Channel Estimation and Equalization for CPM with Application for Aeronautical Communications via a Satellite Link

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***Abstract*—In this paper, we present a generalized polyphase representation for Continuous Phase Modulation (CPM) signals suited to the detection over frequency-selective channels. We first develop two different equalizers based on this representation and relate them to the State of Art. We also derive a Least Squares (LS) channel estimation and an improved LS estimation using a priori on the channel. Simulation results show the equivalence between existing equalizers and also show that our channel estimation leads only to a small degradation in term of Bit Error Rate (BER) in the case of an aeronautical communication over a satellite link.**

I. Introduction

CPM signals are commonly known for their good spectral efficiency and for their constant envelope, a useful feature for embedded amplifiers enabling robustness to non-linearities. They are considered for a wide range of applications as military communication, aeronautical communication by satellite or Machine to Machine applications.
To our knowledge, only a few works consider channel equalization for CPM. The optimal technique consists in a Maximum A Priori Detection (MAP) for joint channel equalization and detection with complexity scaling exponentially with both channel and CPM signal memories, yielding to a prohibitive complexity.
Hence, most proposed solutions consider the separation of channel equalization and CPM detection. Several papers have investigated CPM equalization based on the *Minimum Mean-Squared Error* (MMSE) criterion. First, [1] presents different fractionally spaced MMSE-based equalizers, such as linear block MMSE, a Block-Decision Feedback Equalizer and the corresponding turbo-equalizer. All proposed equalization schemes were derived in the frequency domain capitalizing on the Laurent decomposition. More recently, [2] also develops a slightly different block MMSE-based equalizer structure in the frequency domain using the Laurent decomposition. The main differences will be emphasized in the following. A completely different approach has been proposed by [3]. It chose to work on the over-sampled received signal and presents different filter-based equalizers both in the time and frequency-domain using an MMSE criterion, applying the turbo principle based on soft linear filtering as proposed by Tuchler [4]. [5] presents two different symbol-based equalizers based on a orthogonal representation of the signal or based on the received signal filtered by the matched filters of Laurent decomposition. [6] uses equalizer based on Basis-function. It seems that two orthogonal basis functions are sufficient to represent the signal and perform frequency-domain MMSE equalization like the one in [5] as it considers an orthogonal representation of the signal.
Generally, all those works have been done under the hypothesis of perfect channel knowledge and perfect carrier recovery. Only [2] presents simulation results with channel estimation errors and [6], [7] perform a frequency-domain channel estimation with interpolation (using B-spline functions). To our knowledge, several works deals with channel estimation for CPM. [8] performs frequency-domain channel estimation with superimposed pilots. [9] has developed a joint channel estimation and carrier recovery for M-ary CPM schemes over frequency-selective channels.

In this paper, we develop a generalized polyphase model for circular block-based CPM signals. Based on this general representation, which mainly consists in the generalization of the models used in [2] and [1], we then derive the different equalizer structures used in the literature, showing their equivalence up to a linear transformation in some cases or emphasizing their difference when operating at different sampling rate. Then, LS channel estimation will be derived for the proposed model for both parametric and non-parametric channel model.

The paper is organized as follows. In section II, we derive our polyphase representation for any oversampling factor. Then, in section III, block MMSE equalizers are derived for several models of the existing literature and their similitude, differences and limits are pointed out. In section IV, we will present a LS channel estimation based on this polyphase model with or without a parametric channel model. We will provide some simulation results with an emphasis on the aeronautical channel via a satellite link in section V. Finally, conclusions are drawn in section VI.

II. Polyphase Representation of Circular Block-Based CPM

We consider a block of N coded symbols taken in the M-ary alphabet denoted $\{\alpha_n\}_{0\leq n\leq N-1} \in \{\pm 1, \pm 3, \ldots, \pm M-1\}^N$. The equivalent baseband complex envelope $s_b(t)$ of the transmitted CPM signal is:

$$s_b(t) = \sqrt{\frac{2E_s}{T_s}} \exp\left(j\theta(t, \boldsymbol{\alpha})\right) \tag{1}$$

$$\text{where } \theta(t, \boldsymbol{\alpha}) = \pi h \sum_{i=0}^{N-1} \alpha_i q(t - iT_s) \tag{2}$$

$$\text{and } q(t) = \begin{cases} \int_0^t g(\tau)d\tau, t \leq L_{\text{cpm}} \\ 1/2, t > L_{\text{cpm}} \end{cases} \tag{3}$$

E_s is the symbol energy, T_s is the symbol duration, $\theta(t, \boldsymbol{\alpha})$ is the information phase, $g(t)$ is the frequency pulse, $h = k/p$ is the modulation index with k and p relative prime number and L_{cpm} is the CPM memory. For channel equalization and CPM signal detection, we will consider the Laurent decomposition presented in [10] for binary CPM signals. This decomposition has been then extended in [11] for M-ary CPM signals. Without loss of generality and for ease of presentation, we only consider binary CPMs in this paper, but results can be extented to the non binary case. This decomposition allows us to describe the CPM signal as a sum of P linear Pulse Amplitude Modulation (PAM) $\{l_p(t)\}$ (also called *Laurent Pulses* LP) with complex pseudo-symbols $b_{p,n}$:

$$s(t) = \sum_{p=0}^{P-1} \sum_{n=0}^{N_T-1} b_{p,n} l_p(t - nT_s) \tag{4}$$

It will be useful to derive a model that allows frequency domain equalization, enabling significant gain in complexity at the receiver. To perform frequency domain equalization, we first need to *circularize* the channel, as for linear modulation, enabling the efficient use of FFT operators at the receiver. To do so, we can use several methods such as a Cyclic Prefix (CP) or a known Unique Word (UW) (also called Training Sequence). We suppose the use of UW despite its loss of spectral efficiency compared to CP. This is mainly motivated by the fact that UW can be used to perform parameters estimation as carrier and phase frequency, channel estimation... [12]. It seems important to recall that, due to the CPM memory, some termination symbols must be added at the end of the data block in order to ensure the phase continuity and the uniqueness of the UW. Furthermore, the length of a UW must be larger than the time dispersion of the channel to avoid interference between CPM blocks (as for linear modulations).

In order to differentiate a matrix and a vector, a vector will be represented by an underlined letter ($\underline{\boldsymbol{v}}$) and a matrix by a doubly underlined letter ($\underline{\underline{\boldsymbol{m}}}$). We note $\underline{\underline{\boldsymbol{F}}}_N$ the Fourier matrix of size $N \times N$ which corresponds to a FFT of size N. $\underline{\underline{\boldsymbol{F}}}_{N,M}$ is a block diagonal matrix of size $MN \times MN$ with each block is $\underline{\underline{\boldsymbol{F}}}_N$. The matrix $\underline{\underline{\boldsymbol{I}}}_N$ is the identity matrix of size $N \times N$. We define a circulant matrix as:

$$[\underline{\dot{\boldsymbol{x}}}]_{(n,m)} = [\underline{\boldsymbol{x}}]_{\left(\text{mod}(n-m+1,N),1\right)} \tag{5}$$

The polyphase components of a signal are noted as, for a sampling rate of kR_s where R_s is the symbol rate:

$$x_m^i = x(mT_s + \frac{i}{k}T_s) = x[km + i] \tag{6}$$

We consider a transmission over a frequency-selective channel with impulse response $h_c = \sum_{l=0}^{L-1} a_l \delta(t - \tau_l)$ where L is the number of paths, τ_l and a_l are the delay and the complex attenuation of the l^{th} path. Contrary to [5], there is no hypothesis on the delays τ_l.
Our received signal goes trough a low-pass filter $\Psi(t)$ (assumed here as ideal). Denoting $h(t) = (\Psi * h_c)(t)$, the received signal can be written as:

$$r(t) = h * s(t) + w(t) \tag{7}$$
$$= \sum_m s[m] h\big(t - m\frac{T_s}{k}\big) + w(t) \tag{8}$$

Sampling at rate kR_s, ie. at $t = (kn + i)\frac{T_s}{k}$ with $i \in \{0, 1, \ldots, k-1\}$, the sampled signal $r_n^i \doteq r[kn + i]$ reads as:

$$r_n^i = \sum_m s[m] h[kn + i - m] + w[kn + i] \tag{9}$$

Hence, the polyphase component i can be written as:

$$\begin{aligned} r_n^i = &\sum_{j=0}^{i-1} \sum_m s_m^j h_{n-m}^{i-j} + \sum_m s_m^i h_{n_m}^0 \\ &+ \sum_{j=i+1}^{k-1} \sum_m s_m^j h_{n-m-1}^{k+i-j} + w_n^i \end{aligned} \tag{10}$$

We now define the following vectors and matrices:

$$[\underline{\boldsymbol{l}}_p^i]_{(n,m)} = l_p\Big((n-m)T_s + \frac{i}{k}T_s\Big) \tag{11}$$

$$[\underline{\boldsymbol{h}}^i]_{(n,m)} = h\Big((n-m)T_s + \frac{i}{k}T_s\Big) \tag{12}$$

$$[\underline{\boldsymbol{h}}]_{(n,m)}^{i'} = h\Big((n-m-1)T_s + \frac{i}{k}T_s\Big) \tag{13}$$

$$\underline{\underline{h}} \doteq \begin{bmatrix} \underline{\underline{\dot{h}}}^{0} & \underline{\underline{\dot{h}}}^{(k-1)'} & \dots & \underline{\underline{\dot{h}}}^{1'} \\ \underline{\underline{\dot{h}}}^{1} & \underline{\underline{\dot{h}}}^{0} & \ddots & \vdots \\ \vdots & \ddots & \ddots & \underline{\underline{\dot{h}}}^{(k-1)'} \\ \underline{\underline{\dot{h}}}^{k-1} & \dots & \underline{\underline{\dot{h}}}^{1} & \underline{\underline{\dot{h}}}^{0} \end{bmatrix} \tag{14}$$

$$\text{et } \underline{\underline{l}} \doteq \begin{bmatrix} \underline{\underline{\dot{l}}}_0^{0} & \underline{\underline{\dot{l}}}_1^{0} & \dots & \underline{\underline{\dot{l}}}_{P-1}^{0} \\ \underline{\underline{\dot{l}}}_0^{1} & \underline{\underline{\dot{l}}}_1^{1} & \dots & \underline{\underline{\dot{l}}}_{P-1}^{1} \\ \vdots & \vdots & \vdots & \vdots \\ \underline{\underline{\dot{l}}}_0^{k-1} & \underline{\underline{\dot{l}}}_1^{k-1} & \dots & \underline{\underline{\dot{l}}}_{P-1}^{k-1} \end{bmatrix} \tag{15}$$

We can remark that $\underline{\underline{h}}$ is block Toeplitz matrix and each block is a circulant matrix. Similarly, the blocks of the matrix $\underline{\underline{l}}$ are also circulant matrices. We now obtain a matrix-wise representation of the received signal:

$$\underline{\boldsymbol{r}} = \underline{\underline{\boldsymbol{h}}}\underline{\underline{\boldsymbol{l}}}\underline{\boldsymbol{b}} + \underline{\boldsymbol{w}} \tag{16}$$

In the specific case where $k = 2$, we obtain the same model as [2]. In the Frequency-Domain, we have the following matrix-wise representation:

$$\underline{\boldsymbol{R}} = \underline{\underline{\boldsymbol{F}}}_{N,k}\underline{\boldsymbol{r}} = \underline{\underline{\boldsymbol{H}}}\,\underline{\underline{\boldsymbol{L}}}\,\underline{\boldsymbol{B}} + \underline{\boldsymbol{W}} \tag{17}$$

$$= \underbrace{\underline{\underline{\boldsymbol{F}}}_{N,k}\underline{\underline{\boldsymbol{h}}}\underline{\underline{\boldsymbol{F}}}^H_{N,k}}_{\doteq \underline{\underline{\boldsymbol{H}}}}\underbrace{\underline{\underline{\boldsymbol{F}}}_{N,k}\underline{\underline{\boldsymbol{l}}}\underline{\underline{\boldsymbol{F}}}^H_{N,P}}_{\doteq \underline{\underline{\boldsymbol{L}}}}\underbrace{\underline{\underline{\boldsymbol{F}}}_{N,P}\underline{\boldsymbol{b}}}_{\doteq \underline{\boldsymbol{B}}} + \underbrace{\underline{\underline{\boldsymbol{F}}}_{N,k}\underline{\boldsymbol{w}}}_{\doteq \underline{\boldsymbol{W}}} \tag{18}$$

Note that by FFT property, the blocks of $\underline{\underline{\boldsymbol{H}}}$ and $\underline{\underline{\boldsymbol{L}}}$ are diagonal.

III. CPM Equalization in the Frequency Domain

The optimal approach consists in performing jointly the channel equalization and the detection of the transmitted sequence. However, due to the non-linearity of CPMs, the complexity of this approach is prohibitive. A conventional choice is to perform separately the channel equalization and the CPM detection. We derive here equalizers that are based on the MMSE criterion. After equalization, we will use the MAP detector developed by [13].

A. Channel MMSE Equalizer

This first approach consist in considering only the channel contribution given by $\underline{\underline{\boldsymbol{H}}}$. Hence, by defining $\underline{\boldsymbol{S}} = \underline{\underline{\boldsymbol{L}}}\underline{\boldsymbol{B}}$, $\underline{\underline{\boldsymbol{R}}}_{\text{SS}} = \underline{\underline{\boldsymbol{L}}}\underline{\underline{\boldsymbol{R}}}_{\text{BB}}\underline{\underline{\boldsymbol{L}}}^H$ and $\underline{\underline{\boldsymbol{K}}} = \underline{\underline{\boldsymbol{H}}}\underline{\underline{\boldsymbol{R}}}_{\text{SS}}\underline{\underline{\boldsymbol{H}}}^H + N_0\underline{\underline{\boldsymbol{I}}}_{2N}$, the MMSE equalizer is given by:

$$\underline{\underline{\boldsymbol{G}}}_{\text{MMSE}} = \underline{\underline{\boldsymbol{R}}}_{\text{SS}}\underline{\underline{\boldsymbol{H}}}^H\underline{\underline{\boldsymbol{K}}}^{-1} \tag{19}$$

We can note the correlation matrix $\underline{\underline{\boldsymbol{R}}}_{\text{BB}}$ of the pseudo-symbols vector $\underline{\boldsymbol{B}}$ and the correlation matrix $\underline{\underline{\boldsymbol{R}}}_{\text{SS}}$ of the vector $\underline{\boldsymbol{S}}$ can be precomputed using [10]. This can be extended to the M-ary case. Then the equalized signal is

$$\widehat{\underline{\boldsymbol{S}}} = \underline{\underline{\boldsymbol{G}}}_{\text{MMSE}}\underline{\boldsymbol{R}} \tag{20}$$

We can note that this equalizer extend the one in [2], which as been developed for a sampling rate of $2R_s$. After equalization, $\widehat{\underline{\boldsymbol{S}}}$ is filtered by the oversampled matched filters of the Laurent components, this operation is equivalent to multiply $\widehat{\underline{\boldsymbol{S}}}$ by $\underline{\underline{\boldsymbol{L}}}^H$.

B. Channel and Laurent Pulse MMSE Equalizer

In [1], by analyzing the general polyphase representation, it turns out that the derived structure perform joint equalization of both the channel (given by $\underline{\underline{\boldsymbol{H}}}$) and the Laurent pulses (given by $\underline{\underline{\boldsymbol{L}}}$): each filter linked to the PAM representation is jointly considered with the channel. This equalizer has been only developed for sampling rate of $2R_s$. Here, we propose to extend this equalizer to a sampling rate of kR_s (k is an integer) in order to study it in the same condition as the previous one. Hence, using the generalized polyphase representation, denoting $\underline{\underline{\boldsymbol{P}}} = \underline{\underline{\boldsymbol{H}}}\,\underline{\underline{\boldsymbol{L}}}$, the equalizer is given by:

$$\underline{\underline{\boldsymbol{D}}}_{LE} = \underline{\underline{\boldsymbol{R}}}_{\text{BB}}\underline{\underline{\boldsymbol{P}}}^H[\underline{\underline{\boldsymbol{P}}}\underline{\underline{\boldsymbol{R}}}_{\text{BB}}\underline{\underline{\boldsymbol{P}}}^H + N_0\underline{\underline{\boldsymbol{I}}}_{2N}]^{-1} \tag{21}$$

Hence, the equalized pseudo-symbols are $\widehat{\underline{\boldsymbol{B}}} = \underline{\underline{\boldsymbol{D}}}_{LE}\underline{\boldsymbol{R}}$. The main disadvantage of this equalizer is the used of a non-conventional detector. Indeed, the equalized pseudo-symbols $\widehat{\underline{\boldsymbol{B}}}$ cannot be used in a conventional trellis-based MAP detector as [13] and we have to use the modified detector introduced by [1] and based on [14]. By analogy with the previous equalizer, we can derive a quite simple yet efficient method to compensate for this disadvantage: we can reconstruct the emitted signal using the equalized pseudo-symbols $\widetilde{\underline{\boldsymbol{S}}} = \underline{\underline{\boldsymbol{L}}}\widehat{\underline{\boldsymbol{B}}}$. Indeed, we can remark that

$$\widetilde{\underline{\boldsymbol{S}}} = \underline{\underline{\boldsymbol{L}}}\widehat{\underline{\boldsymbol{B}}} = \widehat{\underline{\boldsymbol{S}}} \tag{22}$$

$$\text{as } \underline{\underline{\boldsymbol{G}}}_{\text{MMSE}} = \underline{\underline{\boldsymbol{L}}}\underline{\underline{\boldsymbol{D}}}_{LE} \tag{23}$$

Hence, by reconstructing the signal, we show that both equalizers are finally strictly equivalent when used with a conventional detector if proper processing is done on the equalized pseudo-symbols.

C. Tan and Stüber's MMSE Equalizer

In [5], the authors propose a symbol rate equalizer based on the received signal which has been filtered by the matched filters of the LPs. Hence, in this case, the signal to equalize using the generalized polyphase representation can be written as:

$$\underline{\boldsymbol{r}}_{\text{f}} = \underline{\underline{\boldsymbol{l}}}^H\underline{\boldsymbol{r}} = \underline{\underline{\boldsymbol{l}}}^H\underline{\underline{\boldsymbol{h}}}\underline{\underline{\boldsymbol{l}}}\underline{\boldsymbol{b}} + \underbrace{\underline{\underline{\boldsymbol{l}}}^H\underline{\boldsymbol{w}}}_{\doteq \widetilde{\underline{\boldsymbol{w}}}} \tag{24}$$

In the frequency domain, we have:

$$\underline{\boldsymbol{R}}_{\text{f}} = \underline{\underline{\boldsymbol{L}}}^H\underline{\underline{\boldsymbol{H}}}\,\underline{\underline{\boldsymbol{L}}}\,\underline{\boldsymbol{B}} + \widetilde{\underline{\boldsymbol{W}}} \tag{25}$$

In their hypothesis, the authors consider that all the channel paths have a delay which is multiple of T_s. Hence, all the polyphase component but the first one are equals to $\underline{\underline{\boldsymbol{0}}}$ and so the matrix $\underline{\underline{\boldsymbol{H}}}$ is diagonal $\underline{\underline{\boldsymbol{H}}} = \text{diag}(\underline{\underline{\boldsymbol{H}}}^{\boldsymbol{0}})$. For ease of presentation and without loss of generality, let us take the case where $k = 2$ (we sample the received signal at $2R_s$). The output $[\underline{\boldsymbol{R}}_{\text{f}}]_k$ of the k^{th} filter is:

$$[\underline{\boldsymbol{R}}_{\text{f}}]_k = \sum_{j=0}^{P-1}\Big(\underline{\underline{\boldsymbol{L}}}_k^{0H}(\underline{\underline{\boldsymbol{L}}}_j^{0H}\underline{\underline{\boldsymbol{H}}}^0 + \underline{\underline{\boldsymbol{L}}}_j^{1}\underline{\underline{\boldsymbol{H}}}^{1'}) + \underline{\underline{\boldsymbol{L}}}_k^{1H}(\underline{\underline{\boldsymbol{L}}}_j^{0}\underline{\underline{\boldsymbol{H}}}^1 + \underline{\underline{\boldsymbol{L}}}_j^{1}\underline{\underline{\boldsymbol{H}}}^0)\Big)\underline{\boldsymbol{B}}_j \tag{26}$$

With the hypothesis on the delays of the frequency-selective channel, we obtain $\underline{\underline{h}}^1 = \underline{\underline{h}}^{1'} = \underline{\underline{0}}$. Hence, our system can be written as:

$$[\underline{\boldsymbol{R}}_{\text{f}}]_k = \sum_{j=0}^{P-1} \left(\underline{\underline{\boldsymbol{L}}}_k^{0H} \underline{\underline{\boldsymbol{L}}}_j^0 \underline{\underline{\boldsymbol{H}}}^0 + \underline{\underline{\boldsymbol{L}}}_k^{1H} \underline{\underline{\boldsymbol{L}}}_j^1 \underline{\underline{\boldsymbol{H}}}^0 \right) \underline{\boldsymbol{B}}_j \tag{27}$$

$$= \underline{\underline{\boldsymbol{H}}}^0 \sum_{j=0}^{P-1} \left(\underline{\underline{\boldsymbol{L}}}_k^{0H} \underline{\underline{\boldsymbol{L}}}_j^0 + \underline{\underline{\boldsymbol{L}}}_k^{1H} \underline{\underline{\boldsymbol{L}}}_j^1 \right) \underline{\boldsymbol{B}}_j \tag{28}$$

$$\text{and so } \underline{\boldsymbol{R}} = \text{diag}(\underline{\underline{\boldsymbol{H}}}^0) \underline{\underline{\boldsymbol{L}}}^H \underline{\underline{\boldsymbol{L}}} \underline{\boldsymbol{B}} + \widetilde{\underline{\boldsymbol{W}}} \tag{29}$$

We obtain the same system as in [5]. The equalizer consists in fact in P parallel diagonal equalizer. The k^{ith} coefficient of the p^{ith} equalizer is:

$$W_{k,p} = \frac{\underline{\underline{\boldsymbol{H}}}[k]^*}{|\underline{\underline{\boldsymbol{H}}}[k]|^2 + N_0 C(p,p;k)} \tag{30}$$

The function $C(p,p;k)$ is the inter-correlation function of the pseudo-symbols. We can remark that this model does not hold when a delay is fractional of T_s as the interference of the second polyphase component of the channel are not taken into account. This is a major drawback of this solution.

IV. Channel Estimation

Several papers consider channel estimation. As we have presented some equalizers under an unified and generalized polyphase model, we will use this model to develop some polyphase channel estimators. In order to perform such estimation, we consider a known sequence of J symbols at the receiver. Hence, we can define a slightly different matrix-wise representation of the received signal:

$$\underline{\boldsymbol{h}} \doteq \begin{bmatrix} \underline{\boldsymbol{h}}^{0T} & \underline{\boldsymbol{h}}^{1T} & \ldots & \underline{\boldsymbol{h}}^{k-1T} \end{bmatrix}^T \tag{31}$$

$$\text{and } \underline{\underline{\boldsymbol{s}}} \doteq \begin{bmatrix} \dot{\underline{\underline{\boldsymbol{s}}}}^0 & \dot{\underline{\underline{\boldsymbol{s}}}}^{(k-1)'} & \ldots & \dot{\underline{\underline{\boldsymbol{s}}}}^{1'} \\ \dot{\underline{\underline{\boldsymbol{s}}}}^1 & \dot{\underline{\underline{\boldsymbol{s}}}}^0 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \dot{\underline{\underline{\boldsymbol{s}}}}^{(k-1)'} \\ \dot{\underline{\underline{\boldsymbol{s}}}}^{k-1} & \ldots & \dot{\underline{\underline{\boldsymbol{s}}}}^1 & \dot{\underline{\underline{\boldsymbol{s}}}}^0 \end{bmatrix} \tag{32}$$

where the matrix $\underline{\underline{\boldsymbol{s}}}$ is of size $kJ \times kJ$ and the vector $\underline{\boldsymbol{h}}$ is the channel vector of length kJ to estimate. Thus, our received signal is:

$$\underline{\boldsymbol{r}} = \underline{\underline{\boldsymbol{s}}}\underline{\boldsymbol{h}} + \underline{\boldsymbol{w}} \tag{33}$$

As we work with block-based CPM, we can notice that in the case where our channel impulse channel has a length larger than the size of our UW, the model does not work (we have inter-block interference). Hence, the maximum length of our estimated channel can be chosen equal to the size of the UW. The size of the block-matrix can so be reduced to $N \times L_{\text{UW}}$. One can also choose a different estimated channel length L_h as long as $L_h < J$. In this paper, we choose $L_h = L_{\text{UW}}$.

A. Least Squares Estimation

A standard approach to estimate the propagation channel in the case of linear modulations is to perform a LS estimation. As the noise in our system model is a white Gaussian noise, the LS estimate of the channel $\underline{\boldsymbol{h}}$ is known as:

$$\widehat{\underline{\boldsymbol{h}}}_{\text{LS}} = (\underline{\underline{\boldsymbol{s}}}^H \underline{\underline{\boldsymbol{s}}})^{-1} \underline{\underline{\boldsymbol{s}}}^H \underline{\boldsymbol{r}} \tag{34}$$

The main difference between this estimation and the classical one for linear modulations is the sampling rate which has to be superior to $2R_s$. The LS operator needs a matrix inversion where the square matrix to inverse is of size $kL_h \times kL_h$. However, it can be pre-computed at the receiver side and so the operator will only consist in a matrix multiplication between a matrix of size $kL_h \times kJ$ and a vector of size kJ.

B. Parametric Least Squares Estimation

We can improve our approach by performing jointly the estimation of the attenuations and the delays of the different path. Such method has been already considered for linear modulations in [15] by using a Parametric Model and is called *Structured LS estimation*. In this paper, we will follow the same approach to improve the previous LS channel estimation. We define the vector $\underline{\boldsymbol{\tau}} = [\tau_0, \ldots, \tau_{L-1}]$ which contains the delay of the different paths and $\underline{\boldsymbol{a}} = [a_0, \ldots, a_{L-1}]^H$ the complex attenuations. For $0 \leq l \leq L-1$, we define the following vector of size N:

$$[\underline{\boldsymbol{\Psi}}^i(\tau_l)]_n = \Psi(nT_s + i\frac{T_s}{k} - \tau_l) \tag{35}$$

In this case, in order to introduce a parametric dependence on the delays, we can introduce the matrix $\underline{\underline{\boldsymbol{P}}}(\underline{\boldsymbol{\tau}})$ of size $kN \times L$ such as:

$$\underline{\underline{\boldsymbol{P}}}(\underline{\boldsymbol{\tau}}) = \begin{bmatrix} \underline{\boldsymbol{\Psi}}^0(\tau_0)^H & \ldots & \underline{\boldsymbol{\Psi}}^0(\tau_{L-1})^H \\ \underline{\boldsymbol{\Psi}}^1(\tau_0)^H & \ldots & \underline{\boldsymbol{\Psi}}^1(\tau_{L-1})^H \\ \vdots & \vdots & \vdots \\ \underline{\boldsymbol{\Psi}}^{k-1}(\tau_0)^H & \ldots & \underline{\boldsymbol{\Psi}}^{k-1}(\tau_{L-1})^H \end{bmatrix} \tag{36}$$

$$\text{and so } \underline{\boldsymbol{h}} = \underline{\underline{\boldsymbol{P}}}(\underline{\boldsymbol{\tau}})\underline{\boldsymbol{a}} \tag{37}$$

Now, by defining $\underline{\underline{\boldsymbol{s}}}_P \doteq \underline{\underline{\boldsymbol{s}}}\underline{\underline{\boldsymbol{P}}}$, our system using this parametric model can be written as

$$\underline{\boldsymbol{r}} = \underline{\underline{\boldsymbol{s}}}\underline{\boldsymbol{h}} + \underline{\boldsymbol{w}} \tag{38}$$

$$= \underline{\underline{\boldsymbol{s}}}_P \underline{\boldsymbol{a}} + \underline{\boldsymbol{w}} \tag{39}$$

In the case of linear modulations, some papers already deal with the estimation of the delays. Some may estimate $\underline{\boldsymbol{\tau}}$ using a ML estimation or a sphere detection [16]–[18]. This problem will not be addressed here and therefore the delays are assumed known at the receiver.

For the aeronautical channel via a satellite link, this vector can be assumed known at the receiver by geometrical consideration given by GPS positioning. Indeed, we can compute the delays with a great accuracy using geometrical consideration as shown in [15]. Hence, our parametric LS channel estimation can be reduced to:

$$\widehat{\underline{\boldsymbol{a}}} = (\underline{\underline{\boldsymbol{s}}}_P^H \underline{\underline{\boldsymbol{s}}}_P)^{-1} \underline{\underline{\boldsymbol{s}}}_P^H \underline{\boldsymbol{r}} \tag{40}$$

$$\text{and } \widehat{\underline{\boldsymbol{h}}} = \underline{\underline{\boldsymbol{P}}}\underline{\boldsymbol{a}} \tag{41}$$

Using this parametric LS operator, we need to inverse the square matrix $\underline{\underline{\boldsymbol{s}}}_P^H \underline{\underline{\boldsymbol{s}}}_P$ of size $L \times L$. The complexity of this operator is low compared to the LS estimation presented in the previous section for any sparse channel such as $L \leq kL_h$. Indeed, we will only estimate a vector of L values instead of a vector of size kL_h. The gain of such method will increase with the sampling factor k. However, we will need to compute this operator as soon as the vector $\underline{\boldsymbol{\tau}}$ changes significantly. In the case of a aeronautical sparse channel where $L = 2$, we only inverse a square matrix of size 2×2.

V. Results

For simulations, we choose two binary CPM schemes with a raised-cosine pulse shape (noted *RC*), a memory of $L_{\text{cpm}} = 3$ and a modulation index $h \in \{\frac{1}{4}, \frac{1}{2}\}$. The block of received signal (with termination and Unique Word included) has a length of 512 symbols and the UW has a length of 16 symbols. We will consider only a transmission over the aeronautical channel via a satellite link. This frequency-selective channel can be modelled with two paths. The received signal is sampled at $2R_s$. In a first subsection, we will show the equivalence between the different equalizers presented in section 3. Then, we will present some results with channel estimation.

A. Equivalence and difference between equalizers

We present in Fig 1 the uncoded BER obtained by simulation for a generic frequency-selective channel . We use the channel (chan 1) proposed in [5]. However, instead of considering delays which are multiples of T_s, we consider the same Power Delay Profile with delays multiples of $T_s/2$. No difference can be seen for the channel and joint channel and LPs equalizers as shown theoretically for both CPM schemes as theoretically proved in this paper. In order to be as much complete as possible, we have also implemented the joint channel and LPs equalizer under the system model of [1] and there is no difference of performance. We can see that the Tan and Stüber's equalizer exhibits a floor as it does not take into account all the interference between signal components due to the frequency-selective channel.

B. Results with channel estimation

In this part, we will consider the 3-RC with $h_{\text{cpm}} = \frac{1}{2}$ CPM signals for the uncoded case. In Fig 2, we show the Mean Square Error (MSE) for channel estimation for different size of training sequence and also with or without a priori information on the second path delay. With no surprise, we can see that the use of a priori information outperforms the classical Least Square estimation. Also, we remark that we obtain performance similar to [7] for the classical Least Squares estimate. When we double the number of symbols in the training sequence, we obtain a gain of almost 6dB which

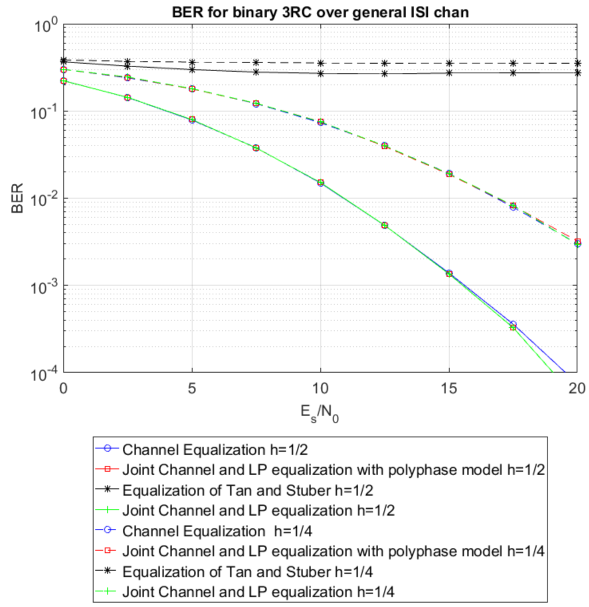

Fig. 1. Uncoded BER over general ISI channel

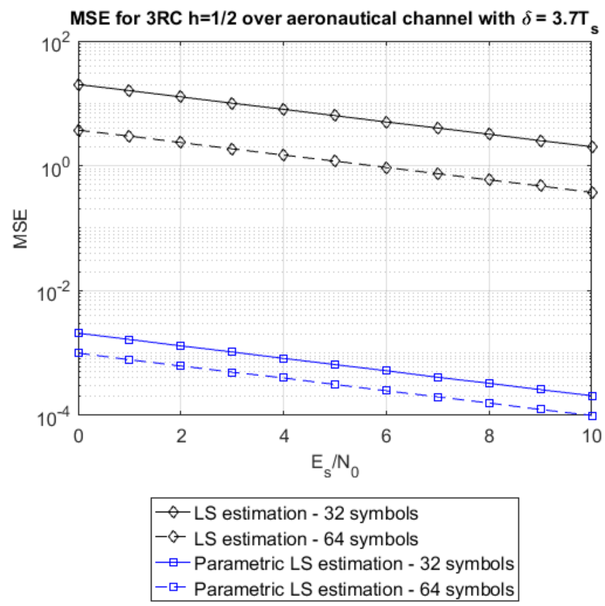

Fig. 2. Channel estimation MSE over the aeronautical channel

seems obvious as for the same numbers of path attenuation to be estimated, we have four times more observations. However, with the structured LS, the gain is only of 3 dB. Fig 3 shows the Bit Error Rate for the channel equalizer with channel estimation. We can see that with the structured Least Squares estimation, our receiver shows a degradation less than 0.5dB. Furthermore, the performance with a classical Least Squares estimation on 64 symbols presents a degradation of 2dB for a BER of 10^{-3}. Finally, Fig 4 shows some results with turbo-detection for the 3-RC CPM with $h = 1/4$. Prior to the CPM modulator, the information bits are encoded using a rate one-half convolutional code with octal representations $(5, 7)_8$

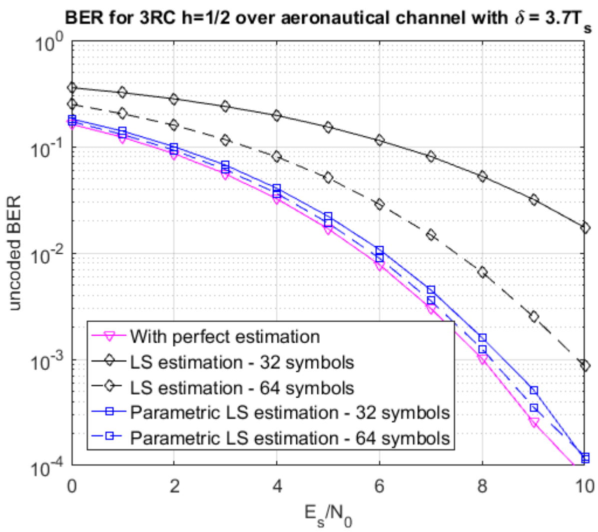

Fig. 3. Uncoded BER over the aeronautical channel with channel estimation

leading to codewords of size 1170. Random interleaving is applied between the channel encoder and the CPM modulator. We then divide these codewords into 5 subframes for which we add a termination and a UW of 16 symbols and of total size 256. We also consider three cases for channel estimation: perfect knowledge, structured LS estimation with 32 or 64 symbols. At a BER of 10^{-3}, the channel estimation over 64 symbols introduces a degradation of 0.75dB. However, with a channel estimation over 32 symbols, the error introduces is more important (degradation of around 1.5dB at a BER of 10^{-2}).

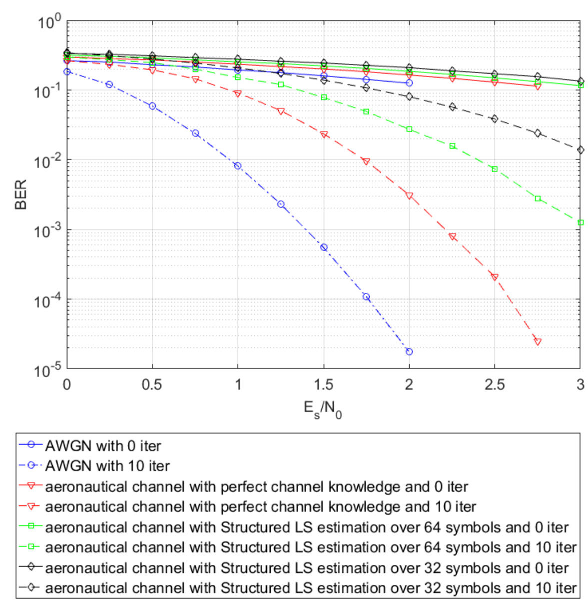

Fig. 4. BER over aeronautical channel with channel estimation and turbo-detection

VI. Conclusion

In this paper, we have presented a generalized polyphase representation of CPM signals. It allowed us to rederive different equalizers of the literature to point out their equivalence and differences. We also used this representation to develop a new Least Squares Channel Estimation which can be enhanced when parametric estimation is possible as in the case of the aeronautical channel over a satellite link. Future works will deal with carrier-recovery for transmission over frequency-selective channels and non-linear equalization schemes.

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