On Nested Property of Root-LDPC Codes

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2 authors:

Lorenzo Ortega Espluga
Institut Polytechnique des Sciences Avancées

Charly Poulliat
Ecole Nationale Supérieure d'Electrotechnique, d'Electronique, d'Informatique, d'H…

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Lorenzo Ortega, Charly Poulliat, *Member, IEEE*

***Abstract*—We investigate on binary Protograph Root-LDPC codes that can embed an interesting property, called *nested* property. This property refers to the ability for a coding scheme to achieve full diversity and equal coding gain for any number of received coded blocks and for any configuration of the received code blocks. One can take advantage of this property for "carousel"-type transmissions broadcasting cyclically coded information. For regular Root-LDPC codes, we show that these codes inherently have both properties over the nonergodic block fading channel and when message passing decoding is used. Then, we show that irregular Root-LDPC structures could not provide equal coding gain except if explicit design rules are enforced to ensure that the nested property is fulfilled when designing irregular Root-LDPC codes. Simulation results show that designed nested Root-LDPC codes achieve good performance and full diversity for coding rates $R = 1/2$, $R = 1/3$ and $R = 1/4$.**

***Index Terms*—MDS, Full diversity, Nested, Root-LDPC codes, Protograph EXIT.**

I. Introduction

The design of error-control codes for the nonergodic block-fading (BF) channel offers a challenging problem, which differs greatly from its counterparts referred to additive white Gaussian noise (AWGN) [1]. Indeed, classical unstructured low-density parity-check codes (LDPC) codes, designed to approach ergodic capacity, cannot generally approach the ideal performance limits of BF channels, and hence code designs suited to the nonergodic nature of the BF channel is needed. Thus, Root-LDPC codes were proposed in [2] as a class of structured LDPC codes efficient over nonergodic block fading channels. For this class of codes, and under iterative decoding, it is possible to operate relatively close to the outage probability bound if irregular structures are considered [2], [3], while achieving full diversity. When the full diversity property is fulfilled, the information bits are decoded with a diversity order equal to the *total* number of received fading blocks [2]. It also holds that these codes are maximum distance separable (MDS), i.e. they are outage-achieving over the noiseless block-erasure channel [2]. The structure of Root-LDPC codes for point-to-point channels has been adapted to other communications scenarios such as the relay channel [4], rate-compatible Root-LDPC codes for the two-block case [5] or spatially coupled structures [6], [7], to cite a few.

In this letter, we are interested in channel coding structures ensuring MDS, full diversity and nested [8] properties over the block fading channel when using a Belief Propagation (BP) decoding algorithm [9]. If the two first properties are met by design for classical Root-LDPC codes, we aim to investigate on the ability of these kind of codes to fulfil the so-called nested property. In this paper, the nested property will refer to the ability for a coding scheme to achieve full diversity **for any number of received coded blocks (up to a maximum of transmitted coded blocks) and for any configuration of the received code blocks**. When this definition is applied to the Root-LDPC structure, this property enforces that the information bits can be decoded with a diversity order equal to the number of received blocks. We further enforce that the nested coding structures must provide equal coding gain or average error probability independently of the received blocks. Note that this property is not always met by design, see for example some designed Protographs in [3]. Root-LDPC codes fulfilling the nested property will be referred to as nested Root-LDPC codes. Nested schemes can be useful for "carousel"-type transmissions [10]. A carousel-type transmission refers to known broadcasting methods used in several protocols of communication such as the GNSS (Global Navigation Satellite System). In such a situation, the information data is repeatedly broadcasted in a continuous cycle, i.e. it is periodically re-transmitted, and different receivers can start to receive the coded blocks at any time and may experience different channel conditions from one block to another. To the authors' knowledge, this property has been first observed and used in [11]. We aim here to provide some additional theoretical proofs and insights of the observed behavior while proposing some practical design to improve the coding gain over regular structures.

The contributions are as follows. For regular Root-LDPC codes [2], we first show that these codes can inherently achieve any diversity order up to the maximum diversity order achievable by their design rate, while providing the so-called nested property over the nonergodic block fading channel. Then, we introduce some rules to design some irregular nested Protograph Root-LDPC codes of rate $R = 1/3$ and $R = 1/4$, to perform better than existing schemes with respect to the nested property.

The paper is organized as follows: Section II introduces the channel model and the notations. In section III, we investigate on the nested property of regular Root-LDPC codes, considering, as an illustrative example, the rate $R = 1/3$. Section IV describes the design of irregular Protograph Root-LDPC codes in order to fulfill the desired nested property, and shows how existing methods may fail to meet this property. Some illustrative results for both regular and irregular structures are analyzed and compared with the state of the art in Section V. Conclusion are finally drawn in Section VI.

II. Root-LDPC codes for block fading channels

The nonergodic BF channel is usually used to characterize slowly-varying fading channels and it can be viewed as an extension of the well-known block erasure channel [2]. For this model, a transmitted codeword is transmitted over a finite

L. Ortega is with TéSA, 31500, Toulouse, France, e-mail: lorenzo.ortega@tesa.prd.fr. C. Poulliat is full professors at INP/ENSEEIHT, Toulouse University, 31000 Toulouse, France, e-mail: charly.poulliat@enseeiht.fr. Grant founded by CNES and TAS.

number n_c of independent block channel realizations. For a block-fading channel with n_c fading blocks, the baseband discrete-time channel output at discrete time index n is given by

$$y_n = h_n x_n + z_n,\ n = 1, \dots, N \tag{1}$$

where N denotes the codeword length, $x_n \in \{-1, +1\}$ is the n-th binary phase shift keying (BPSK) modulated symbol, $z_n \sim \mathcal{N}(0, \sigma^2)$ are centered independent and identically distributed (i.i.d.) real Gaussian noise samples with variance $\sigma^2 = N_0/2$, h_n are real fading coefficients that belong to the set $\mathcal{N} = \{\alpha_1, \alpha_2, \dots, \alpha_{n_c}\}$ such that $h_n = \alpha_{\lceil n_c \cdot n/N \rceil}$, where $\lceil . \rceil$ refers to the ceiling operator. Furthermore, we assume that $\alpha_k, k = 1, \cdots, n_c$, are i.i.d. random variables following a normalized Rayleigh distribution. We also note $L = N/n_c$ the number of symbols per fading block. Since the block fading channel has a zero capacity in the strict Shannon sense [2], the information outage probability is usually used to assess asymptotic performance and the expression in our context is $P_{out}(\gamma) = \text{Prob}\{\overline{I} \le R\}$ where γ denotes the average signal-to-noise ratio (SNR), $\overline{I}$ denotes the instantaneous average mutual information between the BPSK constrained input and the noisy observation at the output of the channel for a particular channel realization $\mathcal{N}$, and R is the transmission rate. Assuming perfect channel state information at the receiver [2], $\overline{I} \triangleq \frac{1}{n_c}\sum_{k=1}^{n_c} I_{AWGN}(\gamma\alpha_k^2)$ where $I_{AWGN}(s)$ is the mutual information associated with binary inputs transmitted over an additive white Gaussian noise (AWGN) channel with a SNR equal to s. $I_{AWGN}(s)$ is also referred to the constrained input AWGN capacity for BPSK inputs. In such nonergodic channel, P_{out} gives a lower bound on the codeword error probability $P_{e,w}$. Moreover, the diversity order d is given by

$$d = -\lim_{\gamma\to\infty}\left(\frac{\log(P_{e,w}(\gamma))}{\log(\gamma)}\right), \tag{2}$$

and is upper-bounded by the intrinsic diversity of the channel, which reflects the slope of the outage limit. Note that to obtain full diversity $d = n_c$ and $R = 1/n_c$ [2].

In order to obtain a channel coding design which achieves full diversity over the BF channel, Root-LDPC codes were proposed in [2]. To illustrate the structure of this family of codes, we consider regular (d_v, d_c) Root-LDPC codes of rate of $R = 1 - d_v/d_c = 1/3$, where d_v (resp. d_c) is the number of edges connected to a variable node (resp. the number of edges connected to a Root-check node). Extensions to lower rates can be easily obtained. In Root-LDPC codes, the variable nodes associated to coded bits are divided into two kind of nodes, called information and redundant bit nodes respectively, that belong to $n_c = 3$ classes, each one associated with an independent fading block. This results in six types of nodes and we denote (i_k, p_k) the couple of nodes associated to a fading block k, $k = 1, \cdots, n_c$. The $m \times n$ base matrix H_B^r associated with the proto-representation is given by

$$H_B^r = \begin{bmatrix} 1 & 0 & 0 & h_{1,4} & h_{1,5} & h_{1,6} & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & h_{2,7} & h_{2,8} & h_{2,9} \\ h_{3,1} & h_{3,2} & h_{3,3} & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & h_{4,7} & h_{4,8} & h_{4,9} \\ h_{5,1} & h_{5,2} & h_{5,3} & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & h_{6,4} & h_{6,5} & h_{6,6} & 1 & 0 & 0 \end{bmatrix}, \tag{3}$$

where $h_{i,j} \in \mathbb{N}^*$, $i \in \{1, \cdots, m\}$ and $j \in \{1, \cdots, n\}$, represent possible parallel edges in the proto representation [9]. In order to decode the Root-LDPC codes, classical algorithms [9] such as the BP can be used. Regarding the encoding, most of existing strategies for encoding LDPC codes can be applied. In particular, as we are mostly dealing with protograph LDPC codes enabling quasi-cyclic structures, existing efficient encoding methods are available (see for example [12]).

III. Structure of Nested Root-LDPC codes

In this section, we investigate on the nested Root-LDPC codes. This family of codes is characterized not only by the inherent properties of the Root-LDPC codes, i.e. the MDS and the full diversity properties [2], but also by an extra property, called nested [8], which enforces that the information bits can be decoded with i) a diversity order equal to the number of received blocks and ii) equal gain or average error probability independently of the received block. Moreover, we claim that some symmetry rules are required in order to derived coding schemes with the nested property. In case of nested Root-LDPC of rate $R = 1/3$ is the following matrix symmetry $h_{3,1} = h_{6,4} = h_{2,7}$, $h_{5,1} = h_{1,4} = h_{4,7}$, $h_{3,2} = h_{6,5} = h_{2,8}$, $h_{5,2} = h_{1,5} = h_{4,8}$, $h_{3,3} = h_{6,6} = h_{2,9}$, $h_{5,3} = h_{1,6} = h_{4,9}$. This condition enforces some "uniform" graph topology from the information bit nodes perspective ensuring similar performance under iterative decoding. This is illustrated in the Tanner graph in Fig. 1, where a symmetrical set of graph connections are required in order to ensure the uniform graph topology. For the regular case, when assigning edges, we did not find cases for which this condition is not met. Note also that, for the regular case, a design rate $R = 1/3$ implies that $\sum_k h_{k,j} = d_v$ is even, $\forall j \in \{1, \cdots, n\}$. In the following, we analyze the nested property resulting from this particular code structure. Proof of this property closely mimic those in [2] in quite a straightforward way, then it has not been included in this letter but it can be found in [13].

1) Diversity order: Let us consider the case where only two blocks (out of the three possible blocks) are retrieved under the general **Rayleigh block fading AWGN channel**. The nodes belonging to classes i_1, i_2 and i_3 should have a diversity order equal to the number of received blocks. Considering the simplest regular case, which is given by a regular $(4, 6)$ Root-LDPC code and the 1-st block erased ($\alpha_1 = 0$), a coded modulated bit $x = +1$ belonging to the class i_1, receives its channel message $\mathcal{L}_0 = 0$ since the associated block is erased. However, it receives 4 extrinsic messages $\mathcal{L}_i^e$, with $i = 1, \dots, 4$, from its 4 neighboring checknodes. The total a-posteriori message is $\mathcal{L}_{app} = \mathcal{L}_1^e + \mathcal{L}_2^e + \mathcal{L}_3^e + \mathcal{L}_4^e$. Let $\mathcal{L}_1^e$ be the extrinsic message generated by the rootcheck of class $1c_2$ connected to x and $\mathcal{L}_2^e$ the extrinsic message generated by the rootcheck of class $1c_3$ connected to x. Then, it is straightforward to verify that thanks to the joint message $\mathcal{L}_1^e + \mathcal{L}_2^e$, the bit x belonging to the class i_1 is decoded with a diversity order equal to 2 [13]. Let us now consider a coded modulated bit $x = +1$ associated to the class i_2, then we have $\mathcal{L}_0 = \frac{2\alpha y}{\sigma^2} = \frac{2}{\sigma^2}(\alpha_2^2 + \alpha_2 z_0)$ [2], where $y = \alpha + z$ and $z \sim \mathcal{N}(0, \sigma^2)$. Moreover, it receives 3 extrinsic messages

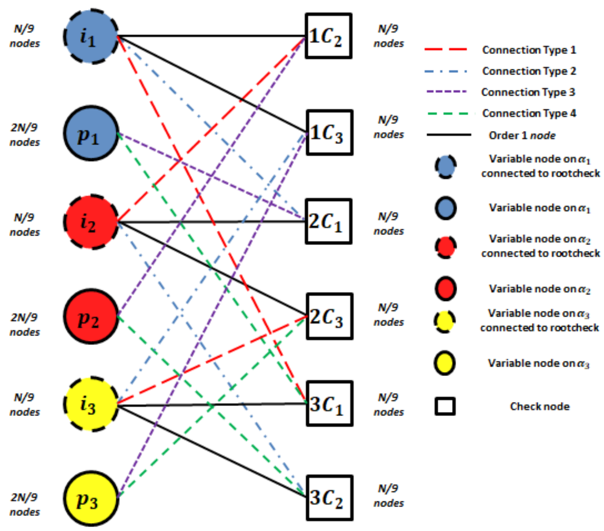

Fig. 1: Tanner graph for nested Root-LDPC code of rate 1/3.

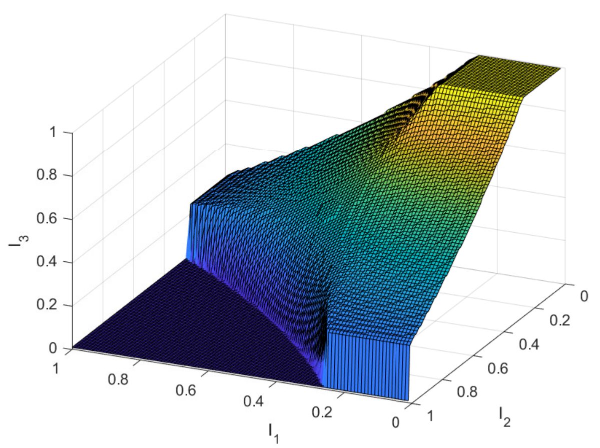

Fig. 2: Mutual information outage boundaries for the regular (4,6) Root-LDPC in a block fading channel with $n_c = 3$.

$\mathcal{L}_i^e$, $i = 1, \ldots, 3$. Since one of its 4 neighboring checknodes ($2c_1$) sends $\mathcal{L}_1^e = 0$, the total a-posteriori message can be computed as $\mathcal{L}_{app} = \mathcal{L}_2^e + \mathcal{L}_3^e + \mathcal{L}_4^e$. Let be $\mathcal{L}_2^e$ the extrinsic messages generated by the rootcheck of class $2c_3$ connected to x. Again, it is straightforward to verify that thanks to the joint message $\mathcal{L}_0 + \mathcal{L}_2^e$ the bit x belonging to the class i_2 is decoded with $d = 2$. The above procedure can be directly applied considering node of the class i_3, obtaining that the diversity is of order $d = 2$ independently of the class i_k.

2) Equal coding gain: In order to ensure the nested property, the information bits are required to be decoded with equal outage probability or average error probability independently of the received blocks. For a given γ and instantaneous fading gains $\alpha = [\alpha_1, \cdots, \alpha_{n_c}]$, based on the update equations of the Protograph EXIT charts [14], the a-posteriori mutual information associated with the information codeword symbols of the k-th fading block, referred to as $I_{APP}^{k,\ell}$, can be computed after ℓ iterations. $I_{APP}^{k,\ell}(\gamma, \alpha)$ is an implicit function of (γ, α). Let $\tilde{\alpha}_{k'}^p = [\tilde{\alpha}_1, \cdots, \tilde{\alpha}_{n_c}]$ be a permutation of the n_c-tuple α such that $\tilde{\alpha}_p = \alpha_{k'}, \forall p = 1, \cdots, n_c$. Then, if for each node type we ensure by design that we have the same local connections for each block k (i.e. the same multi-edge profile from the variable nodes perspective), we claim that for any k', all channel configurations corresponding to a permutation of the type $\tilde{\alpha}_{k'}^k$ will ensure $I_{APP}^{k,\ell}(\gamma, \tilde{\alpha}_{k'}^k) = I_{APP}^{k',\ell}(\gamma, \alpha), \forall k = 1, \cdots n_c$. As an example, we consider the case with 3 different blocks with $\alpha = [\alpha_1, \alpha_2, \alpha_3]$. Then, $\tilde{\alpha}_1^2 = [\alpha_3, \alpha_1, \alpha_2]$ or $\tilde{\alpha}_1^3 = [\alpha_2, \alpha_3, \alpha_1]$ will lead under symmetry condition to $I_{APP}^{1,\ell}(\gamma, \alpha) = I_{APP}^{2,\ell}(\gamma, \tilde{\alpha}_1^2) = I_{APP}^{3,\ell}(\gamma, \tilde{\alpha}_1^3)$. As it holds when ℓ tends to infinity, this would imply a symmetry with respect to the ergodic line for the threshold region also known as fading outage boundaries [15]. This threshold region can be defined as the subset of tuple $\alpha \in \mathbb{R}^{n_c}$, for which we have $I_{APP}^{k,\infty}(\gamma, \alpha) = 1, \forall k = 1, \cdots, n_c$. For $n_c = 2$ received blocks (resp. $n_c = 3$), we have an axial symmetry with respect to the ergodic lines $y = x$ (resp. to $z = y = x$). The threshold region can be also more conveniently represented using the mutual information outage boundaries as a function of $I_k = I_{AWGN}(\gamma\alpha_k^2), k = 1, \cdots, n_c$. This latter case is illustrated in Fig. 2. The mutual information outage boundaries gives the required average mutual information per information block for the regular $(4, 6)$ code to achieve zero error probability with $n_c = 3$. We can observe the symmetric behaviour that is inherited from the symmetry condition. The asymptotic outage probability can be then obtained by averaging the multidimensional fading distribution taking into account the mutual information outage boundaries [15]. From the symmetry property, we claim that the asymptotic outage probability is independent from the received blocks and thus only dependent on the number of received blocks. Note that when the number of blocks is less than n_c, the asymptotic outage probability can be computed by considering the average over the received blocks distribution only.

The same type of reasoning can be applied to the asymptotic average information symbols error probability at iteration ℓ. Since it is directly related to $I_{APP}^{k,\ell}$ due to the symmetry property, when averaging over the fading distribution, the average information symbols error probability [3] can be shown to be only dependent on the number of received blocks. The nested property is derived when assuming that the symmetry condition is fulfilled. This condition does not naturally arise, except for the regular case and it must be ensured by design for the irregular case.

IV. Irregular Nested Protograph Root-LDPC

The preceding regular case is of interest to easily analyze the desired properties of the proposed structure. However, as the variable node degree is even by construction for the regular case, it does not lead to real practical schemes for the following reasons: (a) the actual rate cannot be equal to the design rate, (b) encoding and mapping of information bits to the i_k classes becomes an issue. In this section, we investigate on the design of irregular nested Root-LDPC codes which enables to design codes that lead to practical solutions with optimized demodulation thresholds when two or three blocks are received. Considering a nested Root-LDPC of design rate $R = 1/3$, the base matrix H_B^{ir} with entries $'*'$ to be optimized is given by (4a).

$$H_B^{ir} = \begin{bmatrix} 1&0&0&*&*&*&0&0&0 \\ 1&0&0&0&0&0&*&*&* \\ *&*&*&1&0&0&0&0&0 \\ 0&0&0&1&0&0&*&*&* \\ *&*&*&0&0&0&1&0&0 \\ 0&0&0&*&*&*&1&0&0 \end{bmatrix} \quad (4a) \qquad H_B^{ir} = \begin{bmatrix} 1&0&0&1&0&3&0&0&0 \\ 1&0&0&0&0&0&0&2&2 \\ 0&2&2&1&0&0&0&0&0 \\ 0&0&0&1&0&0&1&0&3 \\ 1&0&3&0&0&0&1&0&0 \\ 0&0&0&0&2&2&1&0&0 \end{bmatrix} \quad (4b)$$

$'*'$ represents the Protograph connection weights $\in \mathbb{N}^*$ to be optimized. As a figure of merit for optimization, we aim to lower some demodulation thresholds. Therefore, we will use Protograph EXIT (PEXIT) Chart [15] to evaluate the different thresholds we need in order to search for the coefficients $'*'$. From the preceding study, we can also note that if the obtained base matrix has not a symmetrical behavior between the different block fading classes, then, the base matrix does not provide the nested property. This fact allows to simplify the search for the $'*'$ coefficients ensuring such a symmetry. For illustration purpose, in this letter, we simply set $'*'$ to be coefficient weights in the set $\{0, 1, 2, 3\}$. Then, we apply the following sequential procedure for optimization:

1-st step - Base matrix set initialization: We enumerate the base matrices with symmetry property.

2-nd step - Threshold computation: For the previous subset, we compute the demodulation thresholds Th_i (expressed in dB) over the ergodic channel when only $i \in (2, \dots, n_c)$ blocks are received, i.e. considering that channel mutual information provided by the erased block is equal to 0.

3-rd step - Protograph selection: We proceed by selecting the Protograph structure that minimizes $\Delta_{dB} = \sum_{i=2}^{n_c} |Th_{capa,i} - Th_i|$ where $Th_{capa,i}$ is the BPSK capacity threshold in dB to achieve $R = 1/i$.

By applying the above procedure, the Protograph structure in (4b) is obtained with $Th_2 = 0.92dB$ and $Th_3 = 0.75dB$. For a design rate of $R = 1/4$, the Protograph in (5) is obtained with $Th_2 = 1.2695dB$, $Th_3 = 1.038dB$ and $Th_4 = 2dB$.

$$H_{B'}^{ir} = \begin{bmatrix} 1&0&0&0&0&0&0&0&0&0&0&0&2&2&0&1 \\ 1&0&0&0&2&1&2&0&0&0&0&0&0&0&0&0 \\ 1&0&0&0&0&0&0&0&2&0&1&2&0&0&0&0 \\ 2&2&0&1&1&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0&0&0&0&2&1&2&0 \\ 0&0&0&0&1&0&0&0&2&2&0&1&0&0&0&0 \\ 2&1&2&0&0&0&0&0&1&0&0&0&0&0&0&0 \\ 0&0&0&0&2&0&1&2&1&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&1&0&0&0&2&0&1&2 \\ 2&0&1&2&0&0&0&0&0&0&0&0&1&0&0&0 \\ 0&0&0&0&2&2&0&1&0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&0&0&2&1&2&0&1&0&0&0 \end{bmatrix} \quad (5)$$

$$H_B^{ir} = \begin{bmatrix} 1&0&0&0&2&0&0&0&2 \\ 1&0&0&1&0&2&0&1&2 \\ 0&0&1&1&0&0&0&0&0 \\ 0&0&2&1&0&0&1&0&2 \\ 0&1&0&0&0&1&1&0&0 \\ 1&0&1&0&0&2&1&0&0 \end{bmatrix} \quad (6)$$

We would like to underline that the previous structures can be also characterized by the rate-compatible property when block puncturing is applied. However, unlike other proposed structures [4], [5], only structures of rate $R = 1/q, \ q \in \{1, 2, 3, 4\}$, can be obtained, as, by design, we only aim to design coding schemes achieving uniformly any diversity order corresponding to the number of received blocks. Thus, mixing the proposed approach with existing methods [4], [5] to achieve intermediate rates is out of scope of this paper.

V. Results and performance analysis

In this section, we present the performance of regular and irregular nested Root-LDPC codes over the Rayleigh block fading channel and we compare the results with Root-LDPC structures presented in the state of the art. The selected regular structures are such that unknown coefficients are set to $h_{ij} = 1$ for $j \in \{1, 4, 7\}$ and to $h_{ij} = 2$ otherwise in equation (3). For the structure of $R = 1/4$, we have $h_{ij} = 1$ for $j \in \{1, 5, 9, 13\}$ and $h_{ij} = 2$ otherwise. In order to construct the practical codes simulated in this section, we have the used the lifting approach described in [16].

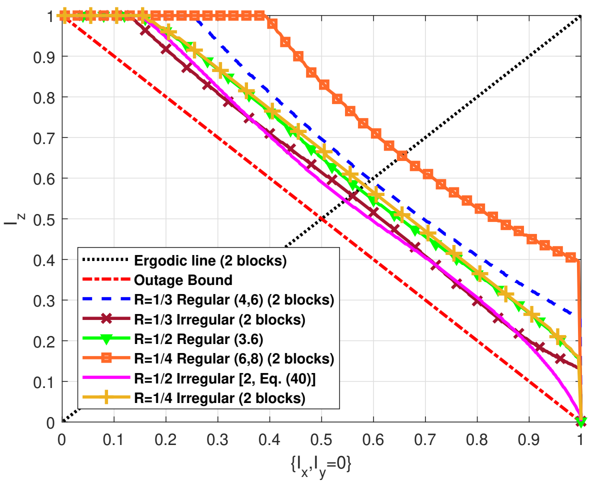

Fig. 3: Mutual information outage boundaries when two block are received. Note that $x, y, z \in \{1, 2, 3\}$.

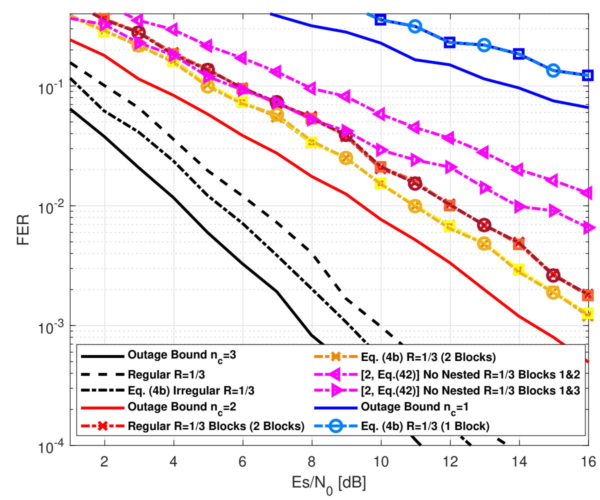

Fig. 4: FER of regular and irregular nested Root codes of rate $R = 1/3$ over the Rayleigh block fading channel

In Fig. 3, we illustrate the asymptotic required block average mutual information performance for the particular case with two received blocks, i.e. the block mutual information of the erased blocks is set 0. The outage bound curve and the ergodic line (in black) are also included. Unlike other Root-LDPC structures in the literature [3, Eq. (40)], we see that structures with nested property are symmetrical with respect to the ergodic line. This means that the code gain is constant regardless of the received block and that frame error rate (FER) is uniform among received blocks. Moreover, we show that the optimized irregular structure provides better performance than the regular one since the outage curve is closer to the outage bound. For the regular cases with $R = 1/3$ and $R = 1/4$, we can still achieve maximal diversity when receiving two blocks, but at the price of a worse region of convergence. For the irregular case with $R = 1/3$, we have improved the performance over the regular $(3, 6)$ Root-LDPC code of rate $R = 1/2$. For the irregular case with $R = 1/4$, we have similar performance with respect to the regular $(3, 6)$ Root-LDPC code of rate $R = 1/2$. In Fig. 4, the FER for a regular and irregular nested Root-LDPC of design rate $R = 1/3$ and $N = 1800$ over a Rayleigh block fading channel is illustrated. Moreover, the outage probabilities P_{out} considering a BPSK modulation with $R = 1/3$, $R = 1/2$ and $R = 1$ and block fading channel with $n_c = 3$, $n_c = 2$ and $n_c = 1$ respectively, are also included. When the entire codeword is received, full diversity is achieved for both structures (regular and irregular), since the FER curves follow the same slope as the outage probability curve for a block fading channel with $n_c = 3$. Note that the irregular structure provides a higher coding gain than the regular one since the FER is closer to the outage probability curve. When one or two codeword blocks have been received, a diversity equal to 1 or 2 is achieved for both structures (regular and irregular). Moreover, equal coding gain is achieved independently of the received blocks, i.e. the nested property is achieved. We would also like to compare the proposed nested Root-LDPC structure with other Root-LDPC structures in the state of the art. Thus, the only proposed Root-LDPC structure of rate $R = 1/3$ and $n_c = 3$ is [3, Eq. (42)]. However, we claim that this structure is not Root at the strict sense. It can be verified over the block erasure channel with two erased blocks, e.g. 2nd and 3rd blocks, that i_2 and i_3 can not be decoded if the first block is correctly received. Therefore, the structure is not MDS and thus is not Root. Moreover, in order to prove that this structure is not nested, we have searched for a good protograph structure following guidelines in [3, Eq. (42)]. We get Eq. (6). Note, that the structure has not a symmetrical pattern. In Fig. 4, the FER is illustrated when 2 blocks are received. Since the structure has not enforced the symmetry condition, the FER depends on the received blocks, i.e. the structure is not nested. Furthermore, we would like to underline that the structure is not MDS, thus if two blocks are erased, the information can not be decoded.

Finally, in Fig. 5, the FER for a regular and irregular nested Root-LDPC of design rate $R = 1/4$ and $N = 2400$ over a Rayleigh block fading channel is illustrated. Moreover, the outage probabilities P_{out} considering a BPSK modulation with $R = 1/4$ and block fading channel $n_c = 4$ is included. Note that since the FER curves follow the same slope as the outage probability curve, the full diversity property is achieved. The same comments can be made for the cases of 1 or 2 erased blocks. Note that equal FER is achieved independently of the received blocks.

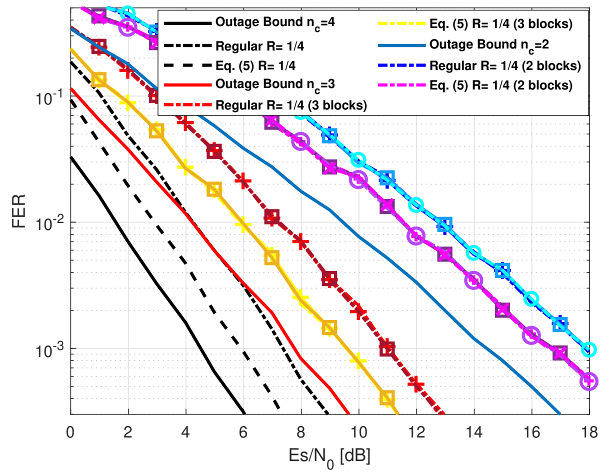

Fig. 5: FER of regular and irregular nested Root codes of rate $R = 1/4$ over the Rayleigh block fading channel

VI. Conclusion

We have investigated on nested Root-LDPC codes which can achieve MDS, full diversity and nested properties over the block fading channel under iterative decoding. The designed nested Root-LDPC codes exhibit good performance compared to existing methods that do not enforce the nested property.

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