On the Time-Delay Estimation Performance Limit of New GNSS Acquisition Codes

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***Abstract*—In previous works, new families of Pseudo-Random Noise (PRN) codes of length 1023 chips were proposed in order to ease the acquisition engine. These studies analyzed several metrics for code design in order to improve the acquisition but no analysis was conducted on the estimation performance, which in turn drives the final position, velocity and timing estimates. The main goal of this contribution is to assess if these new PRN codes designed to improve the acquisition engine lose in achievable time-delay estimation performance with respect to the standard GPS L1 C/A Gold codes. The analysis is performed by resorting to a new compact closed-form Cramér-Rao bound expression for time-delay estimation which only depends on the signal samples. In addition, the corresponding time-delay maximum likelihood estimate is also provided to assess the minimum signal-to-noise ratio that allows to be in optimal receiver operation.**

***Index Terms*—GNSS, time-delay estimation, band-limited signals, Cramér-Rao bound, signal acquistion.**

I. Introduction

Designing new Global Navigation Satellite Systems (GNSS) signals is always a trade-off between improving different performance criteria. Position accuracy, receiver sensitivity (acquisition, tracking or data demodulation thresholds) or the time to first fix (TTFF) are examples of those GNSS receiver design criteria. In previous works [1]–[3], it was shown that a new acquisition signal could help to improve both acquisition and sensitivity of a GNSS receiver. In those studies several aspects were considered in order to improve the performance of the acquisition stage such as designing new spreading modulations [1], [2], or generating a new navigation message structure [3]. Moreover, in [2] new families of Pseudo-Random Noise (PRN) codes of length 1023 chips were proposed in order to ease the acquisition engine. Those families were shown to provide better performance in terms of different acquisition criteria [4] w.r.t. GPS L1 C/A Gold codes [5].

On the other hand, in standard two-step GNSS receiver architectures, the final position, velocity and timing (PVT) estimation performance is directly linked to the corresponding time-delay and Doppler estimation. The optimal estimation performance of a locally unbiased estimator is given by the Cramér-Rao bound (CRB) [6], which provides an accurate lower bound on the mean square error (MSE) sense under certain conditions (for instance, in the high signal-to-noise ratio (SNR) regime). In this article, we aim to evaluate the achievable time-delay estimation performance (i.e. assuming no external errors such as atmospherics delays, orbital or satellite clock errors, or environment-specific effects) of the new PRN codes proposed to improve the acquisition engine, and compare such results to the standard GPS L1 C/A Gold codes used as a performance benchmark. To provide this performance assessment we resort to a recently proposed time-delay estimation compact-form CRB expression which only depends on the signal samples [7]. This new CRB is computed for the three new families of PRN codes of interest (balanced Gold, large Kasami and random sequences) along with the the GPS L1 C/A PRN family. Finally, in order to validate the CRB and obtain the minimum SNR which allows an optimal receiver operation point, we also provide the corresponding time-delay maximum likelihood estimate (MLE), which is known to be asymptotically efficient.

The article is organized as follows: Section II presents the signal model, Section III explains how to implement the three PRN families proposed in [2] to ease the acquisition engine, Section IV introduces the time-delay CRB for band-limited signals along with the MLE, and Section V summarizes the results. Conclusion are drawn in Section VI.

II. Signal Model

We consider the transmission of a band-limited GNSS signal $c(t)$ (bandwidth B), so-called PRN code in the GNSS terminology, over a carrier frequency f_c ($\lambda_c = \frac{c}{f_c}$), from a transmitter (satellite) T at position $\mathbf{p}_T(t) = \mathbf{p}_T + \mathbf{v}_T t$ to a receiver R at position $\mathbf{p}_R(t) = \mathbf{p}_R + \mathbf{v}_R t$. The complex analytic signal at the output of the receiver's antenna can be written as $x_A(t) = \alpha_R c_R(t) + n_A(t)$, with $n_A(t)$ a zero-mean white complex Gaussian noise, and where the gain α_R depends on the transmitted signal power, the transmitter/receiver antenna gains and polarization vectors, and the radial distance between T and R, $\mathbf{p}_{TR}(t)$ [8], [9]. If this radial distance can be approximated by a first order model,

$$\|\mathbf{p}_{TR}(t)\| \triangleq \|\mathbf{p}_R(t) - \mathbf{p}_T(t - \tau(t))\| = c\tau(t) \simeq d + vt,$$

with $\tau(t) = \tau + bt$, $\tau = d/c$ and $b = v/c$. Using the standard narrow-band assumption then

$$c_R(t) = c(t - \tau) e^{-j2\pi f_c \tau} e^{j2\pi f_c (1-b)t}, \quad (1)$$

and the baseband output of the receiver's Hilbert filter is

$$x(t) = \alpha c(t - \tau) e^{-j2\pi f_c b t} + n(t), \quad (2)$$

with $n(t)$ a complex white Gaussian noise within the filter bandwidth with unknown variance σ_n^2, and $\alpha = \alpha_R e^{-j2\pi f_c \tau}$.

The discrete vector signal model is build from $N = N_2 - N_1 + 1$ samples at $T_s = \frac{1}{F_s}$,

$$\begin{aligned}
\mathbf{x} &= \alpha \mathbf{a}(\boldsymbol{\eta}) + \mathbf{n}, \qquad (3)\\
\mathbf{x} &= \left(x(N_1 T_s), \ldots, x(N_2 T_s)\right)^\top,\\
\mathbf{n} &= \left(n(N_1 T_s), \ldots, n(N_2 T_s)\right)^\top,\\
\mathbf{c}(\tau) &= \left(c(N_1 T_s - \tau), \ldots, c(N_2 T_s - \tau)\right)^\top,\\
\mathbf{a}(\boldsymbol{\eta}) &= \left((\mathbf{c}(\tau))_1 e^{-j2\pi f_c b N_1 T_s}, \ldots, (\mathbf{c}(\tau))_N e^{-j2\pi f_c b N_2 T_s}\right)^\top,
\end{aligned}$$

where $\boldsymbol{\eta} = [\tau, b]^T$, $\mathbf{n} \sim \mathcal{CN}\left(\mathbf{0}, \sigma_n^2 \mathbf{I}_N\right)$. Since the transmitter/receiver antenna gains and polarization vectors are in general unknown, α is assumed to be an unknown complex parameter as well [9]–[13]. Thus, the unknown deterministic parameters [14] can be gathered in vector $\underline{\epsilon} = [\sigma_n^2, \tau, b, \alpha, \alpha^*]^T$, where α^* is the complex conjugate of α.

III. Acquisition Code Families

In [2], three families of PRN sequences were proposed as possible candidates for a new acquisition aiding signal. Those sequences are of length 1023 chips since a short time to acquire the signal is required in order to reduce the signal acquisition time:

- PRN Gold family [15, Chapter 2],
- PRN large Kasami family [15, Chapter 2],
- PRN random sequence applying the method in [16].

Notice that the Gold family is already used in the design of GNSS signals (i.e., GPS L1 C/A [5]). The other two families are large Kasami codes and the methodology proposed in [16] to generate efficient memory codes. The latter consist in maximizing a cost function under constraints in order to optimize some properties or design criteria. In the sequel, we present the theoretical background and practical implementation aspects of these three families of codes.

A. *Gold codes*

Gold codes are one important class of periodic sequences which provides reasonably large sets of codes with good periodic cross-correlation and autocorrelation properties. Gold codes have a code period of $2^n - 1$ chips and have $N + 2$ codes in the set. These codes are constructed from selected m-sequences [15, Chapter 2] and particularly by preferred pairs of m-sequences [15, Chapter 2] of length N. The following conditions are sufficient to construct a preferred pair, a and b, of m-sequences of length $N = 2^n - 1$:

- $n \neq 0 \mod 4$ that is to say, n is odd or $n = 2 \mod 4$.
- $b = a[q]$, where q is odd and either has the value $q = 2^k + 1$ or $q = 2^{2k} - 2^k + 1$.
- $\gcd(n, k) = \begin{cases} 1 & for \quad n\ odd. \\ 2 & for \quad n = 2 \mod 4. \end{cases}$

Theorem 1: [15, Chapter 2, Theorem 2] Given a preferred pair of m-sequences a and b of period $N = 2^n - 1$ generated by primitive binary polynomials $f_1(x)$ and $f_2(x)$ with no common factor and where $n \neq 0 \mod 4$. The sequences defined by $G(a, b)$ are called Gold codes, with

$$G(a, b) = \{a, b, a + b, a + Tb, a + T^2 b, \ldots, a + T^{N-1} b\}, \quad (4)$$

where $T^x a$ denotes the operator that produces the sequence whose k-th element is given by a_{k+x}. It should be noted that Gold codes are generated via Linear Feedback Shift Registers as their structure undertakes two binary polynomials [5].

A.1) Balanced Gold codes

A code with odd length is said to be balanced when the number of "ones" exceeds the number of "zeros" by one. This kind of codes have desirable spectral properties, however not all Gold codes are balanced codes. In order to obtain a family of balanced Gold codes, the following procedure must be followed [15, Chapter 2]:

1) First select a preferred pair of m-sequences a and b of length $N = 2^n - 1$.
2) The initial conditions for shift register 2 are obtained by long division of the ratio $\frac{g(x)}{f(x)}$,where $f(x)$ is the characteristic polynomial of sequence b and $g(x)$ is defined as: $g(x) = f(x) + \frac{dxf(x)}{dx}$.
3) The initial conditions for shift register 1 affects only the first tap, which must be 0.
4) The set of Gold codes is formed by modulo-2 addition of the two registers, 1 and 2.

A.2) Designing balanced Gold codes

A balanced Gold code is implemented as follows:

1) Select the first polynomial:
   - $f_1(x) = x^{10} + x^3 + 1 \rightarrow 010000001001 \rightarrow 2011$.
2) Select the second polynomial:
   - $k = 2 \rightarrow q = 2^k + 1 = 5 \rightarrow \gcd(10, 2) = 2$.
   - Find in [17, Annex C], the decimation of m-sequence for $b = a[5]$.
   - $b = a[5] \rightarrow f_2(x) = x^{10} + x^8 + x^3 + x^2 + 1 \rightarrow$ 010100001101 $\rightarrow$ 2415.
3) As we want balanced codes we have to obtain the characteristic phase of the sequence b:
   - $g(x) = f(x) + \frac{dxf(x)}{dx} = x^3$.
   - Compute the long division:
     $x^3 \,\overline{)\, x^{10} + x^8 + x^3 + x^2 + 1} = x^7 + x^5 + 1$.
4) Initial registers:
   - $Init_a$ = 1111111110.
   - $Init_b$ = 0010100001.
5) From sequences a and b we can obtain a balanced Gold code family as given in equation (4).

B. *Kasami codes*

The large Kasami codes [15, Chapter 2], like the Gold codes, are a set of periodic sequences with good correlation properties. Large Kasami codes have a code period of $N = 2^n - 1$ chips under the condition of $\mod(n, 4) = 2$. Moreover the family size is equal to $(N+2)\sqrt{N+1}$. In order to construct large Kasami codes, a small set of Kasami codes

[15, Chapter 2] is required. Small Kasami codes, as well as *No* [18] and *Bent* [19] codes, have the most outperforming correlation properties for a code length of 1023 chips. However the family size is just $\sqrt{N+1} = 32$ codes, which may not be large enough to cope with all the satellites in one GNSS constellation. As in the case of Gold codes, large Kasami codes are constructed from selected m-sequences and particularly by a preferred pair of m-sequences of length N.

Theorem 2: [15, Chapter 2, Theorem 4], Let n be even and let $f_1(x)$ denote a primitive binary polynomial of degree n that generates the m-sequence a. Let $b = a[2^{(n/2)} + 1]$ denote a m-sequence of period $2^{(n/2)} - 1$ generated by the characteristic polynomial $f_2(x)$ of degree $n/2$, and let $f_3(x)$ denote the polynomial of degree n that generates the decimation sequence [15, Chapter 2] $a[q]$. Then, the set of sequences of period N generated by the characteristic polynomial $h(x) = f1(x)f2(x)f3(x)$ is called the large set of Kasami sequences and is denoted by $K_L(a)$.

Note that for the specific code length $N = 1023, (n = 10)$, b is the following decimation sequence:

$$b = a\left[2^{(n/2)} + 1\right] = a[33]. \tag{5}$$

The set of sequences defined by $K_L(a)$ is then the set of Large Kasami sequences:

$$K_L(a) = G(a,c) \bigcup \left(\bigcup_{i=0}^{2^{(n/2)}-1} \{T^i b + G(a,c)\} \right). \tag{6}$$

The family size is equal to $(N+2)\sqrt{N+1} = 32800$ codes. We notice that not all the set of Large Kasami codes have the balanced property [15, Chapter 2], therefore to generate the code subset, we select those which have the balanced property and outperform others in terms of auto/cross-correlation.

B.1) Designing large Kasami codes

The procedure to design large Kasami codes is as follows:

1) Select the first polynomial:
   - $f_1(x) = x^{10} + x^3 + 1 \rightarrow 010000001001 \rightarrow 2011$.
2) Select the second polynomial:
   - $k = 2 \rightarrow q = 2^k + 1 = 5 \rightarrow \gcd(10, 2) = 2$.
   - Find in [17, Annex C], the decimation of m-sequence for $c = a[5]$.
   - $c = a[5] \rightarrow f_2(x) = x^{10} + x^8 + x^3 + x^2 + 1 \rightarrow 010100001101 \rightarrow 2415$.
3) Select the third polynomial:
   - $b = a\left[2^{(n/2)} + 1\right] = a[33]$.
   - Find in [17, Annex C], the decimation of m-sequence for $b = a[33]$.
   - $c = a[33] \rightarrow f_2(x) = x^5 + x^4 + x^3 + x^2 + 1 \rightarrow 000000111101 \rightarrow 0075$.
4) From sequences a, b and c we can obtain a Kasami code family as given in equation (6).

C. Random sequences

A method to create a set of PRN sequences with good correlation properties is provided in [16]. The method consists in building an initial set of random bits pattern, where each bits pattern represents a potential PRN sequence. Then, from the initial set of codes, we apply an iterative algorithm where the updated set of PRN sequences provide enhanced performance compared to the initial set. Following this methodology, the final goal consists in selecting an optimized final set of PRN sequences. A cost function must be defined in order to determine if the current iteration provides a PRN sequence set which is better than the precedent one. Since a new acquisition aiding signal is the design goal, a cost function which penalizes unwanted correlation peaks (those which increase the acquisition error probability) is hence proposed [16]. Then, any correlation value which exceeds the Welch bound [20] represents a system degradation. The cost function used to generate the family of random sequences is,

$$\begin{aligned} F_i &= \sum_{\substack{\tau=1,\\ ACF(\tau)>\Phi_{bound}}}^{N-1} \left(ACF(\tau) - \Phi_{bound}\right)^2 \\ &+ \sum_{j\neq i} \sum_{\substack{\tau=1,\\ CCF(\tau)>\Phi_{bound}}}^{N-1} \left(CCF(\tau) - \Phi_{bound}\right)^2, \end{aligned} \tag{7}$$

where ACF and CCF stand for autocorrelation and cross-corrleation, respectively, and Φ_{bound} represents the Welch bound, defined in (8). The Welch bound represents the theoretical minimum of the maximum value of the autocorrelation and cross-correlation functions, given a subset of L sequences of length N,

$$\Phi_{bound} = N\sqrt{\frac{L-1}{NL-1}}. \tag{8}$$

Each algorithm iteration consist in two steps: a chip flip within the PRN sequence and the cost function evaluation. If the chip flip minimizes the cost function when compared to the precedent iteration, the chip flip is accepted; otherwise the chip flip is rejected. The flow diagram of the algorithm is illustrated in Figure 1.

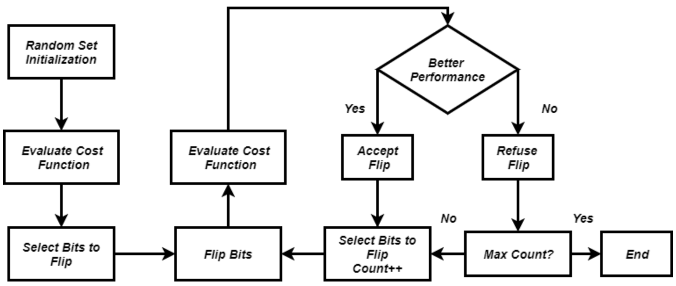

Fig. 1: Diagram flow for optimization of random sequence.

Finally, It is well known that both balanced and minimum ACF side-lobe properties are desired spreading codes characteristics. These properties can be set as initial requirements for the initial set of codes. However, after flipping some of the bits, those qualities may not be preserved. In order to guarantee those desired properties, two pre-required conditions [16] are imposed on the flipping bit step. The first one is the balanced invariance condition [16] where bits are always flipped in pairs, i.e. one bit with null value and one bit with value 1 are flipped to ensure that the code remains balanced. The second condition is to minimize the ACF side-lobe. This property can be ensured by the following equation (9),

$$a_{k-1} + a_{k+1} = a_{j-1} + a_{j+1}, \tag{9}$$

where a_k and a_j are the flipping bits.

IV. Time-delay CRB for Band-limited Signals and Maximum Likelihood Estimation

A. CRB for Time-delay Estimation

In a recent contribution [7] we derived a new compact closed-form CRB for the time-delay estimation of a generic band-limited signal, given by

$$\begin{aligned} F_{\tau|\boldsymbol{\epsilon}}\left(\boldsymbol{\epsilon}\right) &= 2\mathrm{SNR}_{\mathrm{out}} F_s^2 \left(\frac{\mathbf{c}^H \mathbf{V} \mathbf{c}}{\mathbf{c}^H \mathbf{c}} - \left| \frac{\mathbf{c}^H \boldsymbol{\Lambda} \mathbf{c}}{\mathbf{c}^H \mathbf{c}} \right|^2 \right) \\ &\underset{\text{real signal}}{=} 2\mathrm{SNR}_{\mathrm{out}} F_s^2 \left(\frac{\mathbf{c}^H \mathbf{V} \mathbf{c}}{\mathbf{c}^H \mathbf{c}} \right), \end{aligned} \tag{10}$$

where $\mathrm{SNR}_{\mathrm{out}} = \frac{|\alpha|^2 \mathbb{E}}{(\sigma_n^2 / F_s)} = \frac{|\alpha|^2}{\sigma_n^2} \mathbf{c}^H \mathbf{c}$ and $\mathbb{E}$ the energy of the signal. $\boldsymbol{\Lambda}$ and $\mathbf{V}$ are defined as (for $N_1 \leqslant n, n' \leqslant N_2$)

$$(\mathbf{V})_{n,n'} = \left| \begin{array}{l} n' \neq n : (-1)^{|n-n'|} \frac{2}{(n-n')^2} \\ n' = n : \frac{\pi^2}{3} \end{array} \right. \tag{11a}$$

$$(\boldsymbol{\Lambda})_{n,n'} = \left| \begin{array}{l} n' \neq n : \frac{(-1)^{|n-n'|}}{(n-n')} \\ n' = n : 0 \end{array} \right. . \tag{11b}$$

Notice that this CRB expression is especially easy to use because it depends only on the signal samples, the PRN code samples in our case.

B. Maximum Likelihood Time-delay Estimator

Considering the signal model (3), the time-delay MLE is defined as[1] [13]

$$\begin{aligned} \hat{\tau} &= \arg\min_{\tau} \left\{ \mathbf{x}^H \mathbf{\Pi}^{\perp}_{\mathbf{c}(\tau)} \mathbf{x} \right\} = \arg\max_{\tau} \left\{ \frac{\left| \mathbf{c}(\tau)^H \mathbf{x} \right|^2}{\mathbf{c}(\tau)^H \mathbf{c}(\tau)} \right\} \\ &= \arg\max_{\tau} \left\{ \frac{\left| \int_{-\infty}^{+\infty} c(t-\tau)^* x(t)\, dt \right|^2}{\int_{-\infty}^{+\infty} |c(t)|^2 \, dt} \right\}, \end{aligned} \tag{12}$$

which is useful to determine the value of $\mathrm{SNR}_{\mathrm{out}}$ (threshold) which allows to reach the CRB, because it is known that such estimator is asymptotically efficient (e.g., in the high SNR regime) for the conditional signal model of interest [21] [22].

[1]Let $S = span(\mathbf{A})$, with $\mathbf{A}$ a matrix, be the linear span of the set of its column vectors, $S^{\perp}$ the orthogonal complement of the subspace S, $\mathbf{\Pi}_{\mathbf{A}} = \mathbf{A}\left(\mathbf{A}^H \mathbf{A}\right) \mathbf{A}^H$ the orthogonal projection over S, and $\mathbf{\Pi}^{\perp}_{\mathbf{A}} = \mathbf{I} - \mathbf{\Pi}_{\mathbf{A}}$.

V. Results

In this section, we assess the closed-form CRB in (10) for four representative families of codes: i) the standard GPS L1 C/A PRN Gold family with codes of length 1023 chips, ii) a family of balanced Gold codes of length 1023 chips (Section III-A), iii) a family of large Kasami codes of length 1023 chips (Section III-B), and iv) a family of random sequences of length 1023 chips generated by the algorithm described in Section III-C. These signals use a binary phase-shift keying (BPSK) waveform with a chip frequency rate of 1.023 MHz.

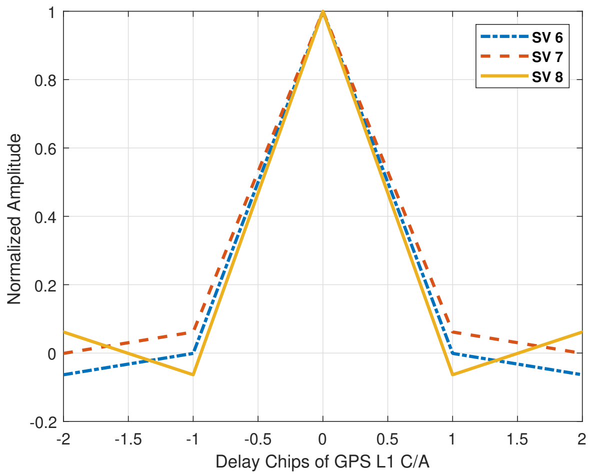

Fig. 2: ACF for satellite with GPS L1 C/A codes 6, 7 and 8.

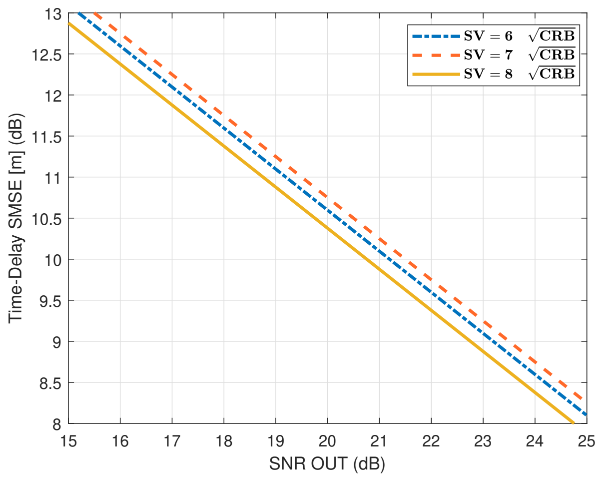

Fig. 3: CRB for satellite with GPS L1 C/A codes 6, 7 and 8, bandwidth equal to 1.023 MHz

In a first step, we aim to evaluate the CRB for the standard GPS L1 C/A PRN Gold family. In [23], it was shown that three different types of autocorrelation function (ACF) with different main lobe behaviours exist in the GPS L1 C/A code family. Those ACFs are illustrated in Fig. 2. Note from these results that the main lobe of the ACF is wider or narrower depending on the the correlation value on both sides of this lobe. The CRB (10) associated with these three types of Gold codes is shown in Fig. 3. As expected, narrower ACFs lead to lower CRBs, thus to better time-delay estimation. Then, not all the codes in a given family provide the same achievable time-delay estimation performance. Now, considering the PRN codes designed to improve the acquisition engine, [4] proposed to select codes whose absolute correlation value on both sides of the main lobe of the ACF was as low as possible (for a length of 1023 i.e. $ACF(1) = ACF(-1) = -1$). That criteria was considered in [2] in order to design PRN codes to improve the acquisition stage. Then, it is expected that for these acquisition codes the time-delay CRB is equal to the GPS L1 C/A code family CRB using PRNs with correlation value on both sides of the main lobe equal to -1.

The CRB and the corresponding MLE in (12) are computed considering $\alpha = (1+j) \cdot \sqrt{SNR_{in}/2}$. The MLE is obtained from 2000 Monte Carlo runs. These results are summarized in Fig. 4, where both time-delay CRB and MLE obtained for the standard GPS L1 C/A PRN Gold family are compared to the three PRN families proposed to ease the acquisition engine. Note that $F_s = 1.023$ MHz and $F_s = 2.046$ MHz are used to compute both CRB and MLE. From these results we can conclude that: i) the new families of acquisition codes do not worsen the estimation capabilities w.r.t. the standard GPS L1 C/A PRNs, because the CRB obtained is equivalent, ii) the optimal receiver operation point (SNR_{out} threshold) is almost the same (i.e. small variations of 1 dB in the case of Kasami codes). Consequently, we can state that the three PRN families proposed to improve the acquisition performance criteria do not penalize the time-delay estimation performance, which is the main driver on the final PVT performance of the receiver. Note also from results in Fig. 4 that the receiver sampling frequency F_s has a direct impact in the final time-delay estimation performance, because this determines the signal bandwidth exploited. Then the higher the F_s is, the better time-delay estimation is obtained, since more energy of the signal is considered. Finally, we underline that the MLE algorithm is efficient from $\text{SNR}_{\text{out}} = 15$ dB, which corresponds to a carrier-to-noise density ratio $C/N_0 = 45$ dB-Hz, since the PRN sequence last 1 ms. To be able to deal with lower C/N_0 we need longer integration times, i.e., $T_I = 10$ ms and $\text{SNR}_{\text{out}} = 15$ dB lead to $C/N_0 \approx 35$ dB-Hz.

VI. Conclusions

In this article we provided the answer to a fundamental question for new GNSS acquisition codes, generated to ease the acquisition stage, regarding their achievable time-delay estimation performance with respect to the standard GPS L1 C/A. In order to compare these families of codes a new compact closed-form CRB expression for time-delay estimation was used. Results show that the same performance is obtained in terms of time-delay estimation independently of the selected family of codes, which confirms that the new acquisition codes do not worsen the final position, velocity and time estimates. Moreover, time-delay MLE results were also provided in order to determine the minimum SNR that allows to be in optimal receiver operation.

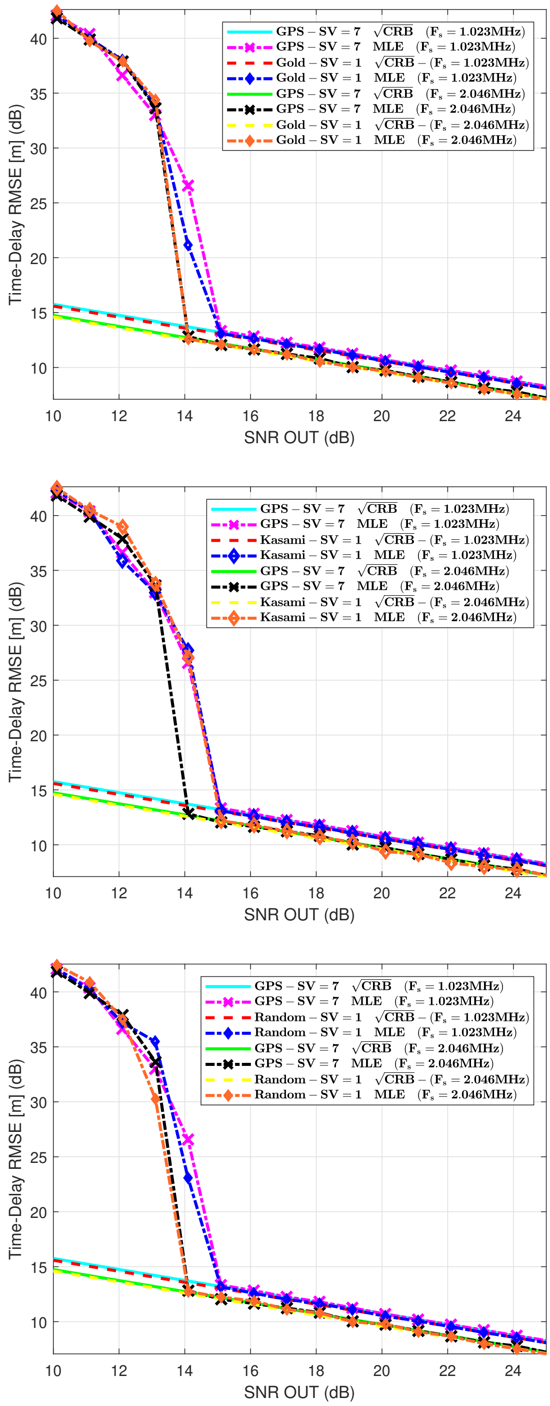

Fig. 4: CRB and MLE for GPS L1 C/A PRN codes, compared to the CRB and MLE for balanced Gold PRN codes (top), large Kasami PRN codes (middle) and Random PRN codes (bottom), for two bandwidths 1.023 MHz and 2.046 MHz.

Acknowledgments

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