Multicarrier Passive Inter-Modulation Prediction from 2-Carrier Measurements

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Passive inter-modulation products between transmitted signals may prevent the correct operation of satellite receivers using the same antenna. In many cases, these passive products do not obey the classical rule of 3 dB/dB slope as a function of carrier input power and they cannot be modeled using the classical theory based on polynomials. This has prevented the exact computation of carrier to inter-modulation ratio in multicarrier conditions from 2-carrier measurements. This has led to the use of higher than necessary margins. We present non-analytic models that generate non-integer inter-modulation slopes identical to that obtained in measurements and permit to predict multicarrier results.

Nomenclature

C	=	carrier power
I	=	inter-modulation product power
C/I	=	carrier to inter-modulation ratio
IM	=	inter-modulation product
PIM	=	Passive Inter-modulation product
f, g, h	=	generic non-linear functions

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I. Introduction

Passive equipment such as antennas filters or waveguides generate harmonics and inter-modulation products when transmitting one or more high power signals. The frequencies of some of these products may be in the reception bandwidth. When a receiver uses the same passive equipment, it will receive these products with little or no isolation. The passive equipment must provide a large dynamic (100 to 150 dB) between carrier power and inter-modulation power in the conditions of operation, C/I in 2-carrier mode or in multicarrier mode.

This has been a problem for telecommunication satellites for a long time [1]. It is worse in modern satellites where the number of transmitted signals, their power and their bandwidth has increased. It is also a problem now for telephony base stations for the same reasons [2]. Older and more recent measurement data has been reviewed to find a non-linear model for passive equipment. The goal is to be able to model the behavior of the equipment and to predict C/I in multicarrier mode and for different signal powers from measurements in 2-carrier mode.

II. Non Analytical Models

We proposed in article [3] a non-analytical behavioral model for passive or active non-linear equipment. It is based on non-linear functions with a discontinuity at origin of the function or its derivatives. The simplest form is given by:

$$y = sign(x) \cdot |x|^p = x \cdot |x|^{p-1} \tag{1}$$

Energy conservation in passive equipment limits the real degree p to 1 or higher values. The non-linear function and its first derivative are always continuous. Only the second or higher derivatives are discontinuous.

The small signal slope of PIM power (in dB) versus input carrier power (in dB) is equal to the real degree p. This result cannot be obtained with an analytical non-linear function where the degree p must be an odd integer value. Only odd integers generate odd IM products. Small signal slope are 3 dB/dB for third order IM, 5 dB/dB for a fifth order IM and so on.

Even functions can also be defined. They don't generate IM products near the fundamental frequencies of carriers but they generate even harmonic products, at sum frequencies and difference frequencies, which may cause interference in receivers.

$$y = |x|^p \tag{2}$$

Real degrees p comprised between 1 and 3 in Eq. (1) permitted to model some passive non-linear equipment and fit measurement data published in [4].

Figure 1 compares simulation and measured data for third to ninth order PIM in a transmission line using Eq. (1) model with degree $p = 1.6$, which is equal to the average slope in dB/dB of the third order IM versus input carrier power. No analytical model has been able to present such a good fit.

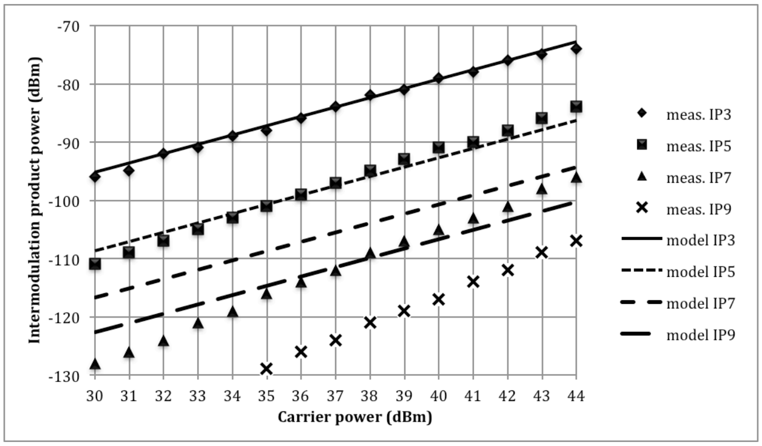

Figure 1: Comparison of measurement data in [4] with results of a model with one term of degree 1.6

Equation (3) shows a more complex function (odd part only) as a sum of terms given by Eq. (1) with different degrees and coefficients:

$$y = sign(x) \cdot \sum_{i=1}^{P} \alpha_i |x|^{p_i} = x \cdot \sum_{i=1}^{P} \alpha_i |x|^{p_i - 1} \tag{3}$$

A classical odd polynomial non-linearity would be obtained by using only odd integers for degrees in Eq. (3).

Figure 2 shows that a much better fit was obtained on the third and fifth order IM products by using two terms with degrees $p = 2$ and $p = 2.5$.

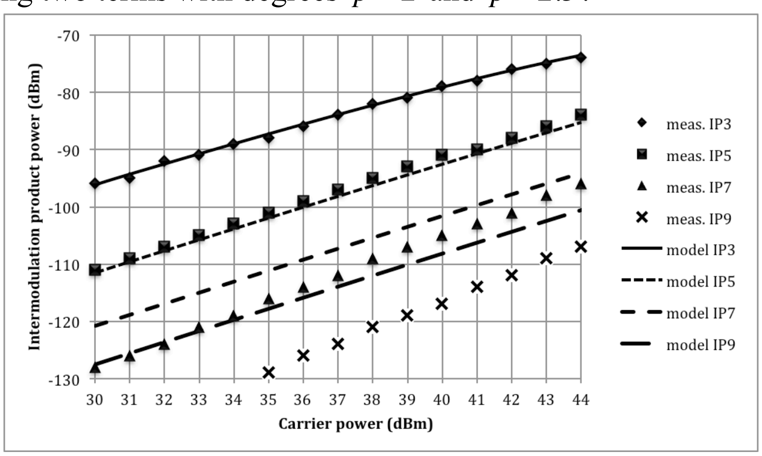

Figure 2: Comparison of measurement data in [4] with results of a model with two terms of degrees 2 and 2.5

A non-analytic Padé-type approximation can also be used under the form:

$$y = sign(x) \frac{\sum_{i=1}^{P} \alpha_i |x|^{p_i}}{1 + \sum_{i=1}^{Q} \beta_i |x|^{q_i}} = x \frac{\sum_{i=1}^{P} \alpha_i |x|^{p_i - 1}}{1 + \sum_{i=1}^{Q} \beta_i |x|^{q_i}} \tag{4}$$

Figure 3 shows results obtained by using such a model with only one non-linear term in the numerator and one on in the denominator, P=1, Q=1. Better results are obtained for seventh and ninth order IM products.

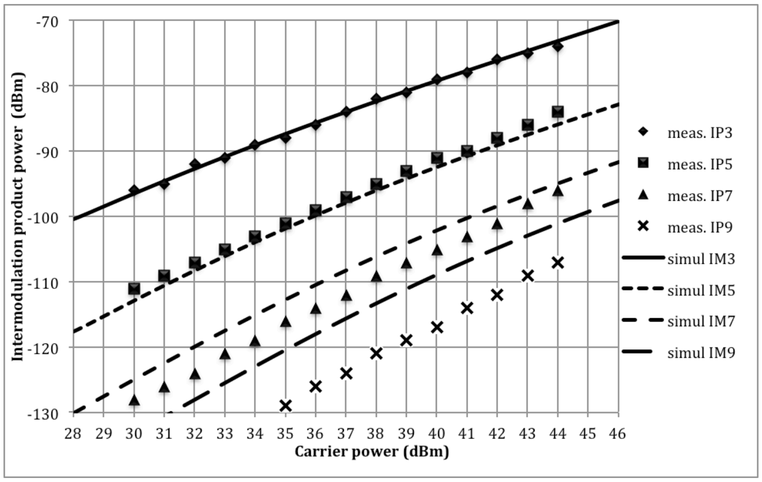

Figure 3: Comparison of measurement data in [4] with results of a non-analytical Padé-type model

A more general form of non-analytical at origin non-linear behavioral model is given in the following equation where *f, g* and *h* are analytical (except at origin) non-linear functions:

$$y = sign(x)\cdot f(|x|) + h(|x|) = x \cdot g(|x|) + h(|x|) \quad \text{with} \quad g(|x|) = f(|x|)/|x| \tag{5}$$

III. Power ratio of IM products of successive orders

Results from models in Eq. (4) and (5) can be obtained only through numerical simulation whereas results from models in Eq. (1), (2) and (3) can be obtained by computation in the same way as polynomial non-linearity results.

The classical computation can be found in Westcott article [5].

The binomial development can be applied also to real degrees and to absolute values. The result is a generalization of results obtained by Westcott and by Blachman [5, 6].

IM products generated by Eq. (1) or (2) are given by a function of same type. These forms of non-analytical functions are invariants of the Chebyshev transformation as was demonstrated by Blachman for polynomials. This invariance characteristic simplifies the computation of IM products power for Eq. (1) to (3) and (5).

In a Chebyshev transformation, the coefficients of the polynomials are multiplied by values that depend on the degree of the polynomial term *n* and on the order *m* of the wanted IM product by the following formula:

$$\frac{n!}{\left(\frac{n+m}{2}\right)!\cdot\left(\frac{n-m}{2}\right)!} \tag{6}$$

The coefficients in Eq. (3) and (5) will be multiplied by a ratio that is formally the same except that the factorial must be replaced by the Γ function as its variable is no longer an integer because the degree *p* is no longer an integer of same parity as order *m*:

$$\frac{\Gamma(p+1)}{\Gamma\left(\frac{p+m}{2}+1\right)\cdot\Gamma\left(\frac{p-m}{2}+1\right)} \tag{7}$$

The order m remains an integer. It is odd or even depending on the symmetry of the non-linear function. Figure 4 shows the variation of these coefficients versus the real degree p for even and odd orders m.

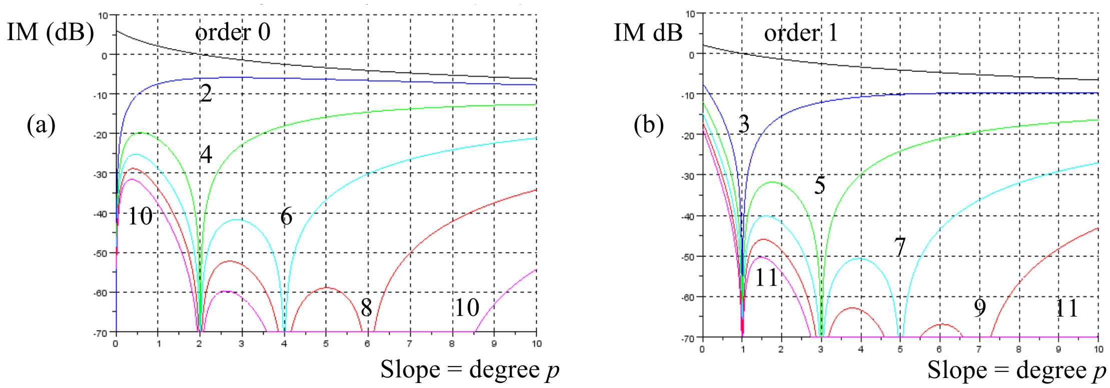

Figure 4: Multiplicative coefficients versus degree (or IM slope) for the first even orders (a) and the first odd orders (b)

IV. Differences with classical models

In addition to the slopes of IM products, it was found that many results that are accepted as approximations are true only for the degree 3 term and are approximations for analytical non-linear models. These approximations may be quite wrong when applied to non-analytical non-linear models.

One of them is the power ratio between a 2-carrier third order product ($2f_1 - f_2$) and a 3-carrier third order product ($f_1 + f_2 - f_3$) in a multicarrier signal. It is not always equal to 6 dB but depends on the degree p as seen in the following table (for an 8-carrier signal):

Table 1: Ratio of 3-carrier type to 2-carrier type IM products versus degree

Degree	1.5	2	2.5	3	3.5	4
Ratio of 3-carrier IM power to 2-carrier IM power (dB)	6.35	6.25	6.15	6	5.85	5.8

This difference, in addition to the non-integer slope, will give quite different results when computing the multicarrier IM for these non-analytical models. This must be taken into account when defining the PIM-test conditions, generally a high power 2-carrier test.

V. Simulation description

Numerical simulation of these non-analytical models is quite identical to polynomial or analytical models simulation. The general model in Eq. (5) shows that for the odd part of the model, the non-linear gain g is computed from the absolute value of the input signal (or the modulus of the complex envelope). Then it is multiplied by the input signal.

This is easier than computing the non-linear function *f* itself. For complex envelopes, the sign function must be replaced by the imaginary exponential of the phase.

For the even part of the model, the continuous part of the output is computed from the absolute value of the input signal (or the modulus of the complex envelope). The second harmonic complex envelope is obtained from a modified equation (which is identical for real signals):

$$h(|x|) = x^2 \cdot h(|x|) / |x|^2 \tag{8}$$

Complex envelopes (given as capital letters) are obtained as:

Continuous part envelope (or harmonic 0) $\quad Y_0 = h(|x|) \quad (9)$

Fundamental envelope (or first harmonic) $\quad Y_1 = X \cdot g(|X|) = X \cdot f(|X|) / |X| = \dfrac{X}{|X|} \cdot f(|X|) \quad (10)$

Second harmonic envelope $\quad Y_2 = X^2 \cdot h(|X|) / |X|^2 = \dfrac{X^2}{|X|^2} \cdot h(|X|) \quad (11)$

This can be generalized to any harmonic. Odd *m* harmonics are obtained from the odd part *f* of the non-linear function:

$$Y_m = X^m \cdot f(|X|) / |X|^m = \left(\frac{X}{|X|}\right)^m \cdot f(|X|) = X \cdot \left(\frac{X}{|X|}\right)^{m-1} \cdot g(|X|) \tag{12}$$

Even *m* harmonics are given by the even part *h* of the non-linear function:

$$Y_m = X^m \cdot h(|X|) / |X|^m = \left(\frac{X}{|X|}\right)^m \cdot h(|X|) \tag{13}$$

Only slight modifications of classical simulation algorithms are necessary. In classical models, functions *g* and *h* are analytical functions of the square of the modulus of the envelope and never of the modulus itself [7, 8] or the phase of the signal. Non-analytical at origin models are not bound by the constraint on the square of the modulus.

VI. Simulation Results

We compare the simulation results for analytical and non-analytical models for a 2-carrier test and an 8-carrier test with one of two test conditions: *at the same carrier power and at the same total power*.

In this presentation we will compare only the third order intermodulation products. The analytical model has one non-linear term of degree 3. The non-analytical models have one non-linear term of degree 1.5 to 3.5.

Non-analytical and analytical models coefficients have been fitted to generate the same 2-carrier third order IM of -84 dBm at an input power of 37 dBm per carrier giving a 121 dB carrier to third order intermodulation ratio C/I_3.

For an 8-carrier signal with the same power per carrier (4 times the total average power), the C/I_3 ratio for both types of IM products ($2f_1 - f_2$ and $f_1 + f_2 - f_3$) is given in the following table:

Table 2: C/I versus number of carriers at constant carrier power

Type of NL	Degree of NL	2-carrier C/I_3 (dB)	8-carrier C/I_3 (dB) type $2f_1 - f_2$	8-carrier C/I_3 (dB) type $f_1 + f_2 - f_3$
Non-analytical	1.5	121	134	127.7
Non-analytical	2	121	129.75	123.5
Non-analytical	2.5	121	125.4	119.3
Analytical	3	121	121	115
Non-analytical	3.5	121	116.5	110.6

For an 8-carrier signal with the same total power, the C/I_3 ratio for both types of IM products ($2f_1 - f_2$ and $f_1 + f_2 - f_3$) is given in the following table:

Table 3: C/I versus number of carriers at constant total power

Type of NL	Degree of NL	2-carrier C/I_3 (dB)	8-carrier C/I_3 (dB) type $2f_1 - f_2$	8-carrier C/I_3 (dB) type $f_1 + f_2 - f_3$
Non-analytical	1.5	121	137	130.7
Non-analytical	2	121	135.8	129.5
Non-analytical	2.5	121	134.4	128.3
Analytical	3	121	133	127
Non-analytical	3.5	121	131.6	125.7

In this case, the total 8-carrier power is 6 dB lower than in the previous case. The C/I_3 values obtained with the analytical model are 12 dB better. This is no longer the case for the non-analytical models. As can be seen, the 8-carrier C/I_3 depends less on the degree of the non-linear term in the case of constant total power. However, in this case, a very high power would be needed for each of the two carriers in the test to get the same total power as in the 8-carrier case.

Table 2 can be modified to obtain, for all degrees, the same C/I_3, in the 8-carrier case and for type $f_1 + f_2 - f_3$ IM product. This gives us the C/I_3 that must be obtained in the 2-carrier test for the same carrier power to obtain the multicarrier C/I_3 required by the system (in our example, 115 dB).

Table 4 shows that a given 8-carrier C/I_3 can be obtained with a lower 2-carrier C/I_3 if the non-linear term degree is lower than 3. This can be used in the definition of the C/I_3 test. The C/I_3 specification can be relaxed by 4 dB each time the slope decreases by 0.5 dB/dB. In such a test, the third order IM slope must be measured with good enough accuracy at the specified carrier power value. A 12 dB relaxation could be obtained in the case of micro-strip lines measured in Fig. 1 to 3 with an average slope of 1.6 dB/dB at 37 dBm carrier power.

A lower relaxation of 4 to 8 dB can be expected from space antennas and reflectors measurements where a slope between 2 and 2.5 dB/dB is generally measured.

Table 4: C/I versus number of carriers at constant carrier power

Type of NL	Degree of NL	2-carrier C/I_3 (dB)	8-carrier C/I_3 (dB) type $2f_1 - f_2$	8-carrier C/I_3 (dB) type $f_1 + f_2 - f_3$
Non-analytical	1.5	108.3	121.3	115
Non-analytical	2	112.5	121.3	115
Non-analytical	2.5	116.7	121.1	115
Analytical	3	121	121	115
Non-analytical	3.5	125.4	120.9	115

Measured data in Fig. 1 to 3 show that the third order IM curve as a function of carrier power is convex. In such a case, a lower slope (and a higher decrease of test C/I_3 specification) can be obtained by measuring C/I_3 at higher carrier power if this is possible.

A measurement at lower than nominal carrier power would give a higher slope and a degree nearer to 3 and so a lower relaxation of the 2-carrier specified C/I_3.

These values show that measuring 2-carrier C/I_3 at the maximum possible power and in the maximum possible carrier power range to obtain a good accuracy on the C/I_3 slope will permit to increase the relaxation on specified C/I_3.

The slope value can be confirmed by the ratio of third to fifth order IM as this ratio depends directly on the degree of the non-analytic non-linear term.

VII. Comparison of models

Table 5 shows that the 3 different behavioral models of micro-strip line measurements that have been shown in figures 1 to 3 result in 8-carrier C/I_3 values with a 3 dB spread (11.9 dB to 14.6 dB better than the result for the analytical model of degree 3).

Table 5: C/I versus number of carriers at constant carrier power for 3 models

Type of NL model	2-carrier C/I_3 (dB)	8-carrier C/I_3 (dB) type $2f_1 - f_2$	8-carrier C/I_3 (dB) type $f_1 + f_2 - f_3$
Analytical, degree 3	121	121	115
Non-analytical shown in Fig. 1	121	133.2	126.9
Non-analytical shown in Fig. 2	121	135.7	129.2
Non-analytical shown in Fig. 3	121	136.1	129.6

The worse result corresponds to the simplest behavioral model with a slope of 1.6 dB/dB and the measured value of 2-carrier C/I_3 at 37 dBm carrier power. The other models give better results because the third order IM slope is lower than 1.6 dB/dB for higher carrier power and also for higher number of carriers at same carrier power.

This worse result should be used if no measurement has been done at higher carrier power. The relaxation of specification is up to 12 dB in that case. More complex models could be used if they can be justified by higher carrier power measurement, giving an additional relaxation of around 2 dB.

In order to use this relaxation, the margins between system specification and 2-carrier C/I_3 specification should be reassessed and eventually modified after system measurements.

This relaxation of specifications is lower (4 to 8 dB) if the measured slope of third order IM is lower (2 to 2.5 dB/dB) such as in antennas and reflectors measurements.

This relaxation may be quite different when other IM orders are compared. The difference between 2-carrier and 8-carrier C/I for the same degree is higher but the corresponding IM slope (and so the degree) may be higher also. In addition more complex types of IM have higher power than 2-carrier types of IM (e.g. for order 5 and degree 5, an IM of type $f_1 + f_2 + f_3 - f_4 - f_5$ has 14 dB more power than an IM of type $3f_1 - 2f_2$).

VIII. Conclusion

A new behavioral model has been proposed for the computation of IM products generated by high power passive components such as transmission lines, filters and antennas.

This model is non-analytical at origin and it explains measured slopes of IM power versus carrier power that are generally different from the classical model slopes of 3, 5 dB/dB and so on. Non-integer values between 1.4 and 2.9 have been reported in the literature.

The non-analytical model can be used to predict more accurately multi-carrier C/ I_3 from 2-carrier measurements at same carrier power or same total power.

A relaxation of 2-carrier test C/ I_3 specifications, typically 4 to 8 dB, and up to 12 dB, depending on the IM slope, can be obtained if the IM power slope is accurately measured and margins are confirmed at their present value by system measurements.

It can also be applied to higher orders of IM products with different relaxation of specifications.

Acknowledgments

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