





# Advanced Signal Processing Methods for GNSS Positioning with NLOS/MP Signals

Nabil Kbayer

# 9 October 2018

Supervisors: Eric Chaumette, Mohamed Sahmoudi (ISAE-SUPAERO)

Reviewers : Paul D. Groves (UCL) Joseph Tabrikian (BGU) Jury Members : Jean-yves Tourneret (ENSEEIHT) Sylvie Marcos (L2S/Centrale Supélec) Juliette Marais (IFSTTAR) Jerome Galy (IUT Béziers)





- 2 3D GNSS Simulator benefits
- 3 Acceptable Level of PR Bias Prediction
- 4 Modified Lower Bounds in non-Gaussian Situations

# 5 Conclusions

3D GNSS Simulator benefits Acceptable Level of PR Bias Prediction Modified Lower Bounds in non-Gaussian Situations Conclusions GNSS Overview GNSS principles GNSS Challenges: Urban Positioning Thesis Objectives

# Plan



- GNSS Overview
- GNSS principles
- GNSS Challenges: Urban Positioning
- Thesis Objectives
- 2 3D GNSS Simulator benefits
- 3 Acceptable Level of PR Bias Prediction
- 4 Modified Lower Bounds in non-Gaussian Situations

# 5 Conclusions

GNSS Overview GNSS principles GNSS Challenges: Urban Positioning Thesis Objectives

### Global Navigation Satellite Systems

GNSS: Satellite radio navigation system used to estimate the position of a receiver by means of constellation of multiple satellites.

Each GNSS system contains 3 major components:

# Space Segment

- Control Segment
- User Segment



GNSS Overview GNSS principles GNSS Challenges: Urban Positioning Thesis Objectives

### **Global Navigation Satellite Systems**

GNSS: Satellite radio navigation system used to estimate the position of a receiver by means of constellation of multiple satellites.

Each GNSS system contains 3 major components:

- Space Segment
- Control Segment
- User Segment



GNSS Overview GNSS principles GNSS Challenges: Urban Positioning Thesis Objectives

### Global Navigation Satellite Systems

GNSS: Satellite radio navigation system used to estimate the position of a receiver by means of constellation of multiple satellites.

Each GNSS system contains 3 major components:

- Space Segment
- Control Segment
- User Segment



3D GNSS Simulator benefits Acceptable Level of PR Bias Prediction Modified Lower Bounds in non-Gaussian Situations Conclusions

# **GNSS** Architecture

GNSS Overview GNSS principles GNSS Challenges: Urban Positioning Thesis Objectives



GNSS Overview GNSS principles GNSS Challenges: Urban Positioning Thesis Objectives

# GNSS quality in urban areas

- Masking effect and Limited satellite visibility ⇒ High DOP
- Signal reflections ⇒ additional Pseudoranges bias



GNSS Overview GNSS principles GNSS Challenges: Urban Positioning Thesis Objectives

### GNSS quality in urban areas

- Masking effect and Limited satellite visibility ⇒ High DOP
- Signal reflections  $\Rightarrow$  additional Pseudoranges bias



GNSS Overview GNSS principles GNSS Challenges: Urban Positioning Thesis Objectives

# GNSS quality in urban areas

- Masking effect and Limited satellite visibility ⇒ High DOP
- Signal reflections ⇒ additional Pseudoranges bias



GNSS Overview GNSS principles GNSS Challenges: Urban Positioning Thesis Objectives

# GNSS quality in urban areas

- Masking effect and Limited satellite visibility ⇒ High DOP
- Signal reflections ⇒ additional Pseudoranges bias



GNSS Overview GNSS principles GNSS Challenges: Urban Positioning Thesis Objectives

# GNSS quality in urban areas

- Masking effect and Limited satellite visibility ⇒ High DOP
- Signal reflections  $\Rightarrow$  additional Pseudoranges bias



Can an external information about the receiver environment of reception be used to assist GNSS in MP/NLOS situations?

Nabil Kbayer

GNSS Positioning with NLOS/MP Signals

3D GNSS Simulator benefits Acceptable Level of PR Bias Prediction Modified Lower Bounds in non-Gaussian Situations Conclusions GNSS Overview GNSS principles GNSS Challenges: Urban Positioning Thesis Objectives

# Objectives

Find out the benefits of using an additional geometrical information of the reception environment to assist the GNSS receiver

 $\Rightarrow$  For this purpose, we have used the 3D GNSS signal propagation Simulator SPRING (provided by the CNES)

- What is the maximum achievable positioning accuracy level reached by GNSS positioning in MP/NLOS settings?
  - $\Rightarrow$  An analysis based on lower bounds computation

BD GNSS signal propagation Simulator overview Positioning by 3D PR Bias Bounding BD Positioning over Candidate Positions Experimental Validation of positioning algorithms

# Plan



2

### 3D GNSS Simulator benefits

3D GNSS signal propagation Simulator overview

Introduction

- Positioning by 3D PR Bias Bounding
- 3D Positioning over Candidate Positions
- Experimental Validation of positioning algorithms
- 3 Acceptable Level of PR Bias Prediction
- 4 Modified Lower Bounds in non-Gaussian Situations

# 5 Conclusions

# SPRING Architecture

3D GNSS signal propagation Simulator overview Positioning by 3D PR Bias Bounding 3D Positioning over Candidate Positions Experimental Validation of positioning algorithms

#### 3D GNSS signal propagation Simulator

3D GNSS simulator = 3D city model + GNSS signal propagation + GNSS receiver model  $\rightarrow$  We have SPRING (CNES)



3D GNSS signal propagation Simulator overview Positioning by 3D PR Bias Bounding 3D Positioning over Candidate Positions Experimental Validation of positioning algorithms

# PR bias prediction using SPRING

### PR bias prediction using 3D GNSS signal propagation simulation

Compute satellite positions based on satellite ephemeris

Introduction

- 2 Determine LOS distance between each satellite and the input position
- Predict received PR measurements using ray-tracing algorithm
- Compute PR bias by difference between predicted PR and LOS distances

#### 3D GNSS Simulator benefits

Acceptable Level of PR Bias Prediction Modified Lower Bounds in non-Gaussian Situations Conclusions

# SPRING Illustration

3D GNSS signal propagation Simulator overview Positioning by 3D PR Bias Bounding 3D Positioning over Candidate Positions Experimental Validation of positioning algorithms

#### 3D GNSS Simulator benefits

Acceptable Level of PR Bias Prediction Modified Lower Bounds in non-Gaussian Situations Conclusions 3D GNSS signal propagation Simulator overview Positioning by 3D PR Bias Bounding 3D Positioning over Candidate Positions Experimental Validation of positioning algorithms

# GNSS/3D Maps Fusion for localization



#### 3D GNSS Simulator benefits

Acceptable Level of PR Bias Prediction Modified Lower Bounds in non-Gaussian Situations Conclusions

# Proposed approach

3D GNSS signal propagation Simulator overview Positioning by 3D PR Bias Bounding 3D Positioning over Candidate Positions Experimental Validation of positioning algorithms



#### 3D GNSS Simulator benefits

Acceptable Level of PR Bias Prediction Modified Lower Bounds in non-Gaussian Situations Conclusions 3D GNSS signal propagation Simulator overview Positioning by 3D PR Bias Bounding **3D Positioning over Candidate Positions** Experimental Validation of positioning algorithms

# Overview of Pattern matching methods



#### 3D GNSS Simulator benefits

Acceptable Level of PR Bias Prediction Modified Lower Bounds in non-Gaussian Situations Conclusions 3D GNSS signal propagation Simulator overview Positioning by 3D PR Bias Bounding 3D Positioning over Candidate Positions Experimental Validation of positioning algorithms

# Proposed Approaches: AML-3D



#### 3D GNSS Simulator benefits

Acceptable Level of PR Bias Prediction Modified Lower Bounds in non-Gaussian Situations Conclusions 3D GNSS signal propagation Simulator overview Positioning by 3D PR Bias Bounding 3D Positioning over Candidate Positions Experimental Validation of positioning algorithms

# Proposed Approaches: Position Matching-3D



Introduction 3D GNSS Simulator benefits Acceptable Level of PR Bias Prediction

Modified Lower Bounds in non-Gaussian Situations Conclusions 3D GNSS signal propagation Simulator overview Positioning by 3D PR Bias Bounding **3D Positioning over Candidate Positions** Experimental Validation of positioning algorithms

### Architecture of the proposed approaches



# Figure: Our AML-3D Block Diagram

### Figure: Our PM-3D Block Diagram

Nabil Kbayer

GNSS Positioning with NLOS/MP Signals

3D GNSS Simulator benefits

Acceptable Level of PR Bias Prediction Modified Lower Bounds in non-Gaussian Situations Conclusions

Experimental settings

General Experimental Setup

Environment: Urban area Toulouse, France Date: 18/03/2015 Mode: Dynamic GNSS Receiver: AsteRx3 Receiver Recording Frequency: 10 Hz Reference: SPAN NOVATEL(IMU-FSAS +RTK)





	GPS 22	GPS 12	GPS 28	GPS 24	GPS 13	GPS 15	GPS 18	GLO 08	GLO 09	GLO 11
Elevation (°)	5.9	21.4	26	47.2	52.4	82.2	26.3	62	35	25.5
<i>C/N</i> 0 (dB-Hz)	35.5	33.5	27.7	46.2	42	47.7	35.2	34	24	28

Nabil Kbayer

GNSS Positioning with NLOS/MP Signals

9 October 2018 18 / 52

3D GNSS signal propagation Simulator overview Positioning by 3D PR Bias Bounding 3D Positioning over Candidate Positions Experimental Validation of positioning algorithms

3D GNSS signal propagation Simulator overview Positioning by 3D PR Bias Bounding 3D Positioning over Candidate Positions Experimental Validation of positioning algorithms

#### **Candidate Point selection**

2D area selection using Q-GIS Software Height extrusion Selection of a grid of point and elimination of indoor points



3D GNSS signal propagation Simulator overview Positioning by 3D PR Bias Bounding 3D Positioning over Candidate Positions Experimental Validation of positioning algorithms

# Results of 3D bias bounding approach



3D GNSS signal propagation Simulator overview Positioning by 3D PR Bias Bounding 3D Positioning over Candidate Positions Experimental Validation of positioning algorithms

# Comparison algo: Shadow Matching SM-3D

Introduction

#### Shadow Matching Principle

Satellite visibility is a valuable information for positioning



1	Define a grid of candidate positions
2	Building Boundaries (BB) computation: For each candidate position, predict building edges using the 3D city model
3	<b>Predict satellite visibility:</b> For each candidate position, predict satellite visibility using the Building Boundaries
4	Measure satellite visibility: Use $C/N0$ ratios to determine the observed satellite visibility
5	Scoring of candidate positions: Based on matching between predicted and measured sat visibility, score each point
6	Final position estimation: Estimate the final user position based on weighting of position having the highest scores

3D GNSS signal propagation Simulator overview Positioning by 3D PR Bias Bounding 3D Positioning over Candidate Positions Experimental Validation of positioning algorithms

#### Performance of Algorithms Based On Candidate Positions Scoring

Introduction



	AML-3D	PM-3D	SM-3D	UBLOX	Least-squares
Mean of HPE (m)	3.18	3.41	4.22	7.27	6.6
HPE AT 95% (m)	5.86	6.36	7.95	11.65	14.66
HPE AT 97% (m)	6.66	6.5	9.15	11.85	15.78

3D GNSS signal propagation Simulator overview Positioning by 3D PR Bias Bounding 3D Positioning over Candidate Positions Experimental Validation of positioning algorithms

### Performance of Algorithms Based On Candidate Positions Scoring



#### **Computational loads**

Proposed methods (with SPRING simulations) have a computation time 3 times greater than the SM-3D

Nabil Kbayer

GNSS Positioning with NLOS/MP Signals

3D GNSS signal propagation Simulator overview Positioning by 3D PR Bias Bounding 3D Positioning over Candidate Positions Experimental Validation of positioning algorithms

# Conclusions of this part

#### Reminder of objectives

- Determine the benefits of using a 3D GNSS propagation simulator to assist the GNSS receiver
- Errors on PR bias prediction using SPRING simulations are unavoidable
- Methods based on PR measurements correction are very sensitive to the PR bias correction provided by SPRING simulations
- Proposed algorithms give comparable results compared to Shadow matching algorithm, but with more computational loads

3D GNSS signal propagation Simulator overview Positioning by 3D PR Bias Bounding 3D Positioning over Candidate Positions Experimental Validation of positioning algorithms

# Conclusions of this part

#### Reminder of objectives

• Determine the benefits of using a 3D GNSS propagation simulator to assist the GNSS receiver

#### Errors on PR bias prediction using SPRING simulations are unavoidable

- Methods based on PR measurements correction are very sensitive to the PR bias correction provided by SPRING simulations
- Proposed algorithms give comparable results compared to Shadow matching algorithm, but with more computational loads

3D GNSS signal propagation Simulator overview Positioning by 3D PR Bias Bounding 3D Positioning over Candidate Positions Experimental Validation of positioning algorithms

# Conclusions of this part

#### Reminder of objectives

- Determine the benefits of using a 3D GNSS propagation simulator to assist the GNSS receiver
- Errors on PR bias prediction using SPRING simulations are unavoidable
- Methods based on PR measurements correction are very sensitive to the PR bias correction provided by SPRING simulations
- Proposed algorithms give comparable results compared to Shadow matching algorithm, but with more computational loads

3D GNSS signal propagation Simulator overview Positioning by 3D PR Bias Bounding 3D Positioning over Candidate Positions Experimental Validation of positioning algorithms

# Conclusions of this part

#### Reminder of objectives

- Determine the benefits of using a 3D GNSS propagation simulator to assist the GNSS receiver
- Errors on PR bias prediction using SPRING simulations are unavoidable
- Methods based on PR measurements correction are very sensitive to the PR bias correction provided by SPRING simulations
- Proposed algorithms give comparable results compared to Shadow matching algorithm, but with more computational loads

3D GNSS signal propagation Simulator overview Positioning by 3D PR Bias Bounding 3D Positioning over Candidate Positions Experimental Validation of positioning algorithms

# Conclusions of this part

PR errors prediction, using GNSS signal propagation simulations, requires:

Introduction

- 3D mapping accurate to cm level (not the case)
- Accurate GNSS signal propagation simulation (very complicated)
- Cm-level accurate receiver position (Impossible)
- Use of the same design information (receiver design, correlator design, pre-correlation bandwidth,...) for the receivers used in the experiments: these information are a trade secret.

 $\rightarrow$  PR errors predicted by SPRING simulator will have a low level of correspondence to those experienced by the actual receiver.

Introduction	Objectives
3D GNSS Simulator benefits	Theoretical acceptable levels of PR bias inaccuracy
Acceptable Level of PR Bias Prediction	
Modified Lower Bounds in non-Gaussian Situations	PR bias estimation using SPRING
Conclusions	Potential applications and conclusions

# Plan

Introduction



# 3D GNSS Simulator benefits

# Acceptable Level of PR Bias Prediction

- Objectives
- Theoretical acceptable levels of PR bias inaccuracy
- Experimental validation
- PR bias estimation using SPRING
- Potential applications and conclusions

### 4 Modified Lower Bounds in non-Gaussian Situations

# Conclusions

Nabil Kbayer

Introduction	Objectives
3D GNSS Simulator benefits	Theoretical acceptable levels of PR bias inaccuracy
Acceptable Level of PR Bias Prediction	
Modified Lower Bounds in non-Gaussian Situations	PR bias estimation using SPRING
Conclusions	Potential applications and conclusions

Previous proposed method based on PR measurements correction from PR bias bounds is very sensitive to the PR correction.

#### Problematic

How much the inaccuracy on the prediction of PR bias impacts the positioning performance of algorithms based on PR measurements?

#### Objectives

- How accurate the PR bias prediction should be to ensure that algorithms based on PR correction give better positioning accuracy compared to conventional positioning algorithms?
- What is the acceptable/allowed level of imprecision on PR bias prediction that any 3D GNSS signal propagation simulators must not exceed?
| Introduction                                     | Objectives  |
|--|---|
| 3D GNSS Simulator benefits                       | Theoretical acceptable levels of PR bias inaccuracy |
| Acceptable Level of PR Bias Prediction           |   |
| Modified Lower Bounds in non-Gaussian Situations | PR bias estimation using SPRING                     |
| Conclusions                                      | Potential applications and conclusions              |

# GNSS Navigation Equation y = Hx + n + b (1) • y: PR innovation measurements • H: Observation matrix

 n: Measurement Noise with a zero-mean Gaussian distribution with covariance matrix R

WLS Estimator: 
$$\mathbf{x}_{WLS} = (\mathbf{H}^T \mathbf{R}^{-1} \mathbf{H})^{-1} \mathbf{H}^T \mathbf{R}^{-1} \mathbf{y} = \mathbf{H}^+ \mathbf{y}$$
  
 $\rightarrow \delta \mathbf{x}_{WLS} = \mathbf{x}_{WLS} - \mathbf{x} = \mathbf{H}^+ (\mathbf{b} + \mathbf{n})$   
Corrected-LS Estimator:  $\mathbf{x}_{CLS} = \mathbf{H}^+_{\mathbf{b}} (\mathbf{y} - \mathbf{c})$   
 $\rightarrow \delta \mathbf{x}_{CLS} = \mathbf{H}^+_{\mathbf{b}} (\mathbf{b} + \mathbf{n} - \mathbf{c}) = \mathbf{H}^+_{\mathbf{b}} (\delta \mathbf{b} + \mathbf{n});$ 

$$\delta \mathbf{b} = \mathbf{b} - \mathbf{c}; \quad \mathbf{H}_{\mathbf{b}}^{+} = (\mathbf{H}^{\mathsf{T}} \mathbf{R}_{\mathbf{b}}^{-1} \mathbf{H})^{-1} \mathbf{H}^{\mathsf{T}} \mathbf{R}_{\mathbf{b}}^{-1}; \quad \mathbf{R}_{\mathbf{b}} \neq \mathbf{R}$$

x: State vector to be estimated

b: MP and NLOS bias (PR bias)

Introduction	Objectives
3D GNSS Simulator benefits	Theoretical acceptable levels of PR bias inaccuracy
Acceptable Level of PR Bias Prediction	
Modified Lower Bounds in non-Gaussian Situations	PR bias estimation using SPRING
Conclusions	Potential applications and conclusions

#### **MSE** Definition

$$\mathsf{MSE}[\mathbf{x}_{LS}] = E[\delta \mathbf{x}_{LS} \delta \mathbf{x}_{LS}^{T}]$$

Acceptable condition for accuracy improvements compared to LS

$$OMSE[\mathbf{x}_{CLS}] = Tr\{MSE(\mathbf{x}_{CLS})\} \le OMSE[\mathbf{x}_{LS}] = Tr\{MSE(\mathbf{x}_{LS})\}$$
(3)

Hypotheses: uncorrelated MP/NLOS bias **b** between satellites, one bias from one satellite *j* and  $\mathbf{R}_{\mathbf{b}} = \mathbf{R}$ :

$$(\boldsymbol{E}\{\delta \mathbf{b} \delta \mathbf{b}^{\mathsf{T}}\})_{j} \leq (\boldsymbol{E}\{\mathbf{b} \mathbf{b}^{\mathsf{T}}\})_{j}$$
(4)

#### First remarks

 For better accuracy by PR measurement correction compared to LS, bias bound prediction errors must have a lower correlation matrix than the true unknown PR bias variation

(2)

Introduction	Objectives
3D GNSS Simulator benefits	Theoretical acceptable levels of PR bias inaccuracy
Acceptable Level of PR Bias Prediction	Experimental validation
Modified Lower Bounds in non-Gaussian Situations	PR bias estimation using SPRING
Conclusions	Potential applications and conclusions

#### Objective: Validation of theoretical acceptable levels of PR bias inaccuracy

- Correction of PR measurements bias of all satellites using method in <sup>a</sup>.
- 2 The error on the prediction of PR bias of the satellite under study  $(\delta \mathbf{b})_j$  is assumed to be Gaussian distribution with a variable mean and variance, i.e.  $(\delta \mathbf{b})_j \sim \mathcal{N}(\mu, \sigma^2)$ .

$$(\boldsymbol{E}\{\delta \mathbf{b} \delta \mathbf{b}^T\})_j = (\sigma^2 + \mu^2)$$

**3** The mean  $\mu$  and the variance  $\sigma^2$  of this Gaussian distribution (of  $(\delta \mathbf{b})_j$ ) are varied until reaching the condition of equality between the positioning errors of LS and CLS solutions, i.e.  $OMSE[\mathbf{x}_{CLS}] = OMSE(\mu, \sigma) = OMSE[\mathbf{x}_{LS}].$ 

<sup>&</sup>lt;sup>a</sup>T. Iwase, N. Suzuki, and Y. Watanabe, Estimation and exclusion of multipath range error for robust positioning, GPS solutions, vol. 17, no. 1, 2013.

Introduction	Objectives
3D GNSS Simulator benefits	Theoretical acceptable levels of PR bias inaccuracy
Acceptable Level of PR Bias Prediction	Experimental validation
Modified Lower Bounds in non-Gaussian Situations	PR bias estimation using SPRING
Conclusions	Potential applications and conclusions



- Region of the maximum acceptable error on PR bias prediction using a 3D GNSS signal propagation simulator
  - $\rightarrow$  Minimum 3D modelling realism to correct PR measurements

Objectives
Theoretical acceptable levels of PR bias inaccuracy
PR bias estimation using SPRING
Potential applications and conclusions

## Determination of Maximum acceptable level of PR bias uncertainty for all satellites and comparison with SPRING PR bias prediction error

	Average Satellite Elevation (°)	Average PR Bias Estimation Error using SPRING Simulation [Meters]	Average Experimental Maximum Acceptable Uncertainty Level on PR bias Estimation [Meters]
GPS 22	5.93	0.83 !!!	44.45
GPS 12	21.47	26.30	37.53
GPS 28	26.35	4.53	9.94
GPS 24	47.23	0.39	1.38
GPS 13	52.47	7.22	8.2
GPS 15	82.29	0.25	0

PR bias prediction error using SPRING simulation is lower than the maximum acceptable on PR bias prediction error in this case

 $\rightarrow$  Confirm the previous result: PR measurement correction using SPRING prediction will slightly enhance the positioning performance

Introduction	Objectives
3D GNSS Simulator benefits	Theoretical acceptable levels of PR bias inaccuracy
Acceptable Level of PR Bias Prediction	
Modified Lower Bounds in non-Gaussian Situations	PR bias estimation using SPRING
Conclusions	Potential applications and conclusions

#### Potential applications of this study / Conclusions

 Identify environments where it is useful to use a 3D GNSS signal propagation simulator to predict PR biases in order to correct PR measurements.

 $\rightarrow$  Map representing the environments where it is useful to use a correction of PR measurements by prediction of PR biases

 production of a requirement metric on PR bias prediction by 3D GNSS signal propagation simulation

 $\rightarrow$  Validation and classification of 3D GNSS signal propagation simulators

ontext roblem formulation ower bounds computation lodified lower bounds computation

## Plan



- 2) 3D GNSS Simulator benefits
- 3 Acceptable Level of PR Bias Prediction

#### Modified Lower Bounds in non-Gaussian Situations

- Context
- Problem formulation
- Lower bounds computation
- Modified lower bounds computation

## 5 Conclusions

#### GNSS observation model with LOS signals only:

$$\mathbf{s}(t) = \sum_{k=0}^{K-1} \alpha_k \cdot \mathbf{c}_k (t - \tau_k) \cdot \mathbf{e}^{2i\pi f_k t} + \mathbf{n}(t)$$

- *α<sub>k</sub>* complex satellite signal amplitude
- $c_k(t)$  transmitted complex baseband navigation signal
- n(t) additive complex zero-mean Gaussian noise
- and *τ<sub>k</sub>*, *f<sub>k</sub>* the delay and Doppler frequency shift of the *k*-th satellite signal

Context Problem formulation Lower bounds computation Modified lower bounds computation

#### GNSS observation model with LOS signals only:

$$\mathbf{s}(t) = \sum_{k=0}^{K-1} \alpha_k . \mathbf{c}_k (t - \tau_k) . \mathbf{e}^{2i\pi f_k t} + \mathbf{n}(t)$$

1

Case *N* snapshots sampled at a  $F_s = \frac{1}{T_s}$  rate from s(t):

$$\mathbf{s} = \mathbf{s}(\boldsymbol{\theta}_d) = \mathbf{A}(\boldsymbol{\eta}) \boldsymbol{\alpha} + \mathbf{n}, \qquad \boldsymbol{\theta}_d = (\boldsymbol{\eta}^T, \boldsymbol{\alpha}^T)^T$$

• 
$$\mathbf{x} = (x(0), ..., x((N-1)T_s))^T$$
,

- $\mathbf{A} = [\mathbf{a}_0, ..., \mathbf{a}_{K-1}]$  is the manifold corresponding to all in-view satellite signals, with  $\mathbf{a}_k = (c_k(-\tau_k), ..., c_k((N-1)T_s \tau_k).e^{2i\pi f_k(N-1)T_s})^T$ ,
- η = (τ<sub>1</sub>, ..., τ<sub>K</sub>, f<sub>1</sub>, ..., f<sub>K</sub>)<sup>T</sup> is the vector of unknown deterministic parameters of primary interest (delays, Doppler-frequency shifts,...).

• 
$$\alpha = (\alpha_0, ..., \alpha_{K-1})^T$$
 and,

**n** = 
$$(n(0), ..., n((N-1)T_s))^T$$

#### GNSS observation model with LOS and MP signals:

$$s(t) = \sum_{k=0}^{K-1} \alpha_k . c_k(t-\tau_k) . e^{2i\pi f_k t} + \sum_{i=1}^{N_{MP}} \alpha_{r,i} . c_k(t-\tau_{r,i}) . e^{2i\pi f_{r,i} t} + n(t)$$

- *α<sub>r,i</sub>* each complex satellite signal amplitude after signal reflections, supposed to be random and unknown,
- and  $\tau_{r,i}$ ,  $f_{r,i}$  the delay and Doppler frequency shift after signal reflections, observed from the receiver, and supposed to be random and unknown.

#### GNSS observation model with LOS and MP signals:

$$s(t) = \sum_{k=0}^{K-1} \alpha_k . c_k (t - \tau_k) . e^{2i\pi f_k t} + \sum_{i=1}^{N_{MP}} \alpha_{r,i} . c_k (t - \tau_{r,i}) . e^{2i\pi f_{r,i} t} + n(t)$$

∜

$$\mathbf{s} = \mathbf{s}(m{ heta}_d,m{ heta}_r) = \mathbf{A}(m{\eta}) \mathbf{lpha} + \mathbf{m}(m{\eta}_r,m{lpha}_r) + \mathbf{n}; \qquad m{ heta}_d =$$

 $(\boldsymbol{\eta}^{\mathsf{T}},\boldsymbol{\alpha}^{\mathsf{T}})^{\mathsf{T}}, \quad \boldsymbol{\theta}_{\mathsf{r}} = (\boldsymbol{\eta}_{\mathsf{r}}^{\mathsf{T}},\boldsymbol{\alpha}_{\mathsf{r}}^{\mathsf{T}})^{\mathsf{T}}$ 

Where,  $\mathbf{m}(\boldsymbol{\eta}_r, \boldsymbol{\alpha}_r) = \sum_{i=1}^{N_{MP}} \mathbf{B}_i(\boldsymbol{\eta}_{r,i}) \boldsymbol{\alpha}_{r,i}$  represents the contribution of all MP and:

- ∀*i* ∈ [1, N<sub>MP</sub>], B<sub>i</sub> manifold corresponding to transmitted signal after signal reflections for the *i*-th multipath (may be different from MP to MP).
- $\eta_r = (\eta_{r,1}, ..., \eta_{r,N_{MP}})^T$  unknown nuisance parameters (delay and the Doppler frequency shift, of each MP after reflections).
- $\boldsymbol{\alpha}_r = (\alpha_{r,1}, ..., \alpha_{r,N_{MP}})^T$  GNSS signal amplitudes of each MP after reflections.

Context Problem formulation Lower bounds computation Modified lower bounds computation

Non-Gaussian errors in MP situations

#### Non-Gaussian p.d.f. in case of MP

In case of MP situations, the p.d.f. of GNSS problem is:

- non-Gaussian
- results from the marginalization of a joint p.d.f. depending on random variables as well (the delays, Doppler frequency shifts and amplitudes of MPs)

$$p(\mathbf{s}; \theta_d) = \int p(\mathbf{s}|\theta_r; \theta_d) p(\theta_r; \theta_d) d\theta_r;$$
  
$$p(\mathbf{s}|\theta_r; \theta_d) = p(\mathbf{s}|\eta_r, \alpha_r; \eta, \alpha) = \mathcal{CN} \left( \mathbf{A}(\eta) \alpha + \mathbf{m}(\eta_r, \alpha_r), \sigma^2 \mathbf{I} \right)$$

#### This will be illustrated with real GNSS data

Nabil Kbayer

r

Context Problem formulation Lower bounds computation Modified lower bounds computation

#### Non-Gaussian errors in MP situations



Nabil Kbayer

Context Problem formulation Lower bounds computation Modified lower bounds computation

Non-Gaussian errors in MP situations

#### Why Lower Bounds?

- **Relative information:** benchmark different estimators
- Absolute information: performance limit of all estimators (mean to characterize the limit of estimation precision)

#### **Raised questions**

- Can standard lower bounds (LBs) be computed in these environments?
- If standard LBs can not be computed, can we derive modified LBs that are computable?

fits Problem formulation Lower bounds computation Modified lower bounds comp

## Problematic

#### Problem formulation

$$p(\mathbf{s}; \theta) = \int_{\Theta_r} p(\mathbf{s}, \theta_r; \theta) \, d\theta_r,$$
$$p(\mathbf{s}, \theta_r; \theta) = p(\mathbf{s}|\theta_r; \theta) \, p(\theta_r; \theta),$$

where 
$$p(\mathbf{s}|\theta_r; \theta)$$
 is the conditional probability density function (p.d.f.) of **s** given  $\theta_r$ , and  $p(\theta_r; \theta)$  is the prior p.d.f. of  $\theta_r$ , parameterized by  $\theta$ .  
We distinguish two subsets:

- the subset of "standard" deterministic estimation problems for which a closed-form expression of p (s; θ) is available
- the subset of "non-standard" deterministic estimation problems for which only an integral form of p (s; θ) is available.

 $\rightarrow$  **Objective:** Compute lower bounds (LBs) on the estimation errors.

Context Problem formulation Lower bounds computation Modified lower bounds computation

## Case of standard estimation

#### McAulay-Seidman bound

The MSE of a particular estimator  $\hat{\theta^0} \in \mathcal{L}_2(\mathcal{S}_S)$  of  $\theta^0$ , i.e.,  $\hat{\theta^0} \triangleq \hat{\theta^0}(\mathbf{s})$ , where  $\theta^0$  is a selected value of the parameter  $\theta$  can be bounded as follows:

$$\begin{split} MSE_{\theta^{0}}\left[\widehat{\theta^{0}}\right] &= \left\|\widehat{\theta^{0}}\left(\mathbf{s}\right) - \theta^{0}\right\|_{\theta^{0}}^{2} \geq \boldsymbol{\xi}\left(\theta^{N}\right)^{T} \mathbf{R}_{\upsilon_{\theta^{0}}}^{-1} \boldsymbol{\xi}\left(\theta^{N}\right);\\ &\left(\mathbf{R}_{\upsilon_{\theta^{0}}}\right)_{n,m} = E_{\mathbf{s};\theta^{0}}\left[\upsilon_{\theta^{0}}\left(\mathbf{s};\theta^{m}\right)\upsilon_{\theta^{0}}\left(\mathbf{s};\theta^{n}\right)\right] \end{split}$$

Where  $\theta^N = (\theta^1, \dots, \theta^N)^T \in \Theta_d^N$  is a vector of *N* selected values of the parameter  $\theta$  (aka test points),

 $\boldsymbol{v}_{\theta^0}\left(\mathbf{s}; \boldsymbol{\theta}^N\right) = \left(v_{\theta^0}\left(\mathbf{s}; \theta^1\right), \dots, v_{\theta^0}\left(\mathbf{s}; \theta^N\right)\right)^T$ : Likelihood ratios associated to  $\boldsymbol{\theta}^N$ ,  $\xi\left(\theta\right) = \theta - \theta^0$  and  $\xi\left(\boldsymbol{\theta}^N\right) = \left(\xi\left(\theta^1\right), \dots, \xi\left(\theta^N\right)\right)^T$ .  $\rightarrow$  Any Lower Bound is a Linear Transformations of the McAulay-Seidman Bound.

Context Problem formulation Lower bounds computation Modified lower bounds computation

## Case of standard estimation

#### Standard LBs in GNSS harsh environments

- Presence of non-Gaussian MP fluctuation p.d.f. in harsh environments → the marginal p.d.f. of the GNSS observations has not an analytical form.
- Standard LBs on MSE cannot be computed in this case.

Solution: embed the initial observation space in a hybrid one where any standard LB can be transformed into a modified one fitted to non-standard deterministic estimation.

Context Problem formulation Lower bounds computation Modified lower bounds computation

## Case of non-standard estimation

Methodology: Look for LBs based on  $p(\mathbf{s}, \theta_r; \theta)$  instead of  $p(\mathbf{s}; \theta)$ . Modified MSB:

$$MSE_{\theta^{0}}\left[\widehat{\theta^{0}}\right] \geq \xi\left(\theta^{N}\right)^{T}\mathbf{R}_{\upsilon_{\theta^{0}}}^{-1}\xi\left(\theta^{N}\right)$$
$$\left(\mathbf{R}_{\upsilon_{\theta^{0}}}\right)_{n,m} = E_{\mathbf{s},\boldsymbol{\theta}_{r};\theta^{0}}\left[\upsilon_{\theta^{0}}\left(\mathbf{s},\boldsymbol{\theta}_{r};\theta^{m}\right)\upsilon_{\theta^{0}}\left(\mathbf{s},\boldsymbol{\theta}_{r};\theta^{n}\right)\right]$$
$$\rightarrow ModifiedLB$$

"Standard" MSB:

$$\begin{split} MSE_{\theta^{0}}\left[\widehat{\theta^{0}}\right] &\geq \xi \left(\theta^{N}\right)^{T} \mathbf{R}_{\upsilon_{\theta^{0}}}^{-1} \xi \left(\theta^{N}\right) \\ \left(\mathbf{R}_{\upsilon_{\theta^{0}}}\right)_{n,m} &= E_{\mathbf{s};\theta^{0}} \left[\upsilon_{\theta^{0}}\left(\mathbf{s};\theta^{m}\right)\upsilon_{\theta^{0}}\left(\mathbf{s};\theta^{n}\right)\right] \\ &\rightarrow StandardLB \end{split}$$

Nabil Kbayer

GNSS Positioning with NLOS/MP Signals

Modified lower bounds computation

## Case of non-standard estimation

- The previous result is general for all LBs
- For example with CRB: MCRB is obtained directly from the CRB:

$$\mathbf{CRB}_{\theta} = E_{\mathbf{s};\theta} \left[ \frac{\partial \ln p(\mathbf{s};\theta)}{\partial \theta} \frac{\partial \ln p(\mathbf{s};\theta)}{\partial \theta^{T}} \right]^{-1}$$
$$\rightarrow \mathbf{MCRB}_{\theta} = E_{\mathbf{s},\theta_{r};\theta} \left[ \frac{\partial \ln p(\mathbf{s},\theta_{r};\theta)}{\partial \theta} \frac{\partial \ln p(\mathbf{s},\theta_{r};\theta)}{\partial \theta^{T}} \right]^{-1}$$

with  $CRB_{\theta} > MCRB_{\theta}$ 

Context Problem formulation Lower bounds computation Modified lower bounds computation

## Case of non-standard estimation

#### Difference between MLBs and LBs

#### Larger vector space:

- LBs are for unbiased estimates belonging to  $\mathcal{L}_2(\mathcal{S}_S)$
- MLBs are for unbiased estimates belonging to L<sub>2</sub> (S<sub>S,Θr</sub>) → Computability of MLBs at the possible expense of tightness

 $\rightarrow$  It is possible to increase the tightness of MLBs by adding constraints in order to restrict the class of viable estimators  $\hat{\theta}^0 \in \mathcal{L}_2(\mathcal{S}_{\mathcal{S},\Theta_r})$ 

MLB is obtained from the LB by substituting d.p.p.

 $\rightarrow$  Any standard LB can be transformed into a modified one fitted to non-standard deterministic estimation

Conclusions Recommendations for future Work Publications

## Plan



- 2 3D GNSS Simulator benefits
- 3 Acceptable Level of PR Bias Prediction

4 Modified Lower Bounds in non-Gaussian Situations

#### 5 Conclusions

- Conclusions
- Recommendations for future Work
- Publications

## Main Conclusions

Conclusions Recommendations for future Work Publications

#### **Objectives/Conclusions**

#### The benefits of 3D GNSS signal propagation simulators:

- PR bias prediction using 3D GNSS simulators is a mere approximation
- Proposed algorithms give comparable results compared to a state-of-the-art method, with more computational loads.

#### The acceptable level of accuracy of 3D GNSS simulators:

- A study to define the allowed level of imprecision on PR bias prediction in order to improve the positioning performance by correcting PR.
- SPRING PR bias prediction is slightly below this acceptable level.

#### Modified lower bounds of estimation accuracy:

 General framework to transform standard LBs into a modified ones at the expense of tightness.

Conclusions Recommendations for future Work Publications

## Future Works

- Evaluation of the proposed approaches in other urban settings and using more measurements
- Reduction of computational loads related to 3D GNSS signal propagation
- 3D Simulator/GNSS fusion at the receiver level: track MP/NLOS signals in the GNSS receiver using the help of 3D GNSS simulators/3D city models.
- 3D Simulator/GNSS fusion using Doppler measurements: use of 3D models/3D GNSS simulators to estimate the Doppler effects and aid the GNSS receiver estimation.

Conclusions Recommendations for future Work Publications

## **Publications**

Journal Papers

[1] N. Kbayer, J. Galy, E. Chaumette, F. Vincent, A. Renaux and P. Larzabal, "On Lower Bounds for Nonstandard Deterministic Estimation," in **IEEE Transactions on Signal Processing**, vol. 65, no. 6, 15 March 2017.

[2] N. Kbayer, M. Sahmoudi, "Performances Analysis of GNSS NLOS Bias Correction in Urban Environment Using a 3D City Model and GNSS Simulator,"

#### in IEEE Transactions on Aerospace and Electronic Systems, 2018.

#### International Conferences

 N. Kbayer, M. Sahmoudi, E. Chaumette, "Robust GNSS Navigation in Urban Environments by Bounding NLOS Bias of GNSS Pseudoranges Using 3D City Model," Proceedings of **ION GNSS+** conference, September 2015.
 N. Kbayer, M. Sahmoudi, "Constructive Use of MP/NLOS Bias of GNSS Pseudoranges: Performance Analysis by Type of Environment," Proceedings of ION ITM conference, January 2017.

[3] N. Kbayer, J. Galy, E. Chaumette, F. Vincent, A. Renaux, P. Larzabal, "Estimation Accuracy Of Non-Standard Maximum Likelihood Estimators," On ICASSP 2017.

[4] N. Kbayer, M. Sahmoudi, H. Ortega-Gonzàlez, C. Rouch, "Approximate Maximum Likelihood Estimation Using a 3D GNSS Simulator for Positioning in MP/NLOS Conditions," Proceedings of **ION GNSS+** conference 2017.

Conclusions Recommendations for future Work Publications

## Thank you for your attention

Conclusions Recommendations for future Work Publications

# **BACK-UP**

Conclusions Recommendations for future Work Publications

## PR bias estimation using reference trajectory



<sup>&</sup>lt;sup>1</sup>T. Iwase, N. Suzuki, and Y. Watanabe, Estimation and exclusion of multipath range error for robust positioning, GPS solutions, vol. 17, no. 1, 2013.

Introduction 3D GNSS Simulator benefits Acceptable Level of PR Bias Prediction Modified Lower Bounds in non-Gaussian Situations Conclusions Non-standard lower bounds (NSLB)

#### Non-standard McAulay-Seidman bound

$$E_{\boldsymbol{\theta}_{r};\boldsymbol{\theta}}\left[\mathbf{C}_{\boldsymbol{\phi}}\left(\widehat{\boldsymbol{\phi}}_{MSB}\right)\right] = \Xi\left(\boldsymbol{\phi}^{N}\right)\mathbf{R}_{\boldsymbol{\upsilon}_{\boldsymbol{\phi}}}^{-1}\left(\boldsymbol{\phi}^{N}\right)\Xi\left(\boldsymbol{\phi}^{N}\right)^{\mathsf{T}} \leq \min_{\widehat{\boldsymbol{\phi}}\in\mathcal{U}_{S}(\mathcal{S}_{S})}\left\{E_{\mathbf{s},\boldsymbol{\theta}_{r};\boldsymbol{\theta}}\left[\left(\widehat{\boldsymbol{\phi}}-\boldsymbol{\phi}\right)\left(\widehat{\boldsymbol{\phi}}-\boldsymbol{\phi}\right)^{\mathsf{T}}\right]\right\}.$$

 $\phi^{N} = \left[\phi^{1} \dots \phi^{N}\right]$  is a vector of *N* selected values of the parameter  $\theta$ (aka test points) and  $\Xi\left(\phi^{N}\right) = \left[\phi^{1} - \phi \dots \phi^{N} - \phi\right]$ 

 $\rightarrow$  **NSMSB** is a LB for  $\widehat{\phi} \in \mathcal{U}_{\mathcal{S}}(\mathcal{S}_{\mathcal{S}})$ , i.e., strict-sense unbiased estimates, whereas the **MMSB** is a LB for  $\widehat{\phi} \in \mathcal{U}_{W}(\mathcal{S}_{\mathcal{X},\Theta_{r}})$ , i.e., wide-sense unbiased estimates.

 $\rightarrow$  Any Non-standard Lower Bound is a Linear Transformations of the McAulay-Seidman Bound.

In general, we cannot compare the NSLB and the MLB

• If  $p(\theta_r; \theta_d)$  does not depend on  $\theta_d$ 

$$\underbrace{\Xi\left(\theta^{N}\right)\mathbf{R}_{\upsilon_{\theta}}^{-1}\left(\theta^{N}\right)\Xi\left(\theta^{N}\right)^{T}}_{\mathsf{MMSB}} \leq \underbrace{E_{\theta_{r};\theta}\left[\Xi\left(\phi^{N}\right)\mathbf{R}_{\upsilon_{\phi}}^{-1}\left(\phi^{N}\right)\Xi\left(\phi^{N}\right)^{T}\right]}_{\mathsf{NSMSB}}.$$

Conclusions Recommendations for future Work Publications

GNSS estimation accuracy depends on:

 Dilution Of Precision (DOP): The geometry between each satellite and the receiver.

 $\Rightarrow$  The DOP is a pure geometrical factor.

- User Equivalent Range Error (UERE): The satellite signal state and health (Presence of errors on measurements).
  - Satellite-based errors: satellite clock errors, orbits errors...
  - Propagation-based errors: ionospheric, tropospheric errors...
  - Receiver-based errors: multipath, receiver noise...

#### Position Error = DOP x UERE

Conclusions Recommendations for future Work Publications

## **SPRING** Architecture





Toulouse 3D model provided by IGN: City model with roof structures and boundary surfaces.

Conclusions Recommendations for future Work Publications

## **SPRING** Architecture





- Ray-tracing algorithm with a propagation model
- Possibility of selecting different propagation phenomena
- Reflection coefficients are set by users
- All building have the same materials

GNSS Positioning with NLOS/MP Signals

Conclusions Recommendations for future Work Publications

## **SPRING** Architecture



Agreter primeters General configuration (in	desidediam Devidual andputs		
	Alter and a second	Vera The second	

- Parameters of the Rx correlator are set by users
- Pseudorange error: error between input LOS pseudorange and computed pseudorange

Conclusions Recommendations for future Work Publications

## Our use of SPRING

- **First step:** Configure SPRING parameters to give the most reliable approximation of GNSS signals propagation:
  - GNSS constellation configuration
  - 3D modelling configuration
  - GNSS propagation parameters
  - GNSS Rx parameters (signal acquisition and tracking)
- Second step: Study the reliability of SPRING simulations.
- Third step: Analyse the output 3D PR bias provided by SPRING simulation.

Conclusions Recommendations for future Work Publications

## Shadow Matching SM-3D



#### Shadow Matching Principle

Use of masked and NLOS Satellite as additional information: Satellite visibility is a valuable information for positioning

<sup>&</sup>lt;sup>2</sup> M. Adjrad and P. D. Groves, Intelligent Urban Positioning: 3D Mapping-Aided GNSS as Solution to the Urban Positioning Problem, GDR-ISIS Seminar, 27/10/2016, Toulouse

Conclusions Recommendations for future Work Publications

## **GNSS** principles

- Multilateration method: find out the user position by knowing ranging to at least three known satellites in view (Pseudoranges).
- Additional measurement to solve synchronization between GNSS receiver clock and satellite clocks.

 $\Rightarrow$  Four ranges (surface of spheres) of four satellites are needed: three for calculating the position in 3D and the fourth one for time synchronization.
Conclusions Recommendations for future Work Publications

# **GNSS** principles

- Multilateration method: find out the user position by knowing ranging to at least three known satellites in view (Pseudoranges).
- Additional measurement to solve synchronization between GNSS receiver clock and satellite clocks.

 $\Rightarrow$  Four ranges (surface of spheres) of four satellites are needed: three for calculating the position in 3D and the fourth one for time synchronization.

Conclusions Recommendations for future Work Publications

# **GNSS** principles

- Multilateration method: find out the user position by knowing ranging to at least three known satellites in view (Pseudoranges).
- Additional measurement to solve synchronization between GNSS receiver clock and satellite clocks.

 $\Rightarrow$  Four ranges (surface of spheres) of four satellites are needed: three for calculating the position in 3D and the fourth one for time synchronization.



Conclusions Recommendations for future Work Publications

## 3D model Overview

### 3D city models

3D city models are digital representations of buildings and other objects presents in cities.

- More and more precise
- Widely available for most big cities
- Free of charge
- Different Levels of details

#### 3D GNSS signal propagation Simulator

3D GNSS simulator = 3D city model + GNSS signal propagation + GNSS receiver model

 $\rightarrow$  We have used the 3D GNSS signal propagation Simulator SPRING (CNES)

Conclusions Recommendations for future Work Publications

### Experimental settings

#### General Experimental Setup

Environment: Urban area Toulouse, France Date: 18/03/2015 Mode: Dynamic GNSS Receiver: AsteRx3 Receiver Recording Frequency: 10 Hz Reference: SPAN NOVATEL(IMU-FSAS +RTK)

