MEMRISTORS AS NON-LINEAR BEHAVIORAL MODELS FOR PASSIVE INTERMODULATION SIMULATION

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I. INTRODUCTION

Leon Chua introduced memristors in 1971 [1] as an ideal two-terminal circuit element in complement to the already known three basic circuit elements: resistor, inductor and capacitor (RLC). Memristors are defined by a non-linear memristance that relates the flux (or integral of voltage across the device) to the charge (or integral of current in the device). Because of this definition the memristor will generate passive intermodulation products and their power will depend on the memory of the past current that is contained in the device.

A. Memristor equations

The memristor permits us to express a relationship between the flux φ and the charge q. This fourth two-terminal circuit element is defined by a curve relating flux to charge. The voltage across a charge-controlled memristor is given by:

$$y(t) = M(q(t)).i(t) \tag{1}$$

$$M(q) = d\varphi(q)/dq \tag{2}$$

The value M is a resistance. If its value is constant, independent of q, then the memristor is equivalent to a fixed resistor. If M varies with q, it cannot be replaced by a resistor. It will then vary as a function of the integral of the current in time, from $t = -\infty$ to the present. Its resistance depends on the past of the component; this justifies the name memristor as a contraction of memory resistor.

A memristor is a passive component if the value of $M(q)$ is non-negative: $M(q) \geq 0$. In that case, it cannot produce power. The completion of the relations between voltage, current, charge and flux by the memristor is shown on the following graph.

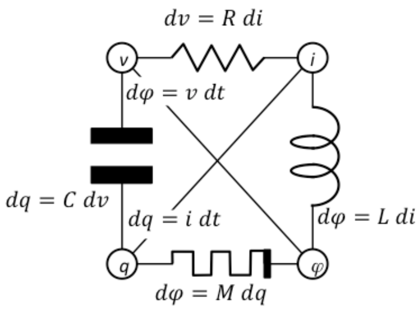

Fig. 1. Graph of relations between voltage, current, charge and flux

Curves of $M(q)$ as a function of q are generally said to be non-linear curves but it is necessary only for theses curves to be non-constant for the memristor to be different from a resistor. However $\varphi(q)$ or $q(\varphi)$ curves must be non-linear for their derivatives to be non-constant. For passive memristors, their slopes must be non-negative. The memristance is

also non-negative in the case of a passive circuit element. The curves can always be inverted and either curve can be used.
In both cases, Chua used flux - charge curves that were piecewise linear curves, such as the dashed curves in Fig. 2.
In that case, the memristance is a piecewise constant curve.

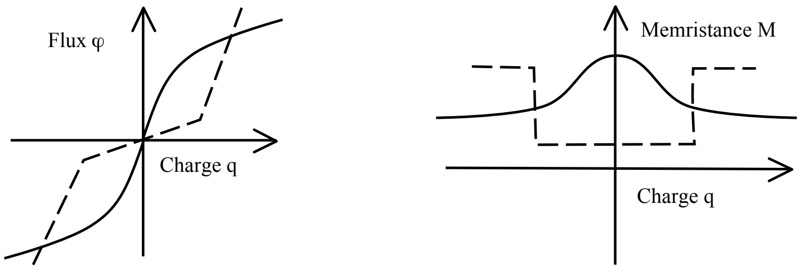

Fig. 2. Typical flux versus charge curves (a, left) and corresponding memristance versus charge curves (b, right), continuous curves (solid lines) or piecewise linear curves (dashed lines)

B. Responses of memristors

In his first paper, Leon Chua proposed to use memristors as behavioral models for “amorphous Ovonic threshold switches” and for “electrolytic E-cells”. Applications to signal processing were also proposed. Leon Chua built memristors from operational amplifiers and common RLC circuits. Responses of devices and systems using memristors were published in 1976 [2]. They show the now well known “pinched hysteresis” or “bow-tie” curves.

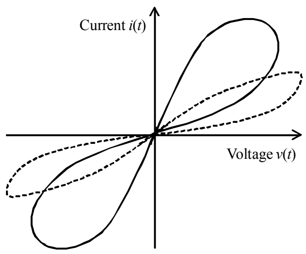

Fig. 3. $i - v$ response curve of a memristor for 2 frequencies (dash and solid)

II. PHYSICAL MODELS

Hewlett-Packard proposed some physical devices in 1998 that exhibited such curves [3, 4]. They attracted great interest particularly as possible memory elements for future computers.
HP memristors are based on the properties of some metal oxides such as TiO2 in a contact between wires. Two grids of wires are deposited at right angles. Each cross over is a memory point. Memristors explain the physical behaviour of HP devices and could also explain some passive intermodulation effects; particularly those that are observed in mesh antennas or carbon fibre antennas.

A. Other passive intermodulation physical models

Many models have been proposed for passive non-linear contacts or impedances in RF transmission lines, connections, filters and antennas. One of them is based on oxidized metals exhibiting non-linear semi-conductor contacts that depend on the pressure applied to the contact [5]. These non-linear models have also a memory of the past e.g. a capacitor in parallel to the non-linear element, see Fig. 4.

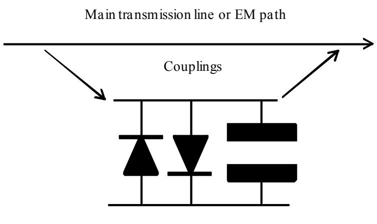

Fig. 4. Schematic of classical physical model of metal contacts

Other models depend on temperature or magnetic hysteresis and are also memory non-linearity [6, 7, 8].
The explanation of the memory and non-linearity of a contact made of TiO2 by a memristor model is quite interesting as another possible cause for some passive RF intermodulation products as it combines in a single circuit element the non-linearity and the memory.

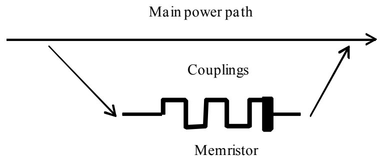

Fig. 5. Schematic of memristor model of metal contacts

TiO2 does not seem to work at microwave frequencies but other oxides have wider bandwidth. Vanadium dioxide (VO2) is non-linear at RF, THz and optic frequencies [9].
Reference [10] shows that Galena crystal, a semi-conductor used as a detector in the first radios can also work as a memristor for some positions of the metal tip on the crystal.

B. Distributed physical models

Reference [11] investigates the non-local nature of passive intermodulation generators distributed along printed transmission lines. Different physical explanations could well be distributed in a given device and they will certainly depend on the materials used, their contacts and environment.
The statistical combination of the output of many physical elements must be taken into account for a given contact, e.g. a flange contact, a connector or a bolt. A simpler behavioral model will permit us to reduce the simulation time for the great number of non-linear circuits that is necessary for a statistical approach.

C. Model fitting

It is not possible to measure directly the non-linear static (input-output current or voltage curve) response or the dynamic response curve with only one carrier (AM/AM and AM/PM curves) as it can be done on an amplifier. All the values of elements in the models are fitted from measurements of harmonics or intermodulation products at the output of the device when the input consists of two or more sinusoidal carriers.
Intermodulation products are generally very low, around 150 dB under the carrier power as the device is mainly passive and linear. Couplings of the non-linear elements to the main power path are adjusted to obtain the required output power.

III. BEHAVIORAL MODEL

In this paper, we do not examine in details all the possible physical explanations. We use the ideal memristor circuit element as a behavioural model for passive non-linearity in filters, antennas and connections.
This memristor is a much simpler memory model than physical models and it can be used for system computation of passive intermodulation behavior.
We will use continuous curves like the ones shown as solid curves in Fig. 2; they are better suited to the simulation of passive intermodulation products behavior as we have shown in [12]. They permit to simulate intermodulation power versus input power with measured slopes, which are different from the classical 3 dB/dB slope [13, 14, 15]. These curves are continuous but their second or higher order derivative is not continuous at origin. The memristance is non-negative for a passive element. It is continuous but its first or higher order derivative may be non-continuous.
This memristor behaves as a resistor of which the resistance value changes, depending on the charge that passed through the memristor since the beginning of the experiment.

A. System simulation models

Fig. 6 shows the behavioral model of the memristors used for simulations.

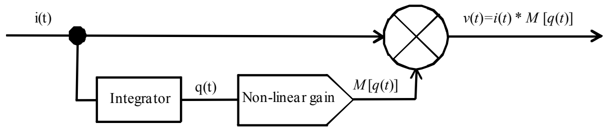

Fig. 6. Proposed behavioral model using a memristor

Integration can be performed either in the time domain or in the frequency domain by multiplying the spectrum by $1/j\omega$. The integrator is mainly a low-pass filter.
Computation of the non-linear gain and multiplication by input signal is performed in the time domain.

Replacing the integrator by a more general filter, filtering the gain and including filters in the RF path gives us a more general behavioral model. We obtain a schematic (Fig.7) similar to a typical first order dynamic Volterra model with a high frequency chain and a low frequency non-linear chain.

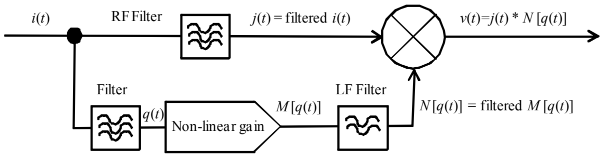

Fig. 7. Extended memristor behavioral model

Depending on the filters used, we would be able to represent memristors but also thermal effects that have been proposed as models in [6] or hysteresis effects simulated and measured in [7, 8].

IV. NON-ANALYTICAL MEMRISTOR

We represent the non-linear flux versus charge in the memristor by non-analytical at origin curves that were introduced in [12]. The memristance $M(q)$ or the gain in the behavioral model is then a function of the modulus of the input. One of the simplest models is obtained by elevating this modulus to a non-integer power. An exponent of 0.6 has been used in the non-linear gain in [12] to fit the measurements given in [15]. Passive intermodulation (PIM) products power expressed in dBm versus input power in dBm increases with a slope of 1.6 dB/dB.
The non-linear flux and memristance equations versus charge are the following ones:

$$\varphi(q) = \left(1 + \alpha \cdot |q|^{0.6}\right) \cdot q \tag{3}$$

$$M(q) = 1 + 1.6 \cdot \alpha \cdot |q|^{0.6} \tag{4}$$

The coefficient α is quite small (around 10^{-6}) and its value is adjusted to obtain the measured value of PIM power, around -100 to -150 dBc (dB with respect to carrier power).
Fig. 8 shows the non-linear part only of flux and memristance curves versus charge. Both are continuous and they are analytic everywhere except at origin. Here, only the second and higher derivatives of flux are discontinuous. The first derivative (memristance) is continuous. Memristance is non-negative. Its non-linear part is zero at origin only and is strictly positive otherwise.

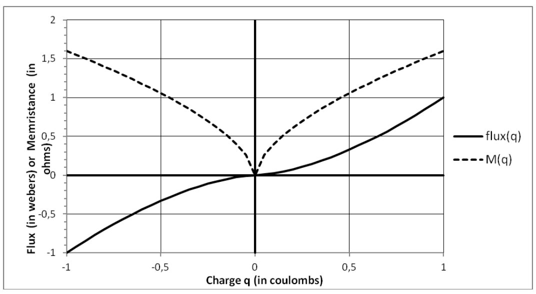

Fig. 8. Typical non-linear non-analytic part of flux versus charge curve used for a passive intermodulation model and corresponding memristance versus charge curve

Either the charge at time origin or the origin of current integration can be independently chosen. The equations linking current and voltage are the following:

$$q(t) = \int_{\tau_0}^{t} i(\tau)d\tau = q_0 + \int_0^t i(\tau)d\tau \tag{5}$$

$$v(t) = \left(1 + 1.6 \cdot \alpha \cdot \left| q_0 + \int_0^t i(\tau) d\tau \right|^{0.6}\right) \cdot i(t) \tag{6}$$

Using a random time origin (e.g. a random phase for the current) or a random integration constant, we get different shapes for the memristance and the voltage versus current response. This will change the non-linear response and the PIM products power. This may explain some of the high variability of PIM measurements.

The flux-charge curve is odd-symmetric with respect to origin. However it is applied to the integral of current, which is not symmetric because of the integration constant so that the model may generate odd and even products and harmonics.

V. RESULTS OF SIMULATION AND VALIDATION

A. Response to a sinusoidal input

Fig. 9 (left) shows a cosine input current and the corresponding charge, non-analytical memristance and voltage for some values of integration constant and memristance linear and non-linear coefficients.

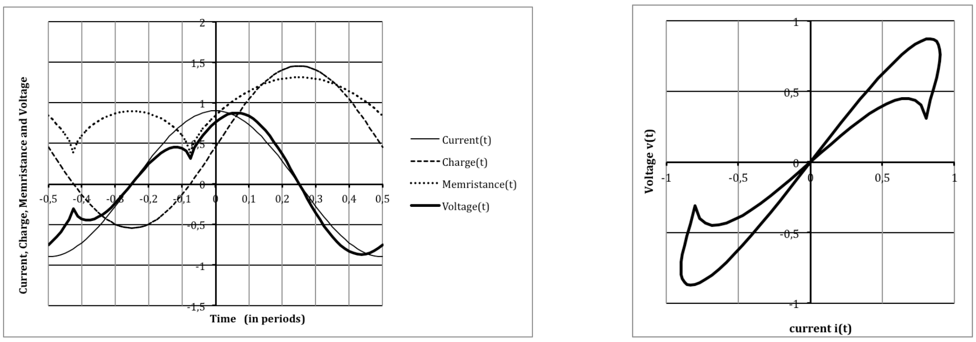

Fig. 9. Exemple of input current, charge, memristance and voltage versus time and current versus voltage response with non-null integration constant

Fig. 9 (right) shows the corresponding voltage versus current response. The "bow-tie" loop presents singularities at the points where the charge is 0 (the integration origin). Then, the memristance is minimum and has singular points.

B. Response with null integration constant

When using a null integration constant (or integrating a signal with null phase from time t=0), this singular point is at time origin where the signal is also symmetric, see Fig. 10 (right). Then, the memristance curve has the same symmetry points as the current and both up-going and down-going branches of the curve are identical. The "bow-tie" loop is reduced to a curve that retraces its steps after cusps at extreme values of input current; see Fig. 10 (right). The loop has a null area.

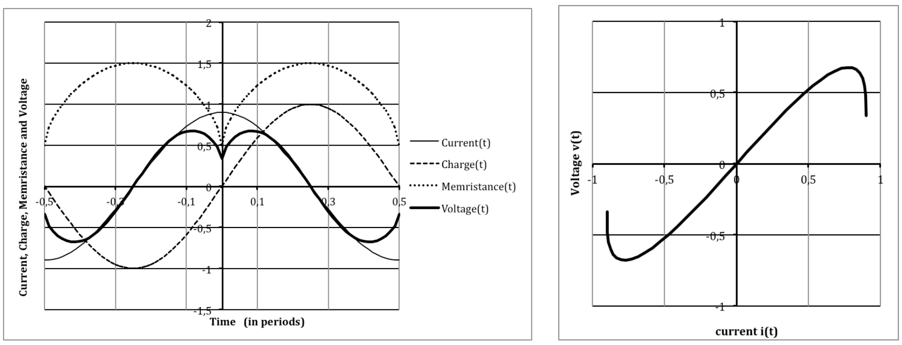

Fig. 10. Input current, charge, memristance and voltage versus time curves and current versus voltage response with null integration constant

A realizable low-pass filter in place of the integrator gives us a "bow-tie" loop similar to that of Fig. 9, even for null integration constant, at frequencies where the phase delay in the filter is different from 90°.

It gives us a curve like Fig. 10 at the frequency where the phase delay of the low-pass filter is exactly 90°.

The change in initial condition in addition to frequency has been proposed in [16] to explain the narrowing of the orbital shape or "bow-tie" loop. Only two shapes were identified. In our work, we show that infinity of shapes can be obtained and we identify the conditions for a completely closed loop and the role of the symmetry of both the signal and the integrated signal around the same point in time. We also show that initial condition or integral constant is the main (and sometimes only) parameter that governs the loop opening or narrowing. The influence of signal frequency comes mainly from the phase delay that occurs in the low-pass filter that may replace the integrator in the memristor circuit and the change in initial condition due to that delay.

C. **Harmonics in response**

Fig. 11 shows a small part of the harmonic spectrum, obtained through a Fourier transform, of the response in Fig. 9 for a given integration constant. Harmonics show the global but not always monotonic decrease that is generally obtained for levels of intermodulation products when the order increases in experiments.

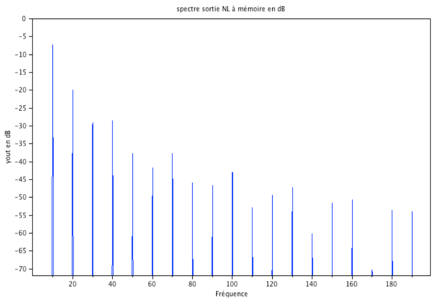

Fig. 11. Spectrum of the first 20 harmonics of voltage obtained in Fig. 9 and 10(a)
for a non-analytical memristor with non-null integration constant. Odd and even harmonics are generated.

As the non-linearity contains only odd terms, even harmonics are not present when the integration constant is null, see Fig. 12 a. Even harmonics power can be more or less important depending on this constant, see Fig 12 a to d.

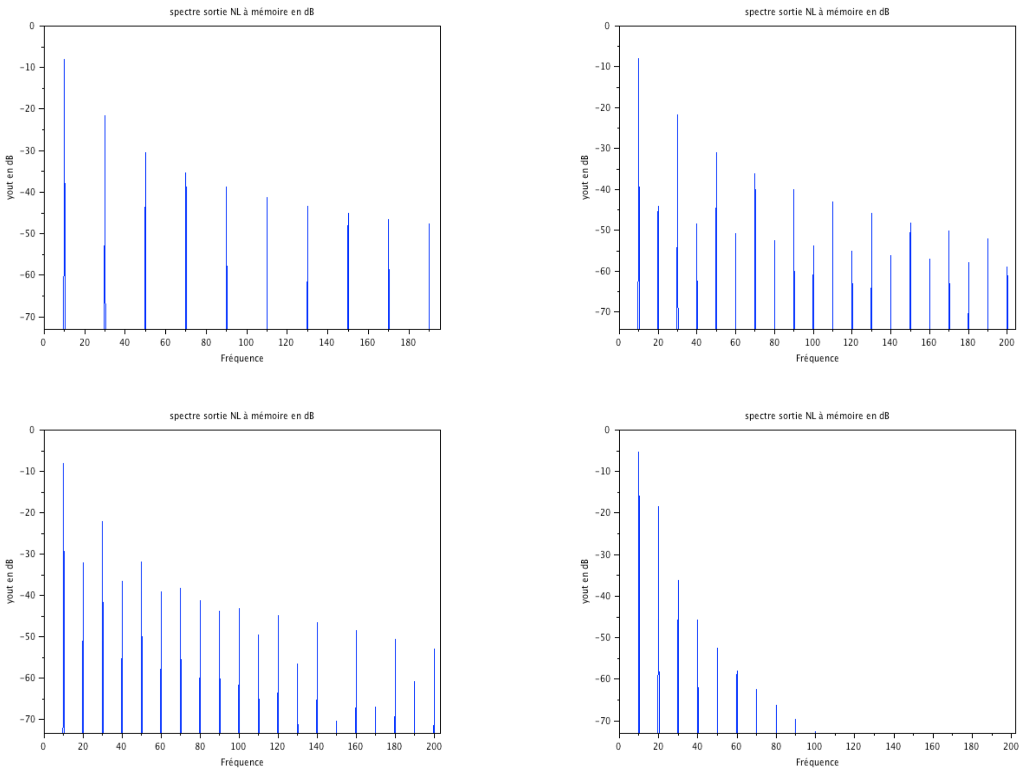

Fig. 12. Spectrum of the first 20 harmonics of voltage obtained for a non-analytical memristor
with different integration constants: 0 (a, upper left), 0.025 (b, upper right), 0.1 (c, lower left) and 1 (d, lower right)

The phase of even harmonics is also distributed depending on this integration constant and they will cancel in average when taking into account a random distribution of positive and negative integration constants.
However Fig. 12 shows that higher values of integration constant give higher values of even harmonics power at low frequencies but also lower level for all harmonics at higher frequencies. The decrease of higher frequency odd harmonics level will be kept after averaging for many different integration constants. The memristor memory acts as a filter on the higher order harmonics.

D. **Intermodulation products**

For a 2-carrier input signal, the memristor will generate intermodulation products around carriers and their harmonics.

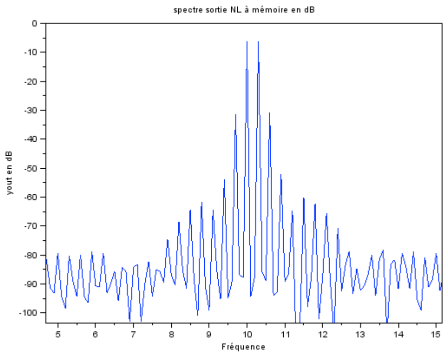

Fig. 13. Spectrum of intermodulation products generated around fundamental signal by a non-analytical memristor model for 2 carriers at 10 and 10.3 GHz.

Fig. 13 shows a zoom of the spectrum around the first harmonic or fundamental signal for 2 carriers at 10 and 10.3 GHz. Because of the closeness of both frequencies, the effect of memory is small; the levels of left and right products are nearly the same.

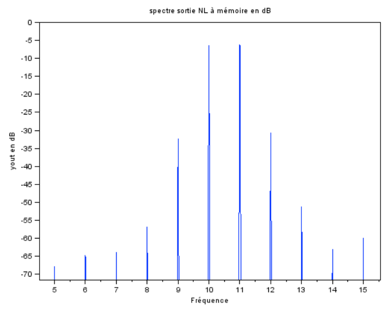

Fig. 14. Spectrum of intermodulation products generated around fundamental signal by a non-analytical memristor model for 2 carriers at 10 and 11 GHz.

Fig. 14 shows the output spectrum for a 10% bandwidth input signal. The classical effect of memory is more important for a larger frequency bandwidth. Carrier levels are nearly equal. Third order PIM levels differ by 1.5 dB and fifth order levels by 6 dB.
The effect on intermodulation products around harmonics varies also with integration constant as can be seen on Fig. 15. Even order intermodulation products levels increase then higher order intermodulation products levels decrease.

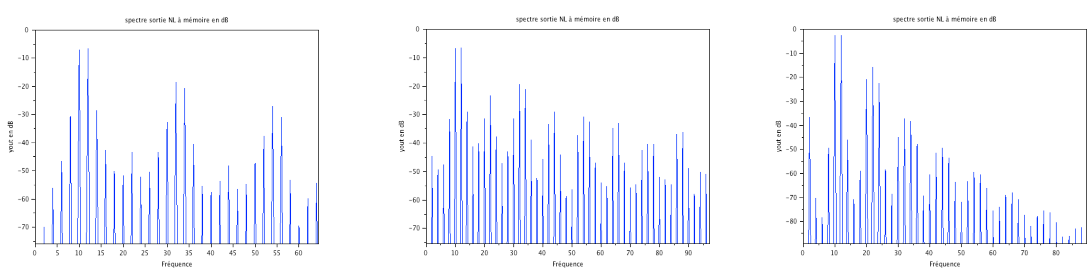

Fig. 15. Spectrum of intermodulation products around fundamental signal and first harmonics for different values of integration constant: 0, 1 and 2.

E. Experimental validation

Simulations results obtained up to now are in good agreement with published experimental results. We keep the non-integer slopes in dB/dB for PIM product levels as a function of input carrier level that have been reported by many authors [8, 9, 13-15] and simulated in [12].
More focused experimental validation is underway and will be presented at the conference.

VI. CONCLUSION

Simulation results obtained with memristor behavioral models of passive non-linearity are in good agreement with published experimental results.
In addition, a generalized behavioral model is easily applicable to other physical effects (thermal and hysteresis) and the memristor could be a physical model for oxidized contacts.
Behavioral models are also computationally efficient and useful for the study of effects coming from the statistical distribution of non-linear generators on the material and the statistical distribution of integration constants or random signal phases in the memory device.

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