Reliability of satellite-to-ground optical transmissions

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Applications of optical links

- Data transfer (telemetry/payload) from scientific or defense spacecraft (LEO)
- Telecommunication (GEO satellites)
- Metropolitan area networks (where fiber optics impractical)
- Deep space probes
...
High to very high data rates
~10 Gbps
>1 Tbps achievable if optical fiber technologies are exploited (WDM, DWDM, Optical amplifiers etc.)

- Decongestion of the RF spectrum
- Enhanced security (high directivity of the beam)
  Stealthy links and jamming capacity reduced
- Very large range (Deep Space applications)
SATELLITE OPTICAL DOWNLINKS - GENERAL CONTEXT

- High to very high data rates
  ~10 Gbps
  >1 Tbps achievable if optical fiber technologies are exploited (WDM, DWDM, Optical amplifiers etc.)

- Decongestion of the RF spectrum

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  Stealthy links and jamming capacity reduced

- Very large range (« Deep Space » applications)

Links highly affected by atmospheric turbulence
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~10 Gbps
>1 Tbps achievable if optical fiber technologies are exploited (WDM, DWDM, Optical amplifiers etc.)

- Decongestion of the RF spectrum

- Enhanced security (high directivity of the beam)
  Stealthy links and jamming capacity reduced

- Very large range (« Deep Space » applications)

Links highly affected by atmospheric turbulence

- Improvement of the link budget and telecom performances:
  + Coupling of incident flux into optical fiber (SMF)
  + Adaptive Optics (AO)
  + Optimisation of digital (coding/interleaving) techniques

Joint optimisation of AO and coding techniques to improve reliability of LEO and GEO downlinks (telemetry and telecoms applications)
PART I : JOINT OPTIMIZATION – PROBLEM OVERVIEW

PART II : AO FOR OPTICAL DONWLINKS

PART III : PHYSICAL LAYER PERFORMANCE ASSESSMENT

PART IV : AO/CROSS-LAYER OPTIMIZATION
Coupling losses and signal fadings
**Principle overview**

- **Opto-mechanical system:**
  Real time correction of phase distortions

- **Three key components:**
  Deformable mirror
  Wavefront sensor
  Real-time computer
ADAPTIVE OPTICS PRINCIPLE

Principle overview

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- **Three key components:**
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  Wavefront sensor
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Limitations?

Minimization of residual phase variance

\[ \sigma^2_{\phi_{res}} = \left\langle \frac{1}{S} \int (\phi(r) - \phi_{corr}(r))^2 \, dr \right\rangle_{\phi} \]
PHYSICAL LAYER DIGITAL MITIGATION TECHNIQUES

Coupled flux attenuation

HIGH PERF. AO
LOW PERF. AO

Time
BURST

Raw data stream

50% Redundancy

CODEWORD

“BURST”

HIGH PERF. AO
LOW PERF. AO

Physiological performance

Code performance limited

Symbol

Coupled flux attenuation

Physical layer digital mitigation techniques
Raw data stream

Interleaved data stream

Deinterleaving

Coupled flux attenuation

HIGH PERF. AO
LOW PERF. AO

SYMBOL

50 % Redundancy

“BURST”

CODEWORD

Time

Physicial Layer Digital Mitigation Techniques

Code performance limited

Code performance increased

Code performance increased

Code performance limited
How to effectively allocate memory (interleaver) and redundancy (code)?
Statistical and temporal characterizations of the channel needed:

**instantaneous coupled optical power after partial AO**
Instantaneous mutual information (~channel capacity)

Theoretical maximal data rate at which information can be transferred over the channel given noise level

The greater the better
Advantages of instantaneous mutual information (MI):

Emulation of interleaving-deinterleaving at Rx

\[ \text{Sliding average window} \]

**Instantaneous mutual information (~channel capacity)**

Theoretical maximal data rate at which information can be transferred over the channel given noise level
Advantages of instantaneous mutual information (MI):

Emulation of interleaving-deinterleaving at Rx

Sliding average window

E[MI(t)]

Ideal infinite interleaver = fundamental limit

Average power losses not recoverable by interleaving
Advantages of instantaneous mutual information (MI):

Emulation of Error Correcting Code (ECC) decoding

Shannon decoding theorem

**ECC coding rate** = amount of non-redundant information in message

\[ R_0^{\text{PHY}} = 0.5 \]
Advantages of instantaneous mutual information (MI):

Emulation of Error Correcting Code (ECC) decoding

Shannon decoding theorem

PHY layer performance metric = Outage probability
CODING possible at both PHY LAYER and HIGHER LAYERS
CROSS-LAYER APPROACH

CODING possible at both PHY LAYER and HIGHER LAYERS

Erasure Code (packets)

INFO Redundancy INFO Redundancy Red.

Error Corr. Code (bits)

INFO Redundancy INFO Redundancy Red. Redundancy

APPLICATION Tx

Higher layers

APPLICATION Rx

TRANSPORT

NETWORK

DATA LINK

PHYSICAL

FSO Channel

PHY Error correcting code redundancy

HL Erasure code redundancy
CROSS-LAYER APPROACH

CODING and INTERLEAVING possible at both PHY LAYER and HIGHER LAYERS

APPLICATION Tx → APPLICATION Rx

Higher layers

Erasure Code & Packet interleaving

Error Corr. Code & Symbol interleaving

PHYSICAL

TRANSPORT

NETWORK

DATA LINK

FSO Channel

Perf. metric: PER

Perf. metric: MI/Outage Prob.
CROSS-LAYER APPROACH

CODING and INTERLEAVING possible at both PHY LAYER and HIGHER LAYERS

Benefits of (optimized) cross-layer coding scheme:
- Lowering Packet Error Rate (PER)
- Alleviate drawbacks required by long interleavers needed on bursty channel
OVERALL SYSTEM OVERVIEW

Application Tx Data

Higher Layers Tx

Channel encoder

Modulator

Atmos. Propagation Channel

Adaptive Optics

Demodulator

Channel decoder

Application Rx Data

Higher Layers Rx

Physical Layer
PART I : JOINT OPTIMIZATION – PROBLEM OVERVIEW

PART II : AO FOR OPTICAL DOWNLINKS

Partial AO Analytic Modeling
Analytic Modeling of Partially Corrected Coupled Flux into SMFs

PART III : PHYSICAL LAYER PERFORMANCE ASSESSMENT

PART IV : AO/CROSS-LAYER OPTIMIZATION
Analytical laws for each one of the errors are known.
Ideal AO correction system: the low order residuals are perfectly corrected

Total residual error

\[ \sigma_{\text{res}}^2 \approx \sigma_{\text{Fitting}}^2 \]

High orders

PARTIAL ADAPTIVE OPTICS

\begin{align*}
\text{Power Spectral Density} \\
\text{Turbulent Spectrum} \\
\text{Fitting Error} \\
\text{Uncorrected high frequencies} \\
\text{Perfectly corrected low frequencies}
\end{align*}

Spatial Frequencies
Partial AO correction system: low order residuals that are highly correlating

$$\sigma_{\text{res}}^2 \approx \underbrace{\sigma_{\text{Fitting}}^2}_{\text{High orders}} + \underbrace{\sigma_{\text{Tempo}}^2 + \sigma_{\text{Alias}}^2 + (\sigma_{\text{Other}}^2)}_{\text{Low orders}}$$

Non negligible impact on communication performances
INSTANTANEOUS COUPLED OPTICAL POWER ATTENUATION

Neglecting amplitude spatial structures influence on coupling fluctuations:

Average coupling losses

Collected power fluctuations

Scintillation

Injection losses

\[ A_{SMF} = \exp(-\sigma^2_\chi) \exp(2\chi_P) \left| \int_{-\infty}^{\infty} W(r) \exp(i\phi(r)) \, dr \right|^2 \]

Rough approx, validated for medium elevation, small perturbations
INSTANTANEOUS COUPLED OPTICAL POWER ATTENUATION

Neglecting amplitude spatial structures influence on coupling fluctuations:

\[ A_{SMF} = \exp\left(-\sigma_X^2\right) \exp(2\chi_P) \left| \int_{-\infty}^{\infty} W(r) \exp(i\phi(r)) \, dr \right|^2 \]

Scintillation

Injection losses

Average coupling losses

Collected power fluctuations

Rough approx, validated for medium elevation, small perturbations

Illustration GEO-Downlink, Drx = 50cm, \( r_0 = 0.069 \, m \), 9 radial orders at 1 kHz

Optical power attenuation probability density distribution

Average fading time against normalized threshold

PART I: JOINT OPTIMIZATION – PROBLEM OVERVIEW

PART II: AO FOR OPTICAL DOWNLINKS

PART III: PHYSICAL LAYER PERFORMANCE ASSESSMENT

Optical Communication Subsystem Overview
- Outage Probability
- Minimum Required Interleaver

PART IV: AO/CROSS-LAYER OPTIMIZATION
PHYSICAL LAYER TRADE-OFF ASSESSMENT
LEO application case $D_{rx} = 0.25 \text{ m} \mid r_0 = 0.056 \text{ m}$

Outage probability for 10 Gbps link coderate $R_0=0.5$

Target: Min required power

- **Med. Perf. AO**
  - 15 actuators 800 Hz
  - 5 dB avg. attenuation

- **Low. Perf. AO**
  - 10 actuators 500 Hz
  - 7 dB avg. attenuation

- DPSK system with -7 dB AO
- DPSK system with -5 dB AO

Req. Rx Power [dBm]
Outage probability for 10 Gbps link coderate $R_0=0.5$

- **Med. Perf. AO**
  - 15 actuators 800 Hz
  - 5 dB avg. attenuation

- **Low. Perf. AO**
  - 10 actuators 500 Hz
  - 7 dB avg. attenuation

**Target:** $10^{-2}$

**Legend:**
- Blue solid line: DPSK system with -5 dB AO
- Green dashed line: DPSK system with -7 dB AO
- Black dash-dotted line: 30 ms interleaver (300 Mb)
Outage probability for 10 Gbps link coderate $R_0=0.5$

**Outage Probability**

- **High Perf. AO**
  - 50 actuators at 2 kHz
  - 3dB avg. attenuation

- **Med. Perf. AO**
  - 15 actuators 800 Hz
  - 5dB avg. attenuation

- **Low. Perf. AO**
  - 10 actuators 500 Hz
  - 7dB avg. attenuation

**Target:** $10^{-2}$

**Required Rx Power [dBm]**

- OOK system with -3db AO
- DPSK system with -5dB AO
- DPSK system with -7dB AO
- 30 ms interleaver (300 Mb)

**Optimizing interleaver at targeted outage probability & required power?**
PHYSICAL LAYER TRADE-OFF ASSESSMENT
LEO application case $D_{rx} = 0.25 \ m$ | $r_0 = 0.056 \ m$ | OOK

Minimum Required PHY interleaving depth
High Perfo. AO | Outage Prob. $10^{-2}$

- Req. Rx Power = $-41 \ dBm$
- Req. Rx Power = $-38 \ dBm$

Min. Required PHY Interleaver Memory [Mb]

Low redundancy

PHY Code Rate
Minimum Required PHY interleaving depth
High Perfo. AO | Outage Prob. 10^{-2}

Req. Rx Power = -41 dBm
Req. Rx Power = -38 dBm

Neglecting lower order residuals underestimate required memory by 50%
Minimum Required PHY interleaving depth
High Perfo. AO | Outage Prob. $10^{-2}$

- Req. Rx Power = -41 dBm
- Req. Rx Power = -38 dBm

Ideal Infinite interleaver limit
Minimum Required PHY interleaving depth

High Perfo. AO | Outage Prob. $10^{-2}$

Req. Rx Power = -41 dBm

Req. Rx Power = -38 dBm

Decrease in redundancy
sharp increase in memory

Transfer to HL
PART I : JOINT OPTIMIZATION – PROBLEM OVERVIEW

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PART III : PHYSICAL LAYER PERFORMANCE ASSESSMENT

PART IV : AO/CROSS-LAYER OPTIMIZATION

Modeling of Cross-layer Coding Scheme
Case Study: LEO Downlink Using DPSK
The PHY and HL coderates inherently related through definition of global coderate:

\[
R_0^{GLOBAL} = R_0^{PHY} \cdot R_0^{HL}
\]

Useful Data Rate = raw data-rate \times R_0^{GLOBAL}
The PHY and HL coderates inherently related through definition of global coderate:

\[ R_{0}^{\text{GLOBAL}} = R_{0}^{\text{PHY}} R_{0}^{\text{HL}} \]

**Fixed global code rate**

**Useful Data Rate** = raw data-rate \( \times R_{0}^{\text{GLOBAL}} \)

**Tx PHY memory** = PHY interleaver depth \( \times \) raw data-rate

**Tx HL memory** = HL interleaver depth \( \times \) raw data-rate \( \times R_{0}^{\text{PHY}} \)
CROSS-LAYER APPROACH: Modeling

\[ R_0^{\text{GLOBAL}} = R_0^{\text{PHY}} R_0^{\text{HL}} = 0.3 \]
CROSS-LAYER APPROACH : Modeling

\[ R_0^{\text{GLOBAL}} = R_0^{\text{PHY}} R_0^{\text{HL}} = 0.3 \]
CROSS-LAYER APPROACH: Modeling

\[ R_0^{\text{GLOBAL}} = R_0^{\text{PHY}} R_0^{\text{HL}} = 0.3 \]

**Mutual Information**

- **\( R_0^{\text{PHY}} = 0.5 \)**
- **\( 1 - R_0^{\text{HL}} = 0.4 \)**

**PHY output PER**

- **GOOD**
- **BAD**
CROSS-LAYER APPROACH: Modeling

$R_0^{\text{GLOBAL}} = R_0^{\text{PHY}} R_0^{\text{HL}} = 0.3$

**Physical Layer**

**Higher Layers**

Data Link | Network | Transport

**Application Layer**

Mutual information

$R_0^{\text{PHY}} = 0.5$

$1 - R_0^{\text{HL}} = 0.4$
CROSS-LAYER APPROACH: Modeling

\[ R_{0}^{GLOBAL} = R_{0}^{PHY} R_{0}^{HL} = 0.3 \]

**PHYSICAL LAYER**

**HIGHER LAYERS**
- DATA LINK
- NETWORK
- TRANSPORT

**APPLICATION LAYER**

**FINAL PERFORMANCE METRIC**: Average of decoded PER

**Graphs**
- **Mutual information**
  - \[ R_{0}^{PHY} = 0.5 \]

- **PHY output PER**
  - \[ 1 - R_{0}^{HL} = 0.4 \]

- **HL output Decoded PER**
CROSS-LAYER APPROACH: Benefits illustration

\[ R_0^{\text{GLOBAL}} = 0.8 \quad \text{| Req. Power} = -38 \text{ dBm} \quad \text{| PHY interleaver 50 Mb} \]

Average decoded PER against size of HL interleaving

\[ R_0^{\text{PHY}} = 0.8 \quad R_0^{\text{HL}} = 1 \]
\[ R_0^{\text{PHY}} = 0.82 \quad R_0^{\text{HL}} = 0.97 \]
\[ R_0^{\text{PHY}} = 0.86 \quad R_0^{\text{HL}} = 0.93 \]
\[ R_0^{\text{PHY}} = 0.90 \quad R_0^{\text{HL}} = 0.89 \]

\(-\)
CROSS-LAYER APPROACH: Benefits illustration

\[ R_0^{\text{GLOBAL}} = 0.8 \mid \text{Req. Power} = -38 \text{ dBm} \mid \text{PHY interleaver 50 Mb} \]

Average decoded PER against size of HL interleaving

No coding on HL

- \( R_0^{\text{PHY}} = 0.8 \), \( R_0^{\text{HL}} = 1 \)
- \( R_0^{\text{PHY}} = 0.82 \), \( R_0^{\text{HL}} = 0.97 \)
- \( R_0^{\text{PHY}} = 0.86 \), \( R_0^{\text{HL}} = 0.93 \)
- \( R_0^{\text{PHY}} = 0.90 \), \( R_0^{\text{HL}} = 0.89 \)
CROSS-LAYER APPROACH: Benefits illustration

\( R_0^{\text{GLOBAL}} = 0.8 \) | Req. Power = -38 dBm | PHY interleaver 50 Mb

Average decoded PER against size of HL interleaving

Optimal allocation of redundancy btw PHY and HL

<table>
<thead>
<tr>
<th>( R_0^{\text{PHY}} )</th>
<th>( R_0^{\text{HL}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.8</td>
<td>1</td>
</tr>
<tr>
<td>0.82</td>
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<td>0.86</td>
<td>0.93</td>
</tr>
<tr>
<td>0.90</td>
<td>0.89</td>
</tr>
</tbody>
</table>

\( R_0^{\text{PHY}} \) vs. \( R_0^{\text{HL}} \) for different values of \( R_0^{\text{PHY}} \) and \( R_0^{\text{HL}} \) showing the mean decoded PER against the size of HL interleaving memory.
RESOURCES OPTIMIZATION USING MINIMUM TOTAL MEMORY
TARGET PER = 10^{-4} | 10 Gbps LEO link
DPSK Receiver

A lot of coding (Redundancy) | Useful Data Rate [Gbps] | No coding (Redundancy)

Min. Total Tx Memory (PHY+HL) [Mb]

- High Perf. AO (-3 dB)
- Med Perf. AO (-5 dB)
- Low Perf. AO (-7 dB)
RESOURCES OPTIMIZATION USING MINIMUM TOTAL MEMORY
TARGET PER = 10^{-4} | 10 Gbps LEO link
DPSK Receiver

Min. Total Tx Memory (PHY+HL) [Mb]

- High Perf. AO (-3 dB)
- Med Perf. AO (-5 dB)
- Low Perf. AO (-7 dB)

Reduction in redundancy compensated by increase in interleaving

Useful Data Rate [Gbps]

A lot of coding (Redundancy) → No coding (Redundancy)
RESOURCES OPTIMIZATION USING MINIMUM TOTAL MEMORY
TARGET PER = 10^{-4} | 10 Gbps LEO link
DPSK Receiver

Unfad. Rx Power [dBm] : -38
PHY mem. [Mb] : 0 | HL mem. [Mb] : 0
PHY Coderate : 0.48 | HL Coderate : 0.62

Unfad. Rx Power [dBm] : -38
PHY Coderate : 0.48 | HL Coderate : 0.62

Unfad. Rx Power [dBm] : -41
PHY Coderate : 0.5 | HL Coderate : 1

Min. Total Tx Memory (PHY+HL) [Mb]

A lot of coding (Redundancy) | Useful Data Rate [Gbps] | No coding (Redundancy)

High Perf. AO (-3 dB)
Med Perf. AO (-5 dB)
Low Perf. AO (-7 dB)

HL interl. exclusively
**RESOURCES OPTIMIZATION USING MINIMUM TOTAL MEMORY**  
TARGET PER = $10^{-4}$ | 10 Gbps LEO link  
DPSK Receiver

- **High Perf. AO (-3 dB)**: HL interl. exclusively
- **Med Perf. AO (-5 dB)**
- **Low Perf. AO (-7 dB)**

- **Unfad. Rx Power [dBm]**: -38  
  - PHY mem. [Mb]: 0 | HL mem. [Mb]: 97  
  - PHY Coderate: 0.48 | HL Coderate: 0.62

- **Unfad. Rx Power [dBm]**: -38  
  - PHY mem. [Mb]: 0 | HL mem. [Mb]: 37  
  - PHY Coderate: 0.36 | HL Coderate: 0.83

- **Unfad. Rx Power [dBm]**: -41  
  - PHY mem. [Mb]: 102 | HL mem. [Mb]: 0  
  - PHY Coderate: 0.5 | HL Coderate: 1

**HL interleaving optimal in less challenging conditions**

Physical turbulence mitigation techniques (AO) can have significant impact on overall system design optimization by driving hardware trade-offs (eg. ASIC vs RAM interleaver)
CONCLUSION
How to ensure reliable optical downlinks?

SIGNIFICANT RESULTS

- First accurate model of partially corrected coupled flux into SMFs
- Detailed investigation of required physical layer interleaving depth and ECC
- First application of cross-layer coding/interleaving scheme to sat. opt. transmissions
- Investigation of overall optimization of AO and data reliability mechanisms
RECOMMENDATION FOR FUTURE WORK

- **Experimental validation** - LEO Downlink planned (DLR’s OSIRIS & ONERA’s LISA)

- Investigation of performance evolution over the duration of a whole link

- Input turbulence conditions not well known (except astron. observation sites)

- Transposition of approach to GEO uplink
Peer-reviewed publication


International conferences


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TéSA
Corinne Mailhes
PHYSICAL LAYER TRADE-OFF ASSESSMENT
Minimum required interleaving depth

Minimum Required PHY interleaving depth
High Perfo. OA | Outage Prob. $10^{-2}$

- Req. Rx Power = -41 dBm
- Req. Rx Power = -38 dBm

Analytic estimation based on statistical and temporal characteristics of coupled flux
MINIMUM MEMORY REQUIRED FOR ERROR-FREE TRANSMISSION
10 Gbps link \( R_0^{\text{GLOBAL}} = 0.6 \) (6 Gbps throughput) | High Perf. AO

MINIMUM MEMORY REQUIRED FOR ERROR-FREE TRANSMISSION
10 Gbps link \( R_0^{\text{GLOBAL}} = 0.6 \) (6 Gbps throughput) | High Perf. AO

Region of PHY Interleaving only

RX Total mem. [Mb] : 678
PHY mem. [Mb] : 502
HL mem. [Mb] : 176

Functioning point requiring both PHY and HL interleaving

NOT ERROR-FREE FOR CONSIDERED INTERLEAVERS RANGE
MINIMUM MEMORY REQUIRED FOR ERROR-FREE TRANSMISSION
10 Gbps link | $R_0^{GLOBAL} = 0.6$ (6 Gbps throughput) | High Perf. AO

Region of HL Interleaving only

RX Total mem. [Mb] : 972
PHY mem. [Mb] : 0
HL mem. [Mb] : 972

RX Total mem. [Mb] : 65
PHY mem. [Mb] : 0
HL mem. [Mb] : 65

Optimal allocation of redundancy

Functioning point requiring minimum overall memory and power

RX Total mem. [Mb] : 88
PHY mem. [Mb] : 0
HL mem. [Mb] : 88

NOT ERROR-FREE FOR CONSIDERED INTERLEAVERS RANGE

Optimal allocation of redundancy

PHY coderate | HL coderate

0.60 | 1.00  0.65 | 0.92  0.70 | 0.92  0.75 | 0.80  0.80 | 0.75  0.80 | 0.80  0.75 | 0.85  0.70 | 0.90  0.67  0.95 | 0.62  1.00 | 0.60
RESOURCES OPTIMIZATION USING MINIMUM TOTAL MEMORY TARGET PER = 10^{-4} OOK Receiver
PHYSICAL MITIGATION TECHNIQUES: APERTURE AVERAGING PLUS AO

Increase in Drx = Aperture averaging of scintillation

Adaptive Optics = Real-time correction of phase fluctuations only

For a fixed Drx scintillation effects are imposed upon the detector

The design of the AO system must be done consequently

Trade-offs scintillation/phase fluctuations mitigation and therefore performances/cost

High perfs AO ➔ Limited by scintillation

Low perfs AO ➔ Limited by phase fluctuations
INSTANTANEOUS COUPLED OPTICAL POWER ATTENUATION

EM field of incoming wave
\[ \Psi(\mathbf{r}) = A_0(\mathbf{r}) \exp(\chi(\mathbf{r}) + i\phi(\mathbf{r})) \]

Unperturbed amplitude

Instantaneous coupled optical power attenuation
\[ A_{SMF} = \left| \int_{-\infty}^{+\infty} P(\mathbf{r}) W(\mathbf{r}) \exp(\chi(\mathbf{r}) + i\phi(\mathbf{r})) \, d\mathbf{r} \right|^2 \]

Log-amplitude

Phase (corrected)

Neglecting amplitude spatial structures influence on coupling fluctuations:

Average coupling losses

Collected power fluctuations

Scintillation

Injection losses

Perfect AO correction

Rustic approx, justified for GEO downlinks (medium elevation, small perturbations)
Phase decomposition into Zernike polynomials
(= Orthonormal basis over a plane and circular domain)

\[ \phi(\mathbf{r}) = \sum_{i=1}^{N} a_i Z_{i} \left( \frac{2r}{D} \right) \]

Zernike coefficient \( a_i \)
Zernike polynomial \( Z_i \)

Set of Zernike polynomials not orthonormal

Retro-propagated SMF mode in pupil plane

Phase decomposition after "re-orthonormalisation"

\[ \phi(\mathbf{r}) = \sum_{i=1}^{N} b_i F_i \left( \frac{2r}{D} \right) \]

Statistical properties (PDF) of each \( a_i \): known analytically
(Independent Gaussian variables BUT not identical)

Temporal properties (PSD) of each \( a_i \): known analytically

Transfer Matrix \{a\} \rightarrow \{b\}: known analytically

"Spatial variance" of the phase fluctuations in the focal plane

Closed-form injection efficiency approximation:

\[ \frac{\rho_\phi}{\rho_0} \sim \exp \left[ -\sigma_{W_0}^2 (\phi) \right] \]
Instantaneous coupled optical power attenuation

\[ A_{SMF} = \exp(-\sigma^2) \exp(2\chi P) \left| \int_{-\infty}^{\infty} W(r) \exp(i\phi(r)) dr \right|^2 \]

Typical Log-normal distribution

Injection losses

"Exponentiated sum of non-identical Gamma variates"

Scintillation

Statistical laws (PDF/CDF)

Temporal properties (PSD, coherence time)

Analytic expressions

![Graph showing normalized probability vs. optical power attenuation](image1)

![Graph showing mean duration of fade events](image2)
RESOURCES OPTIMIZATION USING MINIMUM TOTAL MEMORY
TARGET PER = 10^-4 | 10 Gbps LEO link
DPSK Receiver

Unfad. Rx Power [dBm] : -38
PHY Coderate : 0.36 | HL Coderate : 0.83

Unfad. Rx Power [dBm] : -41
PHY Coderate : 0.5 | HL Coderate : 1

Unfad. Rx Power [dBm] : -39
PHY Coderate : 0.7 | HL Coderate : 1