Experimental Validation of Passive Intermodulation Products Simulation by a Power Law Behavioral Model

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***Abstract* — As analytic nonlinearities cannot simulate correctly passive intermodulation (PIM) behavior, a nonanalytic power law behavioral model has been proposed to explain the non-classical variation of product power versus carrier power. This model has then been applied to many different measurement cases. This has successfully explained some published measures, and it has been used to propose new measurements and predict their results. It seems that the behavioral model may be linked to an underlying physical model of the nonlinearity responsible for the PIM.**

***Index Terms* — Nonlinearity, Intermodulation, Passive intermodulation, Behavioral model, Physical model, Measurement.**

I. INTRODUCTION

Nonlinearities generate harmonics of the input signal and intermodulation (IM) products of signals composed of at least two sinusoidal components. Memoryless active nonlinearities can generally be modeled by polynomials or analytic functions. In the small signal case, the power of active IM products increases with the power of carriers and follows a power law with an exponent that is integer and equal to the order of the IM product. The slope of active IM level versus carrier level in a log/log graph (dB/dB) is equal to this order.

This is not the case for passive harmonics and passive IM (PIM). Their power follows a power law with an exponent that is generally not integer and is not equal to the order of the PIM.

A simple behavioral model has been proposed as a power law model with a real exponent equal to the measured slope of PIM level versus carrier level in a log/log graph (dB/dB). This model is presented in section II.

Quite evidently, it solves the first effect that was observed on PIM, the slope of PIM level versus carrier level. Simulation and mathematical computation also show that all PIM orders follow the same slope, and that it is possible to compute the level of higher order PIM from the slope and the level of order 3 PIM. Section III shows that this model much better explains published discrepancies between model and measurements than the commonly used polynomial model.

In section IV, we present new measurements designed to predict very different levels when the PIM generation follows the proposed model instead of the classical model. The measurement shows that the new model is much better at predicting the results.

II. PIM NONLINEAR MODEL

The power law nonlinear model is first applied to real input (x) and output ($y = f(x)$) signals.

The odd harmonics or odd order PIM of a real signal having the slope s come from the following nonlinear model [1] where α_s is an arbitrary coefficient:

$$f(x) = \alpha_s \operatorname{sign}(x)\,|x|^s = \alpha_s\, x\, |x|^{s-1} \qquad (1)$$

The even order PIM nonlinear model for slope s is given by the following model with a arbitrary coefficient β_s:

$$f(x) = \beta_s |x|^s \qquad (2)$$

The real exponent s is applied to the modulus of the real signal because it is not always possible to apply a real exponent (e.g. 1/2) to a negative real signal and obtain a real signal result.

A nonlinear model cannot be applied separately to the real and imaginary parts of a complex signal but as the nonlinearity depends only on the magnitude of the signal and not its phase, the modulus of the complex signal is also used. This expression for complex signals is formally identical to the expression for real signals. The sign function is classically extended to represent the complex exponential of the phase of the complex signal.

At the origin, these models are non-analytical, meaning that they do not have a Taylor series development. Polynomial approximations can be found but they are of very high degree and valid (have an error lower than a given value) only in limited ranges of the input.

Starting from a real sinusoidal input signal $x(t)$:

$$x(t) = a\cos(\omega t + \varphi) = a\cos\theta \qquad (3)$$

where a is the amplitude, ω the angular frequency and φ the phase, we can compute the harmonics at the output of the nonlinearity given by a function f using a special case of the Fourier transform that is called the Chebyshev transform [2]:

$$f(a\cos\theta) = \frac{1}{2} f_0(a) + \sum_{m=1}^{\infty} f_m(a).\cos(m\theta) \qquad (4)$$

The Chebyshev transforms of function f are given by:

$$f_0(a) = \frac{1}{\pi}\int_{-\pi}^{+\pi} f[a.\cos(\theta)]\,d\theta$$

$$f_m(a) = \frac{1}{\pi}\int_{-\pi}^{+\pi} f[a.\cos(\theta)]\cos(m\theta)\,d\theta \qquad (5)$$

The odd order PIM model (1) gives only odd transforms and harmonics for orders $m = 2p + 1$ [2]:

$$f_{2p+1}(a) = 2\,\alpha_s\,\mathrm{sign}(a)\left(\frac{|a|}{2}\right)^s \frac{\Gamma(s+1)}{\Gamma\left(\frac{s+3}{2}+p\right)\Gamma\left(\frac{s+1}{2}-p\right)} \quad (6)$$

Equation (6) is of the same type as equation (1) and has the same exponent s. All harmonics have the same slope s.

The first order Chebyshev transform ($p = 0$, $m = 1$) is linked to the AM/AM and AM/PM curves.

$$g(a) = f_1(a) = \beta_s\,\mathrm{sign}(a)\,|a|^s \quad (7)$$

It can be applied to bandlimited signals around the fundamental frequency. An equal amplitude 2-carrier signal at radian frequencies ω_1 and ω_2 is:

$$x = \frac{a}{2}[\cos(\omega_1 t + \varphi_1) + \cos(\omega_2 t + \varphi_2)]$$

$$x = a\,\cos\left(\frac{\theta_1+\theta_2}{2}\right)\cos\left(\frac{\theta_1-\theta_2}{2}\right) = a\cos\theta\cos\Omega \quad (8)$$

The Chebyshev transform can then be applied to the function:

$$g(a\cos\Omega) \quad (9)$$

This gives all odd order $m = 2p + 1$ intermodulation products around the fundamental frequencies. As in the case odd harmonics, their levels follow the same slope versus carrier power.

$$g_{2p+1}(a) = \frac{2\,\beta_s\,\Gamma(s+1)}{2^s\,\Gamma\left(\frac{s+3}{2}+p\right)\Gamma\left(\frac{s+1}{2}-p\right)}\mathrm{sign}(a)\,|a|^s \quad (10)$$

When computing the level difference between two consecutive order IM products, the coefficient β_s and the Gamma functions disappear:

$$\frac{g_{2p-1}(a)}{g_{2p+1}(a)} = \frac{g_{2(p-1)+1}(a)}{g_{2p+1}(a)} = \frac{\Gamma\left(\frac{s+3}{2}+p\right)\Gamma\left(\frac{s+1}{2}-p\right)}{\Gamma\left(\frac{s+1}{2}+p\right)\Gamma\left(\frac{s+3}{2}-p\right)} = \frac{s+2p+1}{s-2p+1} \quad (11)$$

III. Validation with Published Measurements

In this section we compare the model predictions with published measurements.

Figure 1 shows order 3 to 9 PIM products levels versus carrier power given in [3] and the model results. The model uses 3 power law terms with exponents 1.5, 2, and 2.5.

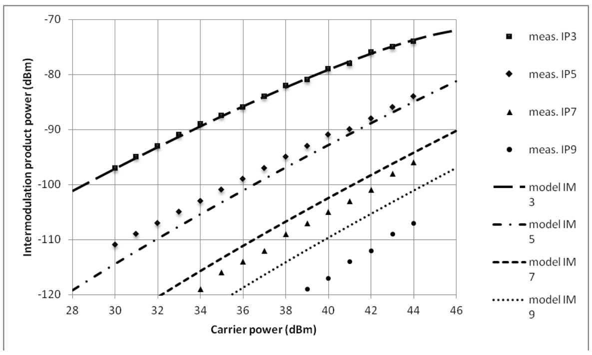

Fig. 1. Measured [3] and modeled levels of PIM products of orders 3, 5, 7, and 9.

Figure 2 shows that the levels of successive order PIM products measured on space qualified components are between the curves obtained using equation (11) of the model for slopes between 2 and 2.5, which are the slopes that are generally measured on these components. The corresponding experimental boundaries are reported as $70\,log(m/n)$ and $90\,log(m/n)$ [4].

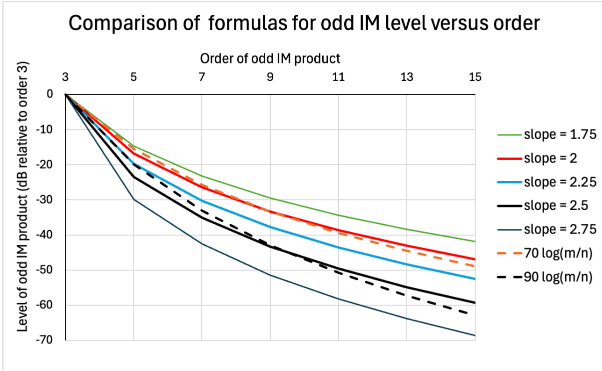

Fig. 2. Experimental boundaries of PIM products of odd orders 3 to 15 [4] and modeled levels obtained for slopes 1.75 to 2.75.

Satellite and antennas industry published "unexplained" multicarrier measurements in 2017 [5, 6]. Their results show that when adding carriers to a 2-carrier signal the level of the PIM products of the first 2 carriers decreases whereas it stays identical in the case of classical models and small signal active IM measurements. This was predicted by the model in 2017 [7]. The decrease in level depends on the slope s and the same decrease is obtained with the behavioral model using the s exponent.

IV. Validation with New Measurements

In this section, a measurement is proposed to show a large discrepancy between predictions of classical and power law models. Some PIM products in the 2-carrier case (identical powers) are measured. Then we add a large third carrier that does not give PIM at the same frequencies and we measure the same PIM products again in this configuration. These PIM products decrease as already shown before, but this decrease becomes very large when the third carrier power is larger than 4 times the power of each of the first 2 carriers.

In these conditions, the signal power is always positive and never crosses the origin. So, the discontinuity at origine never appears in the output signal. It is possible to use a Taylor series development of the nonlinearity in a limited range that does not contain the origin.

Figure 3 shows the nonlinearity and the effect on the signal with and without a third carrier at 4 times the power of the first two carriers.

A waveguide test bench has been defined for this measurement [8]. Three carriers are combined and sent on a nonlinear graphite membrane between two waveguide flanges.

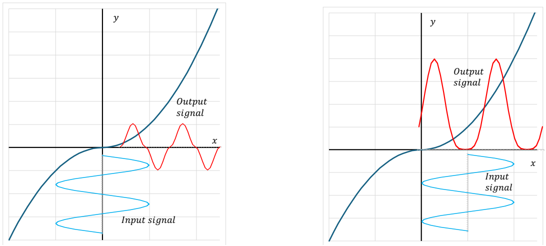

Fig. 3. Nonlinearity (black), input signal (blue) and output signal (red) without third carrier and with third carrier at 4 times the first carriers' power.

PIM products from two identical and fixed power carriers are measured while a third carrier power is varied.

Figure 4 shows the levels of PIM products of order 3, 5 and 7 without third carrier. The slope is around 2.4 for order 3, 3.5 for order 5 and 4.5 for order 7.

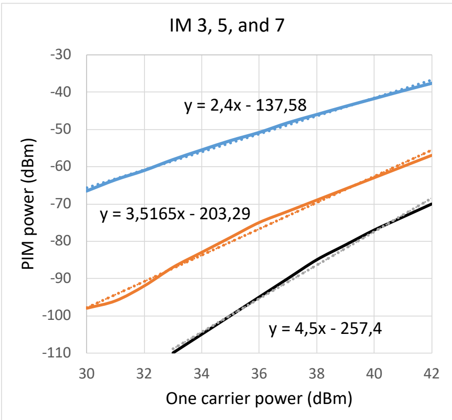

Fig. 4 2-carrier PIM, order 3 (blue), 5 (red), and 7 (black) and their slopes versus carrier power.

Figure 5 shows the PIM levels decrease when the third carrier power is increased.

This decrease is stronger when the third carrier power is 4 times the power of the first 2 carriers or more.

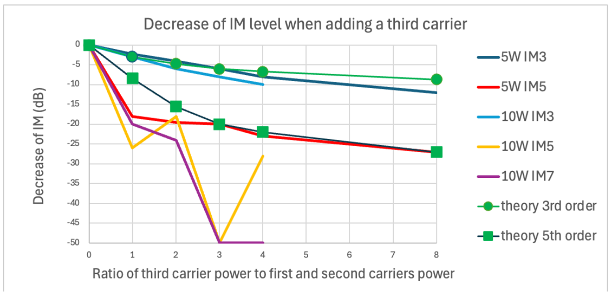

Fig. 5 Decrease of 2-carrier PIM level when adding a third carrier for the two cases of first carriers' power: 5 and 10 watts.

Two cases have been measured:

- one with the two first carriers at 5 watts each.
- one with the two first carriers at 10 watts each.

In both cases, the third carrier is varied from0 to 40 watts.

At 4 times the first carriers the PIM level decrease is around 10 dB for order 3 and 25 to 50 dB for orders 5 and 7.

The order 7 PIM is not measurable in the first case (5 watts carriers) and with a third carrier at 5 watts. Levels of orders 5 and 7 are unstable for small variations of third carrier level.

VI. Conclusion and Future Work

The behavioral model explains or predicts all known present measurements. It seems probable that it is linked to the physical model of the passive nonlinearity. Future work will be in this direction.

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