Robust Statistics for GNSS Positioning

Robust GNSS Day @TéSA

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Institute of Communications and Navigation

Outline

1 GNSS Positioning

- Working Principle

2 Basics on Robust Statistics

- First Notions
- Robust Estimators

3 Robust GNSS Positioning

4 Future Lines of Research

DLR

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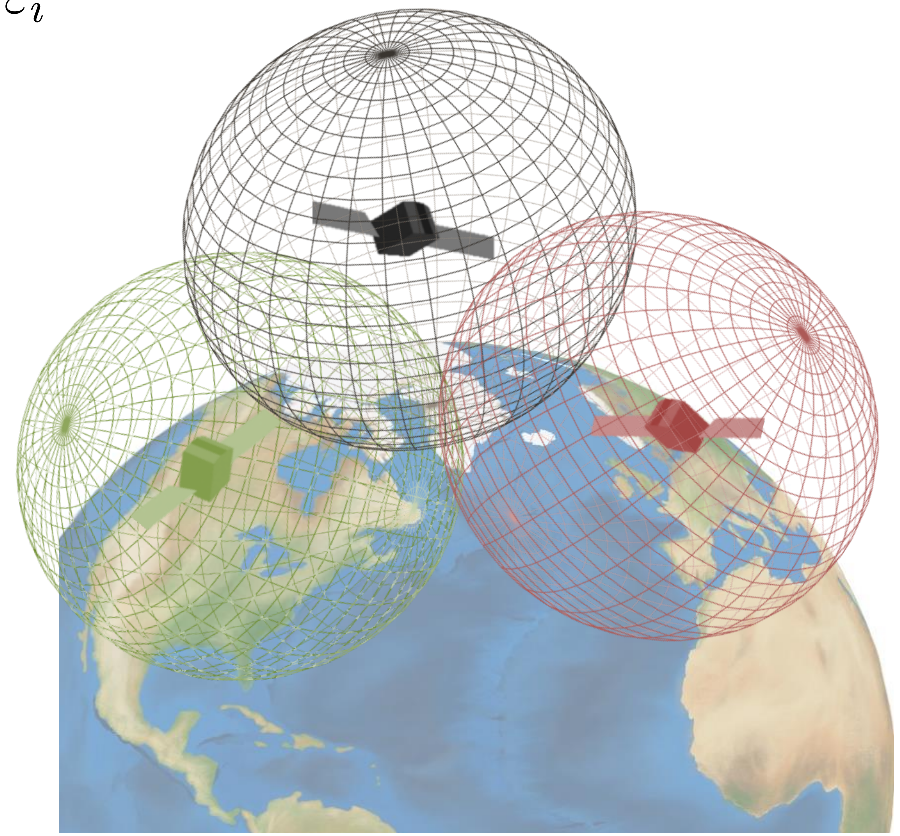
Working Principle

- GNSS positioning consists in solving a geometric problem from the measured ranges to the visible satellites

$$R_i = \|\mathbf{p}_i - \mathbf{p}\|^2 + c\left(\delta t - \delta t_i\right) + I_i + Tr_i + \varepsilon_i$$

$$\mathbf{y} = h(\mathbf{x}) + \boldsymbol{\varepsilon}, \quad \mathbf{x} = \left[\mathbf{p}^\top, c\delta t\right]^\top$$



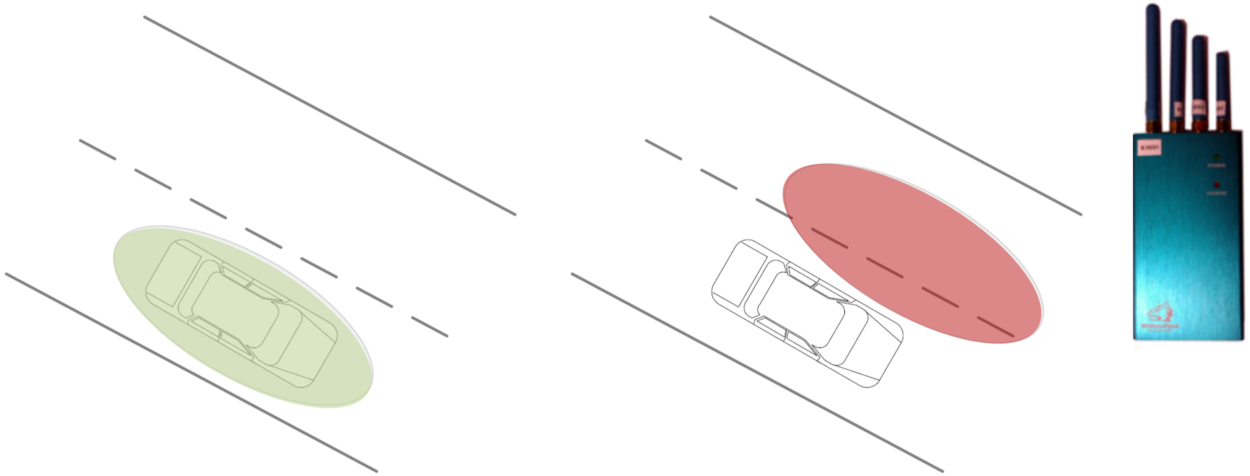
Depending on the equipment and the correction services:

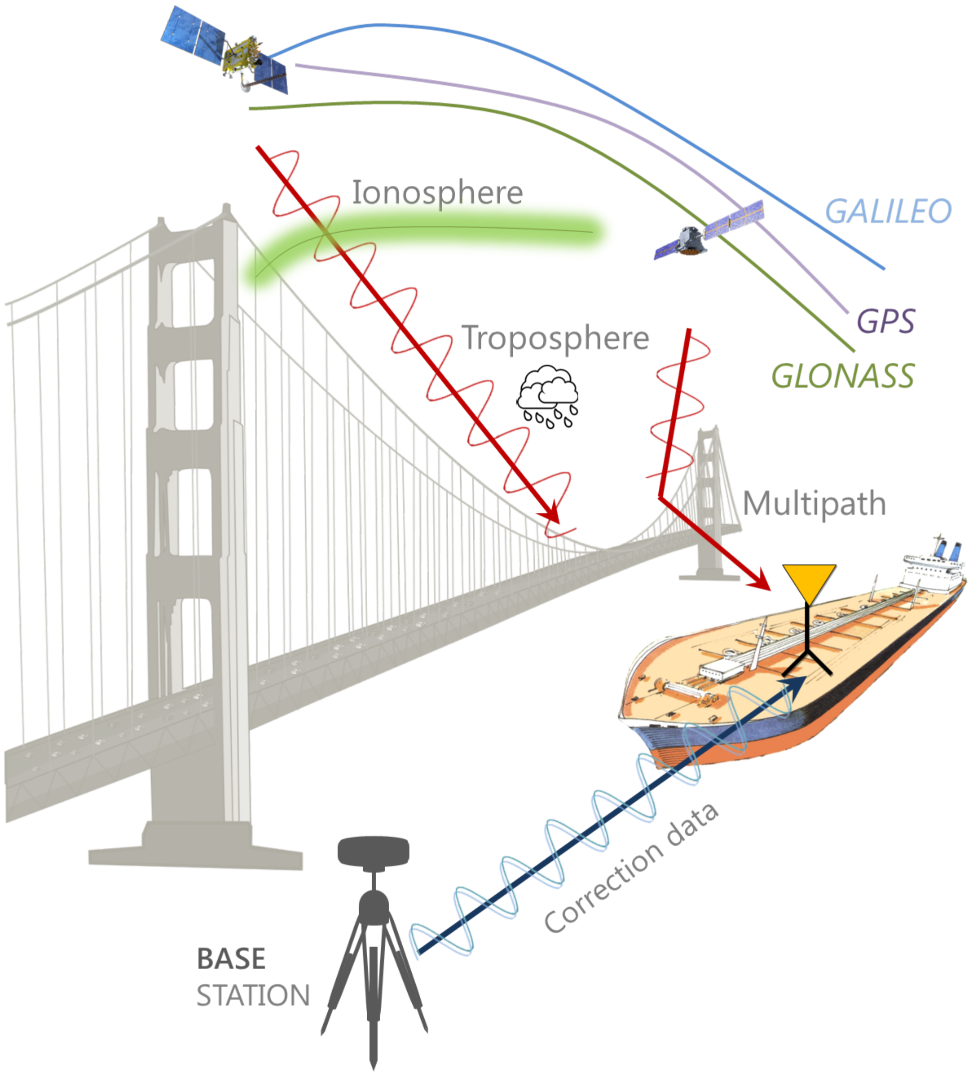
- **S**ingle **P**oint **P**ositioning (SPP)
- **P**recise **P**oint **P**ositioning (PPP)
- **R**eal-**t**ime **K**inematic (RTK)

All navigation techniques have in common:
- The assumption of Gaussian distributed noise
- The use of **L**east **S**quares (**LS**) adjustment

GNSS Challenges

- The performance of satellite – based navigation can be easily disturbed due to space weather events
- External threats (jamming or spoofing) represent a major security concern
- Multipath and none-line-of-sight are the most prominent error during navigation in cities





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Motivation

- Classical Estimation Methods are designed under the Gaussian assumption
  - Relatively easy to derive
  - Optimal when the assumption is hold *exactly*

But… what happens with the Gaussian assumption fails?

- Heavy-tailed noise have been shown in data collection across multiple fields
- The effect of a single *outlier* is unbounded → estimation can be completely spoiled!

Motivation

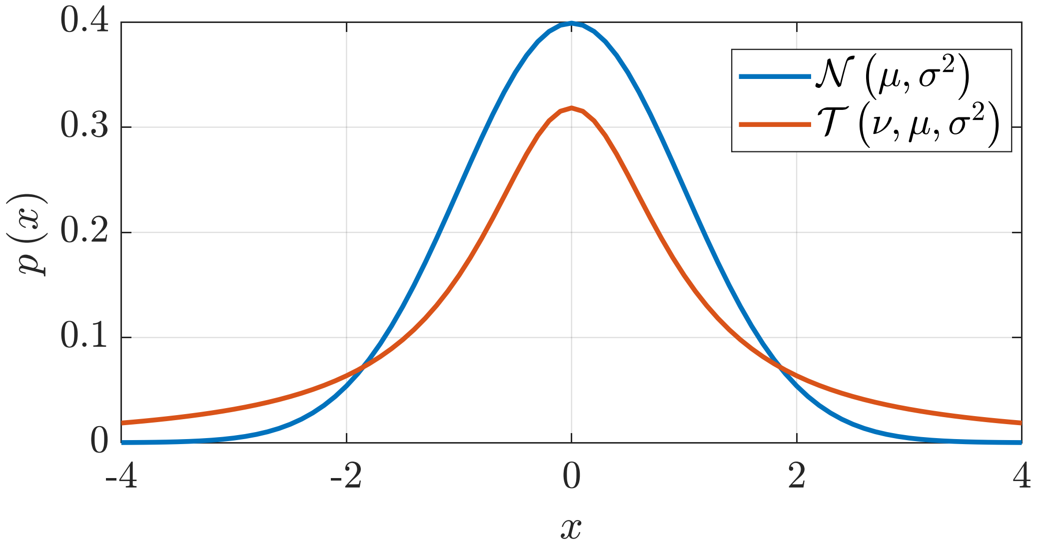
- Robust Statistics aim at deriving estimators which are ***[Maronna19]***:
  - *nearly* optimal when the Gaussian assumption holds
  - *nearly* optimal under heavy-tailed/contaminated distributions

- There are different ways to express noise distributions:

Ideal / exact distribution

$$\varepsilon \sim \mathcal{N}\left(\mu, \sigma^2\right)$$

Heavy-Tailed distribution



Normal Mixture distribution

$$\varepsilon \sim (1-\epsilon)\, G + \epsilon\, H$$

$$G = \mathcal{N}\left(\mu, \sigma^2\right)$$

$$H = ???$$

Robust Statistics Dictionary

- *Robustness* ***[Hampel85, Huber09]*** → Capability of an estimator that:
  i. Does not suffer a large impact under the presence of an erroneous observation, even if it takes an arbitrary value
  ii. Remains without catastrophic effects, even when larger deviations from the model occur

- *Breakdown point* ϵ^* ***[Hampel71]*** → the smallest percentage of contamination that can cause an estimator to take on arbitrarily large aberrant values.

- *Inlier / Outlier* → healthy observations / observations that are well separated from the majority of the data

- *Relative Efficiency* → performance similarity of a method wrt. an optimal method (e.g., the LS) under nominal normal-distributed noise

Working Principle

- Let us consider a linear regression problem:

$$y_i = x_i^\top \boldsymbol{\beta} + \varepsilon_i, \qquad \mathbf{r} = \mathbf{y} - \mathbf{x}^\top \boldsymbol{\beta}$$

Classical Least Squares

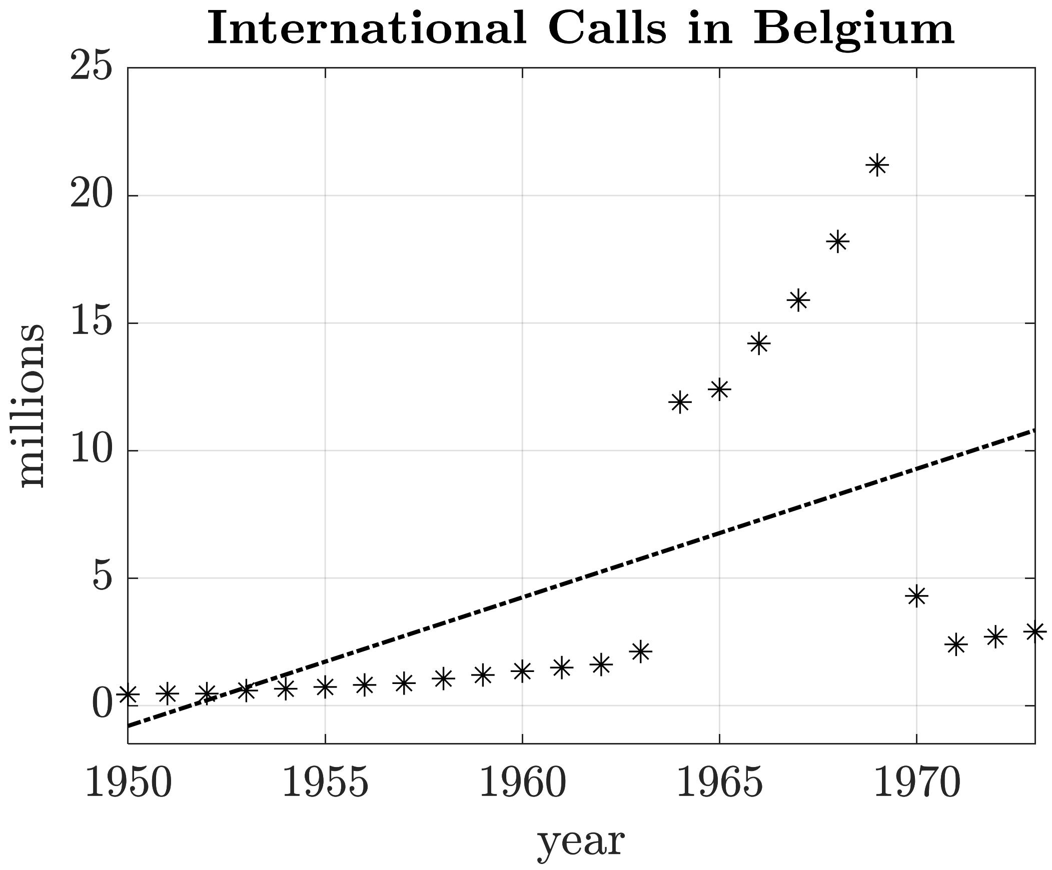
$$\hat{\boldsymbol{\beta}}_{LS} = \arg\min_{\boldsymbol{\beta}} \|\mathbf{y} - \mathbf{x}^\top \boldsymbol{\beta}\|^2 \Rightarrow \arg\min_{\boldsymbol{\beta}} \sum_{i=1}^{n} (r_i(\boldsymbol{\beta}))^2$$

Let's call this *loss function* $\rho_{LS}(\bullet) = r^2$

Least Absolute Deviation

$$\hat{\boldsymbol{\beta}}_{\ell_1} = \arg\min_{\boldsymbol{\beta}} \sum_{i=1}^{n} \left| \frac{r_i(\boldsymbol{\beta})}{\sigma_i} \right|$$

$$\rho_{\ell_1}(\bullet) = |r|$$



[Rousseeuw84]

Robust Estimators (I)

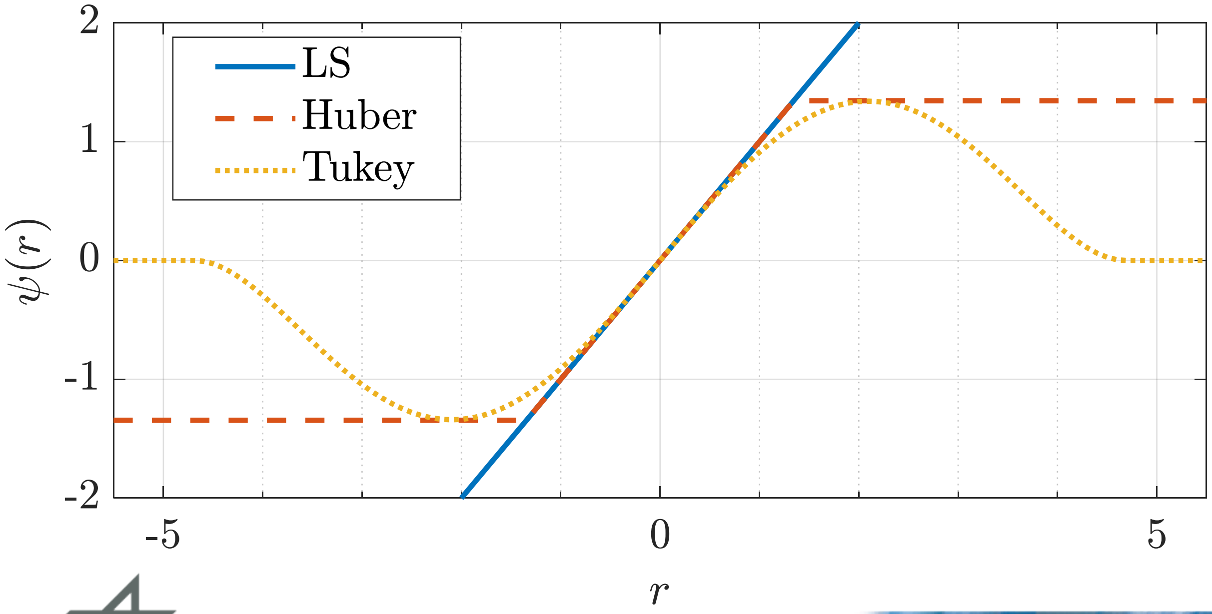
M estimation

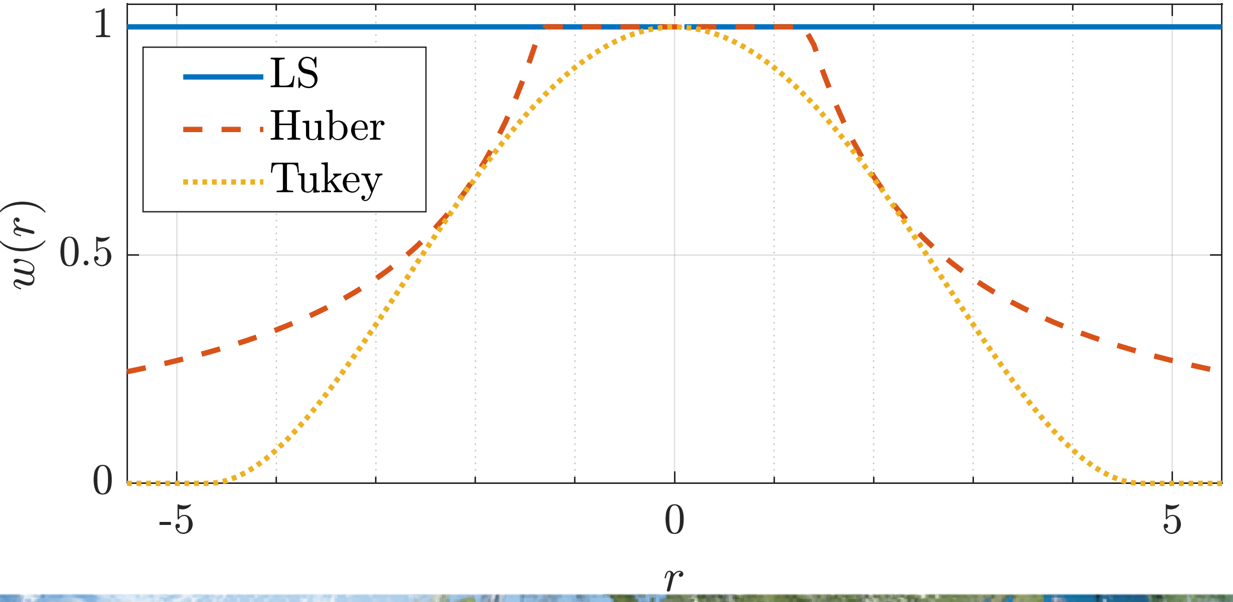
- *[Huber73, Huber81]* proposed replacing the original loss functions for other that bound the influence of contaminated observations

$$\hat{\boldsymbol{\beta}}_M = \arg\min_{\boldsymbol{\beta}} \sum_{i=1}^{N} \rho\left(r_i(\boldsymbol{\beta})\right)$$

$$\textit{influence function} \rightarrow \psi\left(x\right) = \frac{\partial \rho(x)}{\partial x}$$

$$\textit{weighting function} \rightarrow w\left(x\right) = \psi(x)/x$$





Robust Estimators (I)

M estimation

$$\hat{\boldsymbol{\beta}}_M = \arg\min_{\boldsymbol{\beta}} \sum_{i=1}^{N} \rho\left(r_i(\boldsymbol{\beta})\right)$$

$$\textit{influence function} \rightarrow \psi\left(x\right) = \frac{\partial \rho(x)}{\partial x}$$

$$\textit{weighting function} \rightarrow w\left(x\right) = \psi(x)/x$$

How do I solve this…??

$$\sum_{i=1}^{n} \psi\left(r_i\left(\boldsymbol{\beta}\right)\right) \frac{\partial r_i\left(\boldsymbol{\beta}\right)}{\partial \boldsymbol{\beta}} = 0$$

$$\sum_{i=1}^{n} w\left(r_i\left(\boldsymbol{\beta}\right)\right) r_i\left(\boldsymbol{\beta}\right) \frac{\partial r_i\left(\boldsymbol{\beta}\right)}{\partial \boldsymbol{\beta}} = 0$$

This is exactly a weighted least squares!!
The robust estimation turns the problem →
Iteratively **R**eweighted **L**east **S**quares (**IRLS**)

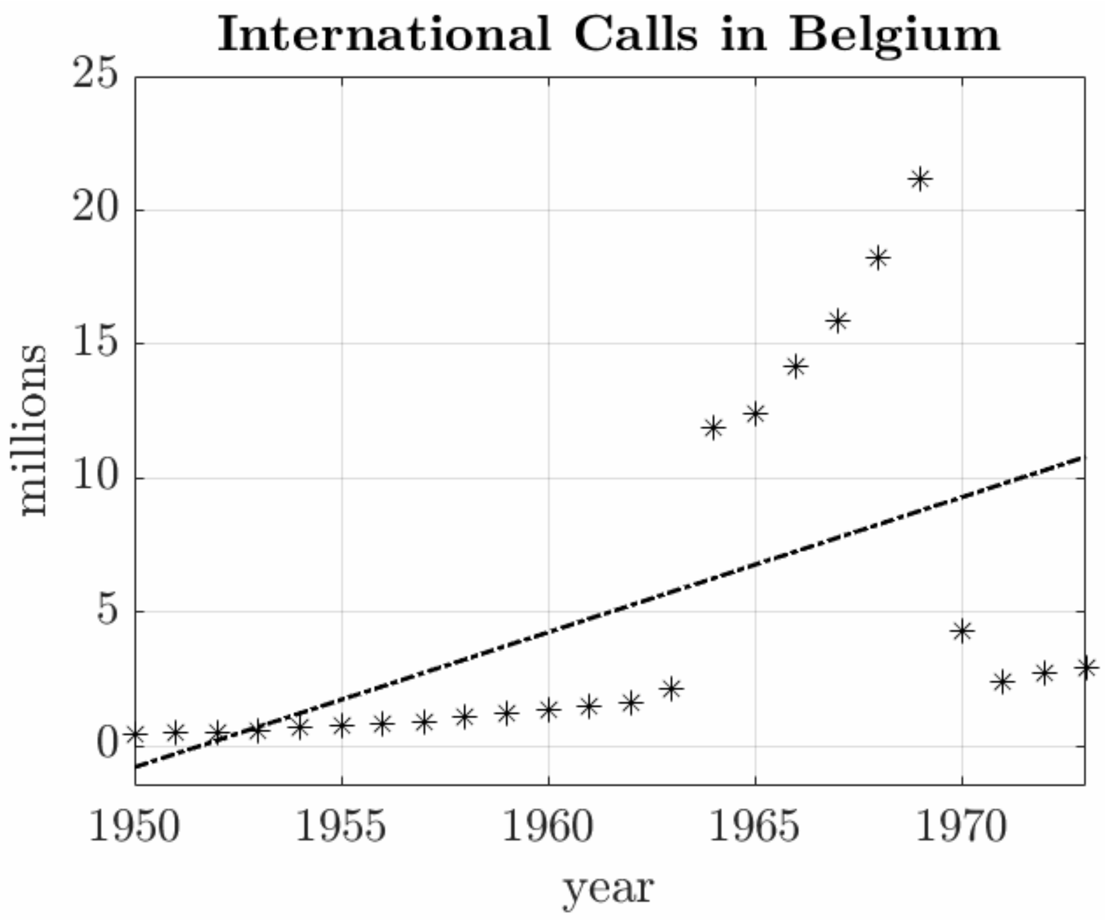
Robust Estimators (I)

M estimation

$$\hat{\boldsymbol{\beta}}_M = \arg\min_{\boldsymbol{\beta}} \sum_{i=1}^{N} \rho\left(r_i(\boldsymbol{\beta})\right)$$

$$\sum_{i=1}^{n} \psi\left(r_i\left(\boldsymbol{\beta}\right)\right) \frac{\partial r_i\left(\boldsymbol{\beta}\right)}{\partial \boldsymbol{\beta}} = 0$$

$$\sum_{i=1}^{n} w\left(r_i\left(\boldsymbol{\beta}\right)\right) r_i\left(\boldsymbol{\beta}\right) \frac{\partial r_i\left(\boldsymbol{\beta}\right)}{\partial \boldsymbol{\beta}} = 0$$



Robust Estimators (II)

S estimator

- *[Rousseeuw84, Croux94, Salibian06]* intend to minimize the robust scale (or dispersion) of the residuals

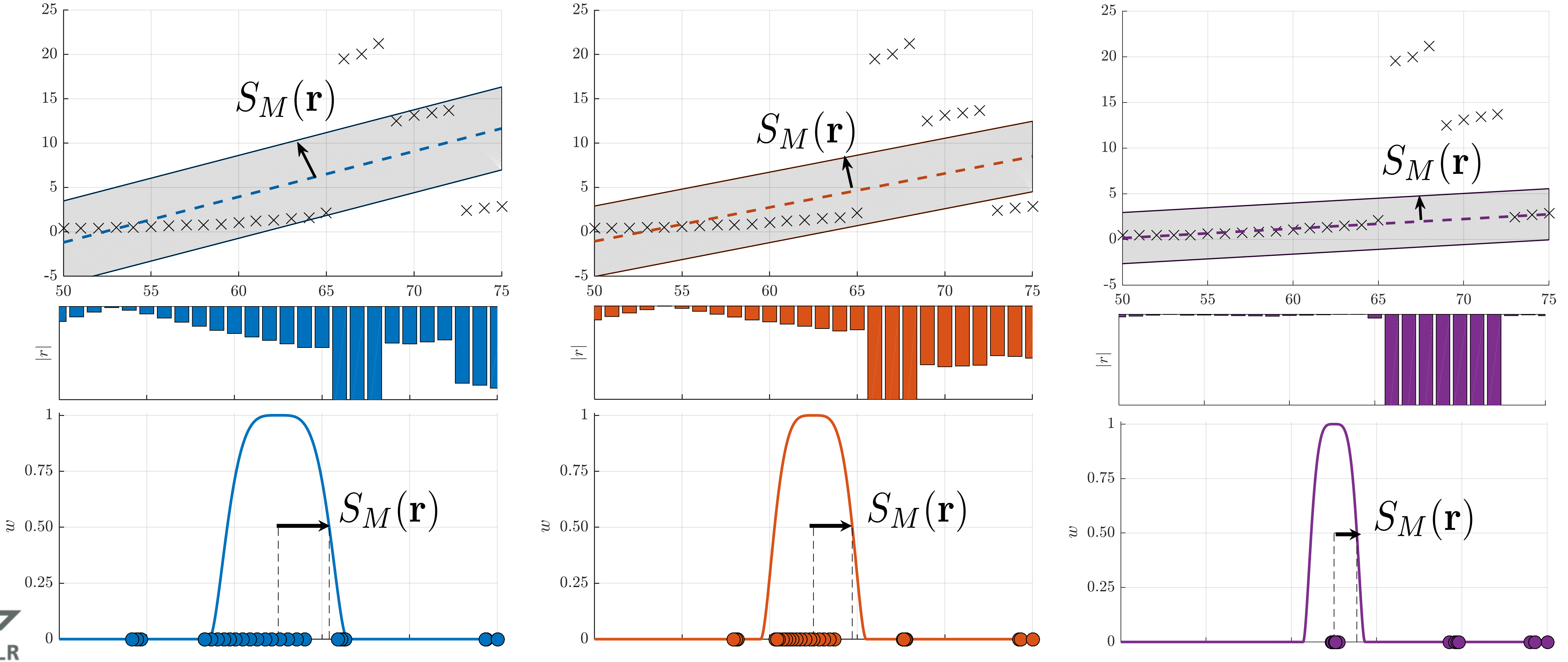
$$\hat{\boldsymbol{\beta}}_s = \arg\min_{\boldsymbol{\beta}} s_M(\mathbf{r}(\boldsymbol{\beta}))$$

$$s_M(\mathbf{r}) = 1.48\ \mathrm{Med}\left(\left|\mathbf{r} - \mathrm{Med}(\mathbf{r})\right|\right)$$

Robust Estimators (II)

S estimator

$$\hat{\beta}_s = \arg\min_{\beta} s_M(\mathbf{r}(\beta)), \quad s_M(\mathbf{r}) = 1.48 \,\mathrm{Med}\left(\left|\mathbf{r} - \mathrm{Med}(\mathbf{r})\right|\right)$$



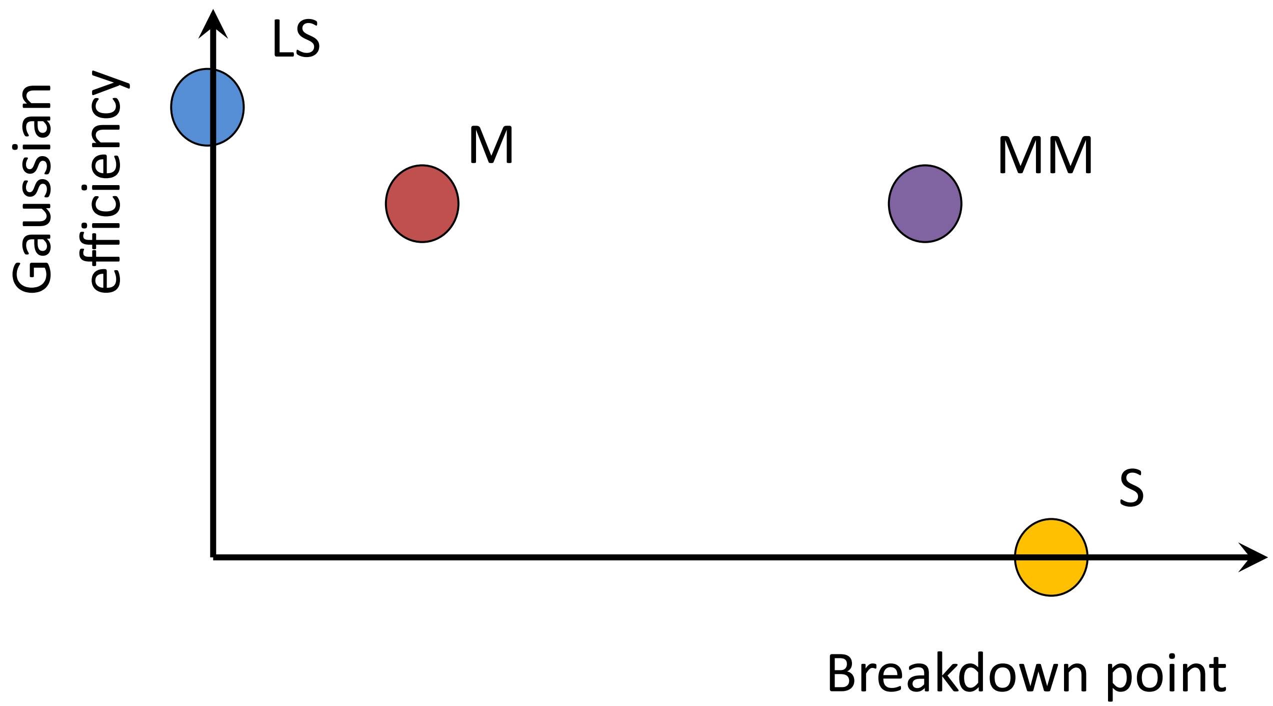
Robust Estimators (III)

MM estimator

- *[Maronna10, Martinez16]* MM seeks for a balance between robustness and efficiency → MM estimates require estimating the error scale & a constant to controls the efficiency

1) An initial estimate is found with a high BP method (S estimator)
2) The dispersion of the residuals is found using a M-scale
3) A low BP with high Gaussian efficiency (M estimator) is applied

Robust Estimators Overview



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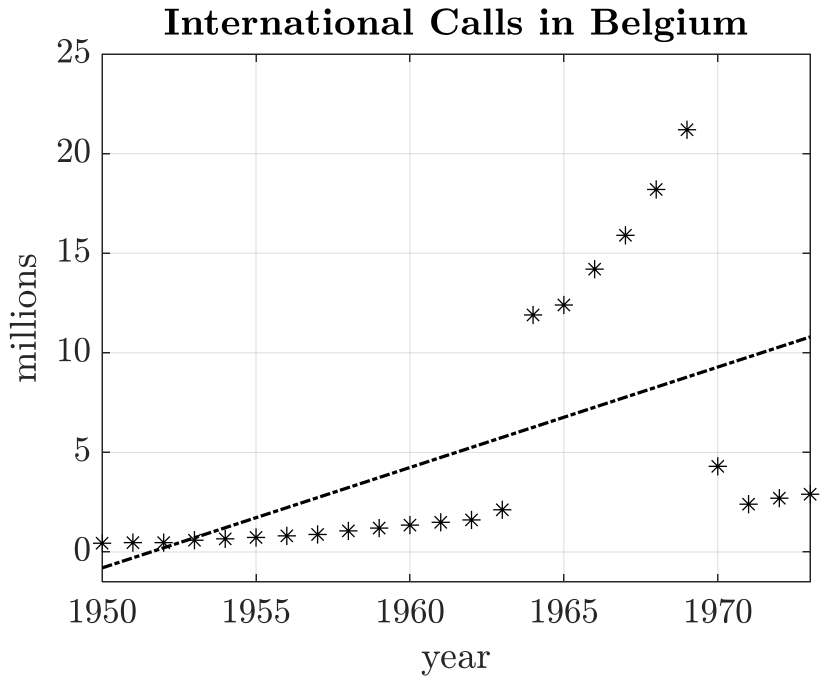
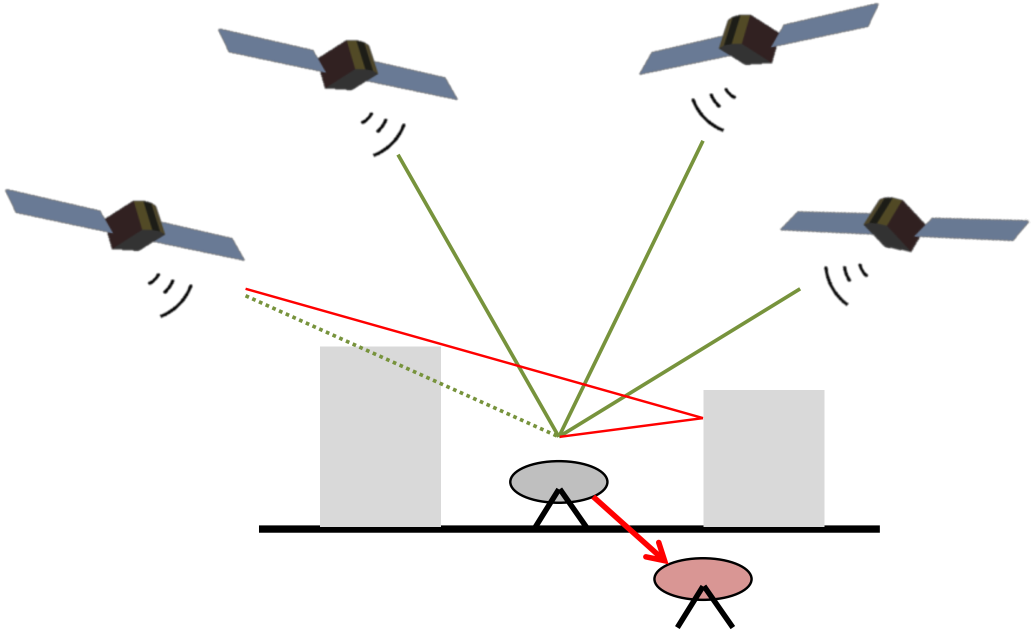
Robust Positioning





$$y_i = x_i^\top \boldsymbol{\beta} + \varepsilon_i$$

$$\mathbf{y} = h(\boldsymbol{\beta}) + \boldsymbol{\varepsilon}$$

$$R_i = \|\mathbf{p}_i - \mathbf{p}\|^2 + c\,(\delta t - \delta t_i) + I_i + Tr_i + \varepsilon_i$$

$$\mathbf{y} = h(\boldsymbol{\beta}) + \boldsymbol{\varepsilon}, \quad \boldsymbol{\beta} = \left[\mathbf{p}^\top, c\delta t\right]^\top$$

$$\boxed{\boldsymbol{\beta} = \arg\min_{\boldsymbol{\beta}} \|\mathbf{y} - h\left(\boldsymbol{\beta}\right)\|^2}$$

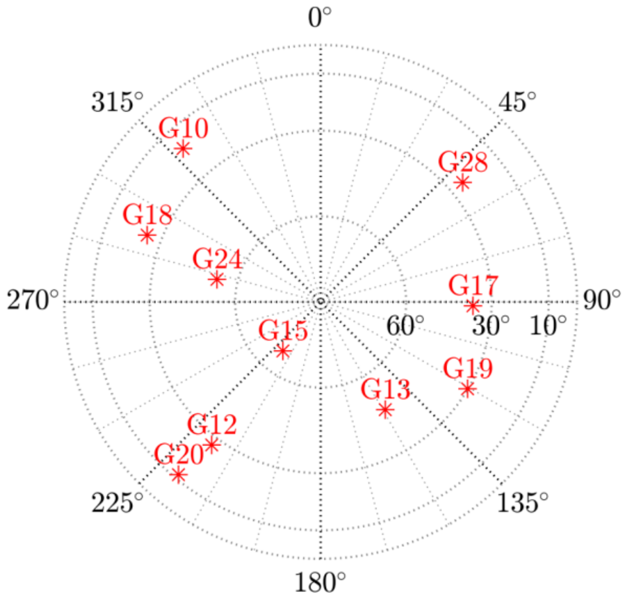
Challenges to Robust Positioning

Applying the principles of Robust Statistics to GNSS might be challenging…

1. Nonlinear regression problem
2. Not i.i.d → independent and identical distributed noise
3. *Fat data samples* → very low redundancy!
4. Theoretical properties of the estimators (e.g., breakdown point, efficiency) are defined for linear problems in affine conditions → <u>we need to assess how robust estimators actually perform</u>

Test and Results

UTC time	15/05/2017 09 : 30 : 00
Location	Koblenz, Germany (50°21’56”N, 7°35’55”E)
Number of satellites n	10
Observation variance noise [m^2]	4
Outlier percentage ϵ	0 - 10 - 20 - 30 - 40
Outlier magnitude α	1 - 3 - 6 - 10 - 30 - 60 -100



Noise distribution

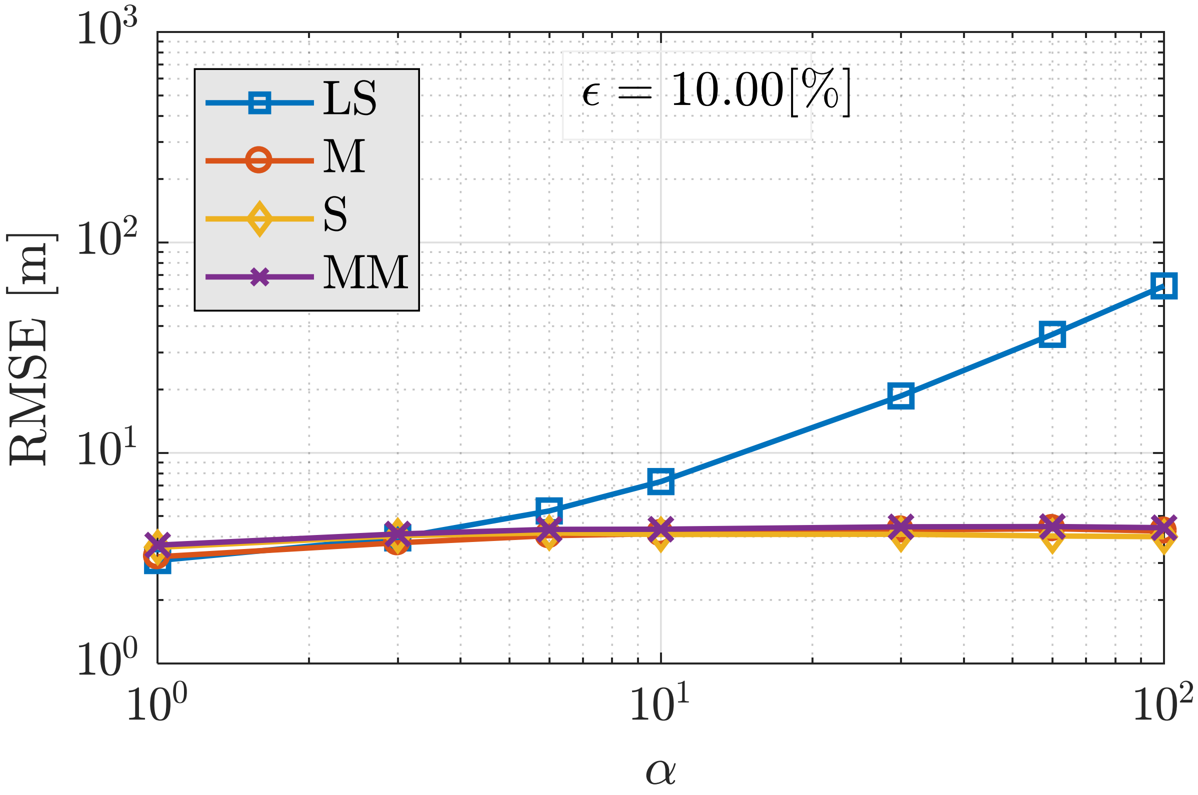
$$\varepsilon \sim (1-\epsilon)\ G + \epsilon\ H$$

$$G = \mathcal{N}\left(0, \sigma^2\right)$$

$$H = \mathcal{N}\left(0, \alpha\ \sigma^2\right)$$

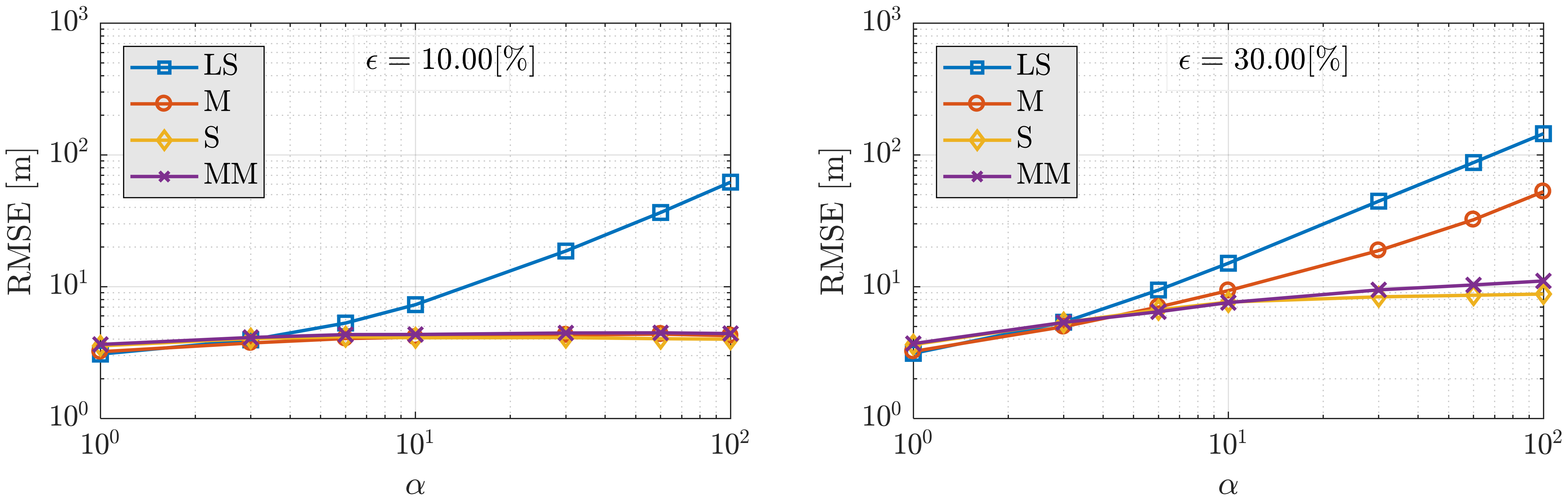
Test and Results

10 observations



Test and Results

10 observations



Test and Results

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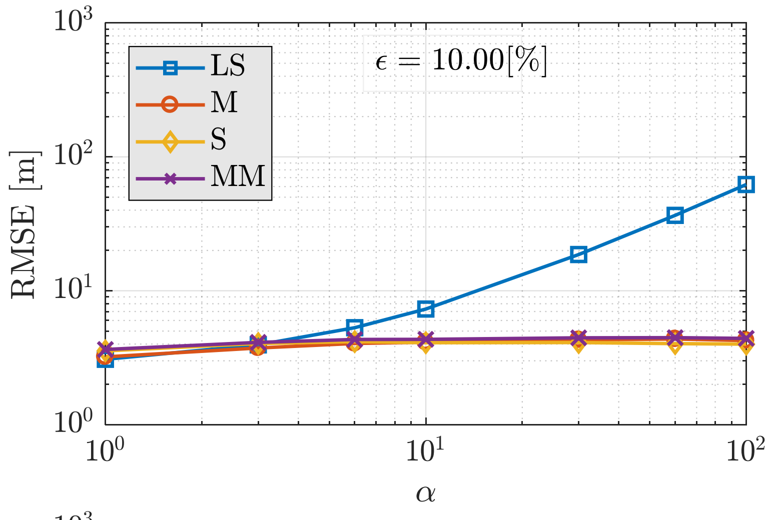
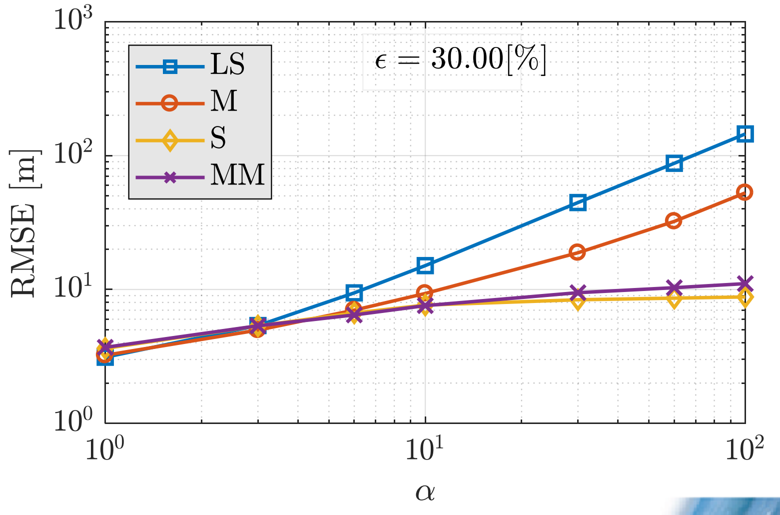
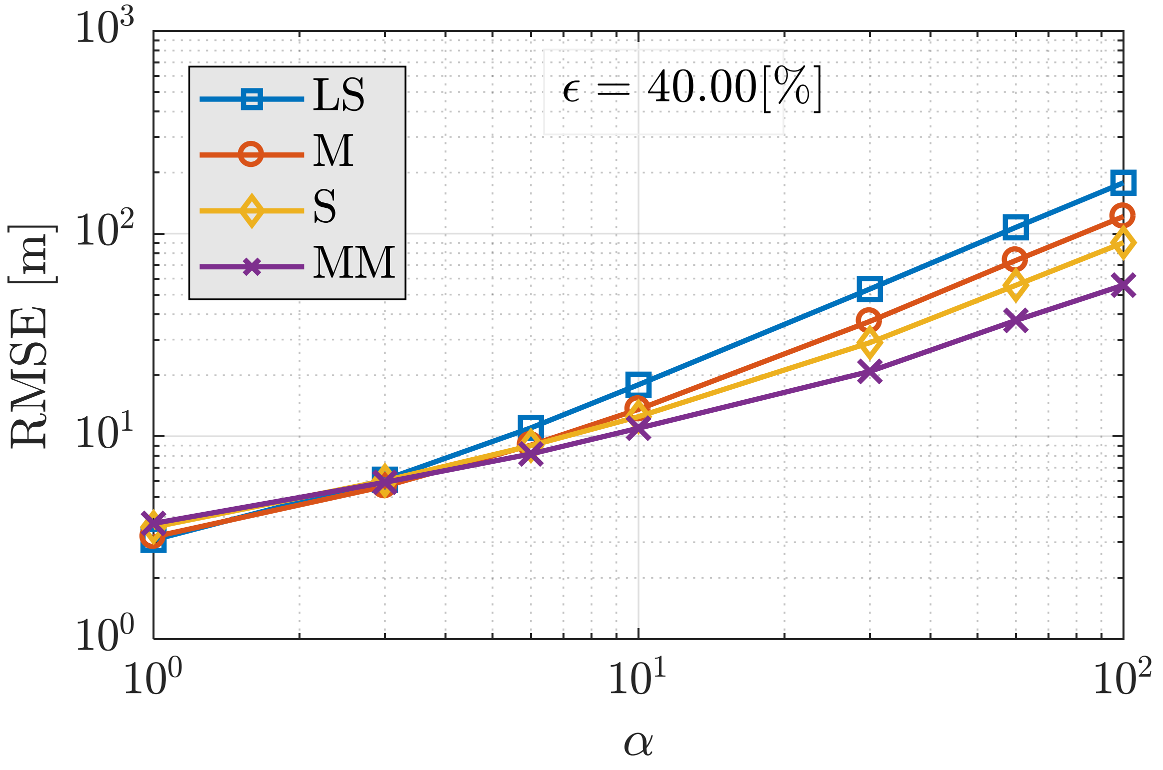






The theoretical breakdown point of the estimators is far away from the empirical ones!

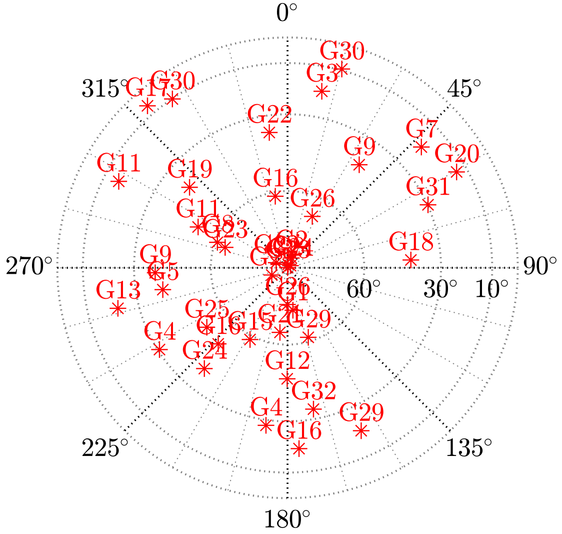
What is the problem??

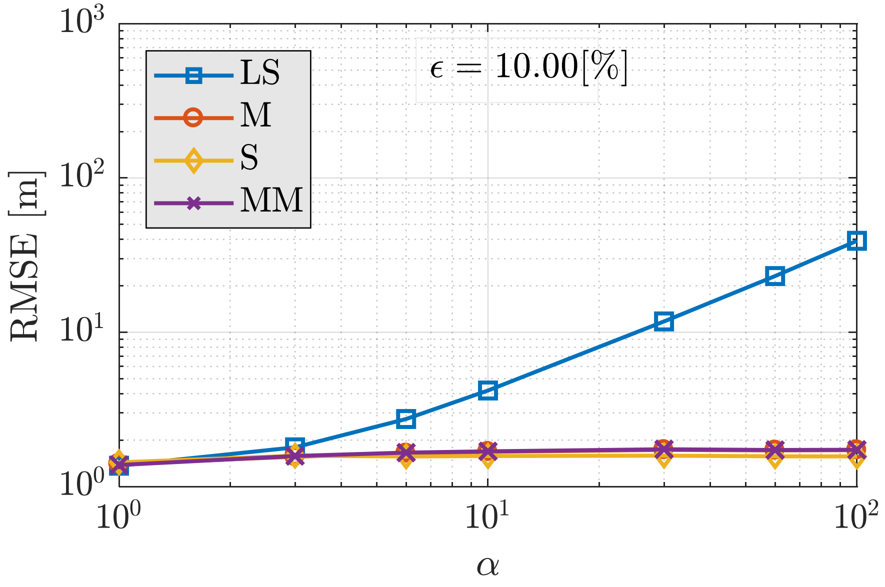
a) Nonlinearity
b) Low redundancy of observations
c) President Donald J. Trump?

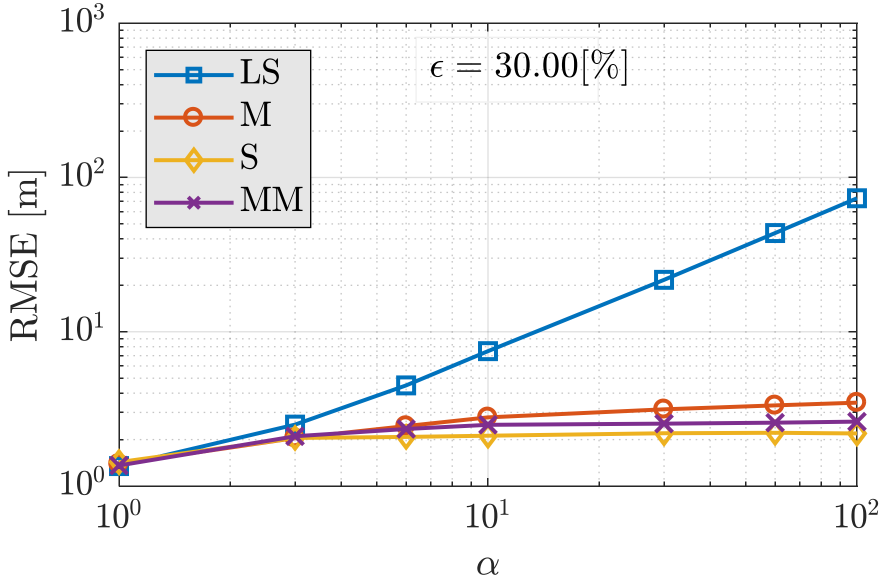
DLR

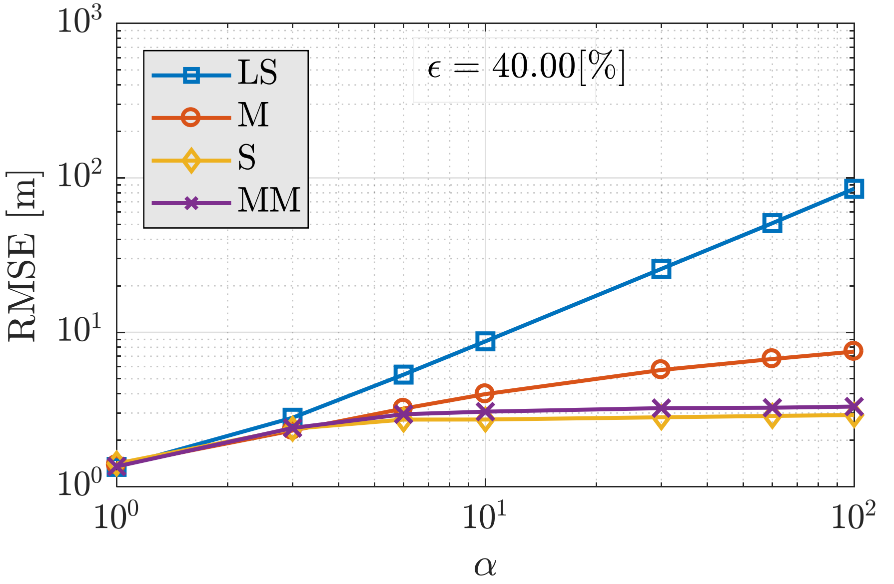
Test and Results

40 observations



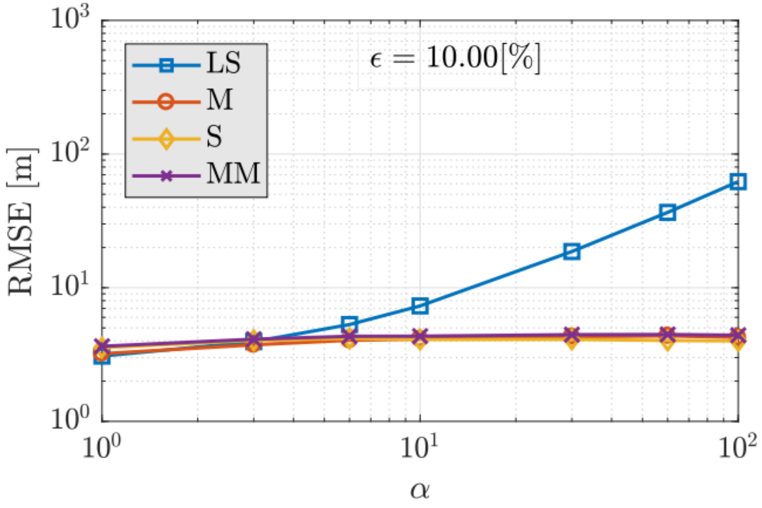




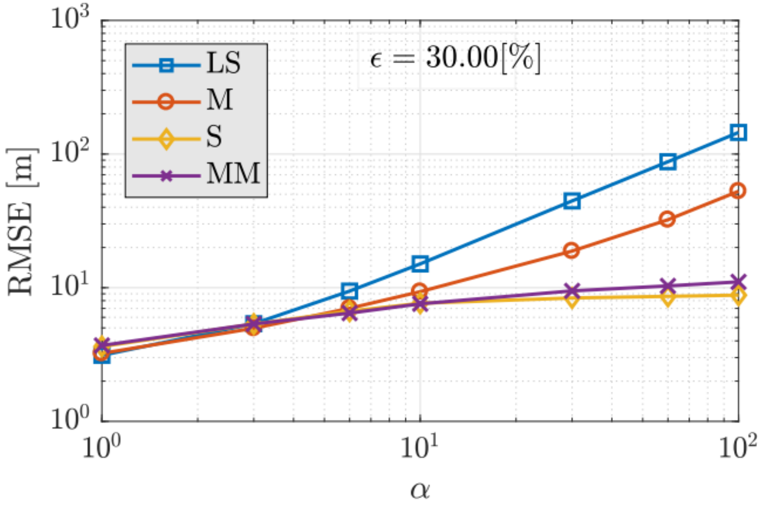


DLR

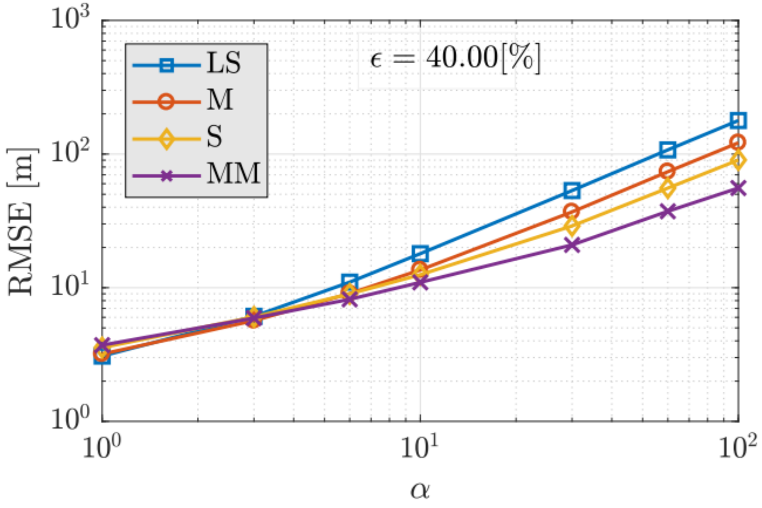
Test and Results



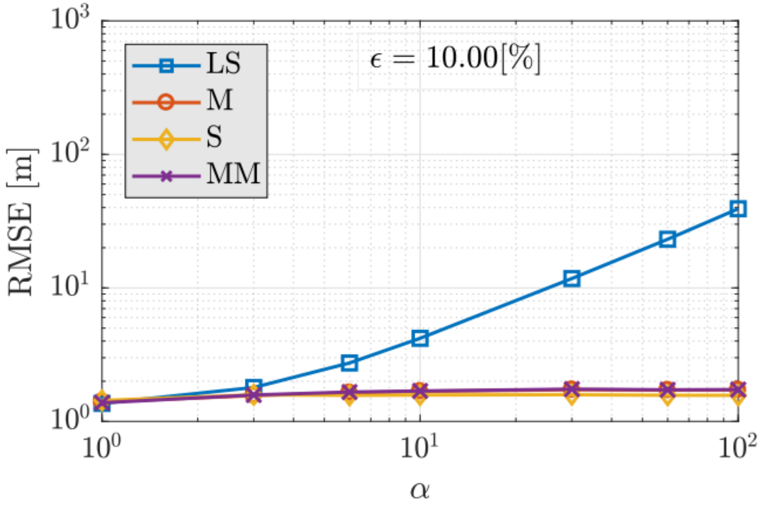
(a) 10% of contaminated data.



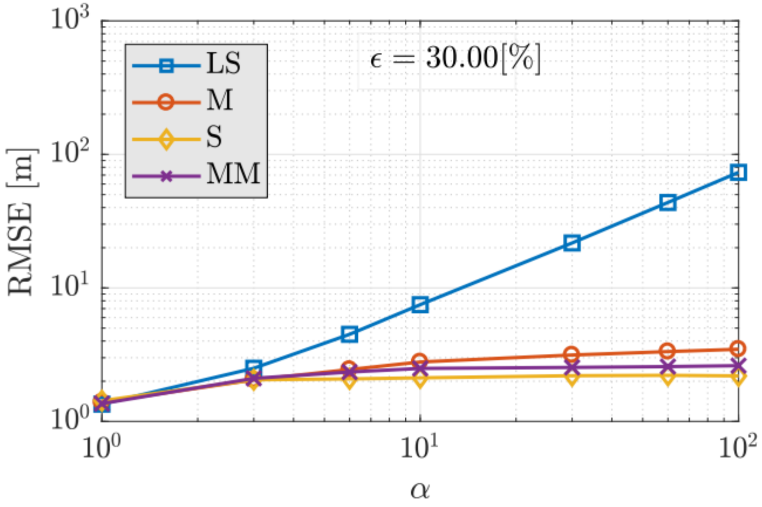
(b) 30% of contaminated data.



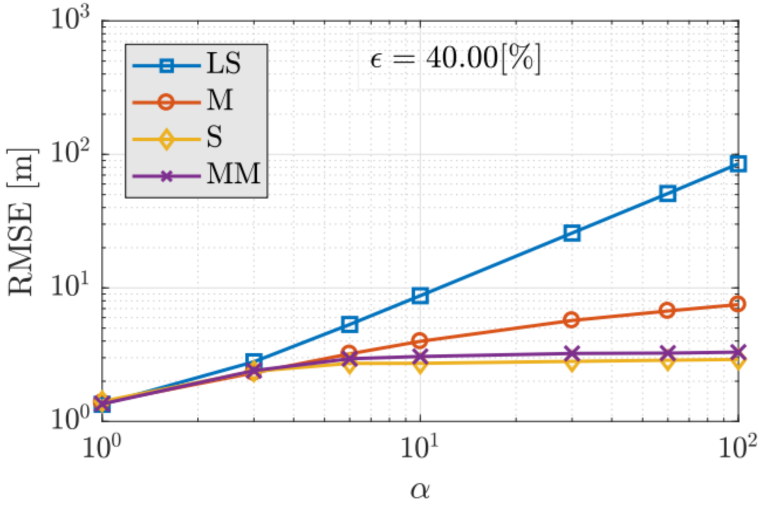
(c) 40% of contaminated data.



(d) 10% of contaminated data.



(e) 30% of contaminated data.

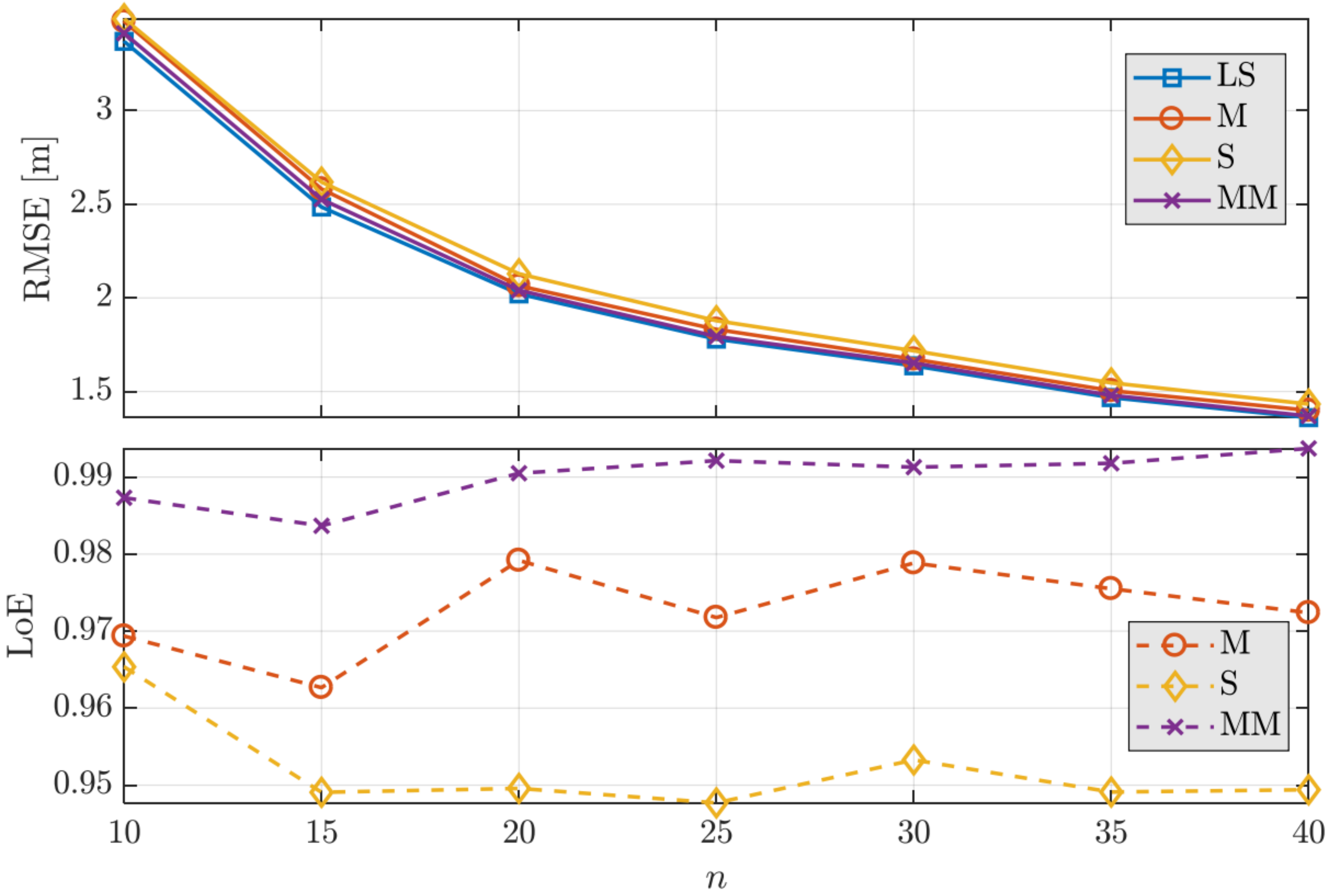


(f) 40% of contaminated data.

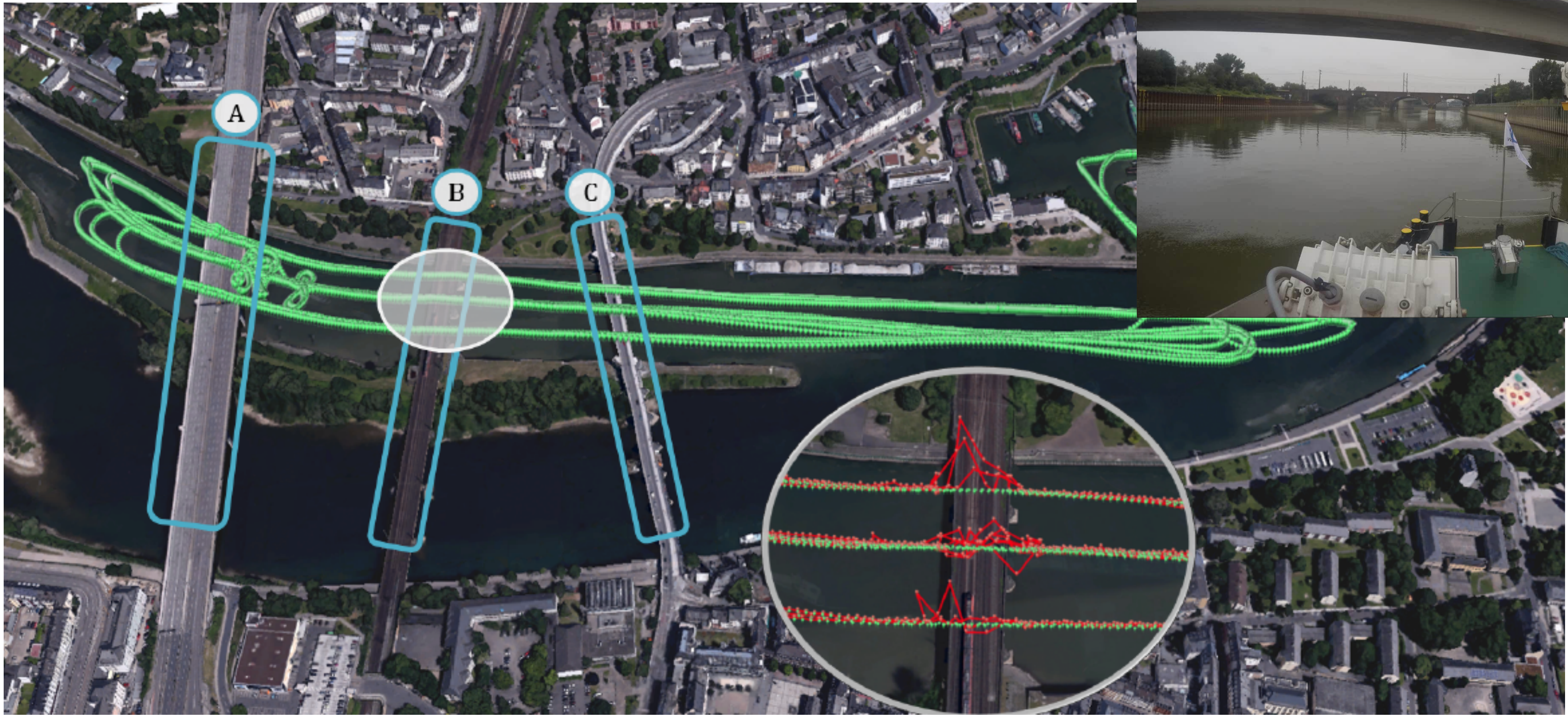
Test and Results

Analysis on Gaussian Efficiency

- Efficiency is here defined as the loss in accuracy of a method in regards to the LS
- Not clear whether there is a relation between efficiency and number of observations



Test and Results: Real Scenario



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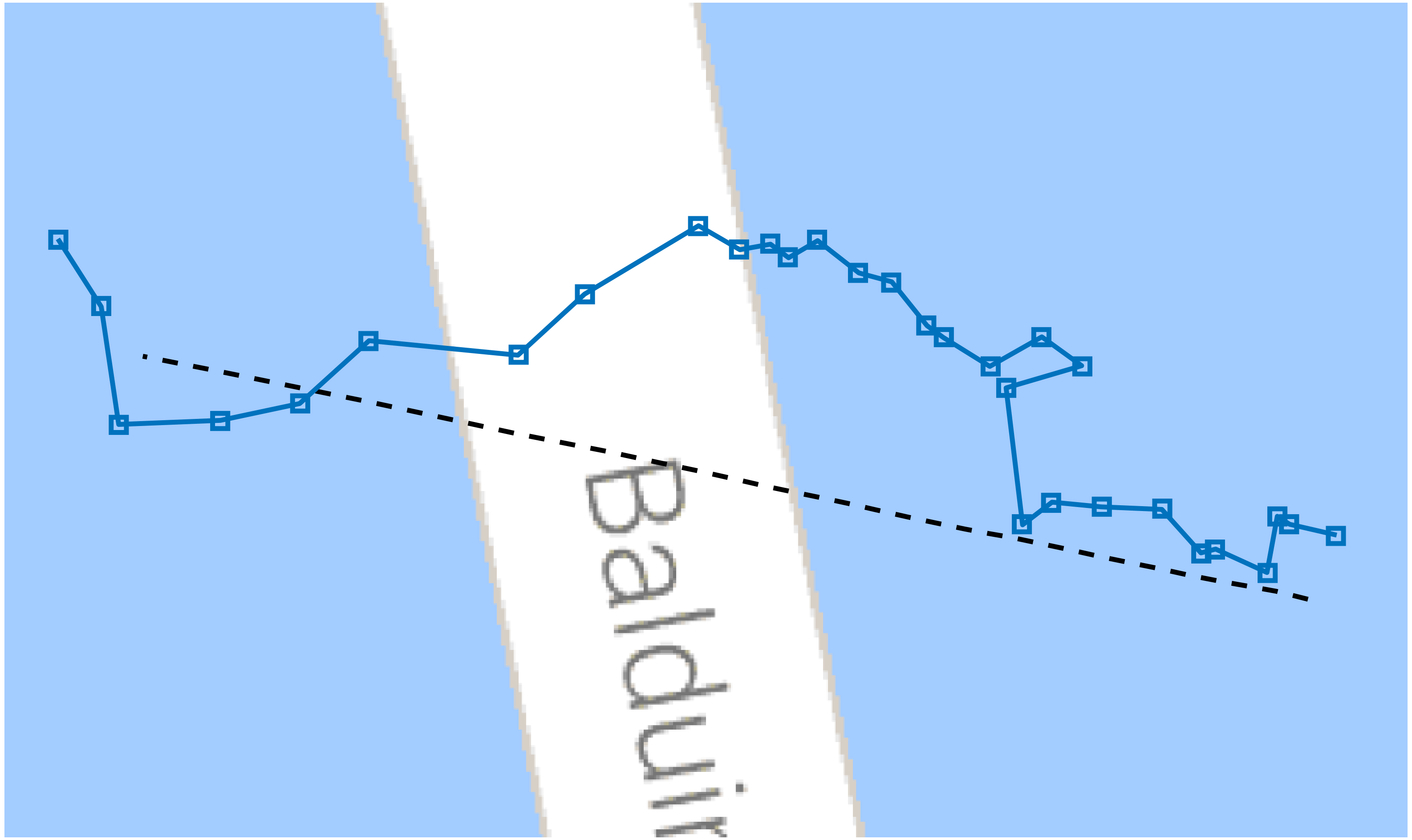
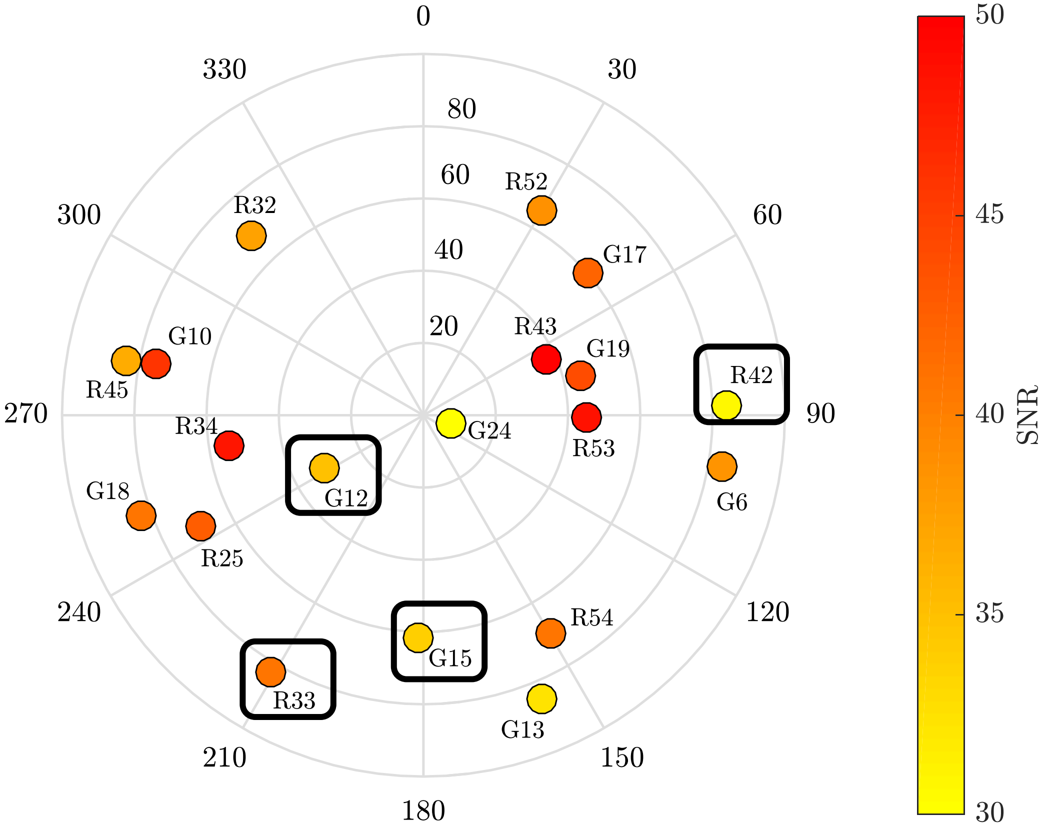
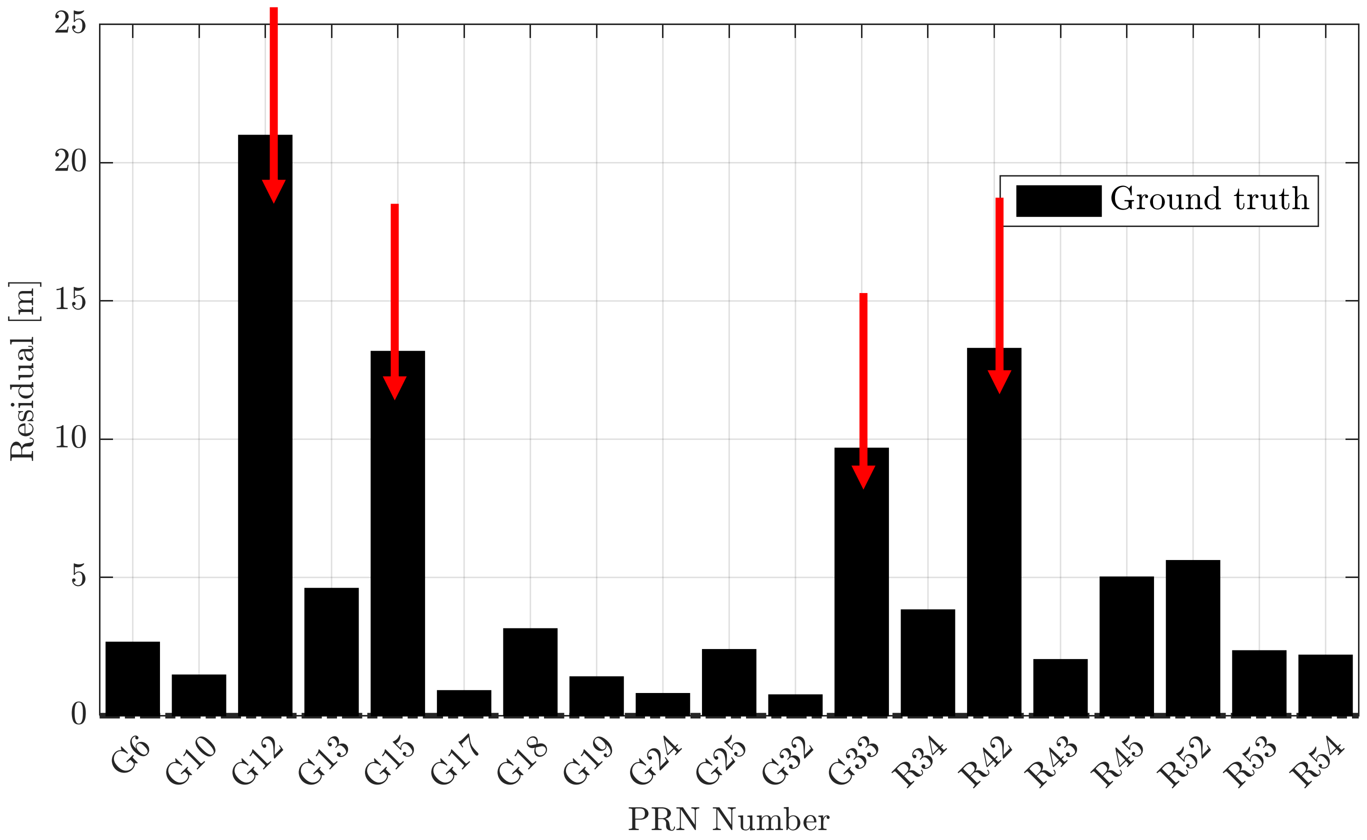


Test and results

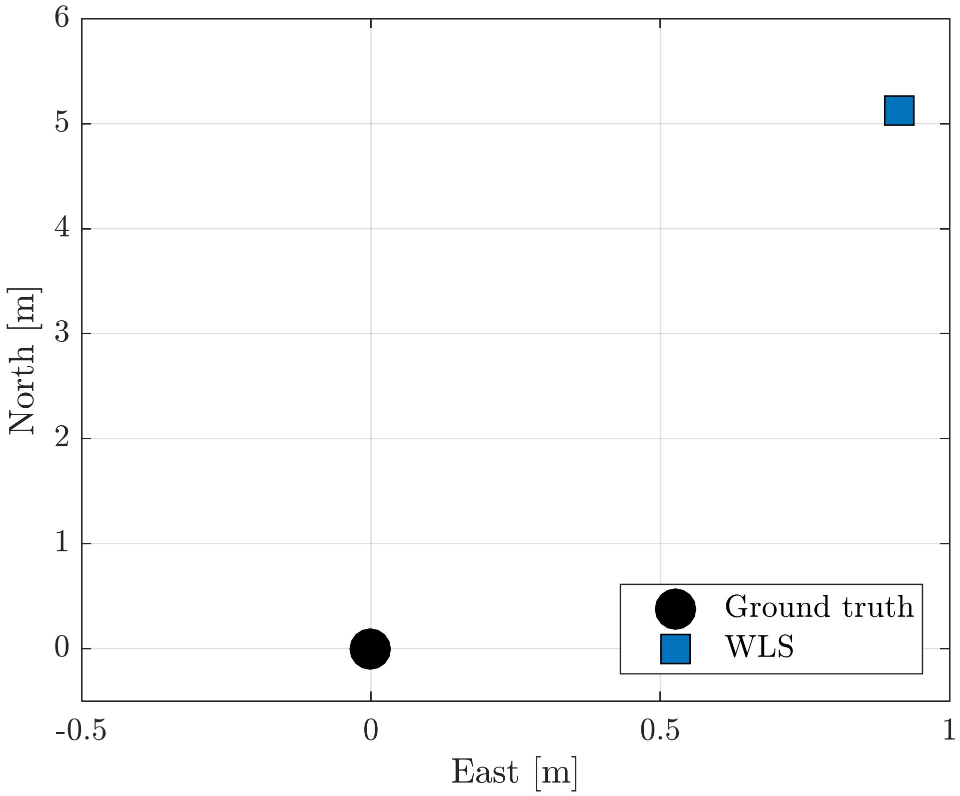


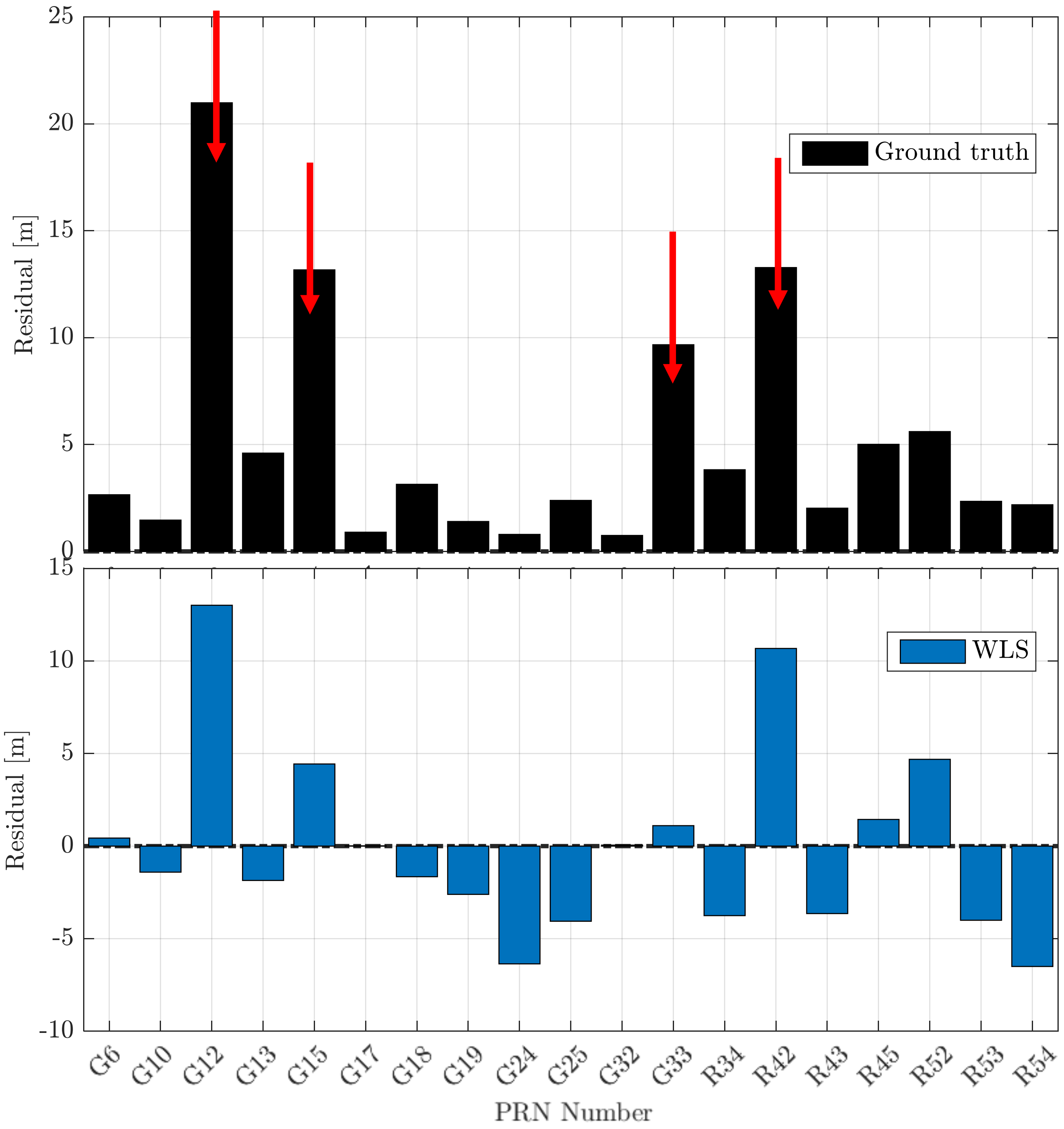


In GNSS-challenging environments…

- Is satellite-elevation a quality indicator?
- Does satellite-SRN matters?

Test and results





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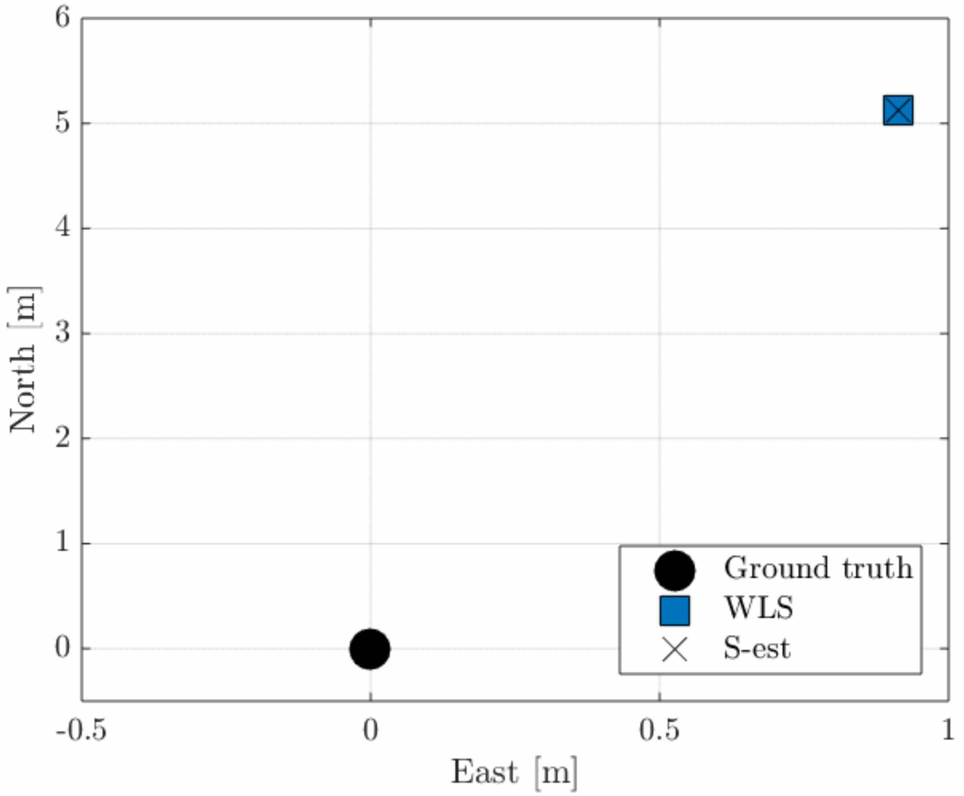

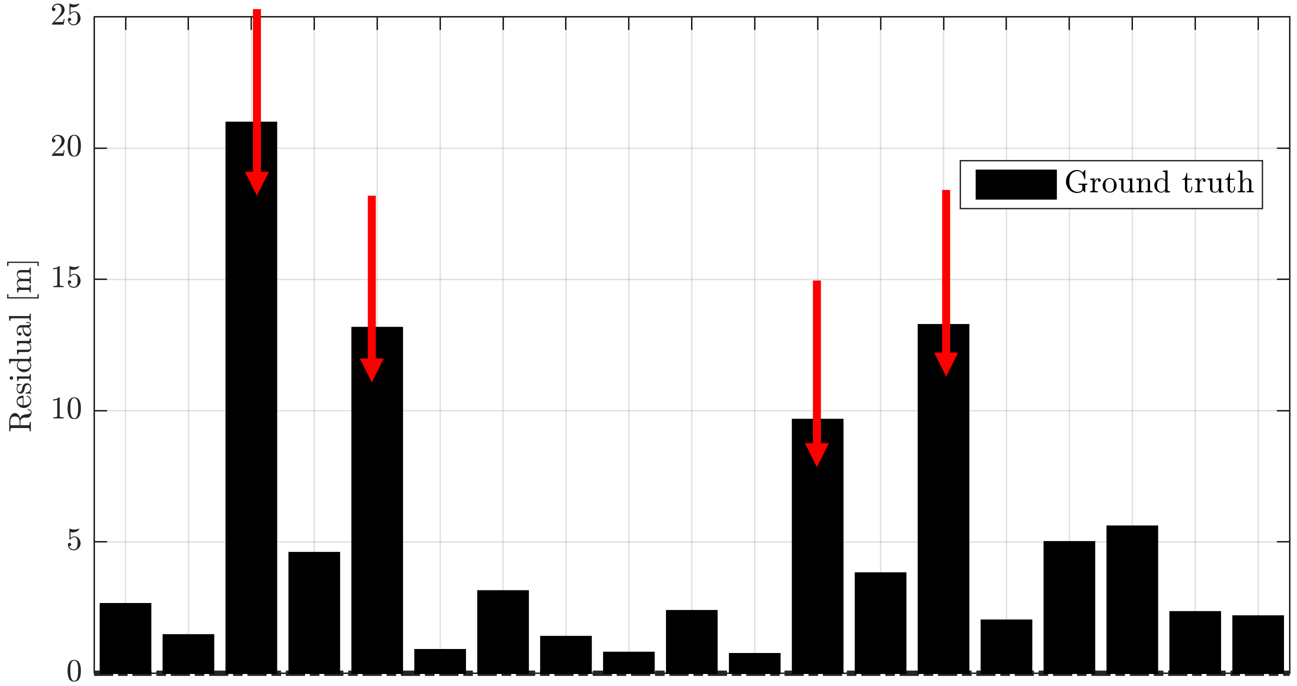

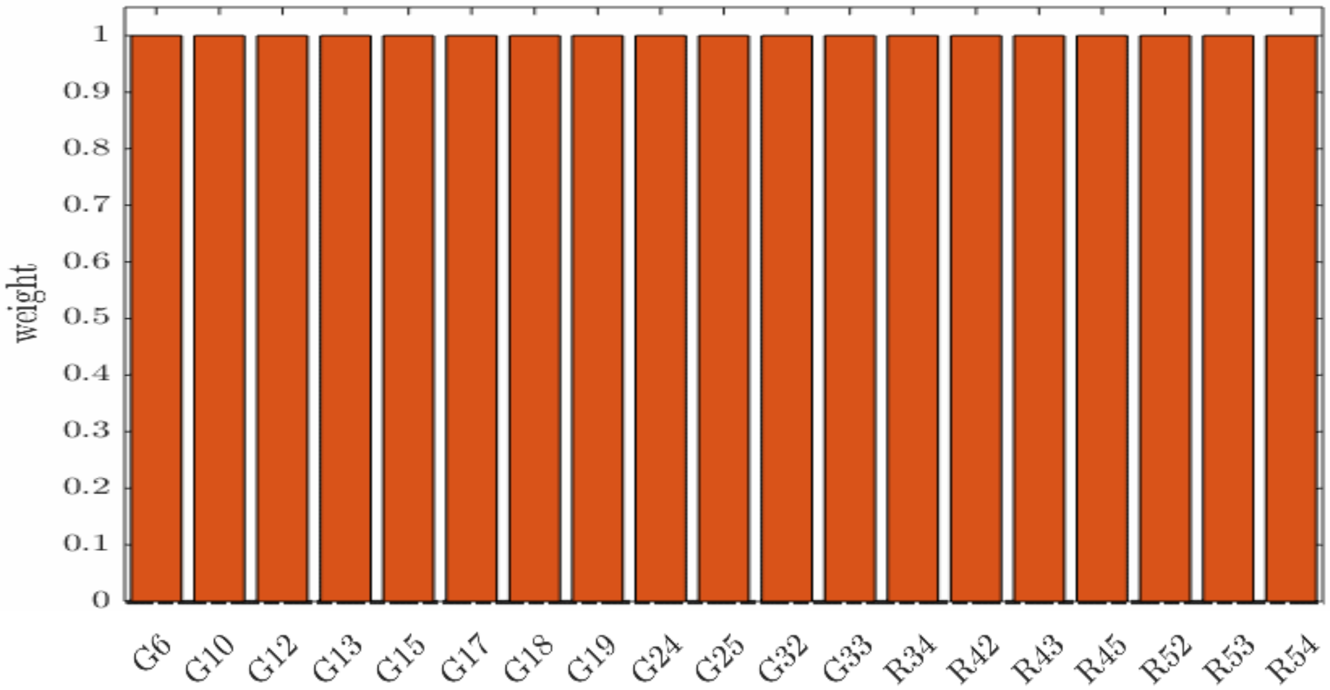

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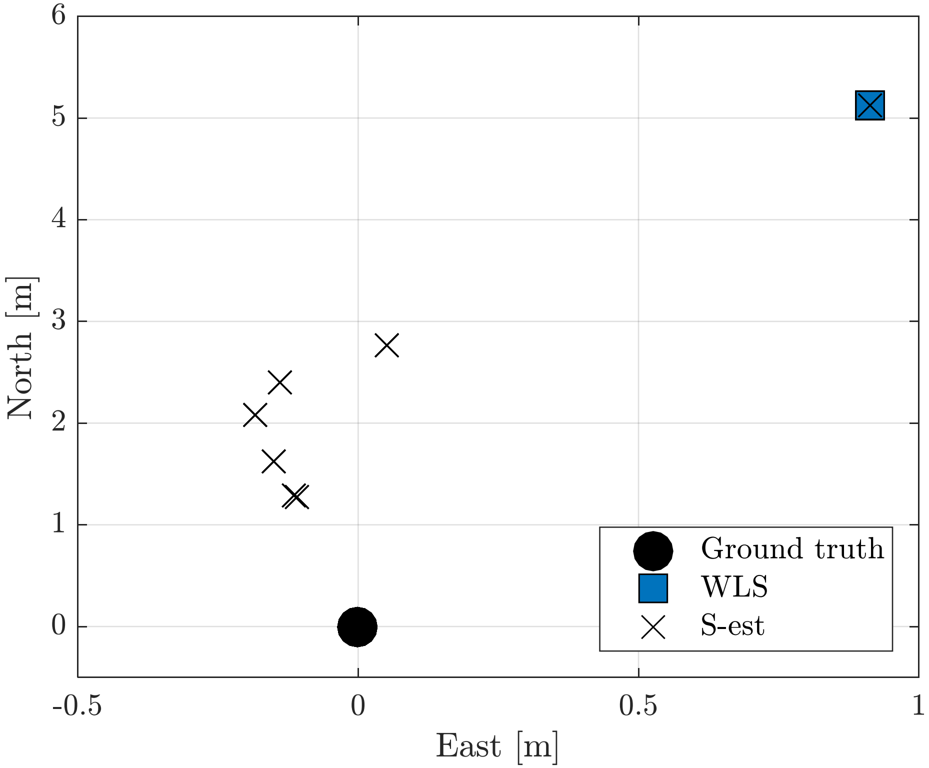

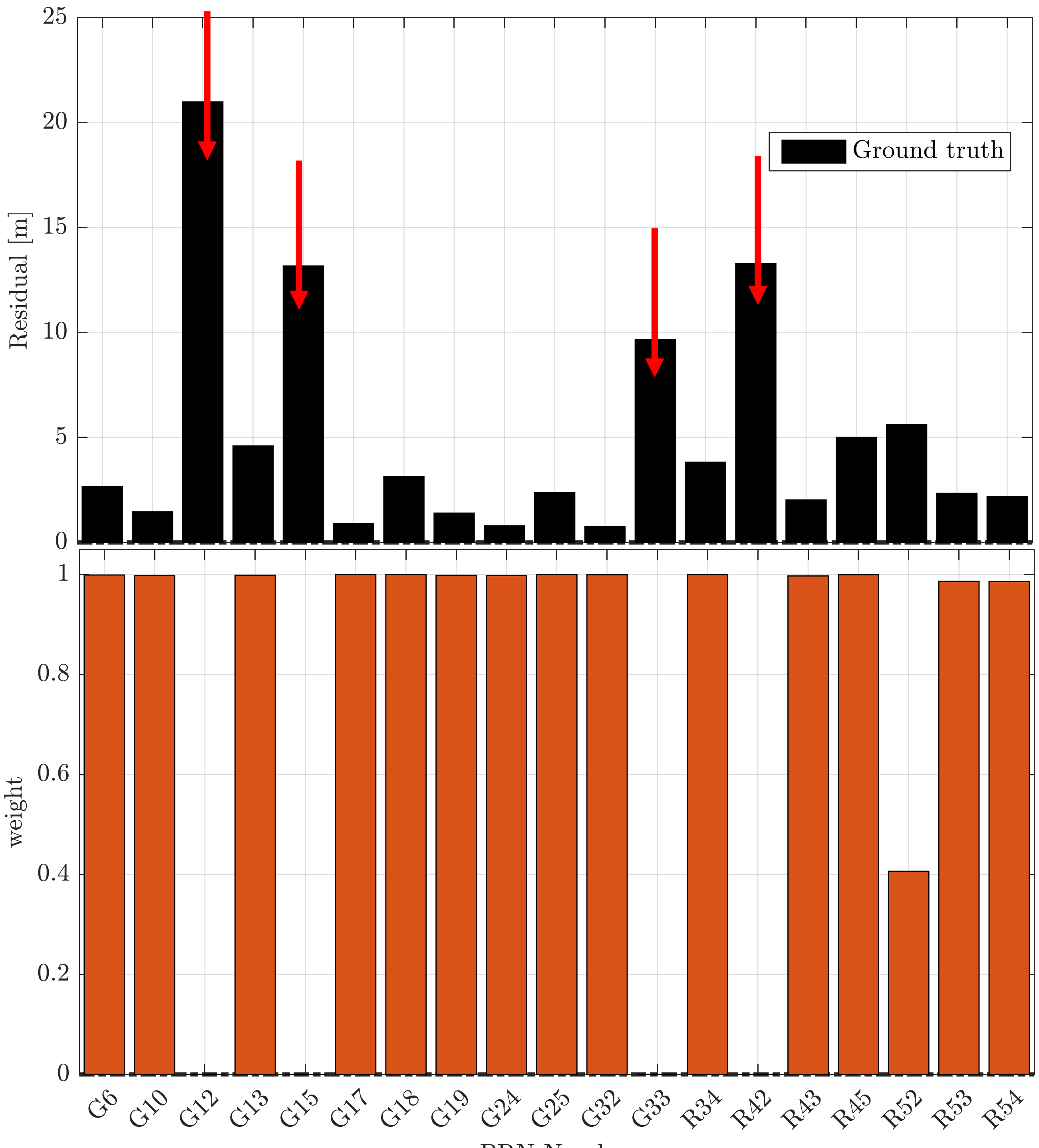

- ✓ Mitigation of 4 simultaneous faults
- ✓ An initial wrong estimation does not compromise the performance

Quick Recap

- Robust Estimators have potential to become the *de facto* estimator for GNSS positioning:
  - Capable of handling multiple simultaneous faults → great for prospective safety-critical applications
  - Scalable with the number of observations → synergy with the deployments of new GNSS and frequencies

- But…
  - Theoretical properties of the methods, bounds on the positioning performance are to be estimated!
  - SPP positioning is just the tip of the *navigation iceberg*

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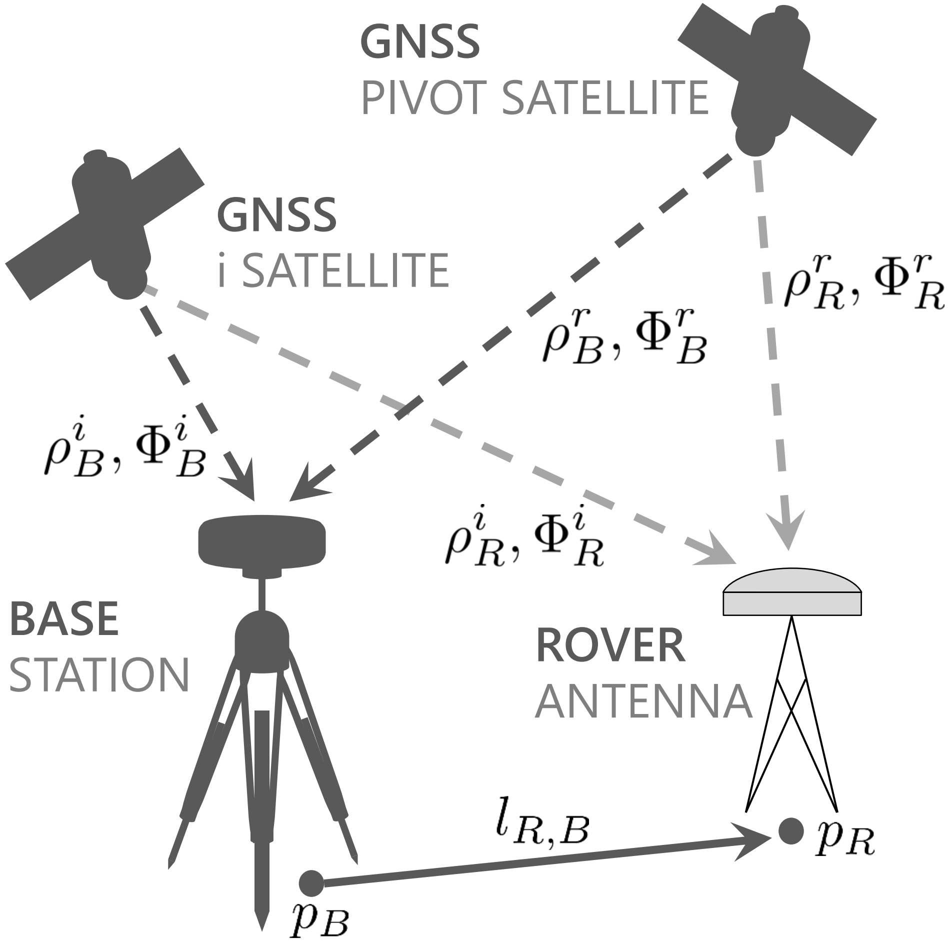
Robust Estimation on RTK

RTK Working Principle



- RTK is a differential phased-based positioning → base station of known coordinates transmits correction data
- Phase observations are very precise but ambiguous
- **Challenges:**
  - ✗ Complexity of ambiguity resolution and dimensionality curse (multi -constellation, -frequency, -antenna)
  - ✗ Integrity of the system: are the ambiguities right?
- **Benefits:**
  - ✓ Instantaneous centimeter-level positioning
  - ✓ Accurate heading (<0.1° for 1 meter baseline)

Robust Estimation on RTK

Code and phase observations

$$\rho_R^i = \|\boldsymbol{p}^i - \boldsymbol{p}\| + I^i + T^i + c\left(dt - dt^i\right) + \varepsilon_R^i$$

$$\Phi_R^i = \underbrace{\|\boldsymbol{p}^i - \boldsymbol{p}\|}_{-\boldsymbol{u}^{i\top}\boldsymbol{p}} - I^i + T^i + c\left(dt - dt^i\right) + \lambda N^i + \epsilon^i$$

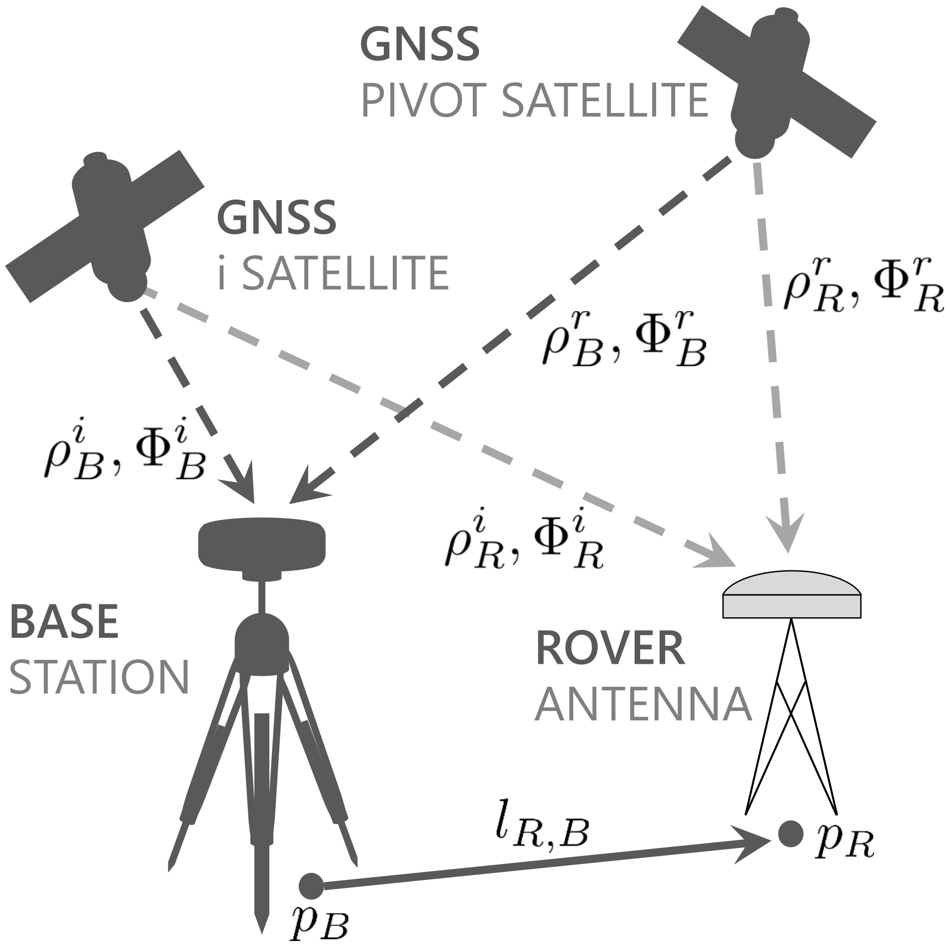
Rover – base observations

$$DD\Phi^{ir} = \Phi_R^i - \Phi_B^i - (\Phi_R^r - \Phi_B^r)$$

$$DD\Phi^{ir} = -\left(\boldsymbol{u}^i - \boldsymbol{u}^r\right)^\top (\boldsymbol{p} - \boldsymbol{p}_B) + \lambda\, \boldsymbol{a}^i + \epsilon^{ir}$$

$$DD\rho^{ir} = \rho_R^i - \rho_B^i - (\rho_R^r - \rho_B^r)$$

$$DD\rho^{ir} = -\left(\boldsymbol{u}^i - \boldsymbol{u}^r\right)^\top (\boldsymbol{p} - \boldsymbol{p}_B) + \varepsilon^{ir}$$

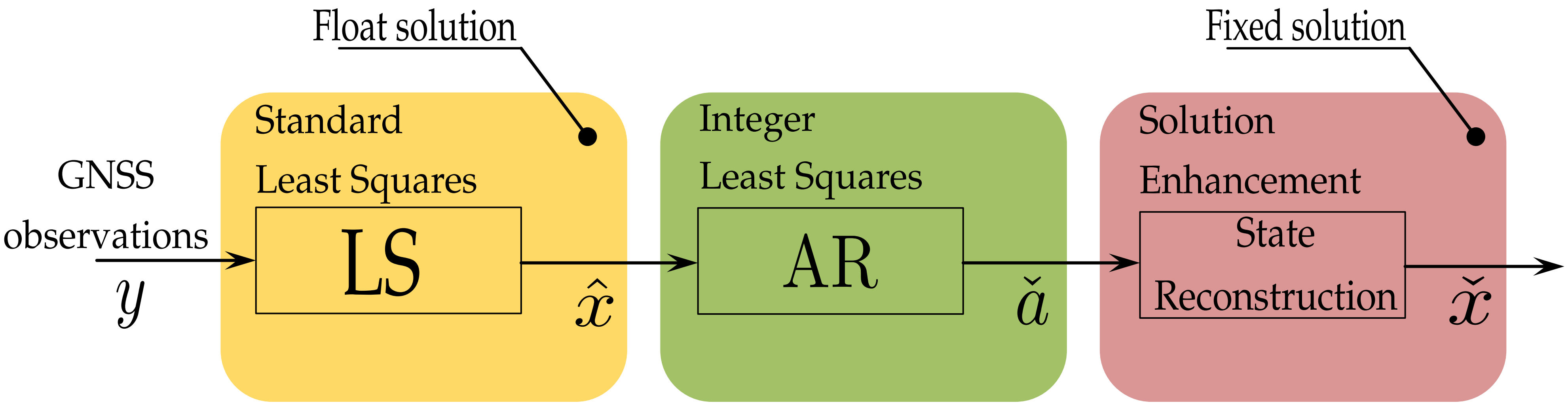


Robust Estimation on RTK

Solving RTK is non-trivial $\rightarrow$ no explicit solution exists to the problem:

$$\boldsymbol{x} \equiv \left[\boldsymbol{p}^\top, \boldsymbol{a}^\top\right]^\top, \quad \boldsymbol{p} \in \mathbb{R}^3, \boldsymbol{a} \in \mathbb{Z}^n$$

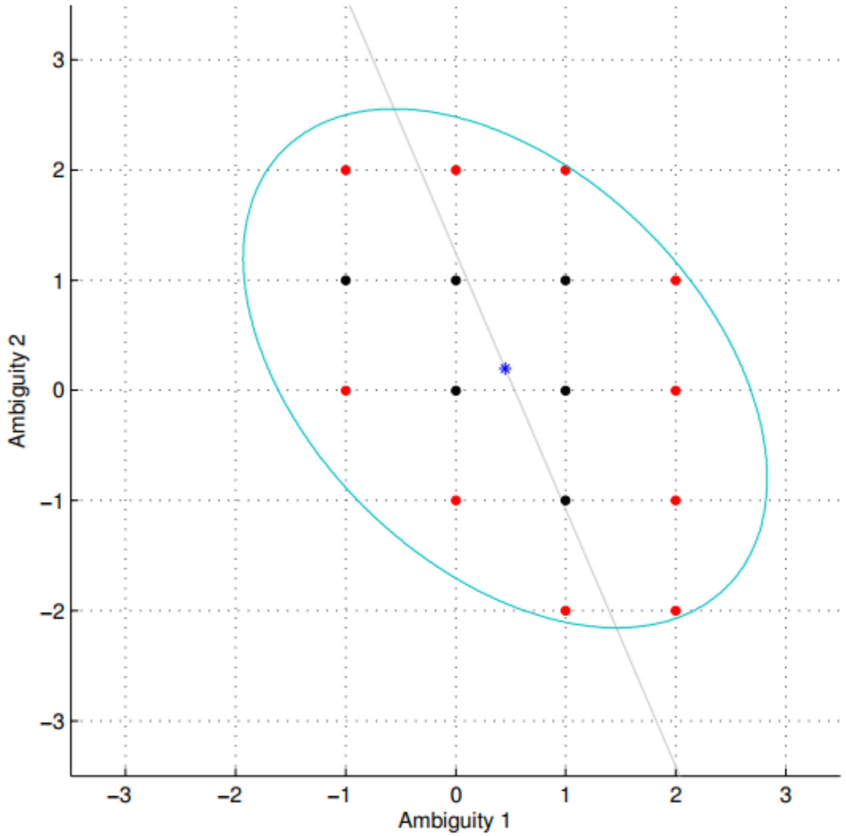
$$\boldsymbol{x} = \min_{\boldsymbol{x}} \|\boldsymbol{y} - h(\boldsymbol{x})\|^2_{\boldsymbol{R}_y}$$



Robust Estimation on RTK

- During the first LS, the effects of the outliers would “leak” to the Ambiguity Resolution
- Adapting a robust estimator for Integer LS adjustment is not an option

New Robust Estimators are to be defined, or the most successful precise navigation will not be possible in challenging scenarios!

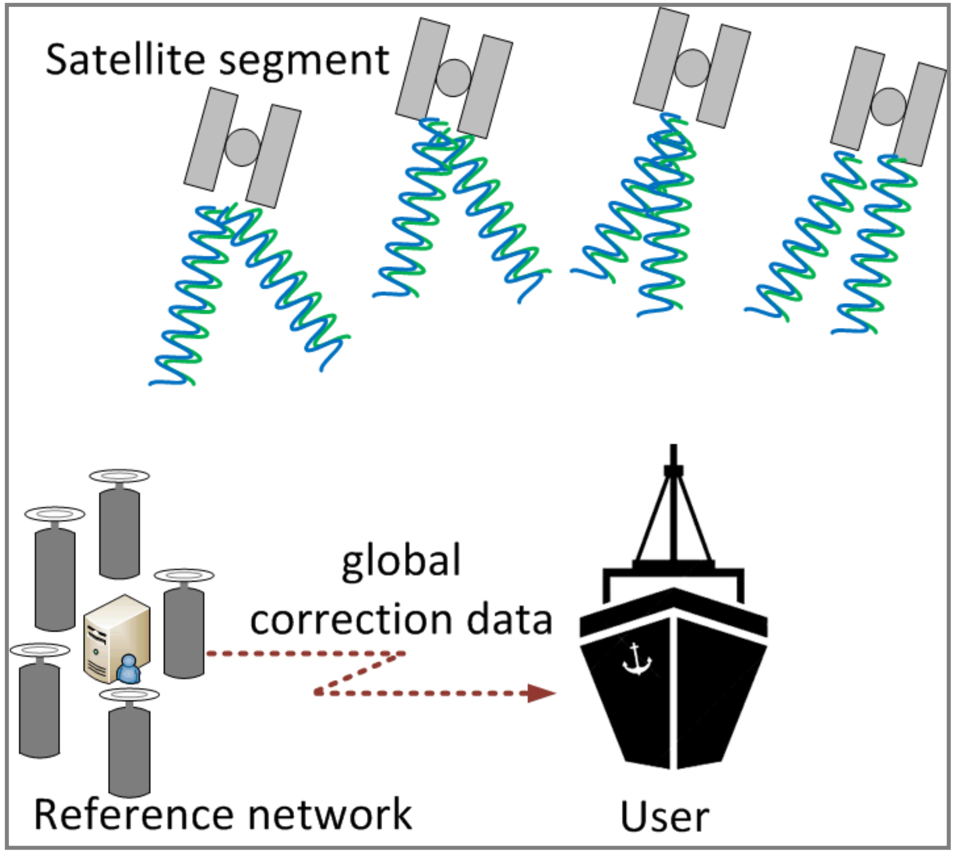


[Verhagen12]

DLR

Precise Point Positioning

- PPP Principle



- “absolute” phased based positioning, user do not need a reference station, but
- reference network is necessary do determine orbit, clocks, bias, atmosphere corrections etc.
- Other corrections: phase center offset, site displacements effects… has to modeled
- for RealTimePPP link to the correction data is necessary (broadcast via satellite, NTRIP, digital radio, AIS/VDES)
- **Challenge:** Fixing of ambiguities (precise and complete correction data are required)
- **Accuracy:** Decimeter up to centimeter after the convergence time (float)

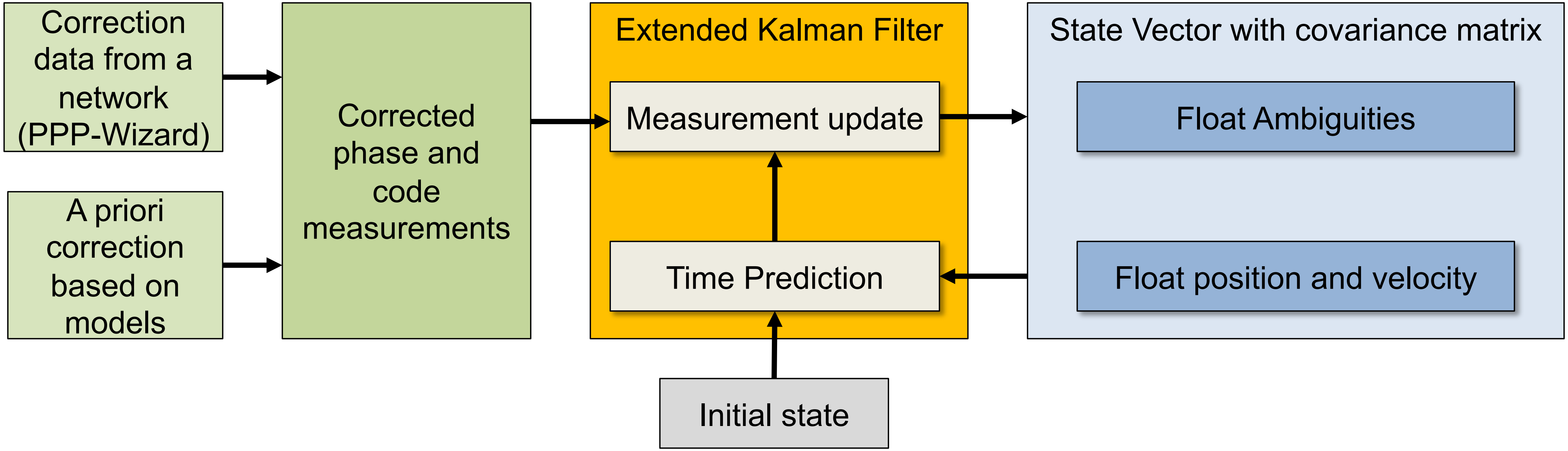
Navigation 4.0: AP 2200 Real Time Precise Point Positioning (PPP)

- Implementation of necessary a priori correction models for PPP

Model component	Description	Magnitude [cm]	Correction of …
Earth Tides	• Sun and moon causes periodic deformation of the solid earth	up to 40	Phase and code observation
Pol Tides	• Polar motion of the earth causes subtle deformation of the earth	2.5	Phase and code observation
Satellite antenna offset	• Precise orbits are referred to the center of mass • Correction to the center of phase is required	50 – 100	Orbit coordinates
Satellite antenna phase center variation	• Deviation between ideal and actual/real phase front	0.5 – 1.5	Phase and code observation
Phase Wind Up	• Measured phase is changed due to the satellite orientation	10	Phase observation
hydrostatic troposphere	• Can be accurately computed a priori from surface pressure, station latitude and height	230	Phase and code observation

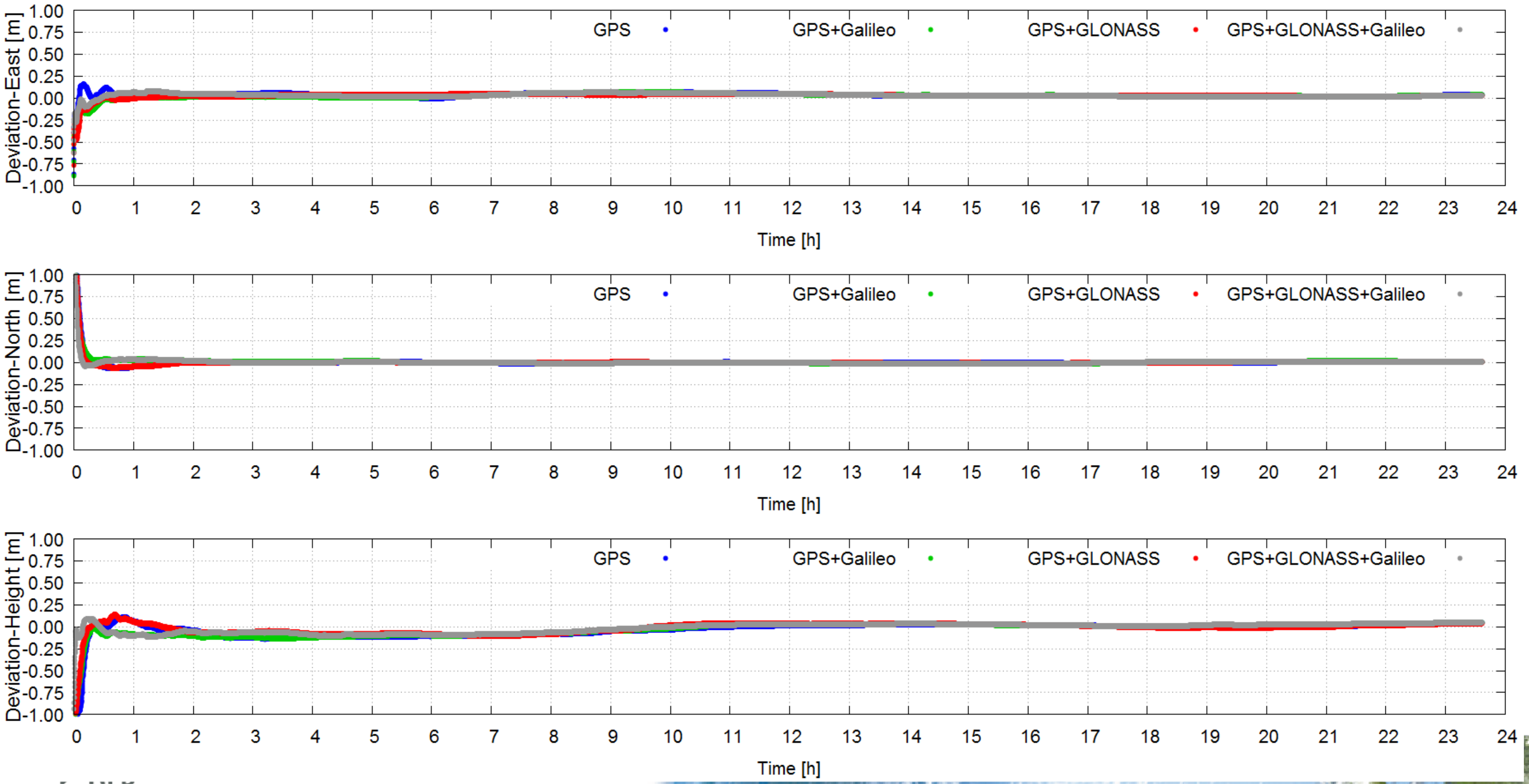
Navigation 4.0: AP 2200 Real Time Precise Point Positioning (PPP)

- Implementation of an Extended Kalman Filter (EKF) to estimate position, velocity and ambiguites



Navigation 4.0: AP 2200 Real Time Precise Point Positioning (PPP)

- First results: deviation to a reference station based (Wetzel) on a float PPP solution



- Using GNSS reduce convergence time (< 10 min)

After convergence time

- East and north accuracy << 1 dm
- Height accuracy 1- 2 dm

References

Principles of Robust Statistics

- [Huber73] Huber, Peter J. "Robust regression: asymptotics, conjectures and Monte Carlo." *The Annals of Statistics* 1.5 (1973): 799-821.
- [Huber11] Huber, Peter J. *Robust Statistics*. Springer Berlin Heidelberg, 2011.
- [Maronna19] Maronna, Ricardo A., et al. *Robust statistics: theory and methods (with R)*. John Wiley & Sons, 2019.
- [Zoubir12] Zoubir, Abdelhak M., et al. "Robust estimation in signal processing: A tutorial-style treatment of fundamental concepts." *IEEE Signal Processing Magazine* 29.4 (2012): 61-80.

References for S-Estimation

- [Rousseeuw84] Rousseeuw, Peter, and Victor Yohai. "Robust regression by means of S-estimators." *Robust and nonlinear time series analysis*. Springer, New York, NY, 1984. 256-272.
- [Croux94] Croux, Christophe, Peter J. Rousseeuw, and Ola Hössjer. "Generalized S-estimators." *Journal of the American Statistical Association* 89.428 (1994): 1271-1281.
- [Salibian06] Salibian-Barrera, Matías, and Víctor J. Yohai. "A fast algorithm for S-regression estimates." *Journal of computational and Graphical Statistics* 15.2 (2006): 414-427.

References for MM-Estimation

- [Maronna10] Maronna, Ricardo A., and Victor J. Yohai. "Correcting MM estimates for “fat” data sets." *Computational Statistics & Data Analysis* 54.12 (2010): 3168-3173.
- [Martinez16] Martinez-Camara, Marta, et al. "The regularized tau estimator: A robust and efficient solution to ill-posed linear inverse problems." *arXiv preprint arXiv:1606.00812* (2016).

References

Robust Statistics on GNSS

- [Kuusnieimi07] Kuusniemi, Heidi, et al. "User-level reliability monitoring in urban personal satellite-navigation." *IEEE Transactions on Aerospace and Electronic Systems* 43.4 (2007): 1305-1318.
- [Knight09] Knight, Nathan L., and Jinling Wang. "A comparison of outlier detection procedures and robust estimation methods in GPS positioning." *The Journal of Navigation* 62.4 (2009): 699-709.
- [Angrisano13] Angrisano, Antonio, et al. "GNSS reliability testing in signal-degraded scenario." *International Journal of Navigation and Observation* 2013 (2013).
- [Medina16] Medina, Daniel Arias, et al. "Robust position and velocity estimation methods in integrated navigation systems for inland water applications." *2016 IEEE/ION Position, Location and Navigation Symposium (PLANS)*. IEEE, 2016.

On Real-Time Kinematic

- [Teunissen93] Teunissen, Peter JG. "Least-squares estimation of the integer GPS ambiguities." *Invited lecture, section IV theory and methodology, IAG general meeting, Beijing, China*. 1993.
- [Verhagen12] Verhagen, Sandra, B. Li, and Mathematical Geodesy. "LAMBDA software package: Matlab implementation, Version 3.0." *Delft University of Technology and Curtin University, Perth, Australia* (2012): 23-35.
- [Medina18] Medina, Daniel, et al. "On the Kalman filtering formulation for RTK joint positioning and attitude quaternion determination." *2018 IEEE/ION Position, Location and Navigation Symposium (PLANS)*. IEEE, 2018.

Thank you for your attention!

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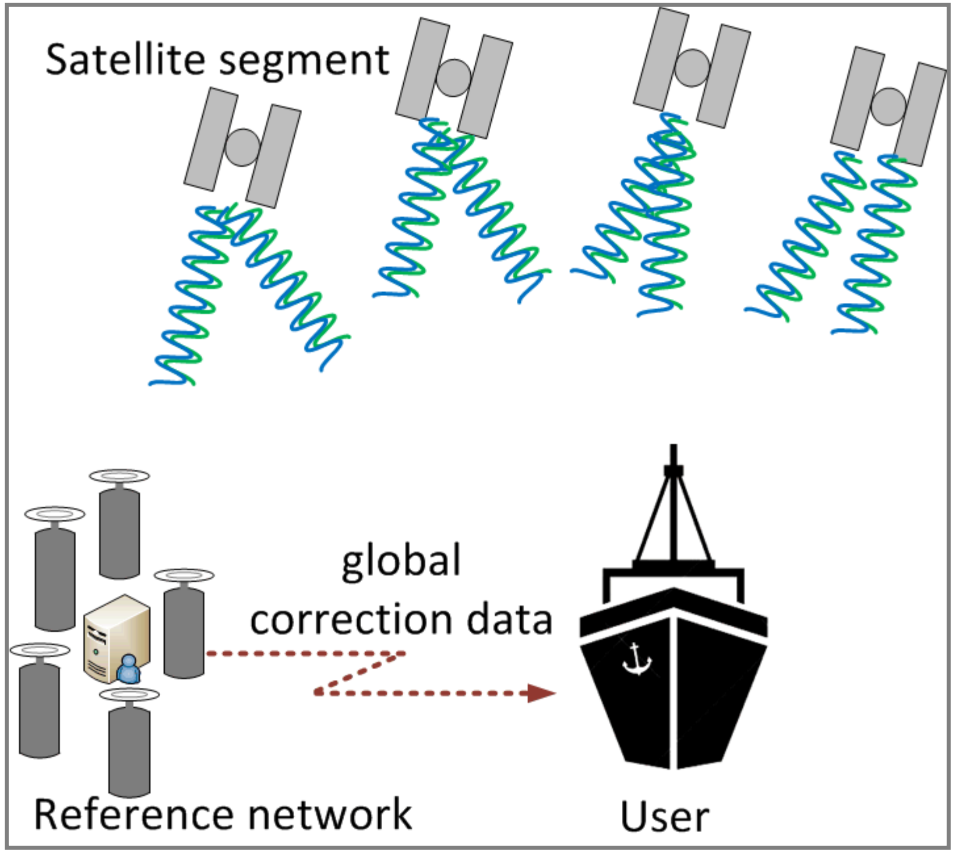
Backup Slides!

Precise Point Positioning

- PPP Principle



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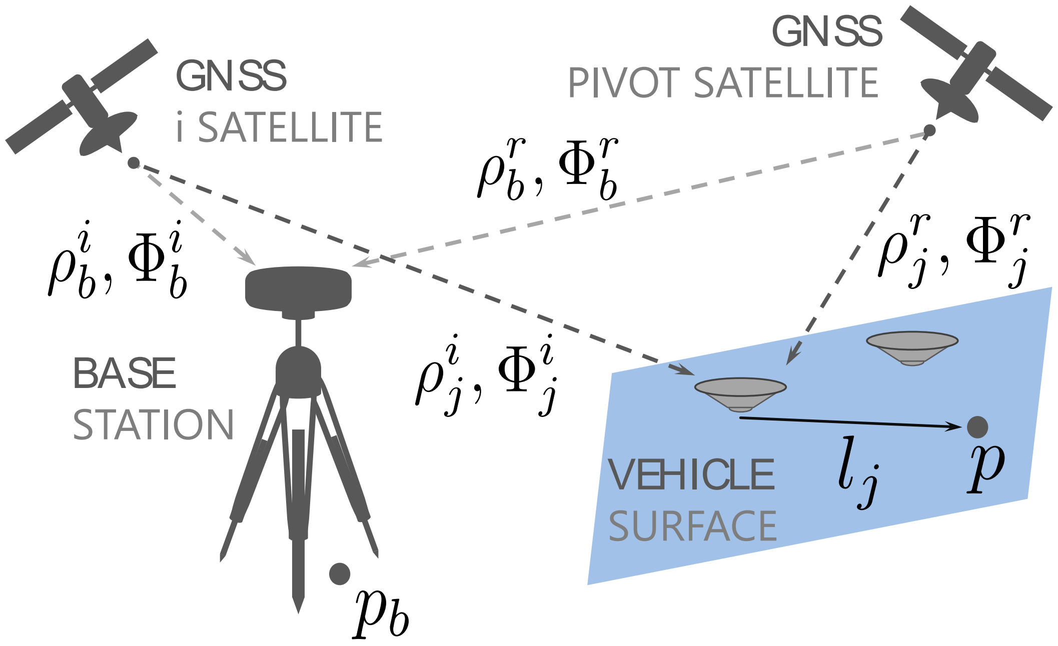
Real-time Kinematic

RTK Working Principle



- Differential phased-based positioning → base station of known coordinates transmits correction data
- Phase observations are more precise but ambiguous
- RTK positioning requires a "broad" communication channel
- **Challenges:**
  - ✗ Complexity of ambiguity resolution and dimensionality curse (multi -constellation, -frequency, -antenna)
  - ✗ Integrity of the system: are the ambiguities right?
- **Benefits:**
  - ✓ Instantaneous centimeter-level positioning
  - ✓ Accurate heading (<0.1° for 1 meter baseline)