On sparse graph coding for coherent and noncoherent demodulation

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***Abstract*—In this paper, we consider a bit-interleaved coded modulation scheme (BICM) composed of an error correcting code serially concatenated with a M-ary non linear modulation with memory. We first compare demodulation strategies for both the coherent and the non coherent cases. Then, we perform an asymptotic analysis and try to show that the design of coding schemes performing well for both the coherent and the non coherent regimes should be done carefully when considering sparse graph based codes such as low-density parity-check (LDPC) codes. It will be shown that optimized coding schemes for the non coherent setting can perform fairly well when using coherent demodulation, while on the contrary, optimized coding schemes for the coherent setting may lead to "non stable" coding schemes in the non coherent setting.**

I. Introduction

Continuous phase modulation (CPM) [1],[2], [3] is a particular modulation characterised by a constant envelop waveform and a phase continuity. The phase of a CPM signal for a given symbol is determined by the cumulative phase of previous transmitted symbols known as the phase memory. Another important feature of CPMs is the modulation index which could restrain, in a particular case, the set of the phase memory to a finite set. A well-known type of CPMs is the CPFSK [4], [5] described by a rectangular phase response and a memory of one, meaning that the new phase is computed only from one previous symbol. In the coherent case, an efficient method to demodulate is proposed by Cheng based on the well-knwon BCJR [7]. Cheng [6] implemented the BCJR algorithm for CPFSK. The trellis consists of Q states where Q is the cardinal of the set composed of all possible values taken by the phase memory. Q is also the denominator of the CPFSK modulation index $h=\frac{P}{Q}$. To get a coherent detector with a finite set of phase memory, it is mandatory having h rational. In noncoherent channel, symbol detection is done through a multisymbol detector well described by Valenti [8].

Multisymbol receiver proposed by Valenti [8][9] does the correlation between the block of received symbols and all existing combinations of same length blocks. The condition required to use this method is the absence of phase shift between symbols belonging to the same block. As well, in this paper perfect frequency and time synchronisation is assumed. We suppose the phase shift is stable over a block of N symbols. It exists M^N possible combinations for a block of size N (with M the modulation order). The process is conducted as follows. The incoming signal is filtered by a bank of M matched filters. Then the conditional probability of the transmitted symbols block of size N under the condition of one of the possible combination is done for each existing combination. The analytical expression of the conditional probability is independent of the initial phase of the block, moreover another extra advantage of the multisymbol receiver is that it can work for any value of h. It reveals the robustness of the detector against untimely channel phase shift.

MWM [10] is strictly equivalent to BCJR in coherent channel but it can also demodulate in noncoherent channel by getting the best from multisymbol. Like BCJR, MWM is a trellis based detector. Block of symbols and accumulated phase are included in states. The aim is to determine the conditional probability of symbols given the observations. The process is done across a forward-backward algorithm just as usual BCJR. The difference lays in the number of states and the correlation done in the branch metric. Indeed the trellis consists of $Q \cdot M^N$ states and the branch metric connecting two consecutive states δ_k and δ_{k+1} is determined from the δ_k accumulated phase and the correlation of symbols belonging into the two consecutive states.

Mutual Information Rate is an efficient tool to evaluate the performances of a detector by estimating its maximum achievable rate. The achievable performance has been studied for coherent CPM, non coherent multisymbol in [11] and [8] respectively. The design of coded scheme is carried out in modern communication systems to reach the maximum achievable rate. However the encountered channel impact the achievable information rate (AIR) and then coherent and non coherent channels fit different "optimal" coded schemes.

In this paper, we consider a bit-interleaved coded modulation scheme (BICM) composed of an error correcting code serially concatenated with a M-ary non linear modulation with memory. Without loss of generality and for ease of presentation, we assume here CPFSK as a modulation scheme. We first compare demodulation strategies for both the coherent and the non coherent cases. Then, we perform an asymptotic analysis and try to show that the design of coding schemes performing well for both the coherent and the non coherent settings should be done carefully when considering sparse graph based codes such as low-density parity-check (LDPC) codes. Indeed, it can be shown that codes designed for the coherent case may be not “stable” for the non coherent case, preventing the use of ”optimal” schemes in the coherent case for the non coherent setting.

The remainder of this paper is organized as follows. In section II, we describe the system model. Then, in section III, we describe and compare different coherent and non coherent receivers. In section IV, we perform an asymptotic analysis and discuss sparse graph based code design for both the coherent and the non coherent case. Finally, section V concludes the paper.

II. System Model

In this paper, we consider a bit-interleaved coded modulation scheme (BICM) composed of an error correcting code serially concatenated with a M-ary non linear modulation with memory. Without loss of generality and for ease of presentation, we assume here CPFSK as a modulation scheme. The study can be extended to any modulation scheme having an EXIT chart converging to the point $(1,1)$ in the case of coherent demodulation. It includes the general class of CPM modulations as well as trellis-coded modulations such as differential modulations. At the emitter, a binary message vector $\mathbf{b}=[b_0,\cdots,b_{K_b-1}]\in GF(2)^{K_b}$ is encoded into a codeword $\mathbf{c}=[c_0,\cdots,c_{N_b-1}]\in GF(2)^{N_b}$ using an error correcting code of rate $R=K_b/N_b$. Each binary codeword $\mathbf{c}$ is then interleaved, mapped into a sequence of N_s M-ary symbols from the considered modulation. Let $u_0^{N_s-1}=\{u_0,...,u_{N_s-1}\}$ be the resulting set of N_s symbols belonging to the M-ary alphabet $\{0,...,M-1\}$. Symbols are then modulated following the CPFSK modulation rule. Using the Rimoldi representation, this can be seen as the serial concatenation of a continuous phase encoder (CPE) and a memoryless modulator. The mapping procedure is given as follows [8]. First, the CPE ensures the continuity between the transmitted continuous-time waveforms by accumulating the phase of each modulated symbol using the the following rule

$$\phi_{k+1}=\phi_k+2\pi h u_k \tag{1}$$

ϕ_k is the accumulated phase at the start of the k^{th} symbol and h is the modulation index. Then, at the k^{th} symbol interval, the memoryless modulator matches the symbol u_k to the continuous waveform signal $x_{u_k}(t)$ corresponding to the u_k^{th} element of $X=\{x_i(t), i=0...M-1\}$

$$x_i(t)=\sqrt{\frac{1}{T}}\cdot e^{j2\pi\frac{iht}{T}}, \quad t\in[0,T)$$

where T is the symbol period. It leads to the CPFSK complex baseband representation of the transmitted continuous-time waveform during the k^{th} symbol observation interval gven by

$$s_k(t)=\sqrt{Es}\cdot x_{u_k}(t)\cdot e^{j\phi_k} \tag{2}$$

The transmitted signal undergoes a phase rotation θ and it is transmitted over a complex circular additive white Gaussian noise (AWGN) channel with noise spectral density N_0. The corresponding complex-baseband received signal is given by $\forall\, t\in[kT;(k+1)T)$,

$$r_k(t)=s_k(t)\cdot a\cdot e^{j\theta}+n(t) \tag{3}$$

where a is a possible channel attenuation which is assumed known by the receiver. Without loss of generality, we assume that a=1. In case of a non coherent receiver, θ is assumed to be unknown, constant over the whole transmission and uniformly distributed between $[0,2\pi[$. In the coherent case, θ is assumed perfectly known and thus can be compensated. $n(t)$ in (3) corresponds to the complex circular AWG noise. At the receiver side, the signal $r_k(t)$ received during the k^{th} symbol interval is passed through a bank of M matched filters whose impulse responses are given by $\bar{x}_i(t)$, i=0, ..., $M-1$ where $\bar{x}_i(t)$ is the complex conjugate of $x_i(t)$. Considering a perfect timing synchronization, $r_{i,k}$ is the element resulting from the correlation between $r_k(t)$ and $\bar{x}_i(t)$.

$$r_{i,k}=\int_0^T r_k(t)\bar{x}_i(t)dt \tag{4}$$

In the sequel we adopt the following notation $\boldsymbol{r}_k=\{r_{0,k},...,r_{M-1,k}\}$ and the set of observations is denoted by $\boldsymbol{r}_0^{N_s-1}=\{\boldsymbol{r}_0,...,\boldsymbol{r}_{N_s-1}\}$.

III. Coherent versus non-coherent detection

A. Coherent detection

For general CPMs (including CPFSK) or differential modulations, optimal coherent maximum a posteriori (MAP) symbol/bit detection is achieved using the BCJR algorithm [6] based on the underlying trellis representation of the modulation scheme [6]. In our case, we aim to evaluate $p(u_k|\boldsymbol{r}_0^{N_s-1})$ for the k^{th} symbol. For the case of CPFSK, the BCJR states are given by the set of accumulated phases $\{\phi_k\}$. The transition $\{\phi_k\to\phi_{k+1}\}$ is done such that $\phi_{k+1}=\phi_k+2\pi h u_k$. The symbol MAP criterion is given by

$$p(u_k|\boldsymbol{r}_0^{N_s-1})\sim\sum_{\{\phi_k\}}\alpha_k(\phi_k)\beta_{k+1}(\phi_{k+1})\gamma(\phi_k\to\phi_{k+1},\boldsymbol{r}_k)p(u_k)$$

where $\gamma(\phi_k\to\phi_{k+1},\boldsymbol{r}_k)\triangleq p(\boldsymbol{r}_k|\phi_k,u_k)$, $\alpha_k(\phi_k)\triangleq p(\boldsymbol{r}_0^{k-1},\phi_k)$ and $\beta_{k+1}(\phi_{k+1})\triangleq p(\boldsymbol{r}_{k+1}^{N_s-1}|\phi_{k+1})$. The two latest quantities can be calculated using the classical forward-backward recursions as follows

$$\alpha_k(\phi_k)=\sum_{\{\phi_{k-1}\}}\alpha_{k-1}(\phi_{k-1})\gamma(\phi_{k-1}\to\phi_k,\boldsymbol{r}_{k-1})p(u_{k-1})$$

$$\beta_{k+1}(\phi_{k+1})=\sum_{\{\phi_{k+2}\}}\beta_{k+2}(\phi_{k+2})\gamma(\phi_{k+1}\to\phi_{k+2},\boldsymbol{r}_{k+1})p(u_{k+1})$$

Moreover, using sufficient statistics at the output of the filter bank [6], we have

$$\gamma(\phi_k\to\phi_{k+1},\boldsymbol{r}_k) = p(\boldsymbol{r}_k|u_k,a,\phi_k) \sim e^{\frac{2a\sqrt{Es}}{N_0}\Re(e^{-j\phi_k}r_{u_k,k})}$$

where $\Re(.)$ is the real part. Based on these expressions, iterative detection and decoding can be performed.

B. Noncoherent detection

1) Multisymbol Receiver: Multisymbol detection is a widely used sub-optimal non-coherent detector enabling complexity versus performance trade-off [9]. Following this approach, soft symbol MAP detection can also be easily derived. We illustrate this detector for the CPFSK case as proposed by [8]. The multisymbol noncoherent detector computes the a posteriori probability of a symbol given a window of symbol observations of length N, denoted by $p(u_k^{k+N-1}|\boldsymbol{r}_k^{k+N-1})$. This conditional a posteriori probability is computed for each possible combination of N symbols $u_k^{k+N-1}{=}\{u_k,...,u_{k+N-1}\}$. We have thus M^N possible combinations. If θ is unknown, averaging over its random phase distribution leads to the following well-known zero-order modified Bessel function based likelihood expression

$$p(\boldsymbol{r}_k^{k+N-1}|u_k^{k+N-1},a){\sim}I_0\left(\frac{2a\sqrt{Es}}{N_0}\left|\mu(u_k^{k+N-1})\right|\right)$$

where
$$\mu(u_k^{k+N-1}){=}\sum_{i=k}^{k+N-1} r_{u_i,i}\cdot e^{-j2\pi h\sum\limits_{n=k}^{i-1}u_n}$$

Based on these likelihoods, soft symbol MAP detection can be done and log-likelihood ratios can be computed to enable soft iterative decoding.

2) Multisymbol With Memory receiver: The multisymbol detector has been recently improved in [10] by including some memory within the decoding process (It has been originally proposed for CPFSK, but can be extended to other modulation schemes). MWM detector is based on a trellis state representation allowing to use a modified version of the BCJR algorithm to compute the conditional probability $p(u_{k+N-1}|\boldsymbol{r}_0^{N_s-1})$. Let $\delta_k{=}\{\phi_k,u_k,...,u_{k+N-2}\}$ denote a state of the MWM detector taking into account the accumulated phase ϕ_k and a series of $N-1$ symbols u_k^{k+N-2}. The transition $\{\delta_k{\to}\delta_{k+1}\}$ must fulfill to the subsequent equation $\phi_{k+1}{=}\phi_k+2\pi h u_k$. This transition corresponds to the emitted symbol u_{k+N-1}. The conditional probability can be written as follows

$$p(u_{k+N-1}|\boldsymbol{r}_0^{N_s-1})\sim\sum_{\{\delta_k\}}\alpha_k(\delta_k)\beta_{k+1}(\delta_{k+1})$$
$$\cdot\gamma(\delta_k{\to}\delta_{k+1},\boldsymbol{r}_k^{k+N-1})p(u_{k+N-1})$$

where $\gamma(\delta_k{\to}\delta_{k+1},\boldsymbol{r}_k^{k+N-1}){\triangleq}p(\boldsymbol{r}_k^{k+N-1}|\delta_k,u_{k+N-1})$, $\alpha_k(\delta_k){\triangleq}p(\boldsymbol{r}_0^{k-1}|\boldsymbol{r}_k^{k+N-2},\delta_k)p(\delta_k)$ and $\beta_{k+1}(\delta_{k+1}){\triangleq}p(\boldsymbol{r}_{k+N}^{N_s-1}|\boldsymbol{r}_{k+1}^{k+N-1},\delta_{k+1})$.

The forward-backward recursions read finally as follows

$$\alpha_k(\delta_k)\quad=\sum_{\{\delta_{k-1}\}}\alpha_{k-1}(\delta_{k-1})\frac{\gamma(\delta_{k-1}{\to}\delta_k,\boldsymbol{r}_{k-1}^{k+N-2})}{p(\boldsymbol{r}_k^{k+N-2}|u_{k+N-2},\delta_{k-1})}p(u_{k+N-2})$$
$$\beta_{k+1}(\delta_{k+1})=\sum_{\{\delta_{k+2}\}}\beta_{k+2}(\delta_{k+2})\frac{\gamma(\delta_{k+1}{\to}\delta_{k+2},\boldsymbol{r}_{k+1}^{k+N})}{p(\boldsymbol{r}_{k+1}^{k+N-1}|u_{k+N},\delta_{k+1})}p(u_{k+N})$$

For CPFSK modulation, the branch metric can be computed as

$$\gamma(\delta_k{\to}\delta_{k+1},\boldsymbol{r}_k^{k+N-1})=p(\boldsymbol{r}_k^{k+N-1}|\delta_k,u_{k+N-1},a)$$

Averaging over the random phase, we have

$$p(\boldsymbol{r}_k^{k+N-1}|\delta_k,u_{k+N-1},a){\sim}I_0\left(\frac{2a\sqrt{Es}}{N_0}\left|\mu(u_k^{k+N-1})\right|\right)$$

which finally gives the following recursions

$$\alpha_k(\delta_k)\sim\sum_{\{\delta_{k-1}\}}\alpha_{k-1}(\delta_{k-1})\frac{I_0\left(\frac{2a\sqrt{Es}}{N_0}\left|\mu(u_{k-1}^{k+N-2})\right|\right)}{I_0\left(\frac{2a\sqrt{Es}}{N_0}\left|\mu(u_k^{k+N-2})\right|\right)}p(u_{k+N-2})$$
$$\beta_k(\delta_k)\sim\sum_{\{\delta_{k+1}\}}\beta_{k+1}(\delta_{k+1})\frac{I_0\left(\frac{2a\sqrt{Es}}{N_0}\left|\mu(u_k^{k+N-1})\right|\right)}{I_0\left(\frac{2a\sqrt{Es}}{N_0}\left|\mu(u_k^{k+N-2})\right|\right)}p(u_{k+N-1})$$

Based on these BCJR like equations, soft iterative detection and decoding can be performed.

IV. Asymptotic Analysis and code design

In this section, we analyse the asymptotic performance of the different receivers, namely the coherent receiver and the non coherent multisymbol receiver with and without memory. First asymptotic AIRs are derived. Then we perform an EXIT charts analysis showing that the classical area theorem [12], [13] seems to remain valid in our case. Then, we discuss properties of the three detectors based on their EXIT characteristics. By referring to state-of-the optimization methods for sparse-graph based codes, we will see that optimized sparse graph coding schemes for the coherent case can no longer be considered for the non-coherent case. This is due to the fact that threshold optimized code profiles for LDPC codes in the coherent case can be not stable in the non-coherent case. But, on the contrary, an optimized profile for the non coherent case is always stable for the coherent case and optimization results tend to show that it performs fairly well compared to constrained profiles in the coherent case.

A. Symmetric Information Rate

The mutual information rate of finite-state machine channels have been studied by several authors, among them [14], [15], [16]. Following the general expression given by [16] enabling the conditioning over an unknown initial channel state, the mutual information rate between the channel input source $\mathcal{U}$ and the channel output $\mathcal{R}$ is given by

$$I(\mathcal{U},\mathcal{R}){=}\lim_{N_s\to\infty}\frac{1}{N_s}I(u_0^{N_s-1},\boldsymbol{r}_0^{N_s-1}|\delta_0)$$

where $I(u_0^{N_s-1},\boldsymbol{r}_0^{N_s-1}|\delta_0)$ is the mutual information between the input process $u_0^{N_s-1}$ and the output process $\boldsymbol{r}_0^{N_s-1}$ conditioned on the initial state δ_0. The expression of $I(u_0^{N_s-1},\boldsymbol{r}_0^{N_s-1}|\delta_0)$ can be derived as follows for the three considered receivers enabling to evaluate *the maximum achievable rate for a given detector*.

1) Coherent BCJR Algorithm:

$$I(u_0^{N_s-1},\boldsymbol{r}_0^{N_s-1}|\phi_0){=}N_s\cdot\log_2(M)$$
$$+\sum_{k=0}^{N_s-1}E\left[\log_2\left(p(u_k|\boldsymbol{r}_0^{N_s-1},\phi_0^k)\right)\right]$$

Computation of $p(u_k|\boldsymbol{r}_0^{N_s-1},\phi_0^k)$ is performed following [11] based on BCJR forward-backward recursions.

2) Multisymbol With Memory Detector [10]:

$$I(u_0^{N_s-1}, \boldsymbol{r}_0^{N_s-1}|\delta_0) = \big(N_s - (N-1)\big) \cdot \log_2(M) + \sum_{k=0}^{N_s-N} E\left[\log_2\left(p(u_{k+N-1}|\boldsymbol{r}_0^{N_s-1}, \delta_0^k)\right)\right]$$

Calculation of $p(u_{k+N-1}|\boldsymbol{r}_0^{N_s-1}, \delta_0^k)$ is also performed following [11] but using δ_0^k.

3) Multisymbol Receiver [8]: Let $\mathcal{B}$ be the set of M^N possible combinations for a symbols block of size N, the mutual information can be directly expressed as follows [8]

$$I(u_k^{k+N-1}, \boldsymbol{r}_k^{k+N-1}) = N \cdot log(M) + E\left[\log_2\left(\frac{p(\boldsymbol{r}_k^{k+N-1}|u_k^{k+N-1}, a)}{\sum_{\hat{u}\in\mathcal{B}} p(\boldsymbol{r}_k^{k+N-1}|\hat{u}_k^{k+N-1}, a)}\right)\right]$$

Information rate is obtained dividing the above quantity by N.

B. EXIT charts and code design

For all the considered receivers, we also have performed an EXIT charts analysis [17], [13]. An example is shown in Fig. 2 for E_s/N_0=0 dB. It is shown that only the coherent receiver is converging to the point $(1, 1)$, which is a common feature to non linear modulations with memory such as CPM/CPFSK or differential modulations (if no specific precoding is used) or also for precoded linear channels with memory. This issue will strongly impact the code design when iterative decoding is used. We have calculated the area under the EXIT curves which is often conjectured as a good approximation for the maximum achievable coding rate in various situations [13] for a serially concatenated system. It has only been proved for concatenated codes over the erasure channel [12]. An approximation of the information rate is finally given by

$$\mathcal{C}_{approx.} \simeq \log_2(M) \cdot \int_0^1 I_e(x)\mathrm{d}x \qquad (5)$$

where I_e is the mutual information measured at the output of the soft receiver. As shown in Fig. 1 for the case of CPFSK based detectors, the approximation is relatively accurate when compared to direct calculation of the mutual information rate. This suggests that good coding schemes under iterative decoding can be designed by state-of-the-art curve fitting methods to approach these maximum achievable rates. When considering sparse graph based binary coding schemes such as binary LDPC codes, these methods are generally derived from some multi-edge type extensions of the original approach proposed by [18]. A recent example is given by [19]. The remaining question is how to design good codes that can perform fairly well in both regimes (coherent and non coherent). Usually, the problem is solved sub-optimally by first considering the design of a good coding scheme for the coherent regime and then, it is applied to the non coherent case relying on some robustness arguments. We will see that it may be not the best option. We first consider the non-coherent case. As in most serially concatenated systems, EXIT curves do not reach the point $(1, 1)$ and so methods as in [18][19] can be used for LDPC code design. This leads to an upper bound on the degree 2 nodes associated with the Tanner graph of the code. This is due to the so-called *stability condition*. We now consider the coherent case. For this scheme, the EXIT curves reach the point $(1, 1)$. Several approaches are possible. Mainly, non systematic low-density generator matrix based coding schemes have been considered as in [6], [20] using some analogies with irregular-repeat accumulate coding schemes. For the case of LDPC codes, it has been shown in [21], [22] that some good LDPC codes can be designed if some degree one nodes are carefully introduced satisfying a stability condition constraining degree one nodes only. In that case, degree 2 nodes are not anymore constrained. It follows from this simple fact that coding scheme designed for the coherent case cannot be used for the non coherent setting since the resulting profiles cannot be stable. On the contrary, a code designed for the non coherent case is always stable under coherent decoding. Of course, for the coherent case, one can also avoid degree one nodes in the design. Unfortunately, based on our experiments, it appears that, for the same AIR, the fraction of degree two nodes for code profiles optimized in the coherent case is always greater than the fraction of degree two nodes for a code profile optimized in the non coherent case. Here, again the code may be not stable under non coherent decoding. To have good coding schemes operating in both regimes, it seems reasonable to design first codes for the non coherent case and then assess the performance in the coherent case. In Fig. 3, we have compared the bit error rate (BER) performances of different optimized coding schemes for rate one-half codes with N_b=4096 coded bits following [21][19]. For fair comparison, we consider the optimization in the coherent case without degree one nodes. The best scheme is the optimized coherent case with no degree one nodes, followed by the scheme optimized for the non coherent case with the MWM detector but operating with the coherent receiver. It shows that we have a relatively small loss compared to the optimal coherent case with no degree one node allowed. The two last curves correspond to the optimized non coherent cases with both MWM and multi-symbol detectors respectively. As we can see, good performances can be achieved with the improved MWM detection approach.

Table 1 : Degree distribution of the code used Fig.3.

	d_c	d_v	ρ	λ
Coherent/MWM	$\{4, 5\}$	$\{2, 8\}$	$\{0.54, 0.46\}/\{0.34, 0.66\}$	$\{0.87, 0.13\}/\{0.81, 0.19\}$
Multisymb	$\{5, 6\}$	$\{2, 8\}$	$\{0.94, 0.06\}$	$\{0.715, 0.285\}$

V. CONCLUSION

In this paper, we have discussed the design of sparse graph based coding schemes serially concatenated with some non linear modulations such as CPFSK considering both coherent and non coherent demodulations. We have performed an asymptotic analysis computing information rates. Based on EXIT analysis, we have shown that optimized coding schemes for the coherent setting may be not at all suited for the non-coherent case. On the contrary, it seems possible to design

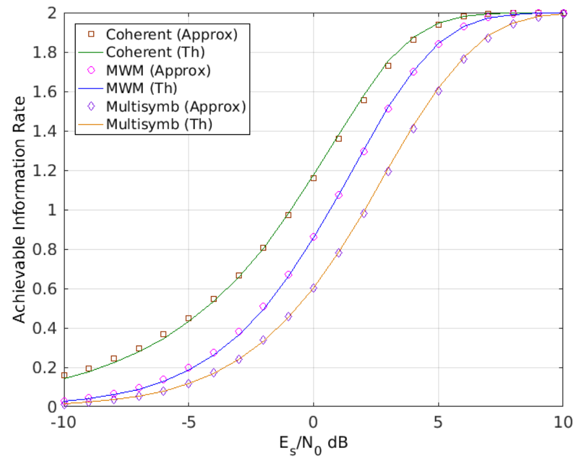

Fig. 1. Approximate (Approx) and theoretical (Th) AIR: coherent BCJR, noncoherent MWM and Multisymbol of 4-CPFSK with $h{=}\frac{5}{7}$ and $N{=}3$

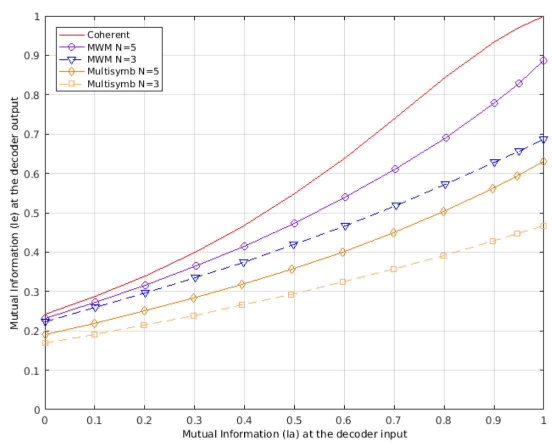

Fig. 2. Exit charts: coherent BCJR, noncoherent MWM and Multisymbol of 4-CPFSK with $h{=}\frac{5}{7}$ for $\frac{E_s}{N_0}{=}0$dB.

coding schemes for the non coherent setting that perform fairly well when using coherent demodulation. The study could be extended to any modulation scheme having an EXIT chart converging to the point $(1, 1)$ in the coherent case.

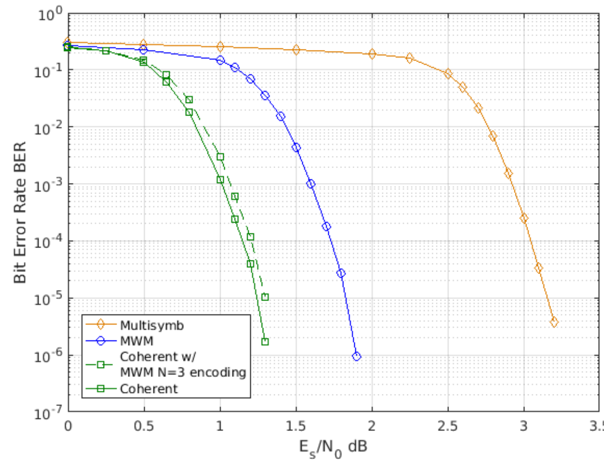

Fig. 3. BER: coherent BCJR, noncoherent MWM and Multisymbol of 4-CPFSK with $h{=}\frac{5}{7}$, $N{=}3$ and $R{=}\frac{1}{2}$.

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