MULTIPATH MITIGATION IN GLOBAL NAVIGATION SATELLITE SYSTEMS USING A BAYESIAN HIERARCHICAL MODEL WITH BERNOULLI LAPLACIAN PRIORS

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ABSTRACT

A new sparse estimation method was recently introduced in a previous work to correct biases due to multipath (MP) in GNSS measurements. The proposed strategy was based on the resolution of a LASSO problem constructed from the navigation equations using the reweighted-ℓ_1 method. This strategy requires to adjust the regularization parameters balancing the data fidelity term and the involved regularizations. This paper introduces a new Bayesian estimation method allowing the MP biases and the unknown model parameters and hyperparameters to be estimated directly from the GNSS measurements. The proposed method is based on Bernoulli-Laplacian priors, promoting sparsity of MP biases.

Index Terms— GNSS, multipath, sparse representation

1. INTRODUCTION

Satellite navigation consists in estimating the position of a receiver using satellite measurements such as pseudoranges and Doppler measurements (also referred to as pseudorange rates) [1, ch. 7] leading to the following observation model

$$\rho_{i,k} = f_1(\boldsymbol{r}_k) + \varepsilon_{i,k}, \quad \dot{\rho}_{i,k} = f_2(\boldsymbol{v}_k) + \dot{\varepsilon}_{i,k} \tag{1}$$

for $i = 1, ..., s_k$, where s_k is the number of visible satellites at instant k, $\rho_{i,k}$ and $\dot{\rho}_{i,k}$ are the pseudorange and pseudorange rate (which is colinear to the Doppler [1, ch. 7]) for satellite $\#i$ at time k, $\boldsymbol{r}_k = (x_k, y_k, z_k, b_k)^T$ contains the receiver position and clock bias, $\boldsymbol{v}_k = (\dot{x}_k, \dot{y}_k, \dot{z}_k, \dot{b}_k)^T$ gathers the receiver velocity and clock drift, f_1 and f_2 are two nonlinearities (defined for instance in [1, pp. 203 and 205]) and $\varepsilon_{i,k}$ and $\dot{\varepsilon}_{i,k}$ are measurement errors, e.g., due to atmospheric delays, multipath (MP) or relativity. The sequential estimation of the states $\boldsymbol{r}_k$ and $\boldsymbol{v}_k$ from (1) has received a considerable attention in the literature. A classical solution is to linearize the observation equations and to determine the unknown vector by using the least squares method or the extended Kalman filter (EKF) [1, ch. 3]. More precisely, denote as $\boldsymbol{y}_k = (y_{1,k}, ..., y_{2s_k,k})^T \in \mathbb{R}^{2s_k}$ the vector of pseudorange errors, where $y_{i,k} = \rho_{i,k} - \hat{\rho}_{i,k}$ is the difference between the ith pseudorange at time k and its estimation, $y_{s_k+i,k} = \dot{\rho}_{i,k} - \hat{\dot{\rho}}_{i,k}$ is the difference between the ith pseudorange rate at time k and its estimation, $i = 1, ..., s_k$ where s_k is the number of pseudorange measurements at time k. We introduce the state vector $\boldsymbol{x}_k = (\boldsymbol{x}_{1,k}^T, \boldsymbol{x}_{2,k}^T)^T$ at time instant k with $\boldsymbol{x}_{1,k} = \boldsymbol{r}_k - \tilde{\boldsymbol{r}}_k$, $\boldsymbol{x}_{2,k} = \boldsymbol{v}_k - \tilde{\boldsymbol{v}}_k$ (where $\tilde{\boldsymbol{r}}_k$ and $\tilde{\boldsymbol{v}}_k$ will be explicited later), the Jacobian matrix $\boldsymbol{H}_k \in \mathbb{R}^{s_k \times 4}$ associated with the non-linear transformation at time instant k (which is the same for the two functions f_1 and f_2 [1, ch. 7]), the residual error vector $\boldsymbol{\xi}_k = (\xi_{1,k}, ..., \xi_{2s_k,k})^T \in \mathbb{R}^{2s_k}$ containing the residual error for the ith pseudorange at time instant k denoted as $\xi_{i,k}$ and the residual error for the ith pseudorange rate at time instant k denoted as $\xi_{s_k+i,k}$, for $i = 1, ..., s_k$. All the notations introduced before lead to the following observation equation

$$\boldsymbol{y}_k = \begin{bmatrix} \boldsymbol{H}_k & \boldsymbol{0}_{s_k} \\ \boldsymbol{0}_{s_k} & \boldsymbol{H}_k \end{bmatrix} \boldsymbol{x}_k + \boldsymbol{\xi}_k \tag{2}$$

where $\boldsymbol{0}_{s_k}$ is the $s_k \times s_k$ zero matrix, which is complemented by a state equation to be processed by the EKF.

A new estimation method was recently introduced by the authors of this paper to estimate and correct additive biases, e.g., due to MP, possibly affecting the observed measurements defined in (2) thanks to sparse regularization [2]. The proposed strategy was based on the resolution of a LASSO problem constructed from the navigation equations using the reweighted-ℓ_1 method. It required to adjust the regularization parameters balancing the data fidelity term and the involved regularizations. This paper introduces a new Bayesian estimation method allowing the MP biases and the unknown model parameters and hyperparameters to be estimated directly from the GNSS measurements.

The paper is organized as follows. Section 2 introduces the statistical model based on Bernoulli-Laplacian priors used to mitigate MP biases in GNSS measurements. Section 3 studies a Markov chain Monte Carlo method to sample the posterior distribution of this statistical model and to build estimators of the unknown model parameters. Section 4 presents simulation results allowing the performance of the proposed estimation method to be appreciated. Conclusions are finally reported in Section 5.

2. MUTIPATH MITIGATION

The proposed MP mitigation strategy assumes that residual errors affecting the pseudoranges and pseudorange rates are essentially due to MP and can be modeled by additive biases leading to

$$\boldsymbol{\xi}_k = \boldsymbol{m}_k + \boldsymbol{n}_k \tag{3}$$

where $\boldsymbol{m}_k = (m_{i,k})_{i=1,...,s_k}$ contains the MP biases for the linearized pseudoranges and pseudorange rates at time instant k, and $\boldsymbol{n}_k = (n_{i,k})_{i=1,...,s_k}$ is an additive zero mean Gaussian noise vector whose covariance matrix is denoted as $\boldsymbol{R}_k$. More precisely, $m_{i,k} = 0$ when there is no MP affecting the pseudorange relative to satellite $\#i$, and $m_{i,k} \neq 0$ when there is an MP affecting this pseudorange (the same definition applies to $m_{i+s_k,k}$, i.e., to the variable associated with the pseudorange rate relative to satellite $\#i$ at time instant k). A last assumption is that some satellites are not affected by MP, which will be taken into account by considering a Bernoulli-Laplace prior for $m_{i,k}$, allowing the navigation problem to be solved using a Bayesian framework. This section defines the different parts of the hierarchical Bayesian model that will be considered to solve the estimation problem defined by (2) and (3).

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2.1. State model

We consider a random walk state model for the state vector $(\boldsymbol{r}_k^T, \boldsymbol{v}_k^T)^T$ defined by the propagation equation

$$\begin{bmatrix}\boldsymbol{r}_{k+1}\\ \boldsymbol{v}_{k+1}\end{bmatrix} = \boldsymbol{F}_k \begin{bmatrix}\boldsymbol{r}_k\\ \boldsymbol{v}_k\end{bmatrix} + \boldsymbol{u}_k \quad \text{with} \quad \boldsymbol{F}_k = \begin{bmatrix}\boldsymbol{I}_4 & (\Delta t_k)\boldsymbol{I}_4\\ \boldsymbol{0}_4 & \boldsymbol{I}_4\end{bmatrix} \quad (4)$$

where $\boldsymbol{I}_4$ is the $\mathbb{R}^{4\times4}$ identity matrix, $\boldsymbol{0}_4$ is the $\mathbb{R}^{4\times4}$ zero matrix, Δt_k is the time between instants k and $k+1$, and $\boldsymbol{u}_k$ is a zero-mean Gaussian noise of covariance matrix $\boldsymbol{Q}_k \in \mathbb{R}^{8\times8}$, leading to

$$\boldsymbol{u}_k \sim \mathcal{N}(\boldsymbol{0}_8, \boldsymbol{Q}_k) \quad (5)$$

where $\boldsymbol{0}_8$ is the zero vector of $\mathbb{R}^8$. This state model has been used in several navigation solution, including the EKF.

2.2. Observation model

The s_k pseudorange and Doppler measurements can be classically expressed as

$$\boldsymbol{y}_k = \bar{\boldsymbol{H}}_k \boldsymbol{x}_k + \boldsymbol{m}_k + \boldsymbol{n}_k \quad (6)$$

where $\boldsymbol{y}_k = (y_{i,k})_{i=1,\ldots,2s_k} \in \mathbb{R}^{2s_k}$ is defined in Section 1, $\bar{\boldsymbol{H}}_k \in \mathbb{R}^{2s_k\times8}$ is a block diagonal matrix with two blocks equal to $\boldsymbol{H}_k$, and $\boldsymbol{n}_k$ is an additive white Gaussian noise with covariance matrix $\boldsymbol{R}_k$. In order to account for different noise variances for the pseudoranges and pseudorange rates, we assume that $\boldsymbol{R}_k = \mathrm{diag}(\sigma_{i,k}^2) \in \mathbb{R}^{2s_k\times2s_k}$ is a diagonal matrix, whose elements

$$\sigma_{i,k}^2 = \begin{cases} c_{1,k}\mu_{i,k}, \ i = 1,\ldots,s_k, \\ c_{2,k}\mu_{i,k}, \ i = s_k+1,\ldots,2s_k \end{cases}, \mu_{i,k} = 10^{-\frac{(C/N_0)_{i,k}}{10}} \quad (7)$$

are related to the signal to noise ratio in the ith channel at time instant k (denoted as $(C/N_0)_{i,k}$, provided by standard receivers). Note that this formulation was proposed in [3] with $c_{1,k} = 1.1\times10^4$ m^2. In this paper, based on the analysis of various, real datasets, we will use $c_{2,k} = 1.1\times10^2$ m^2.s^{-2}, in order to have a pseudorange variance 100 times larger than the pseudorange rate variance. Assuming that the different measurement vectors are independent, the joint likelihood of $\boldsymbol{y}_k$ is

$$f(\boldsymbol{y}_k|\boldsymbol{\theta}_k) = \prod_{i=1}^{s_k} f(y_{i,k}|\boldsymbol{\theta}_{1,k}) \prod_{i=s_k+1}^{2s_k} f(y_{i,k}|\boldsymbol{\theta}_{2,k}) \quad (8)$$

where the first term is related to the pseudoranges and the second one to the pseudorange rates. By denoting as $\boldsymbol{h}_{i,k}^T$ the i-th row of the matrix $\boldsymbol{H}_k$, we obtain $y_{i,k} \sim \mathcal{N}(\boldsymbol{h}_{i,k}^T\boldsymbol{x}_{1,k} + m_{i,k}, \sigma_{i,k}^2)$ and $y_{i+s_k,k} \sim \mathcal{N}(\boldsymbol{h}_{i,k}^T\boldsymbol{x}_{2,k} + m_{i+s_k,k}, \sigma_{i+s_k,k}^2)$, for $i = 1,\ldots,s_k$. Moreover, we use the notation $\boldsymbol{\theta}_k = (\boldsymbol{\theta}_{1,k}^T, \boldsymbol{\theta}_{2,k}^T)^T$ with $\boldsymbol{\theta}_{1,k} = (\boldsymbol{x}_{1,k}^T, \boldsymbol{m}_{1:s_k,k}^T)^T$ and $\boldsymbol{\theta}_{2,k} = (\boldsymbol{x}_{2,k}^T, \boldsymbol{m}_{s_k+1:2s_k,k}^T)^T$. After defining the observation equations for our navigation model, we need to define the priors associated with the unknown model parameters, that are classically used in any Bayesian inference. These priors will be used to determine the posterior distribution $p(\boldsymbol{\theta}_k|\boldsymbol{y}_k)$ and to define Bayesian estimators of $\boldsymbol{\theta}_k$.

2.3. Priors

2.3.1. *State vector*

We introduce a state vector $\boldsymbol{x}_k$ defined as

$$\boldsymbol{x}_k = \begin{bmatrix}\boldsymbol{r}_k\\ \boldsymbol{v}_k\end{bmatrix} - \begin{bmatrix}\tilde{\boldsymbol{r}}_k\\ \tilde{\boldsymbol{v}}_k\end{bmatrix} \quad (9)$$

where $(\tilde{\boldsymbol{r}}_k, \tilde{\boldsymbol{v}}_k)$ is a point around which (1) has been linearized. According to the EKF theory, we have

$$\begin{bmatrix}\tilde{\boldsymbol{r}}_k\\ \tilde{\boldsymbol{v}}_k\end{bmatrix} = \boldsymbol{F}_{k-1} \begin{bmatrix}\hat{\boldsymbol{r}}_{k-1}\\ \hat{\boldsymbol{v}}_{k-1}\end{bmatrix} \quad (10)$$

where $(\hat{\boldsymbol{r}}_{k-1}^T, \hat{\boldsymbol{v}}_{k-1}^T)^T$ is the state vector estimated at time instant $k-1$, leading to the following prior for $\boldsymbol{x}_k$

$$\boldsymbol{x}_k \sim \mathcal{N}(\boldsymbol{x}_k; \boldsymbol{0}_8, \boldsymbol{F}_{k-1}\boldsymbol{P}_{k-1|k-1}\boldsymbol{F}_{k-1}^T + \boldsymbol{Q}_{k-1}) \quad (11)$$

where $\boldsymbol{P}_{k-1|k-1}$ is the state covariance matrix estimated at the previous time instant $k-1$. Note that this prior depends on all the measurements acquired before time instant k via the covariance matrix $\boldsymbol{P}_{k-1|k-1}$ (notations involving conditioning on the previous measurements $\boldsymbol{y}_{1:k-1}$ are omitted for brievity, see the technical report [4] for more details).

2.3.2. *MP vector $\boldsymbol{m}_k$*

The components of the MP vector $\boldsymbol{m}_k$ can be equal to zero when the corresponding channel is not affected by MP, or different from zero when there is MP corrupting this channel. In order to promote sparsity, we assign Bernoulli-Laplace priorr to these vectors. Note that this kind of prior has been used successfully in different applications [5, 6, 7]. Based on these works, the following probability density function (pdf) is chosen as prior for $m_{i,k}$

$$f(m_{i,k}|a_{1,k}, z_{i,k}) \propto \begin{cases} \delta(m_{i,k}) & \text{if } z_{i,k} = 0 \\ \exp\left(-\frac{a_{1,k}w_{i,k}}{\sqrt{c_{1,k}}}|m_{i,k}|\right) & \text{if } z_{i,k} = 1 \end{cases} \quad (12)$$

for $i = 1,\ldots,s_k$, where $z_{i,k}$ is a binary random variable indicating the presence or absence of MP in the ith measurement at time instant k. A similar prior is used for $m_{i,k}$ for $i = s_k+1,\ldots,2s_k$ by changing $(a_{1,k}, c_{1,k})$ to $(a_{2,k}, c_{2,k})$. Note that the hyperparameters $a_{1,k}$ and $a_{2,k}$ control the amplitudes of the non-zero MP components in the pseudoranges and pseudorange rates and that $w_{i,k}$ is a weight defined as an increasing function of $(C/N_0)_{i,k}$ (as in [8]) and of the ith satellite elevation. Indeed, the higher $(C/N_0)_{i,k}$, the better. Similarly, the higher the elevation, the better. In order to simplify the analysis and finish the description of the proposed model, we propose to consider the completion procedure initially suggested in [9]. This completion consists of introducing one latent variable $\tau_{i,k}^2$ for each MP bias $m_{i,k}$, in order to obtain simpler conditional distributions. These conditional distributions will be used in the Gibbs sampler considered to sample the posterior of interest. Thus, as in [9], we assign the following prior to $(\tau_{i,k}^2, m_{i,k})$, for $i = 1,...,s_k$

$$\tau_{i,k}^2|a_{1,k} \sim \mathcal{E}\left(\tau_{i,k}^2; \frac{2}{w_{i,k}^2 a_{1,k}^2}\right) \quad (13)$$

$$m_{i,k}|z_{i,k}, \tau_{i,k}^2 \sim \begin{cases} \delta(m_{i,k}) & \text{if } z_{i,k} = 0 \\ \mathcal{N}(m_{i,k}; 0, c_{1,k}\tau_{i,k}^2) & \text{if } z_{i,k} = 1 \end{cases} \quad (14)$$

where $\mathcal{E}(.)$ denotes the exponential distribution. A similar prior is used for $i = s_k+1,...,2s_k$, by replacing $(a_{1,k}, c_{1,k})$ with $(a_{2,k}, c_{2,k})$.The indicator variable $z_{i,k}$ is classically assigned a Bernoulli prior. We assume that the MP probabilities are different for pseudoranges and pseudorange rates. Thus, $z_{i,k}$ is assigned a Bernoulli prior with parameter $p_{1,k} \in]0,1[$ for $i = 1,...,s_k$, and parameter $p_{2,k} \in]0,1[$ for $i = s_k+1,...,2s_k$

$$\begin{aligned} z_{i,k}|p_{1,k} &\sim \mathcal{B}(z_{i,k}; p_{1,k}), \quad i = 1,...,s_k \\ z_{i,k}|p_{2,k} &\sim \mathcal{B}(z_{i,k}; p_{2,k}), \quad i = s_k+1,...,2s_k \end{aligned} \quad (15)$$

with $\boldsymbol{z}_k = (z_{1,k}, ..., z_{2s_k,k})^T$ and $\boldsymbol{p}_k = (p_{1,k}, p_{2,k})^T$. Assuming a priori independence for the variables $z_{i,k}$, the following indicator prior is obtained

$$f(\boldsymbol{z}_k|\boldsymbol{p}_k) = \prod_{i=1}^{s_k} f(z_{i,k}|p_{1,k}) \prod_{i=s_k+1}^{2s_k} f(z_{i,k}|p_{2,k}). \tag{16}$$

Similarly, assuming $m_{i,k}, \tau_{i,k}|\boldsymbol{a}_k, \boldsymbol{z}_k$ are independent leads to

$$f(\boldsymbol{m}_k, \boldsymbol{\tau}_k|\boldsymbol{a}_k, \boldsymbol{z}_k) = \prod_{i=1}^{2s_k} f(m_{i,k}|z_{i,k}, \tau_{i,k}^2) f(\tau_{i,k}^2|\boldsymbol{a}_k) \tag{17}$$

with $\boldsymbol{\tau}_k = (\tau_{1,k}, ..., \tau_{2s_k,k})^T$ and $\boldsymbol{a}_k = (a_{1,k}, a_{2,k})^T$.

2.3.3. Joint prior distribution

Combining (11), (16) and (17) and assuming prior independence between the different parameters, the following prior is obtained

$$f(\boldsymbol{\theta}_k, \boldsymbol{z}_k|\boldsymbol{\varphi}_k) = f(\boldsymbol{x}_k) f(\boldsymbol{m}_k, \boldsymbol{\tau}_k|\boldsymbol{a}_k, \boldsymbol{z}_k) f(\boldsymbol{z}_k|\boldsymbol{p}_k) \tag{18}$$

where $\boldsymbol{\varphi}_k = (\boldsymbol{a}_k^T, \boldsymbol{p}_k^T)^T$ is the hyperparameter vector.

2.4. Hyperpriors

The priors defined in the previous section depend on hyperparameters forming the vector $\boldsymbol{\varphi}_k = (\boldsymbol{\varphi}_{1,k}^T, \boldsymbol{\varphi}_{2,k}^T)^T$, with $\boldsymbol{\varphi}_{j,k} = (p_{j,k}, a_{j,k}^2)^T$. Independent uniform priors are assigned to the probabilities $p_{j,k}$ expressing the absence of knowledge about the probability of having an MP in a given channel, i.e.,

$$p_{j,k} \sim \mathcal{U}_{[0,1]}(p_{j,k}), \quad j = 1, 2. \tag{19}$$

The hyperpriors for the MP amplitudes $a_{1,k}$ and $a_{2,k}$ are defined using non-informative Jeffreys priors leading to

$$f(a_{j,k}^2) \propto 1/a_{j,k}^2, \quad j = 1, 2 \tag{20}$$

and to the following joint hyperprior

$$f(\boldsymbol{\varphi}_k) = \prod_{j=1}^{2} \left[f(p_{j,k}) f(a_{j,k}^2) \right]. \tag{21}$$

2.5. Posterior distribution

The posterior distribution of the proposed Bayesian model can be derived using the hierarchical structure between the observation model, the model parameters and hyperparameters, leading to

$$f(\boldsymbol{\theta}_k, \boldsymbol{z}_k, \boldsymbol{\tau}_k^2, \boldsymbol{\varphi}_k|\boldsymbol{y}_k) \propto f(\boldsymbol{y}_k|\boldsymbol{\theta}_k) f(\boldsymbol{\theta}_k, \boldsymbol{z}_k|\boldsymbol{\varphi}_k) f(\boldsymbol{\varphi}_k) \tag{22}$$

where the likelihood $f(\boldsymbol{y}_k|\boldsymbol{\theta}_k)$ has been defined in (8), the parameter prior $f(\boldsymbol{\theta}_k, \boldsymbol{z}_k|\boldsymbol{\varphi}_k)$ in (18) and the hyperprior $f(\boldsymbol{\varphi}_k)$ in (21).

3. GIBBS SAMPLER

Obtaining closed-form expressions of Bayesian estimators (such as the minimum mean square error (MMSE) estimator or the maximum a posteriori (MAP) estimator) of the unknown model parameters associated with the posterior (22) seems to be very complicated. Therefore we propose to draw samples from the posterior distribution (22) and to use these samples to compute estimators of the model parameters. More precisely, we consider a Gibbs sampler whose principle is to sample the different variables according to their conditional distributions [10] that are provided below.

3.1. Conditional distributions

Mathematical details allowing the conditional distributions of the proposed Bayesian model to be computed are omitted here for brevity (but can be found in the technical report [4]). Table 1 summarizes the different results, where $\mathcal{E}$, $\mathcal{N}$, $\mathcal{GIG}$, $\mathcal{IG}$ and $\mathcal{B}e$ are the exponential, normal, generalized inverse Gaussian, inverse gamma and beta distributions. Note that different definitions of the $\mathcal{GIG}$ distribution can be found in the literature. Here, $\mathcal{GIG}(x|p, a, b)$ denotes the distribution whose probability density function is

$$f(x) \propto x^{p-1} \exp\left[-\frac{1}{2}\left(ax + \frac{b}{x}\right)\right] 1_{\mathbb{R}^+}(x)$$

where $1_{\mathbb{R}^+}(x)$ denotes the indicator function on $\mathbb{R}^+$. Furthermore, denoting as $\|\boldsymbol{z}_k\|_{0,j}$ the ℓ_0 pseudo-norm of the vector containing the element $z_{k,i}, i \in I_j$, the following notations have been used

$$\mu_{m_{i,k}} = \begin{cases} \frac{\tau_{i,k}^2}{\mu_{i,k}+\tau_{i,k}^2}(y_{i,k} - \boldsymbol{h}_{i,k}\boldsymbol{x}_{1,k}), & i = 1, ..., s_k \\ \frac{\tau_{i,k}^2}{\mu_{i,k}+\tau_{i,k}^2}(y_{i,k} - \boldsymbol{h}_{i-s_k,k}\boldsymbol{x}_{2,k}), & i = s_k+1, ..., 2s_k \end{cases} \tag{23}$$

$$\sigma_{m_{i,k}}^2 = \begin{cases} \frac{c_{1,k}\mu_{i,k}\tau_{i,k}^2}{\mu_{i,k}+\tau_{i,k}^2}, & i = 1, ..., s_k \\ \frac{c_{2,k}\mu_{i,k}\tau_{i,k}^2}{\mu_{i,k}+\tau_{i,k}^2}, & i = s_k+1, ..., 2s_k \end{cases} \tag{24}$$

$$u_{i,k} = \begin{cases} 1 - p_{1,k}, & i = 1, ..., s_k \\ 1 - p_{2,k}, & i = s_k+1, ..., 2s_k \end{cases} \tag{25}$$

$$v_{i,k} = \begin{cases} \frac{p_{1,k}}{\sqrt{c_{1,k}}}\sqrt{\frac{\sigma_{m_{i,k}}^2}{\tau_{i,k}^2}} \exp\left(\frac{\mu_{m_{i,k}}^2}{2\sigma_{m_{i,k}}^2}\right), & i = 1, ..., s_k \\ \frac{p_{2,k}}{\sqrt{c_{2,k}}}\sqrt{\frac{\sigma_{m_{i,k}}^2}{\tau_{i,k}^2}} \exp\left(\frac{\mu_{m_{i,k}}^2}{2\sigma_{m_{i,k}}^2}\right), & i = s_k+1, ..., 2s_k \end{cases} \tag{26}$$

$$\boldsymbol{\Sigma}_{\boldsymbol{x}_k}^{-1} = \bar{\boldsymbol{H}}_k^T \boldsymbol{R}_k^{-1} \bar{\boldsymbol{H}}_k + (\boldsymbol{F}_k \boldsymbol{P}_{k|k} \boldsymbol{F}_k^T + \boldsymbol{Q}_k)^{-1} \tag{27}$$

$$\boldsymbol{\mu}_{\boldsymbol{x}_k} = \boldsymbol{\Sigma}_{\boldsymbol{x}_k} \bar{\boldsymbol{H}}_k^T \boldsymbol{R}_k^{-1} (\boldsymbol{y}_k - \boldsymbol{m}_k). \tag{28}$$

and thus we update the state covariance matrix as $\boldsymbol{P}_{k+1|k+1} = \boldsymbol{\Sigma}_{\boldsymbol{x}_k}$.

$\tau_{i,k}^2$	$\mathcal{E}\left(\tau_{i,k}^2; \frac{2}{a_{1,k}^2 w_{i,k}^2}\right)$	if $z_{i,k} = 0, i \in I_1$
	$\mathcal{GIG}\left(\tau_{i,k}^2; \frac{1}{2}, w_{i,k}^2 a_{1,k}^2, \frac{m_{i,k}^2}{c_{1,k}}\right)$	if $z_{i,k} = 1, i \in I_1$
	$\mathcal{E}\left(\tau_{i,k}^2; \frac{2}{a_{2,k}^2 w_{i,k}^2}\right)$	if $z_{i,k} = 0, i \in I_2$
	$\mathcal{GIG}\left(\tau_{i,k}^2; \frac{1}{2}, w_{i,k}^2 a_{2,k}^2, \frac{m_{i,k}^2}{c_{2,k}}\right)$	if $z_{i,k} = 1, i \in I_2$
$m_{i,k}$	$\delta(m_{i,k})$	if $z_{i,k} = 0$
	$\mathcal{N}\left(m_{i,k}; \mu_{m_{i,k}}, \sigma_{m_{i,k}}^2\right)$	if $z_{i,k} = 1$
$z_{i,k}$	$\mathcal{B}\left(z_{i,k}; \frac{v_{i,k}}{u_{i,k}+v_{i,k}}\right)$	
$\boldsymbol{x}_k$	$\mathcal{N}\left(\boldsymbol{x}_k; \boldsymbol{\mu}_{\boldsymbol{x}_k}, \boldsymbol{\Sigma}_{\boldsymbol{x}_k}\right)$	
$a_{j,k}^2$	$\mathcal{G}\left(a_{j,k}^2; s_k, \frac{1}{2}\sum_{i \in I_j} w_{i,k}^2 \tau_{i,k}^2\right)$	
$p_{j,k}$	$\mathcal{B}e\left(p_{j,k}; \|\boldsymbol{z}_k\|_{0,j} + 1, s_k - \|\boldsymbol{z}_k\|_{0,j} + 1\right)$	

Table 1: Conditional distributions of the parameters and hyperparameters, where $I_1 = (1, ..., s_k)$ and $I_2 = (s_k+1, ..., 2s_k)$.

3.2. Estimators

Once the different samples have been generated by the Gibbs sampler, the unknown model parameters are estimated as

$$\hat{\boldsymbol{z}} = \underset{\boldsymbol{z}\in\{0,1\}^{2n}}{\arg\max}\ \#\mathcal{M}(\boldsymbol{z}) \tag{29}$$

$$\hat{\boldsymbol{p}} = \frac{1}{\#\mathcal{M}(\hat{\boldsymbol{z}})} \sum_{m\in\mathcal{M}(\hat{\boldsymbol{z}})} \boldsymbol{p}^{(m)} \quad \text{where} \quad \boldsymbol{p} \in \{\boldsymbol{\theta}, \boldsymbol{\tau^2}, \boldsymbol{\varphi}\} \tag{30}$$

where $\#A$ denotes the cardinal of the set A, $[\![a, b]\!]$ denotes the set of integers in $[a, b]$ and

$$\mathcal{M}(\boldsymbol{z}) = \{m \in [\![n_{\text{burn-in}} + 1, n_{\text{iter}}]\!], \boldsymbol{z}^{(m)} = \boldsymbol{z}\} \tag{31}$$

where $n_{\text{burn-in}}$ is the burn-in period (containing the first observations of the chain that are not considered in the estimation) and n_{iter} is the total number of iterations. Note that the Gibbs sampler convergence was accelerated using Metropolis-Hastings moves as in [4].

4. EXPERIMENTAL VALIDATION

The proposed algorithm was validated using synthetic data, allowing its performance to be assessed with controlled ground truth. For this validation, GNSS measurements associated with 8 satellites were generated according to (6), with satellite and receiver positions extracted from real data. These measurements were contaminated by additive Gaussian noise with known C/N_0 values. MP vectors $\boldsymbol{m}_k$, referred to as biases, were finally generated (with known locations) and added to the noisy GNSS measurements. More precisely, we considered a scenario using a real trajectory. Fixed biases were then generated between time instants 50 and 130 in 3 channels (corresponding to 3 satellites). Noise variances were finally generated in agreement with the values of C/N_0 as in (7) and the weights $w_{i,k}$ appearing in the MP prior were computed following [8] (with the corresponding C/N_0 values). More details about the simulation scenario including the trajectory and the parameters of interest can be found in [4].

The Gibbs sampler was run with 10000 iterations including a burn-in period of 1000 samples, to make sure the sampler has converged. This convergence was confirmed by computing the so-called potential scale reduction factor (PSRF) [11] at each time instant for each parameter. We checked that this PSRF does not exceed 1.2 as recommended in [12]. The estimated pseudorange biases and the corresponding areas representing $\pm$ standard deviations (computed using 100 Monte Carlo runs) are displayed in Fig. 1. Note that there is no false detection in absence of MP ($k \in \{1, ..., 49, 131, ..., 200\}$) and that the bias amplitudes are correctly estimated for $k \in \{50, 130\}$. Note also that peaks in the standard deviations correspond to missed detections for a given Monte Carlo run (the higher the theoretical bias, the higher the standard deviation). The corresponding planar and altitude position errors are displayed in Fig. 2 and compared to the EKF errors. We can observe that the bias estimation allows the position errors to be reduced in the presence of MP. More details including estimated pseudorange rate biases or posterior distributions of the unknown parameters can be found in [4].

5. CONCLUSION

This paper studied a new Bayesian model for estimating the multipath biases potentially affecting GNSS measurements. This model was based on Bernoulli-Laplace priors exploiting the potential presence of MP biases in the different satellite channels. Interesting properties of the proposed algorithm are 1) it does not require to adjust regularization parameters as in [2], 2) it only requires the prior knowledge of two hyperparameters, namely c_1 and c_2, corresponding to average noise variances. The results obtained on realistic data (with controlled ground truth) are globally very promising.

Our future work will be dedicated to study the performance of the proposed algorithm in more constrained environments with different levels of sparsity for the multipath biases. Another prospect would be to reduce the computational complexity of the proposed sampler, e.g., by considering variational Bayesian approaches.

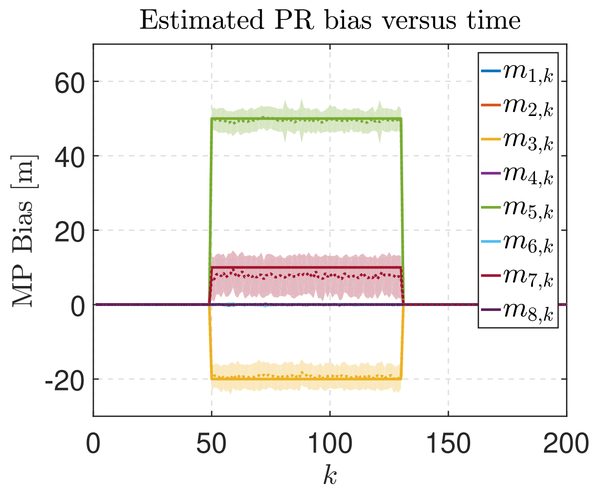

Fig. 1: Ground truth (plain) and estimated biases (dotted) for pseudoranges versus time (100 Monte Carlo runs).

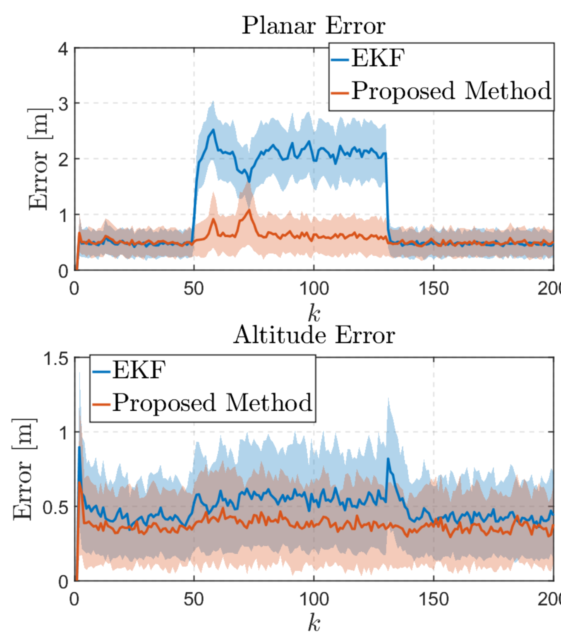

Fig. 2: Planar and altitude errors versus time (for the EKF and proposed method).

6. REFERENCES

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