/3 to the rate 1/4. The family of rate compatible root LDPC codes of rate 1/4 is an error correcting structure of rate 1/4 which its information and redundant data are divided into 4 block fading data units. The family of codes is also characterized by MDS property, full diversity and rate-compatibility properties.

The information and the redundant data are divided in 4 block fading data units (equal to the coding rate). Each block fading data unit includes 1/4 of the information data 1/4 of the redundant data. Since the coding rate is 1/4, the redundant data is three times the information data.

Let's present the rate compatible root LDPC parity check matrix for a code rate 1/4, considering the systematic form of the matrix:

$$H_\alpha = \begin{pmatrix} I_{1,1} & 0 & 0 & H_{4,1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & H_{14,1} & H_{15,1} & H_{16,1} \\ I_{1,2} & H_{2,2} & 0 & 0 & 0 & 0 & 0 & H_{8,2} & H_{9,2} & H_{10,2} & 0 & 0 & 0 & 0 & 0 & 0 \\ I_{1,3} & 0 & H_{3,3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & H_{11,3} & H_{12,3} & H_{13,3} & 0 & 0 & 0 \\ H_{1,4} & I_{2,4} & 0 & 0 & H_{5,4} & H_{6,4} & H_{7,4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & I_{2,5} & 0 & H_{4,5} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & H_{14,5} & H_{14,5} & H_{14,5} \\ 0 & I_{2,6} & H_{3,6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & H_{11,6} & H_{12,6} & H_{13,6} & 0 & 0 & 0 \\ H_{1,7} & 0 & I_{3,7} & 0 & H_{5,7} & H_{6,7} & H_{7,7} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & H_{2,8} & I_{3,8} & 0 & 0 & 0 & 0 & H_{8,8} & H_{9,8} & H_{10,8} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & I_{3,9} & H_{4,9} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & H_{14,9} & H_{15,9} & H_{16,9} \\ H_{1,10} & 0 & 0 & I_{4,10} & H_{5,10} & H_{6,10} & H_{7,10} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & H_{2,11} & 0 & I_{4,11} & 0 & 0 & 0 & H_{8,11} & H_{9,11} & H_{10,11} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & H_{3,12} & I_{4,12} & 0 & 0 & 0 & 0 & 0 & 0 & H_{11,12} & H_{12,12} & H_{13,12} & 0 & 0 & 0 \end{pmatrix} \quad (14)$$

where $I_{i,j}$ are identity matrices, $H_{i,j}$ are a combination of QC matrices with different Hamming weights and subindexes i and j represent the column and row position inside the matrix. As in equation *(6)*, we have re-structured the parity check matrix by inserting the information data in the four first columns. Therefore, columns 1, 5, 6 and 7 represent the data sent in the first block fading data unit, columns 2, 8, 9 and 10 represent the data sent in the second block fading data unit, columns 3, 11, 12 and 13 represent the data sent in the third block fading data unit and columns 4, 14, 15 and 16 represent the data sent in the fourth block fading data unit.

Moreover, in order to compute the generator matrix, the submatrix corresponding to the redundant data must be full rank. Therefore equation *(14)*:

$$H_\alpha = \begin{pmatrix} A & B \end{pmatrix} \quad (15)$$

where A represents the 4 first columns of the matrix and B represents the reminder columns of the matrix (corresponding to the redundant data).

In figure 4, it is represented the mapping of the information and redundant/parity bits by using the bipartite Tanner protograph representation that also shows how the different information and parity bits are connected to rootchecks ($1c_2$, $1c_3$, $1c_4$, $2c_1$, $2c_3$, $2c_4$, $3c_1$, $3c_2$, $3c_4$, $4c_1$, $4c_2$ and $4c_3$).

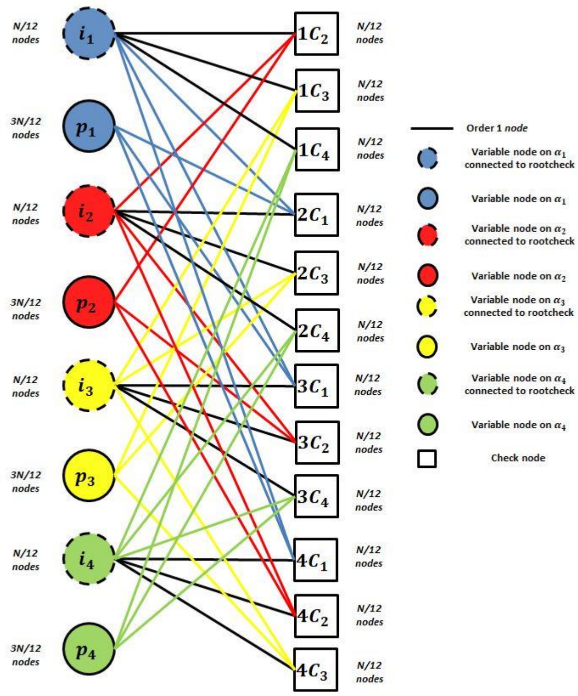

Figure 4 Tanner graph for a Root LDPC code of rate 1/4

In order to verify that the structure in equation 4.36 accomplishes the MDS, full diversity and rate compatible properties, the steps in section III can be followed.

Now, we present the construction of the rate compatible Root LDPC code of rate 1/4. Considering the structure in equation (14), the submatrices $I_{i,j}$ are QC submatrices of Hamming weight 1. The submatrices $H_{1,4}$, $H_{1,7}$, $H_{1,10}$, $H_{2,2}$, $H_{2,8}$, $H_{2,11}$, $H_{3,3}$, $H_{3,6}$, $H_{3,12}$, $H_{4,1}$, $H_{4,5}$ and $H_{4,9}$ are QC submatrices of Hamming weight 2. Finally, the remainder subset of submatrices $H_{i,j}$, generate the submatrices H_1, H_2, H_3 and H_4 of Hamming weight 3.

$$H_1 = \begin{pmatrix} H_{5,4} & H_{6,4} & H_{7,4} \\ H_{5,7} & H_{6,7} & H_{7,7} \\ H_{5,10} & H_{6,10} & H_{7,10} \end{pmatrix}, H_2 = \begin{pmatrix} H_{8,2} & H_{9,2} & H_{10,2} \\ H_{8,8} & H_{9,8} & H_{10,8} \\ H_{8,11} & H_{9,11} & H_{10,11} \end{pmatrix},$$
$$H_3 = \begin{pmatrix} H_{11,3} & H_{12,3} & H_{13,3} \\ H_{11,6} & H_{12,6} & H_{13,6} \\ H_{11,12} & H_{12,12} & H_{13,12} \end{pmatrix}, H_4 = \begin{pmatrix} H_{14,1} & H_{15,1} & H_{16,1} \\ H_{14,5} & H_{15,5} & H_{16,5} \\ H_{14,9} & H_{15,9} & H_{16,9} \end{pmatrix} \quad (22)$$

Optimal Message Structure

First, the message is split in two parts: the first part carries the CED (information data) and redundant data, the second part carries additional data. The first part can carry two different messages, described as follows (considering as an example that the CED represent 600 bits):

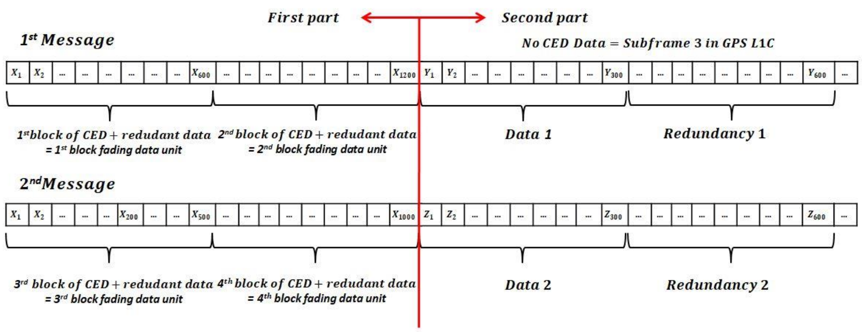

Figure 5 Navigation Message Structure Rate Compatible Root LDPC of Rate 1/3

- The 1st message sends the first and the second blocks (equivalent to the first block fading data unit and the second fading block data unit) and contains 2/4 of the CED and ephemerides data (300 bits) and 2/4 of the redundant data (900 bits). With this information, we are able to retrieve the CED with the same resilience as a toot LDPC code of rate 1/2.
- The 2nd message sends the third and the fourth blocks (equivalent to the third block fading data unit and the fourth fading block data unit) and contains 2/4 of the CED and ephemerides data (300 bits) and 2/4 of the redundant data (900 bits). With this information we are capable to retrieve the CED with the same resilience as a root LDPC code of rate 1/2. Moreover, if we have already received half of the 1st message or the entire 1st message, we can enhance the resilience by using structures of rate 1/3or 1/4 respectively.

RESULTS

AWGN Channel

The Additive White Gaussian Noise (AWGN) channel is the model used to estimate the background noise on the transmission channel. This model does not include fading or interferences coming from other sources. The model follows *(23)* [15]:

$$y_k = x_k + n_k \qquad (23)$$

where y_k is the received sample, x_k is the transmitted symbol and n_k is the AWGN noise. Moreover $n_k \sim \mathcal{N}(0, \theta^2)$ where $\theta^2 = N_0/(4T_i)$ and T_i= 1ms.

Results for AWGN Channel

We have computed the results for the rate-compatible root LDPC code of rate *1/3* (Figure 6). Complete state Information (CSI) is considered at the receiver:

- In case that one block fading data unit has been received (cyan, green and yellow lines), thanks to the MDS property, we can retrieve the information data (CED) when the C/N_0 is high. As a consequence, we reduce the TTD compare with the legacy GPS L1C subframe 2, since we do not have to wait to retrieve all the data (That involves reduction of the 50% of the time).
- In case that two block fading data units have been received (black, blue and magenta lines), we can retrieve the information data (CED) with an error correction capability of almost the level of the legacy GPS L1C sub frame 2 (red line). Just a small gap of 0.55 dBHz for an error probability of 10^{-2}. In any case, thanks to the MDS property for C/N_0 higher than 25 dBHz, the TTD is reduced.
- In case that three block fading data units have been received (red star line), the error correcting capabilities enhance the legacy GPS L1C subframe 2 in 1.2dBHz for an error probability of 10^{-2}. Thanks to that, receiver devices capable of operating in the rank of 22-24 dBHz (low sensitivity devices) could retrieve the CED.

We have computed the results for the rate-compatible root LDPC code of rate *1/4* (Figure 7). CSI is considered at the receiver:

- In case that one block fading data unit has been received (cyan, green, black and yellow lines), thanks to the MDS property, we can retrieve the information data (CED) when the C/N_0 is high. As a consequence, we reduce the TTD compare with the legacy GPS L1C subframe 2, since we do not have to wait to retrieve all the data.
- In case that two block fading data units have been received (black, blue, green and magenta start lines), we can retrieve the information data (CED) with an error correction capability of almost the level of the legacy GPS L1C subframe 2 (red line). Just a small gap of 0.55/0.65 dBHz for an error probability of 10^{-2} is illustrated. In any case, thanks to the MDS property for C/N_0 higher than 25 dBHz, the TTD is reduced. The reason that we don't have the same error correcting capabilities in

all the cases is due to the size of the block of the matrix. Since the size of the CED is 600 bits, each submatrix inside the parity check matrix is not big enough to achieve the perfect correction performance.

- In case that three block fading data units have been received (black, blue, cyan and magenta dash lines), the error correcting capabilities enhance the legacy GPS L1C subframe 2 in 1.1/1.2dBHz for an error probability of 10^{-2}. Thanks to that, receiver devices capable of operating in the rank of 22-24 dBHz (low sensitivity devices) could retrieve the CED.
- In case that three block fading data units have been received (red star line), the error correcting capabilities enhance the legacy GPS L1C subframe 2 in 2.2dBHz for an error probability of 10^{-2}. Thanks to that, receiver devices capable of operating in the rank of 21-24 dBHz (low sensitivity devices) could retrieve the CED.

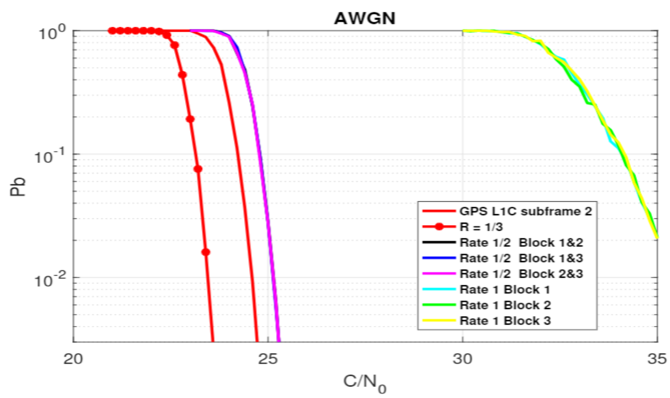

Figure 6 Rate Compatible Root LDPC of Rate 1/3 vs GPS L1C subframe 2 AWGN Channel

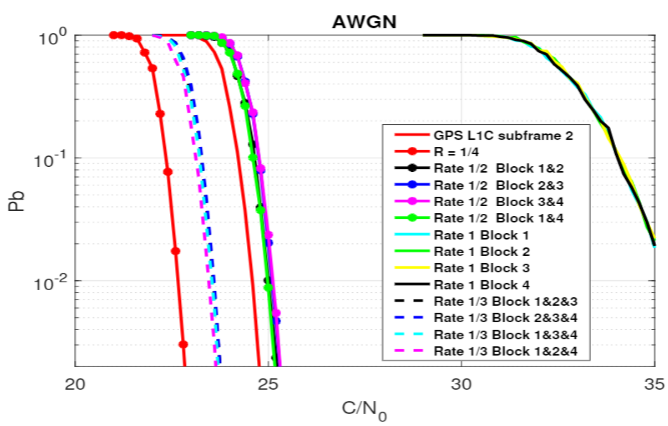

Figure 7 Rate Compatible Root LDPC of Rate 1/4 vs GPS L1C Subframe 2 AWGN Channel

Urban Channel

To model the urban environment, we have used a 2-state Prieto model for a vehicle speed of 40 km/h and an elevation angle of 40 degrees. The model follows *(24)* [14]:

$$y_k = h_k x_k + n_k \qquad (24)$$

where y_k is the received sample, x_k is the transmitted symbol, h_k is the multiplicative term due to the multipath (2-state Prieto model [16]) and n_k is the AWGN noise. Moreover $n_k \sim \mathcal{N}(0, \theta^2)$ where $\theta^2 = N_0/(4T_i)$ and T_i= 1ms.

Results for Urban Channel

We have computed the results for the rate-compatible root LDPC code of rate *1/3* (Figure 8). CSI is considered at the receiver:

- In case that two block fading data units have been received (black, blue and magenta lines), we can retrieve the information data (CED) with an error correction capability of almost the level (blue and black lines) of the legacy GPS L1C sub frame 2 (red star line) or the same level (magenta line).

- In case that three block fading data units have been received (red line), the error correcting capabilities enhance the legacy GPS L1C subframe 2 in almost 2.5dBHz for an error probability of 10^{-2}. Moreover, thanks to the fact that the system converges faster, a reduction of the TTD is achieved in urban environments.

We have computed the results for the rate-compatible root LDPC code of rate *1/4* (Figure 9). CSI is considered at the receiver:

- In case that four block fading data units have been received (red star line), the error correcting capabilities enhance the legacy GPS L1C subframe 2 in almost 4dBHz for an error probability of 10^{-2}. Moreover, thanks to the fact that the system converges faster, a reduction of the TTD is achieved in urban environments.

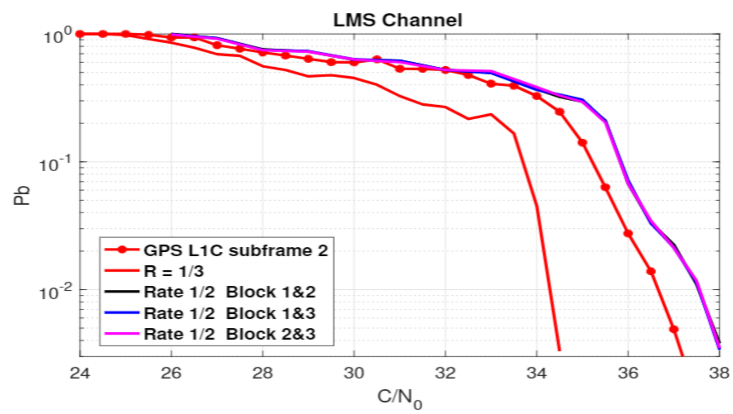

Figure 8 Rate Compatible Root LDPC of Rate 1/3 vs GPS L1C Subframe 2 Urban Channel

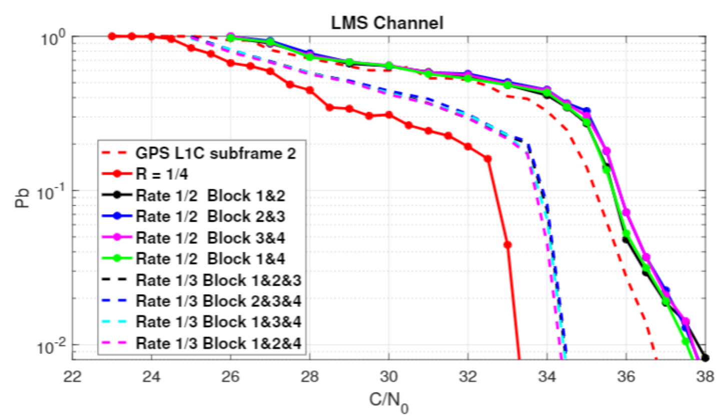

Figure 9 Rate Compatible Root LDPC of Rate 1/4 vs GPS L1C subframe 2 Urban Channel

Pulsed Jamming Channel

The pulsed jamming channel models an environment where a jammer device introduces selective interference [17] in some of the symbols of the message. The model follows *(25).*

$$y_k = \begin{cases} x_k + n_k & with\ probability\ (1-p) \\ x_k + n_{k+i} & with\ probability\ p \end{cases} \qquad (25)$$

where y_k is the received sample, x_k is the transmitted symbol, n_k is the AWGN noise with $n_k \sim \mathcal{N}(0, \theta^2)$ where $\theta^2 = N_0/(4T_i)$ and T_i= 1ms and $n_{k+i} \sim \mathcal{N}\left(0, {\theta_i}^2\right)$ where ${\theta_i}^2$ depends of the thermal noise θ^2 and the intentional interference. Remark that the power of the interference depends of the probability *p*. For a fixed interference power, a higher *p* involves less interference power per symbol. In the following simulation, *p* is fixed to 0.1 and the extra interference power is fixed to 2dB per symbol in case of Gaussian jamming interference (case where all the symbols are harmed by the same interference power). Therefore the pulsed jamming harms each symbol with an extra power:

$$Power_{extra} = 10\ log_{10}\left((10^{Power}/_{10})/p\right) \qquad (26)$$

Results for Pulsed Jamming Channel

We have computed the results for the rate-compatible root LDPC code of rate *1/3* (Figure 10). CSI is considered at the receiver,

- In case that two block fading data units have been received (black, blue and magenta start lines), we can retrieve the information data (CED) with an error correction capability of almost the level the legacy GPS L1C sub frame 2 (red line). Gap of 0.8 dBHz for an for an error probability of 10^{-2} is illustrated. In any case, for C/N_0 higher 26 dBHz, thanks to the MDS property, the TTD is reduced.
- In case that three block fading data units have been received (red dash line), the error correcting capabilities enhance the legacy GPS L1C subframe 2 in almost 1.4dBHz for an error probability of 10^{-2}. Since more data block fading data units are used to decode, for the same interference an enhancement of the performance are accomplished. Thanks to the enhancement of the performances, the CED can be retrieved in a higher rank of C/N_0.

We have computed the results for the rate-compatible root LDPC code of rate *1/4* (Figure 11). CSI is considered at the receiver:

- In case that four block fading data units have been received (red star line), the error correcting capabilities enhance the legacy GPS L1C subframe 2 in almost 2.5dBHz for an error probability of 10^{-2}. Since more data block fading data units are used to decode, for the same interference an enhancement of the performance are accomplished.

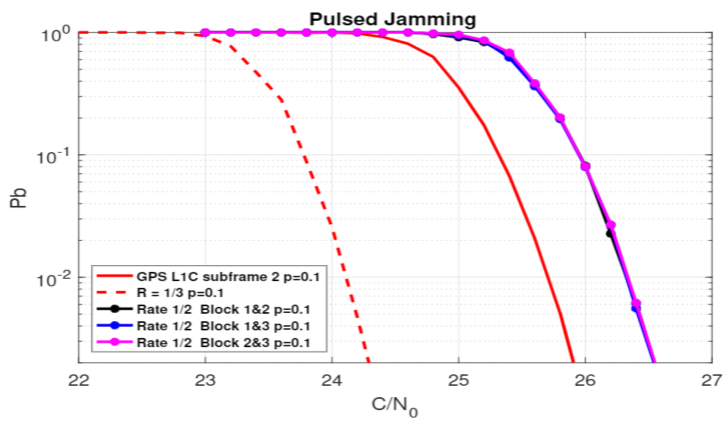

Figure 10 Rate Compatible Root LDPC of Rate 1/3 vs GPS L1C subframe 2 Pulsed Jamming Channel

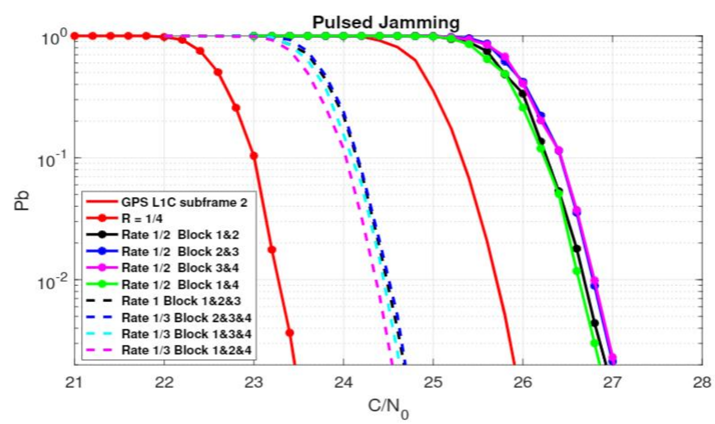

Figure 11 Rate Compatible Root LDPC of Rate 1/4 vs GPS L1C Subframe 2 Pulsed Jamming Channel

CONCLUSION

In this paper, we have provided the methodology to construct two error correcting structures for GNSS systems, which are capable to provide Maximum Distance Separable (MDS), full diversity and rate-compatible properties. The first property allows the receiver to reduce the TTD in good environments, since with *k* block fading data units of information we can retrieved the CED.

Moreover, thanks to the full diversity and the rate-compatible properties and enhancement in the error correction capabilities and in the data demodulation threshold are achieved. Thanks to the improvement of the data demodulation, these structures provide a reduction of the TTD under harsh environments such as low carrier to Noise ratio environments, urban environments and pulsed jamming environments. The proposed error correcting structures are then simulated and compared with the GPS L1C subframe 2 error correcting scheme under the precedent transmission environments. Simulations show an outstanding improvement of the error correction capabilities when three of four block fading data units are retrieved:

- Under AWGN environment when three of four block fading data units are retrieved, an enhancement of 1.2/2.2 dBHz respectively are accomplished.
- Under urban environment when three of four block fading data units are retrieved, an enhancement of 2.5/4 dBHz respectively are accomplished.
- Under pulsed jamming environment when three of four block fading data units are retrieved, an enhancement of 1.4/2.5 dBHz respectively are accomplished.

ACKNOWLEDGMENTS
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