

On the Time-Delay Estimation Accuracy Limit of GNSS Meta- Signals

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Context

GNSS Goal:

$$\|\mathbf{p}_{TR}(t)\| \triangleq \|\mathbf{p}_R(t) - \mathbf{p}_T(t - \tau(t))\| = c\tau(t) \simeq d + vt,$$
$$\tau(t) \simeq \tau + bt, \quad \tau = \frac{d}{c}, \quad b = \frac{v}{c}, \quad c = 299,792,458 \text{ m/s},$$

Time-Delay Accuracy Limit :

- *Cramer Rao Bound*

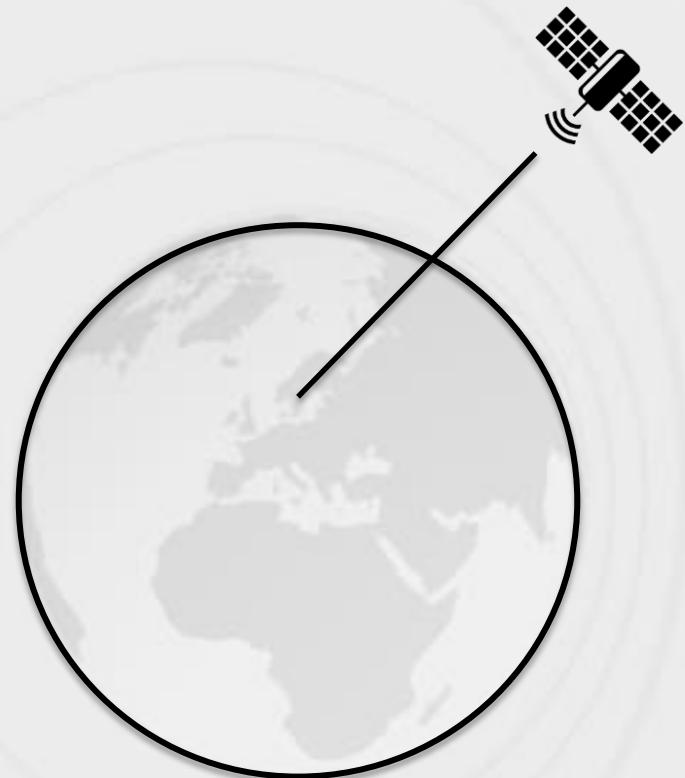
GNSS Meta-Signals

- *GPS*
- *Galileo*
- *Beidou*

Extended Work:

[1] Das, P.; Ortega, L.; Vilà-Valls, J.; Vincent, F.; Chaumette, E.; Davain, L. Performance Limits of GNSS Code-Based Precise Positioning: GPS, Galileo & Meta Signals. *Sensors* **2020**, *20*, 2196.

[2] Ortega, L.; Medina, D.; Vilà-Valls, J.; Vincent, F.; Chaumette, E. Positioning Performance Limits of GNSS Meta-Signals and HO-BOC Signals. *Sensors* **2020**, *20*, 3586.



Content

- ***1-Signal Model***
- ***2-GNSS Meta-Signals***
- ***3-Time-delay CRB for Band-limited Signals and Maximum Likelihood Estimation***
- ***4-Results and Conclusions***

1-Signal Model

$$x_A(t) = \alpha_R c ((1 - b) (t - \tau)) e^{j2\pi f_c(1-b)t} e^{-j2\pi f_c \tau} + n_A(t),$$

$$x_A(t) \approx \alpha_R c (t - \tau) e^{j2\pi f_c(1-b)t} e^{-j2\pi f_c \tau} + n_A(t),$$

and the baseband output of the receiver's Hilbert filter is

$$x(t) = \alpha c (t - \tau) e^{-j2\pi f_c b t} + n(t),$$

The discrete vector signal model is build from $N = N_2 - N_1 + 1$ samples at $T_s = \frac{1}{F_s}$,

$$\mathbf{x} = \alpha \mathbf{a}(\boldsymbol{\eta}) + \mathbf{n},$$

$$\mathbf{x} = (x(N_1 T_s), \dots, x(N_2 T_s))^{\top},$$

$$\mathbf{n} = (n(N_1 T_s), \dots, n(N_2 T_s))^{\top},$$

$$\mathbf{c}(\tau) = (c(N_1 T_s - \tau), \dots, c(N_2 T_s - \tau))^{\top},$$

$$\mathbf{a}(\boldsymbol{\eta}) = ((\mathbf{c}(\tau))_1 e^{-j2\pi f_c b N_1 T_s}, \dots, (\mathbf{c}(\tau))_N e^{-j2\pi f_c b N_2 T_s})^{\top},$$

2-GNSS Meta-Signals

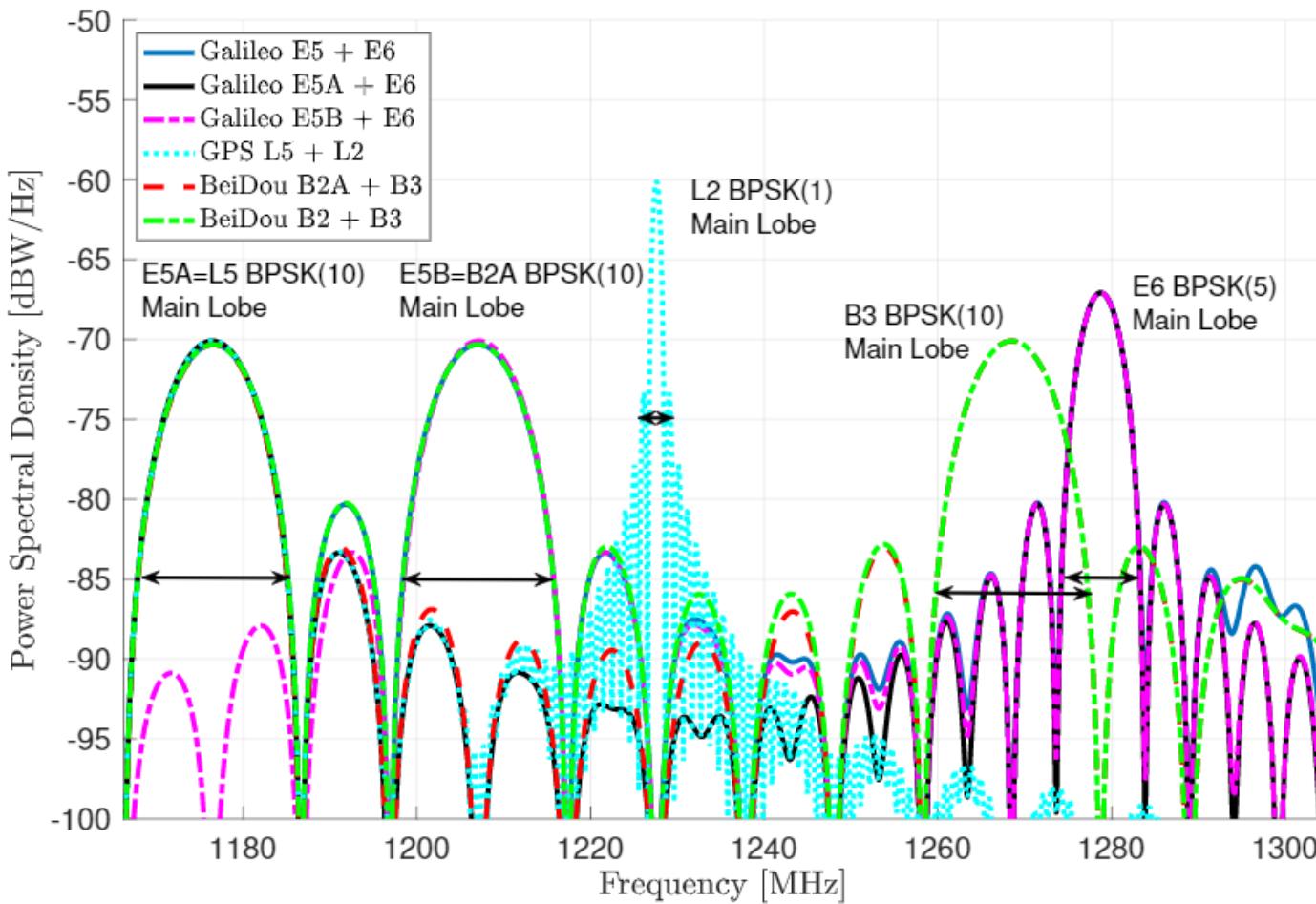
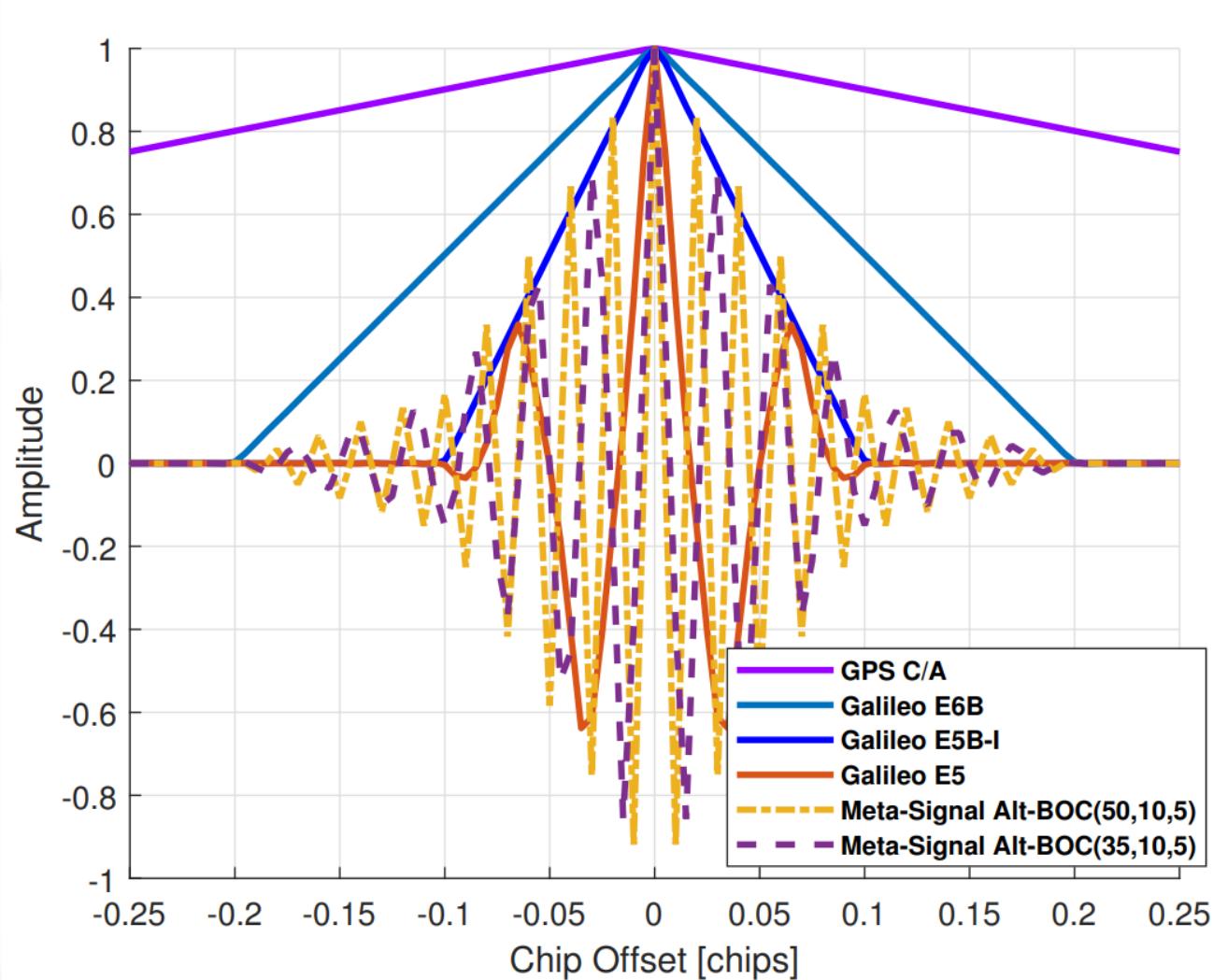
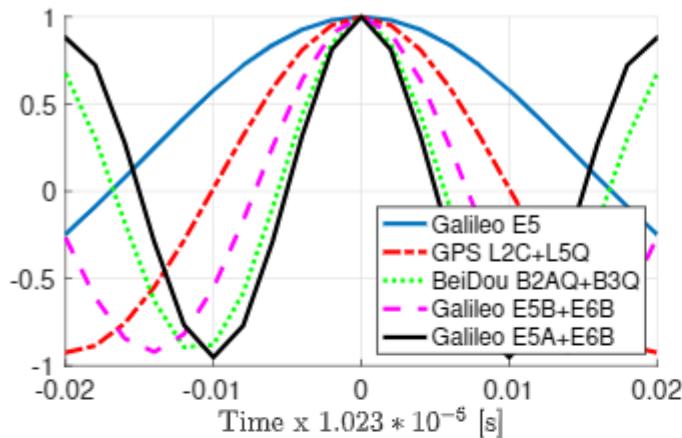


Figure 1. PSD for the different GNSS meta-signals.

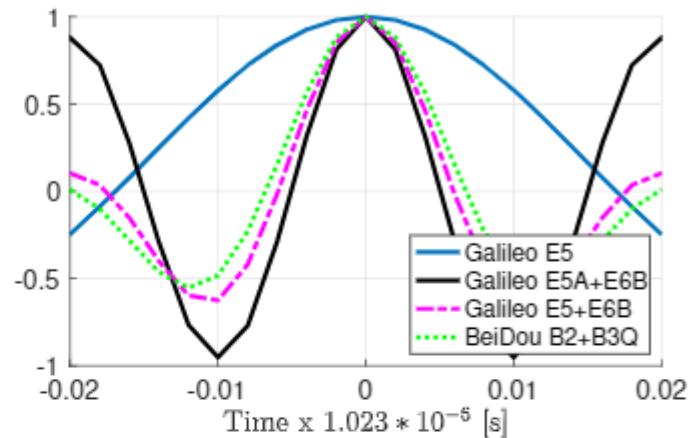
2-GNSS Meta-Signals



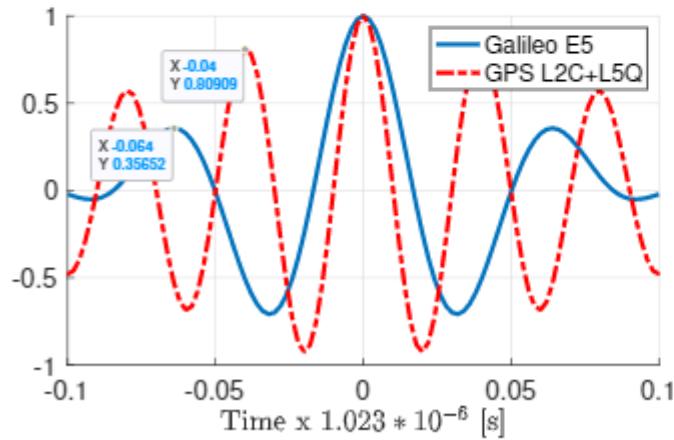
2-GNSS Meta-Signals



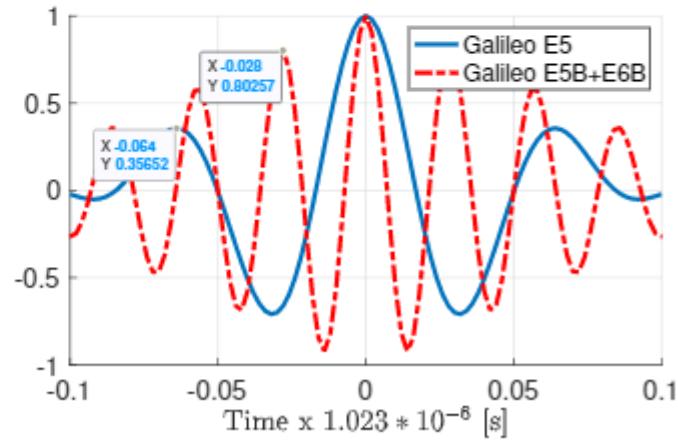
(a)



(b)

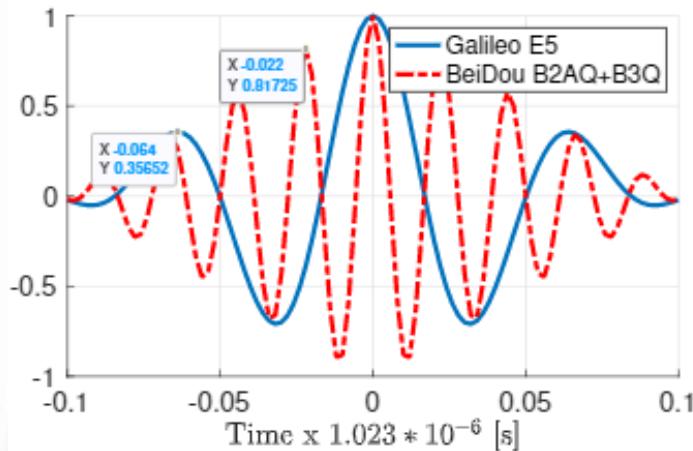


(c)

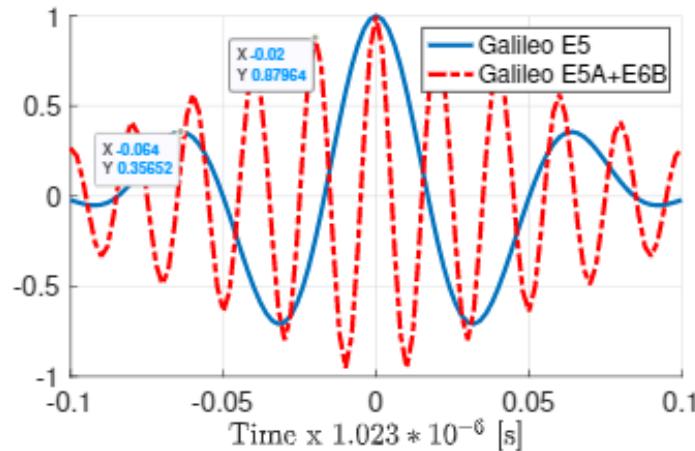


(d)

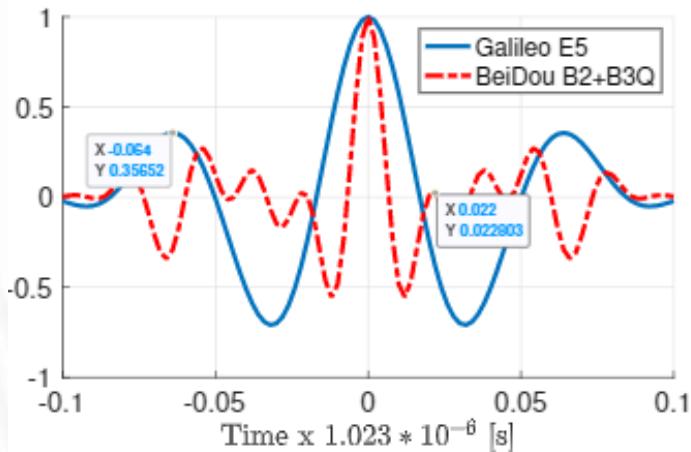
2-GNSS Meta-Signals



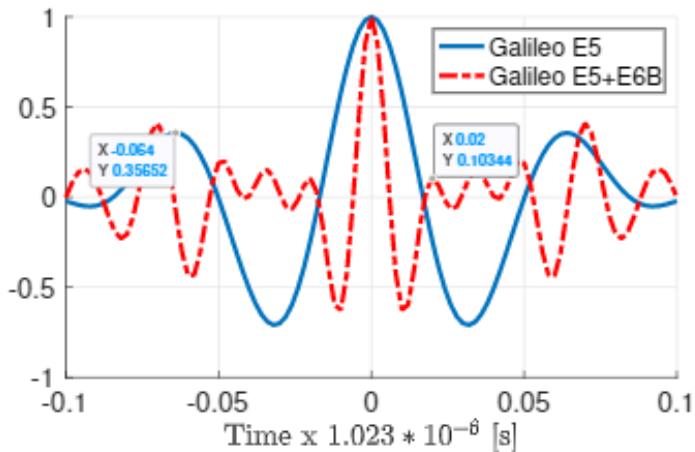
(e)



(f)



(g)



(h)

3-Time-delay Cramer Rao Bound for Band-limited Signals and Maximum Likelihood Estimation

Cramer Rao Bound for Band-limited Signals

$$F_{\tau|\underline{\epsilon}}(\underline{\epsilon}) = 2\text{SNR}_{\text{out}} F_s^2 \left(\frac{\mathbf{c}^H \mathbf{V} \mathbf{c}}{\mathbf{c}^H \mathbf{c}} - \left| \frac{\mathbf{c}^H \boldsymbol{\Lambda} \mathbf{c}}{\mathbf{c}^H \mathbf{c}} \right|^2 \right) \underset{\text{real signal}}{=} 2\text{SNR}_{\text{out}} F_s^2 \left(\frac{\mathbf{c}^T \mathbf{V} \mathbf{c}}{\mathbf{c}^T \mathbf{c}} \right),$$

where $\text{SNR}_{\text{out}} = \frac{|\alpha|^2 \mathbb{E}}{(\sigma_n^2 / F_s)} = \frac{|\alpha|^2}{\sigma_n^2} \mathbf{c}^H \mathbf{c}$ and \mathbb{E} is the energy of the signal. $\boldsymbol{\Lambda}$ and \mathbf{V} are defined as (for $N_1 \leq n, n' \leq N_2$)

$$(\mathbf{V})_{n,n'} = \begin{cases} n' \neq n : (-1)^{|n-n'|} \frac{2}{(n-n')^2} \\ n' = n : \frac{\pi^2}{3} \end{cases}; \quad (\boldsymbol{\Lambda})_{n,n'} = \begin{cases} n' \neq n : \frac{(-1)^{|n-n'|}}{(n-n')} \\ n' = n : 0 \end{cases}.$$

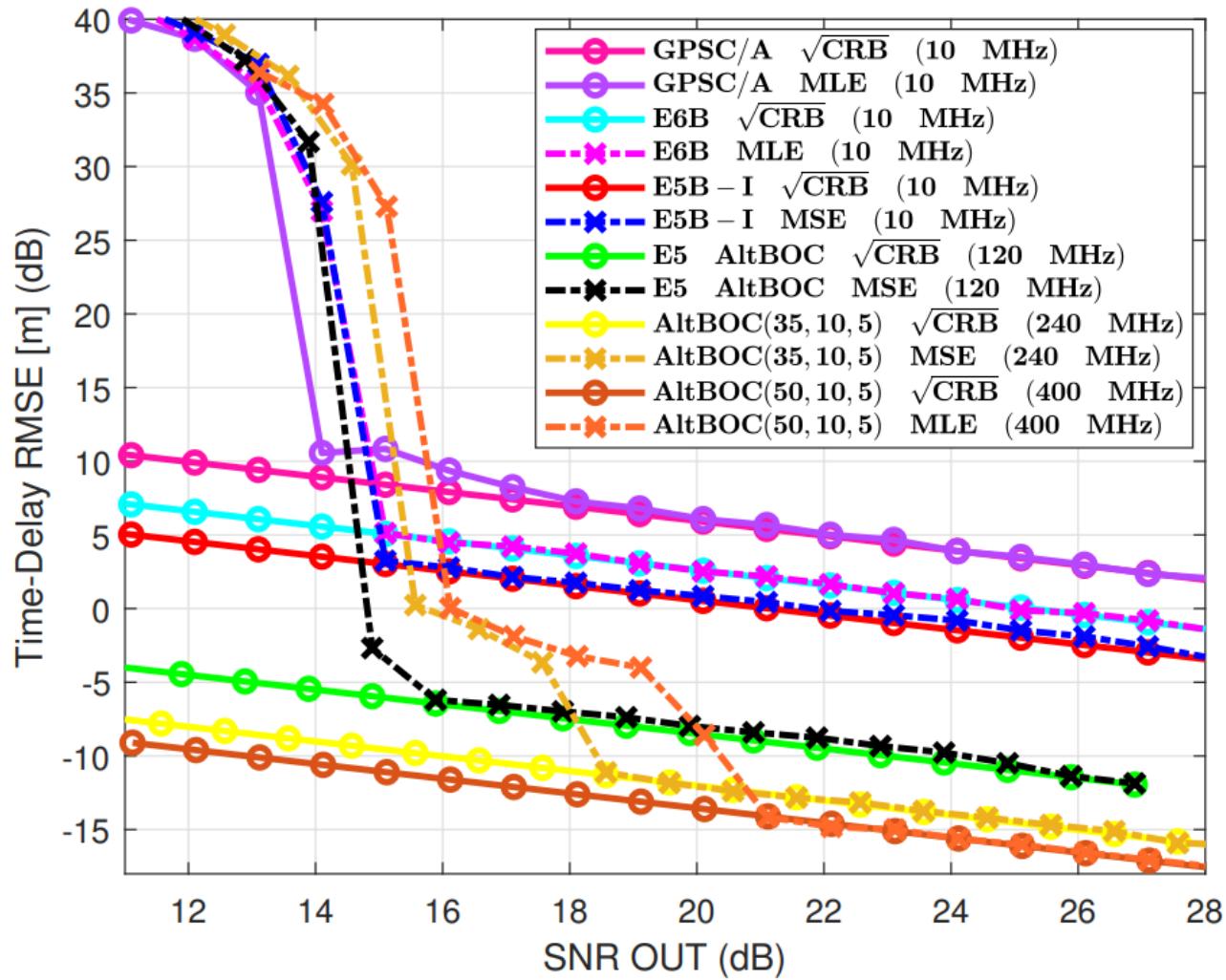
From the FI, the time-delay CRB is written as

$$\text{CRB}_{\tau|\underline{\epsilon}}(\underline{\epsilon}^0) = \left(F_{\tau|\underline{\epsilon}}(\underline{\epsilon}^0) \right)^{-1},$$

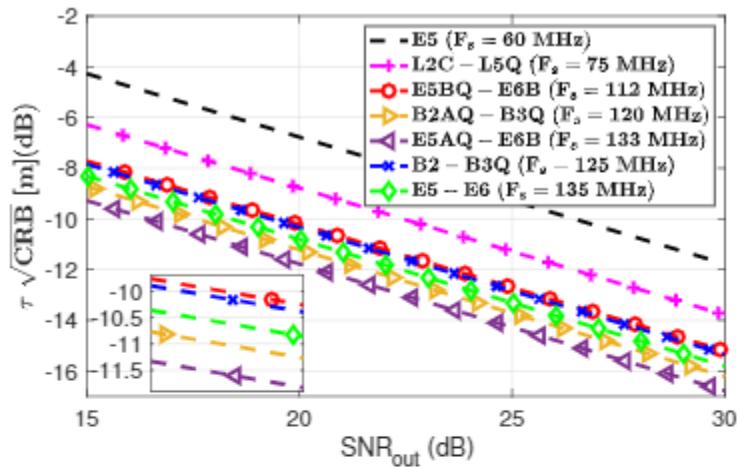
Maximum Likelihood Estimation

$$\hat{\tau} = \arg \min_{\tau} \left\{ \mathbf{x}^H \boldsymbol{\Pi}_{\mathbf{c}(\tau)}^\perp \mathbf{x} \right\} = \arg \max_{\tau} \left\{ \frac{|\mathbf{c}(\tau)^H \mathbf{x}|^2}{\mathbf{c}(\tau)^H \mathbf{c}(\tau)} \right\},$$

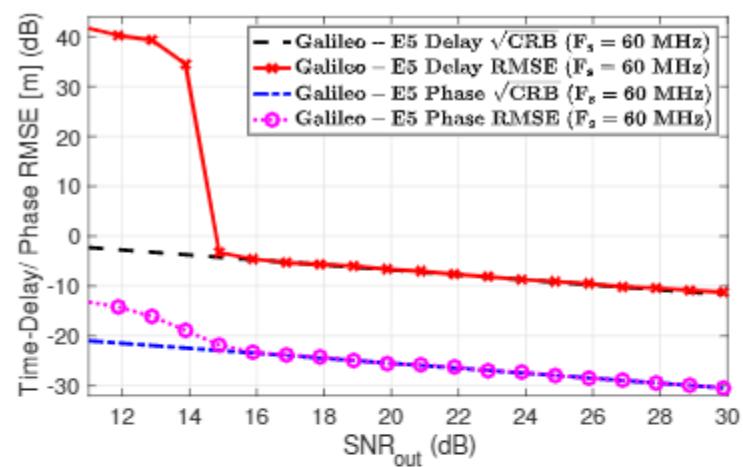
4-Results



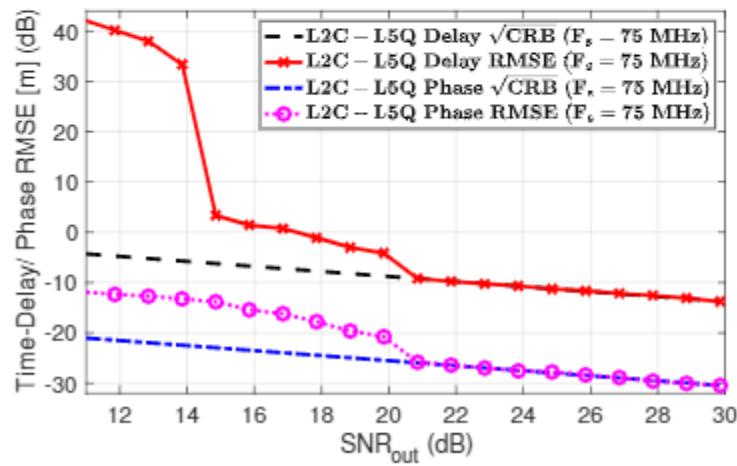
4-Results



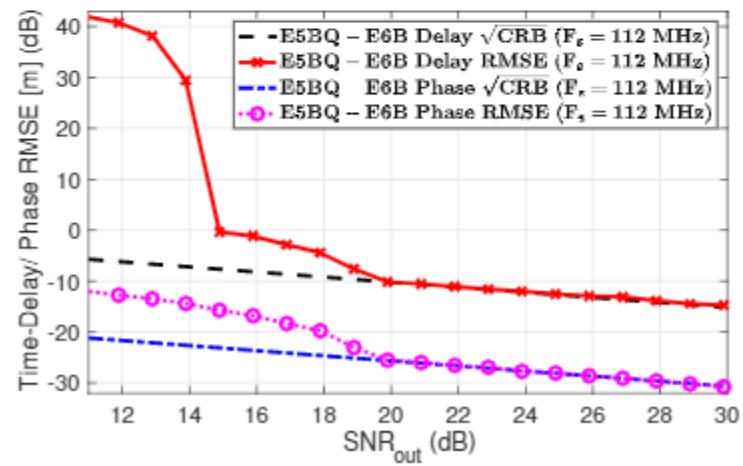
(a)



(b)

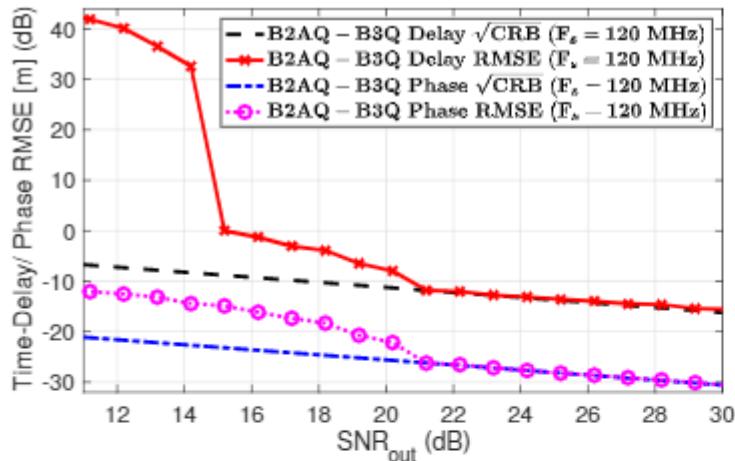


(c)

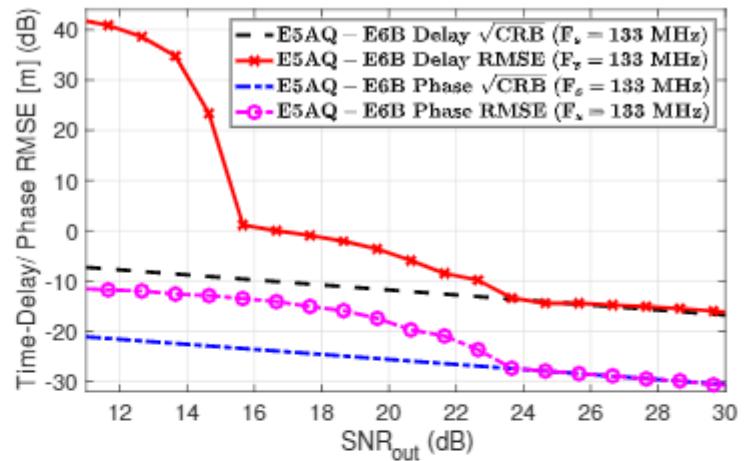


(d)

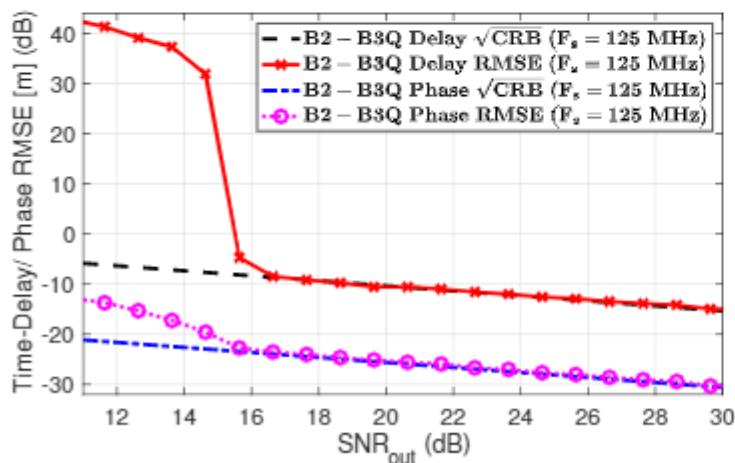
4-Results



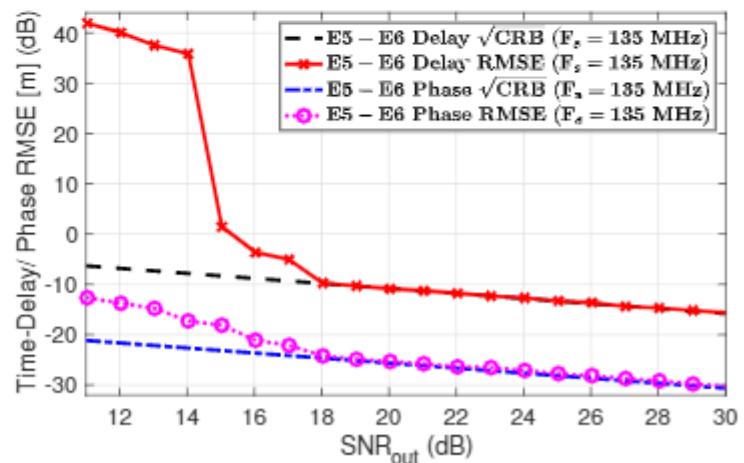
(e)



(f)



(g)



(h)

Conclusions

- *Evaluation of the time-delay estimation of the GNSS Meta-signals.*
- *Formula of the Cramer Rao bound (depends on the samples of the signal).*
- *Evaluate the threshold through the MLE.*
- *Large Bandwidth GNSS Meta-Signals can have possible false locks due to high secondary correlation peaks. This issue can degraded the time-delay estimation performance.*