2 and 3-carrier Passive Intermodulation Products in a waveguide nonlinearity: Theory and Experiments

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INTRODUCTION

This paper presents some theoretical results and measurements of passive intermodulation (PIM) products made on a waveguide flange contact nonlinearity with 2 and 3 carriers at different power levels.
The dependence of the passive intermodulation (PIM) products power on the carrier power is the main difference between active and passive intermodulation (IM) products.
For easy computation, memoryless active nonlinearities are generally modelled by polynomials or by analytical mathematical functions (e.g. hyperbolic tangent) [1, 2]. These functions are continuous, have continuous derivatives of all orders and can be approximated by their Taylor series developments, at least in "small signal" conditions where the IM power is much less than the fundamental carrier power, mathematically in a domain around the origin.
In these conditions, the power of each active IM products depends on the carrier power elevated to an exponent equal to the order of the IM product, e.g. exponent 3 for order 3, exponent 5 for order 5 [1, 2]. On a dB/dB graph, the slopes of IM levels versus carrier level are equal to their order 3, 5, …
This is not the case for passive IM products where the level of IM products depends on the carrier level with a slope that is different from the order, generally a non-integer value between 2 and 3 and about the same for all orders on a dB/dB graph [3 - 8]. A model based on a non-analytical power function has been proposed [9]. It has been used to predict the behavior of the passive IM products such as the level of high order of products, the nearly equal slope for all IM orders, and the decrease of a 2-carrier IM product power when a third carrier is added [9, 10].
The experiment is carried out on a nonlinear graphite material introduced between two waveguide flanges in a PIM test bench in Ku band using two low PIM triplexers. Two carriers with power up to 40 watts per carrier or three carriers with power up to 40 watts per carrier can be transmitted on the test bench.
The main theoretical and experimental results are presented in this paper and validate the theoretical results: a slope of 2.4 dB/dB is measured on the third order PIM product and the levels of higher order PIM products are correctly obtained in different configurations of carriers and power by using the non-analytical model based on a power law nonlinearity with an exponent of 2.4 for order 3.
Measurement with 3 carriers (with the same or different power) are compared with the theory. A particular combination of 3 carrier powers shows that some third order PIM can be nearly eliminated in concordance with the theory.

PIM POWER DEPENDENCE THEORY

The levels in dB of many passive intermodulation products depend on carrier level in dB through a linear relationship with a slope generally between 1 and 3 for the third order PIM [3-8]. This linear relationship can happen for ranges up to 30 dB, from 1 watt to 1 kW as shown in one of the oldest articles on this phenomenon [3].
No analytical function has been found to be able to reproduce correctly this behavior even with a polynomial of degree 49 [4]. Such a polynomial would give invalid results if extrapolated even by 1 dB.
The most evident function that allows to model this behavior is a power function with an exponent equal to the slope in the dB/dB graph. The following antisymmetric function generates odd order IM products with a slope equal to s:

$$f(x) = \alpha_s \,\mathrm{sign}(x)\, |x|^s = \alpha_s \, x\, |x|^{s-1} \tag{1}$$

Even order IM products can be obtained by using the symmetric function:

$$f(x) = \beta_s |x|^s \tag{2}$$

The derivatives of orders higher than s of these functions are infinite or indefinite at origin. So, the functions have no Taylor series development at origin and cannot be correctly modelled by polynomials of low degree except when s is either an odd integer in (1) or an even integer in (2). In the general case, approximations can be found but they are of high degree and are valid only on a very limited range.

Computation of harmonics

The levels of harmonics of the fundamental sinusoidal signal can be obtained through the Chebyshev transform [1]. This transform is a direct application of Fourier transform of the nonlinear function output when the input signal x is a pure cosine carrier. It depends on integration and not on the derivatives of the nonlinear function, which can be non-analytical.

$$x = a\cos(\omega t + \varphi) = a\cos\theta \tag{3}$$

The nonlinearity gives a series of integer harmonics:

$$f(x) = f(a\cos\theta) = \frac{1}{2}f_0(a) + \sum_{m=1}^{\infty} f_m(a).\cos(m\theta) \tag{4}$$

The order m Chebyshev transform of function f is:

$$f_m(a) = \frac{1}{\pi}\int_{-\pi}^{+\pi} f[a.\cos(\theta)]\cos(m\theta)\,d\theta \tag{5}$$

The non-analytical nonlinearity (like any nonlinear function) generates only harmonics at frequencies that are integer multiples of the fundamental frequency. As $f(a+b) \neq f(a) + f(b)$, it would be wrong to apply the nonlinearity to a part of the input signal, such as $a\exp(i\theta)/2$ to obtain signals such as $(a/2)^s\exp(is\theta)$ that are not integer harmonic of the input signal.

In the case of (1) we get only odd order harmonics and odd order transforms for $m = 2p+1$:

$$f_{2p+1}(a) = 2\,\alpha_s\,\mathrm{sign}(a)\left(\frac{|a|}{2}\right)^s \frac{\Gamma(s+1)}{\Gamma\left(\frac{s+3}{2}+p\right)\Gamma\left(\frac{s+1}{2}-p\right)} = \alpha_s\,a\left(\frac{|a|}{2}\right)^{s-1} \frac{\Gamma(s+1)}{\Gamma\left(\frac{s+3}{2}+p\right)\Gamma\left(\frac{s+1}{2}-p\right)} \tag{6}$$

This transform is a function of the same type as the initial function and with the same exponent s. The order 1 ($p = 0$) Chebyshev transform gives the fundamental response and is linked to the complex gain and AM/AM and AM/PM curves:

$$f_1(a) = 2\,\alpha_s\,\mathrm{sign}(a)\left(\frac{|a|}{2}\right)^s \frac{\Gamma(s+1)}{\Gamma\left(\frac{s+3}{2}\right)\Gamma\left(\frac{s+1}{2}\right)} = g(a) = \beta_s\,\mathrm{sign}(a)\,|a|^s = \beta_s\,a\,|a|^{s-1} \tag{7}$$

Computation of intermodulation products

A bandwidth limited signal with 2 equal amplitude carriers can be put into the form:

$$x = \frac{a}{2}[\cos(\omega_1 t + \varphi_1) + \cos(\omega_2 t + \varphi_2)]$$

$$x = a\,\cos\left(\frac{\omega_1 t + \omega_2 t + \varphi_1 + \varphi_2}{2}\right)\cos\left(\frac{\omega_1 t - \omega_2 t + \varphi_1 - \varphi_2}{2}\right) = a\cos\theta\cos\Omega \tag{8}$$

Then the IM products of odd orders $m = 2p+1$ at radians frequencies $(p+1)\omega_1 - p\omega_2$ and $(p+1)\omega_2 - p\omega_1$ are obtained with the order m Chebyshev transforms of the function $g = f_1$:

$$g_{2p+1}(a) = 2\,\beta_s\,\mathrm{sign}(a)\left(\frac{|a|}{2}\right)^s \frac{\Gamma(s+1)}{\Gamma\left(\frac{s+3}{2}+p\right)\Gamma\left(\frac{s+1}{2}-p\right)} = \beta_s a\left(\frac{|a|}{2}\right)^{s-1} \frac{\Gamma(s+1)}{\Gamma\left(\frac{s+3}{2}+p\right)\Gamma\left(\frac{s+1}{2}-p\right)} \tag{9}$$

This is a generalization of the computation for polynomials, where s is an odd integer in (1), that would give:

$$g_{2p+1}(a) = 2\,\beta_s\,\mathrm{sign}(a)\left(\frac{|a|}{2}\right)^s \frac{s!}{\left(\frac{s+1}{2}+p\right)!\,\left(\frac{s-1}{2}-p\right)!} = 2\,\beta_s\left(\frac{a}{2}\right)^s \frac{s!}{\left(\frac{s+1}{2}+p\right)!\,\left(\frac{s-1}{2}-p\right)!} \tag{10}$$

In the polynomial case, when s is an odd integer in (10), if $\frac{s-1}{2} - p$ is negative, that is for odd IM orders $m = 2p+1$ higher than the odd degree s of the polynomial, the factorial $\left(\frac{s-1}{2} - p\right)!$ in the dividend is infinite, and the IM level is 0. This is the classical result that polynomial functions generate IM only up to a maximum order that is equal to the degree of the polynomial.

MAIN CONSEQUENCES OF THE THEORY

Simple model with only 2 experimental parameters

The exponent s of the function is equal to the measured slope of the IM3 level versus carrier level.
The coefficient of the function is computed to minimizes the quadratic error with the measured IM3 level.
All the modelled higher IM products have the same slope s versus carrier power in this model.

Model easy to use in simulation software

The model can be used in bandpass-limited simulation with complex envelope signal X around a fundamental carrier, using the combined AM/AM and AM/PM curve given in (7) as:

$$g(X) = \beta_s \operatorname{sign}(X)\,|X|^s = \beta_s\, X\,|X|^{s-1} \qquad (11)$$

The nonlinear gain applied to the fundamental signal in (11) depends only on the amplitude of the envelope signal.

High Harmonic Generation and High Order IM Generation

The difference in level between two IM orders is fixed and depends only on the slope and the orders of the IM products. For successive odd orders $2p-1$ and $2p+1$ we have:

$$\frac{g_{2p-1}(a)}{g_{2p+1}(a)} = \frac{\Gamma\left(\frac{s+3}{2}+p\right)\Gamma\left(\frac{s+1}{2}-p\right)}{\Gamma\left(\frac{s+1}{2}+p\right)\Gamma\left(\frac{s+3}{2}-p\right)} = \frac{s+2p+1}{s-2p+1} = -\frac{2p+1+s}{2p-1-s} \qquad (12)$$

As an example, a slope $s = 2$ in (10) would result in a level amplitude ratio of 7 or a level difference of 17 dB between orders 3 and 5 with $p = 2$ in (12) and an amplitude ratio of 3 or a difference of 9.5 dB between orders 5 and 7 with $p = 3$ in (12).
For lower values of IM orders, these level differences between successive IM orders depend on the slope s and the nature of the material.
For higher values of IM orders, (12) gives a level difference that is approximately: $20\{\log(2p+1) - \log(2p-1)\}$, which converges to 0 for higher orders.
Because this level difference becomes very low for higher values of orders, many IM products of higher orders have measurable levels and may desensitize or jam a receiver at receive frequencies far from the transmit frequencies.
The non-analytic nonlinear functions given by (1) and (2) generate high harmonics (HHG), a property that has been found in passive materials up to THz and optic frequencies [11].
As the nonlinear function at fundamental frequency follows the same type of equation as the one in baseband, it generates high order IM products around the fundamental frequency instead of high order harmonics. It explains problems that we may encounter at frequencies far removed from the transmit frequencies. High orders PIM levels at receive frequencies may be high enough to jam receivers.
A simple example is the case $s = 0$ in (1), the hard limiter nonlinearity that transform a sinusoidal signal into a square signal having odd harmonics with amplitudes that decrease as $1/f$ (and level difference as $20\log(m/n)$ between harmonics m and n) as shown in fig. 1.

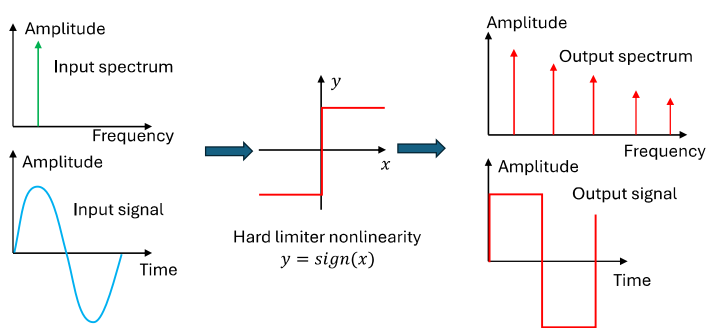

Fig. 1. Spectrum and signal before and after a hard limiter.

The successive sum of differences between IM levels of adjacent orders given by (12) result in the curves in fig. 2 for slopes between 2 and 2.5.
They are compared with the experimental "rule of thumb" limits proposed by MDA in ESA PIM contract report [8]: $70\log(m/n)$ and $90\log(m/n)$.They show a very good agreement.

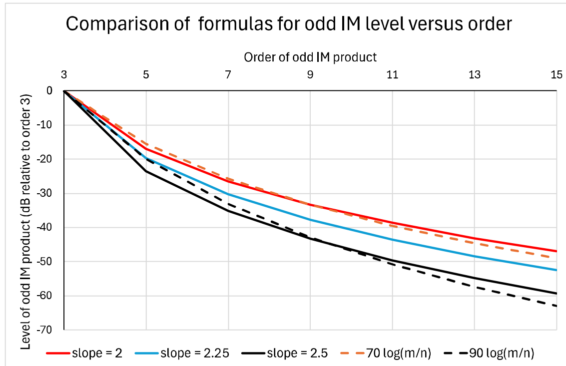

Fig. 2. Level of odd order IM products relative to third order IM product for 3 slopes and MDA experimental "rule of thumb" limits.

Odd and even orders of IM may come from two different nonlinear functions obeying (1) and (2), there may not be a correlation between the coefficients or the exponents of these two functions, so the level difference between odd and even orders can differ widely for different materials. However, they may be equal if the nonlinearity is identically null on one side of the origin.

These consequences have been experimentally described in ESA PIM contract report [8].

Relaxation of 2-carrier IM specifications

A simulation using the proposed model also shows that the order 3 PIM levels decrease when carriers are added to a 2-carrier signal. This is not the case in small signal for the polynomial or analytical functions models and for active IM.

For classical (polynomial or analytic function) nonlinearities, in small signal conditions, the level of a third order IM product generated by two carriers e.g. $2f_2 - f_1$ (with a small signal slope of 3 versus carrier level) does not change when other carriers are added (provided they do not generate IM products at the same frequency as the first one).

This is not the case when using non-analytical functions given in (1) and (2) [10].

If the slope is lower than the order of the IM product e.g. $s = 2$ or $s = 2.5$ for IM order 3, the level of the IM decreases when adding carriers; This decrease is 4 dB for $s = 2.5$ and 8 dB for $s = 2$ when adding 8 or more carriers.

This effect can be used to relax the 2-carrier specification on devices that will work under multi-carrier operation compared to the classical computation.

Note that if the slope is higher than 3, the addition of carriers will increase the level of the third order IM product compared to the 2-carrier case.

Such an increase of IM3 when adding carriers is also obtained with a degree 5 polynomial that gives a third order IM with a slope of 5.

Addition of a higher power third carrier

This decrease is more important when adding one higher power third carrier because the effect may prevent the signal from going through the discontinuity of the nonlinear function.

Fig. 3 shows an odd nonlinear function $y = \text{sign}\,x\ |x|^2$ with a slope $s = 2$.

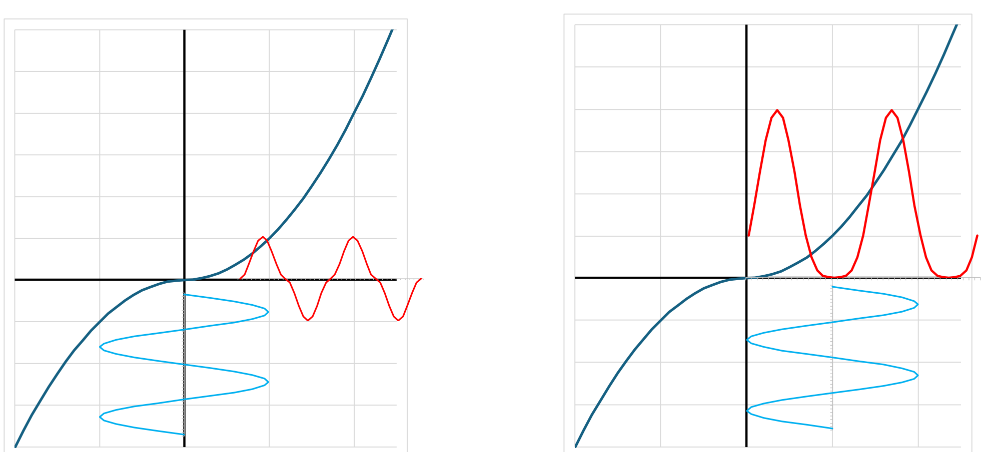

Fig. 3. Signals at input (blue) and output (red) of nonlinearity without bias (left) and with bias (right)

First, a sinusoidal signal with a peak amplitude of 1 is applied at the input and the graph of the output is given. It shows only the fundamental frequency and odd harmonics.
In a second time, a bias of amplitude 1 is added to the input signal. The input signal is always positive and never crosses the discontinuity of the nonlinear function. It is then possible to use a Taylor series development of the nonlinearity around the point (1, 1) in the range $[0, 2]$ and compute the harmonics using the classical method [1, 2] or a simulation of the nonlinearity.
The third harmonic amplitude is much lower in this case.
In the case of $s = 2$, the nonlinear curve is a branch of parabola. All its derivatives are 0 except for the first and second ones. So, this nonlinearity generates neither odd harmonics nor even harmonics higher than 2.
For slopes different from 2, some residual odd and even harmonics are generated with levels that can be computed from the derivatives of the function and its Taylor series development
It would be difficult to apply a bias and measure harmonics in a waveguide test bench. However, the same experiment can be done in a bandwidth around the fundamental frequency by replacing the signals by their envelope.
Two identical power carriers at frequencies f_1 and f_2 have a sinusoidal envelope that crosses the discontinuity of the nonlinearity at fundamental frequency. Intermodulation products replace the harmonics and can be measured in an IM waveguide test bench.
If we add a third carrier with four times the power, the envelope is always on one side of the discontinuity and the odd order IM products of the f_1 and f_2 carriers nearly disappear. In the case of $s = 2$, the nonlinear curve is a branch of parabola. All its derivatives are 0 except for the first and second ones. So, it generates neither odd IM products nor even IM products higher than 2. For slopes different from 2, some residual odd and even IM products are generated with levels that can be computed from the derivatives of the function and its Taylor series development [1, 2] or by a simulation.

Variation of power ratio between two carriers

If we vary the power ratio between two carriers while keeping the total power fixed, we see a variation of third order IM products levels along slopes equal to ± 2 or ∓ 1 versus power ratio.
This does not imply that the degree of the nonlinearity is 3.
The same slopes are obtained for the order 3 IM with a polynomial of degree 5 and with any non-analytical function in the form of (1) with any slope.
There is a small difference in the positions of the maxima, but it is not sensitive enough to determine the value of the slope or degree. The ± 2 or ∓ 1 slopes of IM level versus power ratio imply only that the measured IM is effectively of the form $2f_2 - f_1$ or $2f_1 - f_2$.

IM POWER DEPENDENCE MEASUREMENTS

Two-carrier test bench description

A 2-carrier test bench has been used to measure IM products, see Fig 4.
Low PIM triplexers combining two transmit filters and one receive filter are used to combine the carriers and to filter them out of the measured IM. They were designed and manufactured by Cobham (now Exens Solutions).
The two transmit filters bandwidths are respectively 11.34 to 11.67 GHz for f1 and 12.24 to 12.64 GHz for f2, the receive filter bandwidth is 13.79 to 14.29 GHz. The available power is up to 40 watts for each of the 2 carriers.
Frequencies f_1 in filter 1 and f_2 in filter 2 are chosen so that at least one IM3 or IM5 or IM7 frequency is in receive filter bandwidth and can be measured.

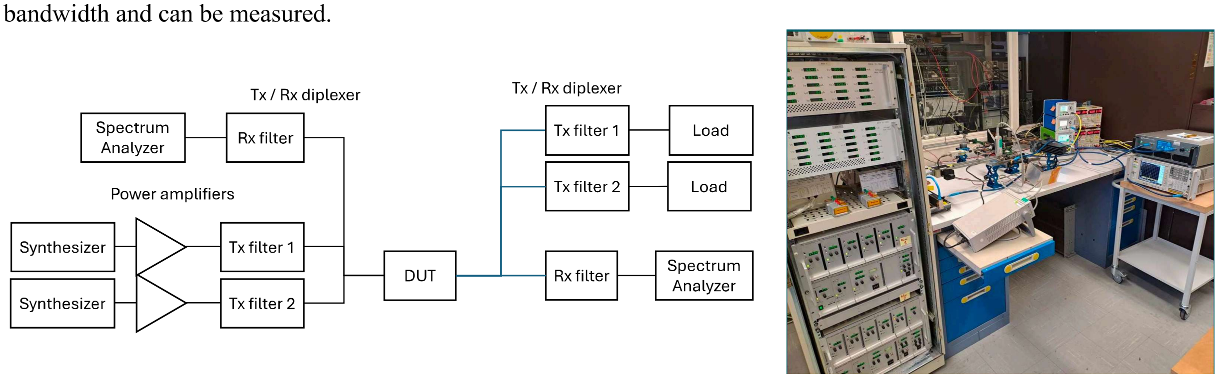

Fig. 4. Device under test in 2-carrier PIM waveguide test bench.

It is possible to measure the level of IM that is transmitted in the direction of the receive triplexer and the level of the same IM that is reflected in the direction of the transmission triplexer.

3-carrier test bench description

The 3-carrier test bench is obtained by using a waveguide hybrid 90° 3 dB coupler on one transmission chain to add two carriers. Because of this passive combination we add 3 dB losses in the chain. The third carrier is at a frequency f3 in the filter 2 bandwidth. Its frequency is chosen so that it does not generate IM products near the one between f1 and f2 that is measured.

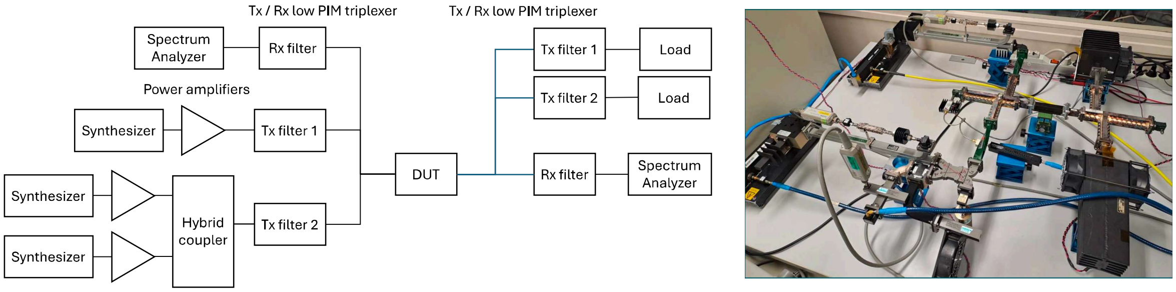

Fig. 5. Device under test in 3-carrier PIM waveguide test bench.

Device under test

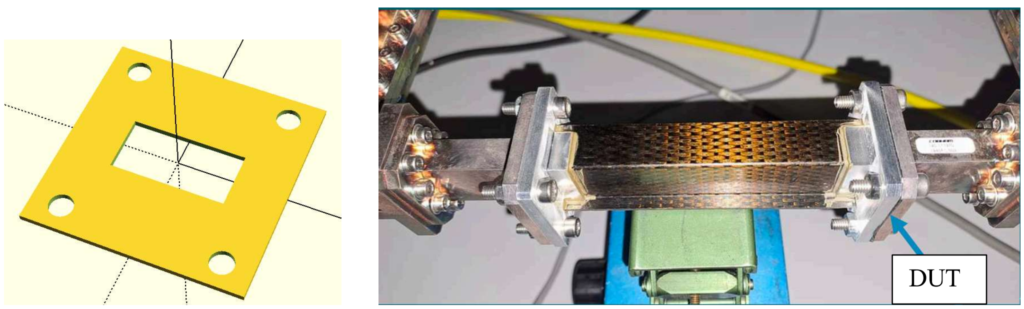

Fig. 6. Device under test.

All waveguide flanges in the test bench are low PIM 6-hole flanges used for space equipment except for the DUT.
The device under test is a gasket in the shape of the standard waveguide WR 75 flange, see fig. 6. It is cut in a graphite material (Panasonic EYGS121803DP). Two 25 microns thickness sheets are inserted between two standard WR 75 waveguides with flat flanges.
A reference low-PIM waveguide flange and a flat WR75 waveguide flange were also measured.

Measurement results

Measurements of 2-carrier PIM products of orders 3, 5, and 7

Using the 2-carrier test bench, we measure PIM products of orders 3, 5, and 7.
We use the following frequencies for the carriers and measured PIM:

Table 1. Carriers and IM frequencies for orders 3, 5 and 7.

Frequency f1 (GHz)	Frequency f2 (GHz)	PIM order	PIM frequency (GHz)
11.406	12.606	3	13.806
11.406	12.3	5	14.088
11.65	12.3	7	14.25

Both carriers have the same power, which is varied from 30 to 42 dBm.
Reference low PIM flange and WR75 flat flange give IM products levels less than -160 dBc and cannot be measured in the test bench noise.
For the DUT, we get habitual PIM slopes of 2.4 for order 3, 3.5 for order 5, and 4.5 for order 7.
Results are shown in fig. 7.

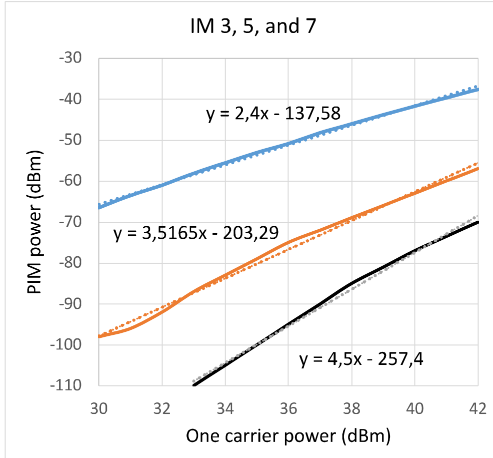

Fig. 7. 2-carrier PIM orders 3, 5, and 7.

Measurement of 3-carrier PIM products of orders 3, 5, and 7

Using the 3-carrier test bench, we add a third carrier in transmit filter Tx2. The frequency of this third carrier is: f3=12.506 GHz. Its power is varied from 5 W to 40 W while the other two carriers are kept at either 5 or 10 W.
We measure the decrease of the 2-carrier measured PIM versus third carrier power.
The results are given in table 2 and fig. 8.

Table 2. Decrease of 2-carrier IM level for orders 3, 5 and 7 when a third carrier is added.

F1 and F2 power (W)	F3 power (W)	Power ratio	IM 3 decrease (dB)	IM 5 decrease (dB)	IM 7 decrease (dB)
5	5	1	2 to 2.5	18	< noise
5	10	2	4	19.5	< noise
5	15	3	6	20	< noise
5	20	4	8	23	< noise
5	40	8	12	27	< noise
10	10	1	3	26	20
10	20	2	6	18	24
10	30	3	8	/	< noise
10	40	4	10	28	< noise

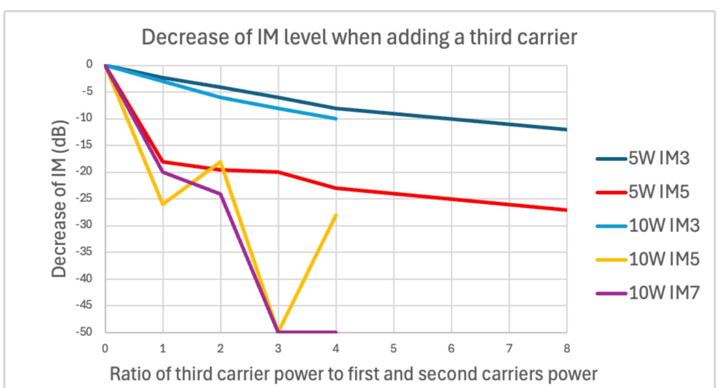

Fig. 8. Decrease of 2-carrier IM level for orders 3, 5 and 7 when a third carrier is added.

CONCLUSION

The theory of passive IM presented in this paper is based on a behavioral model that is non-analytic instead of polynomial or analytical nonlinear functions.

The proposed model is easy to fit to measurements with only two parameters, the slope and the level of one measured IM product.
It is easy to simulate as a memoryless bandpass nonlinearity with a nonlinear gain depending only on the amplitude of the envelope of the signal applied to the nonlinearity.

This model explains many differences between active and passive intermodulation behavior:

- IM products level variation versus carrier level variation with nearly constant slopes.
- High harmonics and high order IM products generation with levels high enough to be measured.
- Decrease of IM level versus order depends on the slope, between $70\log(order)$ and $90\log(order)$ for slopes between 2 and 2.5.
- Curves of 2-carrier IM3 products levels versus power ratio between the 2 carriers do not change much for slopes between 2 and 5. The slopes of 2 and -1 prove only that the frequency coefficients of the measured third order products are 2 and -1. The change is not large enough to determine the slope of the IM products levels versus carrier level.
- Decrease of third order IM product generated by 2 carriers when a third carrier is added if the slope is less than 3.
- Near elimination of this 2-carrier IM3 product if the third carrier is at 4 times the power of the first 2 ones and the slope is around 2.
- Possible relaxation of 2-carrier PIM specifications of devices that will be used in multi-carrier operation with respect to specifications obtained by using classical polynomial computation of ratio between 2-carrier and multi-carrier PIM level.

All these consequences of the theory have been validated by measurements made in CNES and by many measurements available in published papers.

Measurements show a slight increase of the slope with the increase of order instead of the exact equality of slopes that is a result of this simple model. It does not seem to change too much the consequences in other measurements.
Some on-going work is aimed at a model explaining this increase of IM slopes with order.

ACKNOWLEDGMENT

The research presented in this paper has been financed by CNES (French Space Agency) under its R&D program R-S22/TC-0007-150-01.
The test bench has been built, and measurements have been performed by CNES microwave department DTN/TP/HY in Toulouse.

REFERENCES

[1] N. M. Blachman, "Detectors, Bandpass nonlinearities and their optimization: inversion of the Chebyshev transform", IEEE Transactions on Information theory, IT-17, No. 4, July 1971, pp. 398-404
[2] R. J. Westcott, "Investigation of multiple FM/FDM carriers through a satellite TWT operating near to saturation", Proc. IEE, Vol. 114, No. 6, June 1967, pp 726-740
[3] R. C. Chapman, J. V. Rootsey, I. Poldi, and W. W. Davison: "Hidden threat – Multicarrier passive component IM generation", AIAA/CASI 6th Communications Satellite Systems Conference, Paper 76-296, Montreal, Canada, April 5-8, 1976
[4] J. Henrie, A. Christianson, W. J. Chappell, "Prediction of Passive Intermodulation from Coaxial Connectors in Microwave Networks", IEEE Trans. MTT, vol. 56, no. 1, January 2008, pp. 209-216
[5] A. Shitvov, D. Zelenchuk, A. Schuchinsky, "Carrier-power dependence of passive intermodulation products in printed lines", 2009 Loughborough Antennas and Propagation Conference, 16-17 November 2009, pp. 177-180
[6] R. Hartman, "Passive intermodulation (PIM) testing moves to the base station", Microwave Journal, May 11, 2011
[7] R. Hartman, T. Bell, "PIM Test Power Levels for Mobile Communication Systems", Microwave Journal, March 14, 2013
[8] "Final review presentation (ESA-TRP PIM Interference)", ESA Contract No.: 4000126012/18/NL/HK
[9] J. Sombrin, "Non-analytic at the origin, behavioral models for active or passive non-linearity", International Journal of Microwave and Wireless Technologies, 2013, 5(2), pp. 133–140.
[10] J. Sombrin, G. Soubercaze, I. Albert, "Relaxation of the multicarrier passive intermodulation specifications of antennas", EuCAP 2014
[11] H. Hafez et al., "Extremely efficient terahertz high-harmonic generation in graphene by hot Dirac fermions", Nature, Vol. 561, 27 September 2018, pp. 507-511, https://doi.org/10.1038/s41586-018-0508-1