2 and 3-carrier Passive Intermodulation Products in a waveguide nonlinearity: Theory and Experiments

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Outline

1. Introduction
2. PIM nonlinear model
3. Predictions of nonlinear model
4. Measurement of waveguide flange PIM
5. Conclusion

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1. Introduction

- The slope of passive intermodulation products levels versus signals level is about constant for all orders
- A simple nonlinear model based on this fact has been proposed for PIM behavior
- It predicts many other differences and “odd” behaviors of PIM
- All measurement results are explained by this simple model
- A measurement has been defined to explore the model in particular conditions and a waveguide flange has been measured with 2 and 3 carriers

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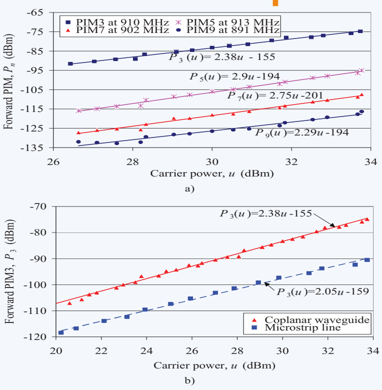
How to explain measured curves?



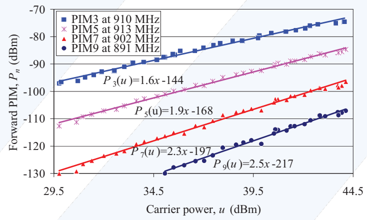


Fig. 4 Measured forward PIM3, PIM5, PIM7 and PIM9 products vs. carrier power on the microstrip line with central section of width W_c=7.44 mm with the tapered matching transformers.

- Reference Shitvov 2009
- 3rd, 5th, 7th and 9th order IM   Slopes 1.6 to 2.9 dB/dB

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2. PIM nonlinear model

- The simplest nonlinear behavioral model proposed to explain the constant slope α of harmonics and PIM products of all orders is:

  $y = \text{sign}(x)\,|x|^s$ for odd harmonics and products

  $y = |x|^s$ for even harmonics and products
- Only two parameters that are easily measured:
  - For odd PIM: level of order 3 PIM and slope of order 3 PIM
- If x and y are real signals, harmonics and intermodulation products are generated with slopes α (in dB/dB) versus signal level
- Their frequencies are integer combinations of the input signal frequencies and identical to frequencies obtained with polynomials

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3. Predictions of nonlinear model (1)

- Only integer harmonics frequencies
  - Odd for the odd model, even for the even model
- For a real input signal $x = a\cos(2\pi\omega t + \varphi) = a\cos\theta$

$$y = \alpha_s \text{sign}(x)\,|x|^s = \alpha_s x|x|^{(s-1)} = \alpha_s a|a|^{(s-1)}\cos\theta\,(\cos^2\theta)^{\left(\frac{s-1}{2}\right)}$$

Slope = s

$$y = \alpha_s \frac{a|a|^{(s-1)}}{2^{\left(\frac{s-1}{2}\right)}}\cos\theta\,(1+\cos 2\theta)^{\left(\frac{s-1}{2}\right)}$$

Infinite number of harmonics

$$y = \alpha_s \frac{a|a|^{(s-1)}}{2^{\left(\frac{s-1}{2}\right)}}\cos\theta\left\{1 + \frac{s-1}{2}\cos 2\theta + \left(\frac{s-1}{2}\right)\left(\frac{s-3}{2}\right)\cos^2 2\theta + \cdots\right\}$$

Integer harmonics only

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3. Predictions of nonlinear model (2)

- The model can be applied to the complex envelope of an analytic signal around a carrier in a simulation software
- The function sign (x) is extended to $e^{i\varphi}$ if $x = \rho e^{i\varphi}$
- The odd model will give odd intermodulation products near the odd harmonics
- The even model will give even products near the even harmonics
- For a 2-carrier input signal at frequencies $f1$ and $f2$, harmonics and intermodulation products of order $|m| + |n|$ have a frequency $mf1 + nf2$ with m and n in $\mathbb{Z}$ and are around the harmonic $|m + n|$ with the phase $|m + n|\varphi$

3. Predictions of nonlinear model (3)

- Chebyshev transform can be used for analytical computations
- It is a special case of the Fourier transform. It uses only integrals
- No Taylor series development of the model is needed
- The model discontinuities are not a problem

$$f(x) = f(a\,\cdot) = \frac{1}{2} f_0(a) + \sum_{m=1}^{\infty} f_m(a).\cos(m\theta)$$

$$f_m(a) = \frac{1}{\pi} \int_{-\pi}^{+\pi} f[a.\cos(\theta)] \cos(m\theta)\, d\theta$$

3. Predictions of nonlinear model (4)

- The odd nonlinear model gives the $m = 2p + 1$ harmonic as:

Slope = s

$$f_{2p+1}(a) = 2\alpha_s \operatorname{sign}(a)\left(\frac{|a|}{2}\right)^s \frac{\Gamma(s+1)}{\Gamma\left(\frac{s+3}{2}+p\right)\Gamma\left(\frac{s+1}{2}-p\right)} = \alpha_s a\left(\frac{|a|}{2}\right)^{s-1} \frac{\Gamma(s+1)}{\Gamma\left(\frac{s+3}{2}+p\right)\Gamma\left(\frac{s+1}{2}-p\right)}$$

Infinite number of harmonics: for polynomials of degree s, if m > s this Γ term is infinite

Levels of all orders are linked

- The two nonlinear models are eigenvalues of the Chebyshev transform, which means that the intermodulation products of a 2-carrier signal behave exactly as the harmonics of a sinusoidal signal
- The nonlinear model applied to the complex envelope of the analytic signal is the first Chebyshev transform ($m = 1, p = 0$)

$$f_1(a) = 2\alpha_s \operatorname{sign}(a)\left(\frac{|a|}{2}\right)^s \frac{\Gamma(s+1)}{\Gamma\left(\frac{s+3}{2}\right)\Gamma\left(\frac{s+1}{2}\right)} = \beta_s \operatorname{sign}(a)\,|a|^s = \beta_s\, a\,|a|^{s-1}$$

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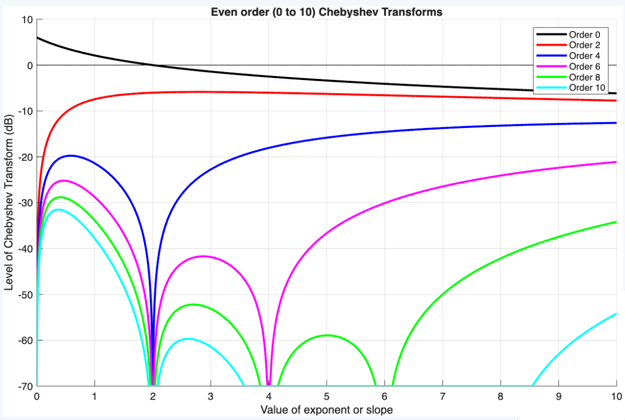
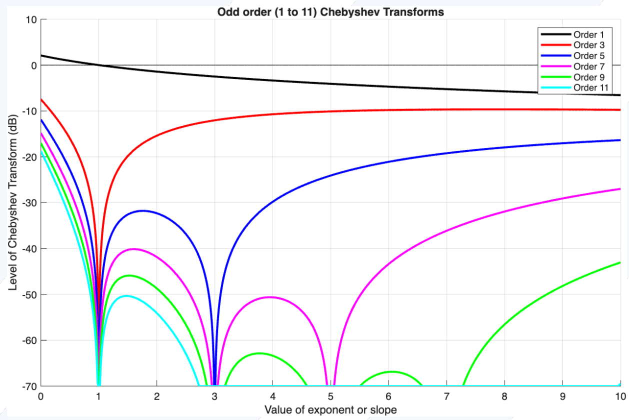
3. Predictions of nonlinear model (5)





Levels of even and odd harmonics or PIM products versus slope s

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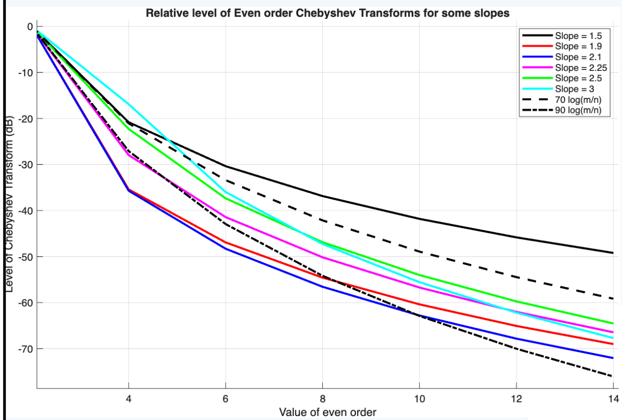
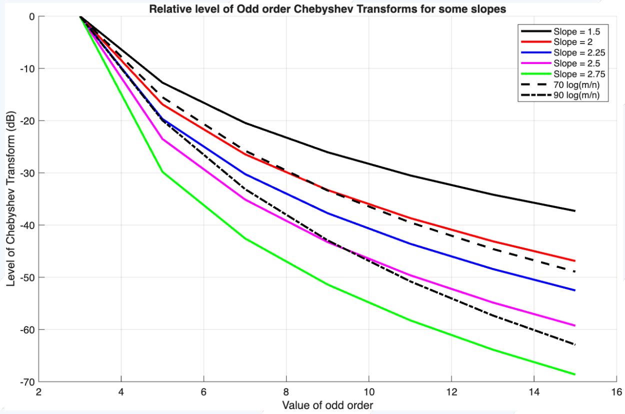
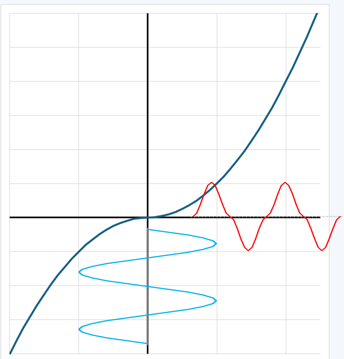
3. Predictions of nonlinear model (6)

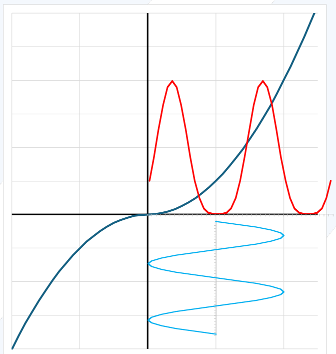




Levels of even and odd harmonics or PIM products versus order

3. Predictions of nonlinear model (7)



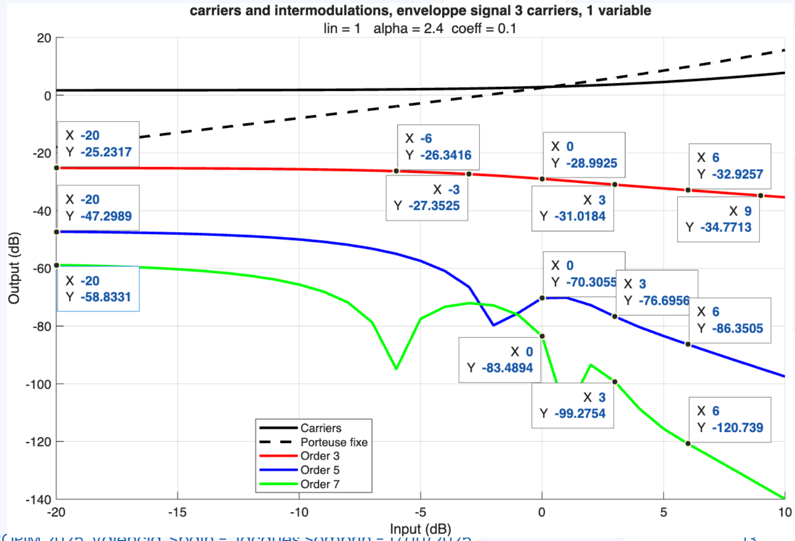


Decrease of harmonic or PIM level when adding an offset to the input signal

When the input signal no longer crosses the discontinuity, a Taylor series development exist. Back to the classical case with much lower PIM levels

3. Predictions of nonlinear model (8)

- Prediction of 2-carrier PIM products level when a third carrier with variable power is added for a nonlinearity with slope 2.4



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4. Measurement of waveguide flange PIM (1)

- Adding an offset and measuring the harmonics is not feasible in waveguide
- Adding a third carrier and measuring the PIM products generated by carriers 1 and 2 is easier
- A third carrier with 4 times the power of each of carriers 1 and 2 strongly decreases the levels of PIM products
- For a slope of 2 (parabolic shape) these products disappear
- For other slopes, a simple simulation is used to predict the PIM levels

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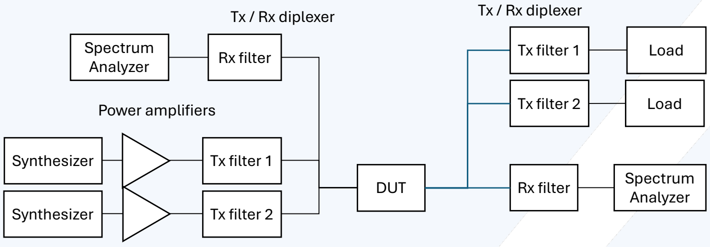
4. Measurement of waveguide flange PIM (2)

Test bench

The DUT is a resistive sheet between 2 waveguide flanges



4. Measurement of waveguide flange PIM (3)

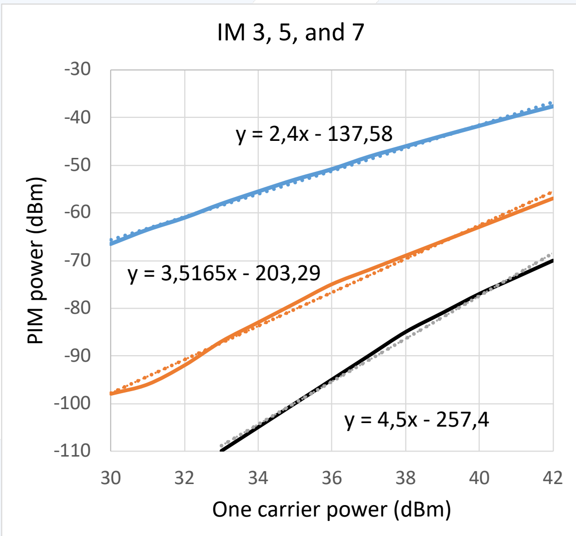
Measurement results with 2 carriers

Slopes:

2.4 for 3rd order

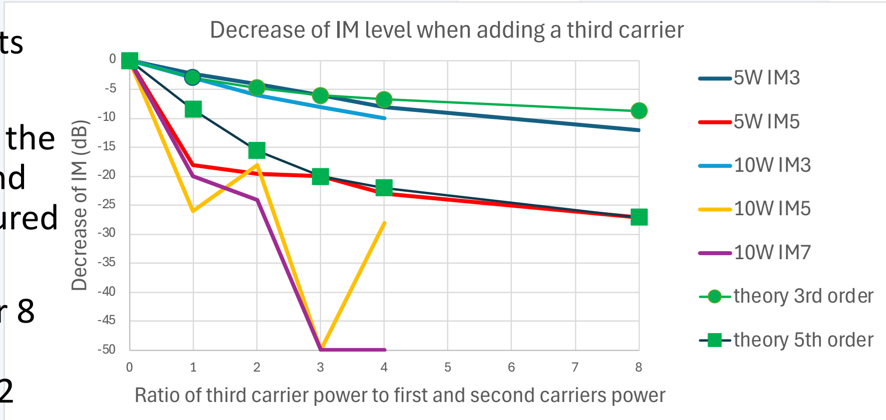
3.5 for 5th order

4.5 for 7th order



4. Measurement of waveguide flange PIM (4)

- Measurement results with 3 carriers
- Carriers 1 and 2 are the same (5 or 10 W) and their PIM are measured
- Third carrier power varies from 0 to 4 or 8 times the power of each carriers 1 and 2



Theoretical slope 2.4 for order 3 and 3.5 for order 5

5. Conclusion

- Most measurement results are predicted or explained with a simple nonlinear model using only 2 parameters
- The model can be easily simulated with complex multi-carrier and modulated signals
- Left unexplained up to now: slope varies slightly with carrier level (saturation) and increases with order

- I will happily discuss measurement results that pose you problems

- Questions ?