Graph Laplacian-based Regularization Approach for Detecting Abnormal Ship Behavior on Trajectories

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Project Team

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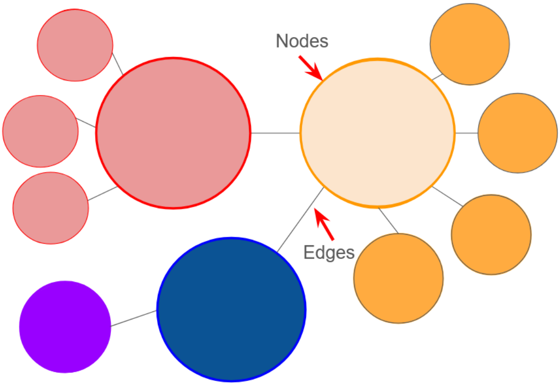
Outline

Context

A graph is a collection of objects/entities (nodes) that are all interconnected (by edges).



In graph signal processing, signal processing tasks are generalized to signals living on non-Euclidean domains.

Context

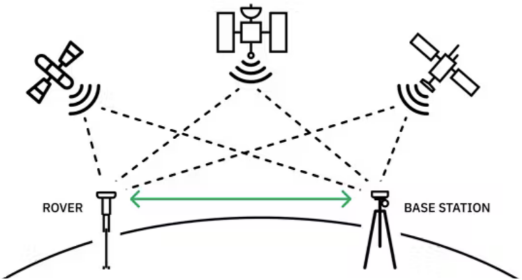
Examples of Graphs

Social Network (TéSA)

Context

Examples of Graphs

Global navigation satellite system (GNSS)



Applications

→ Graphs provide a structural/relational representation of the data.

Clustering

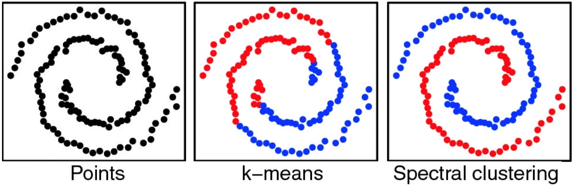
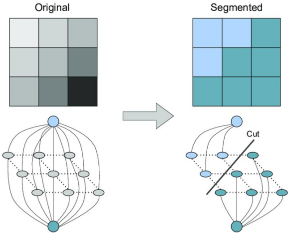
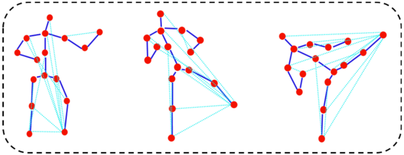


Image Segmentation



Action Recognition



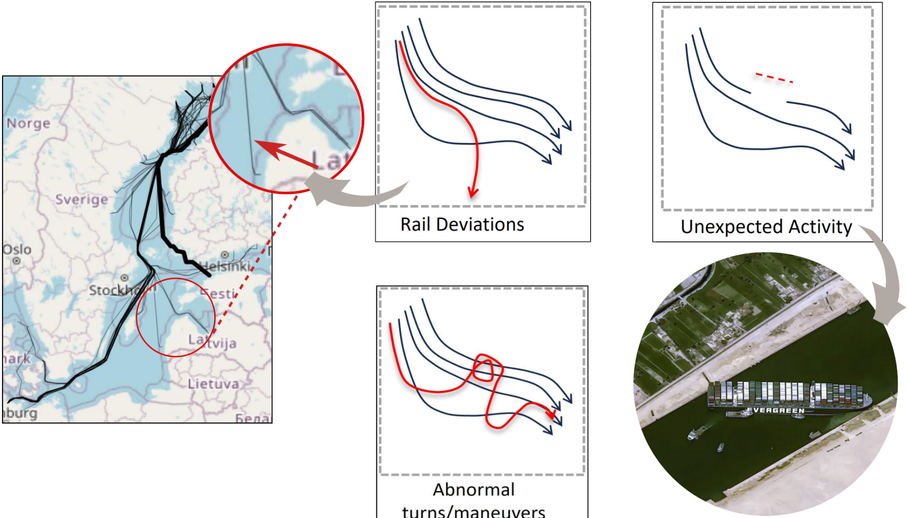
Ship Trajectories

Automatic Identification System (AIS)

Source (modified) : [AIS guide]

- $\{\boldsymbol{x}_1, ..., \boldsymbol{x}_t, ..., \boldsymbol{x}_T\}$: AIS trajectory.
- $\boldsymbol{x}_t = [\text{lon}, \text{lat}, \text{cog}, \text{sog}, \text{time}]^\top$, $\boldsymbol{x}_t \in \mathbb{R}^d$, where d is the number of features.

Project Goal

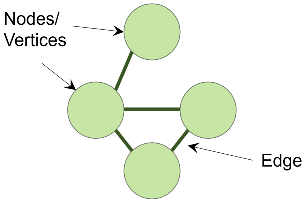


Cargo ship stuck in the Suez Canal

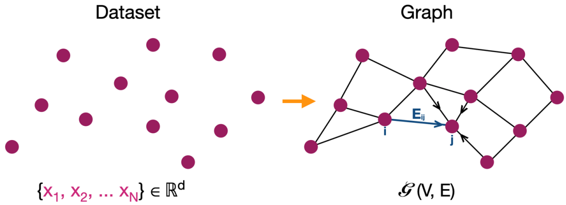
Introduction to Graphs

A graph is represented as $G = (V, E)$

- V is the set of nodes (vertices)
- E is the set of edges (links, connections)



Graph Data



the edge weights can represent the degree of association/influence/similarity between two nodes

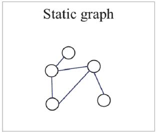
Type of Graphs

Graph Type

PLAIN (static) Graph

$$G = (V, E)$$

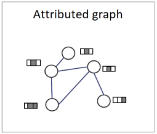
- the only information about it is its **structure**



ATTRIBUTED Graph

$$G = (V, E, X)$$
$$X \in \mathbb{R}^{N \times d}$$

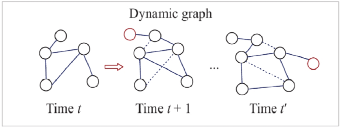
- nodes and/or edges have **features** associated with them



DYNAMIC Graph

$$G(t) = (V(t), E(t), X_v(t), X_e(t))$$

- $X_v(t), X_e(t)$: node/Edge attributes
- graph structure and node attributes **evolve at the time**

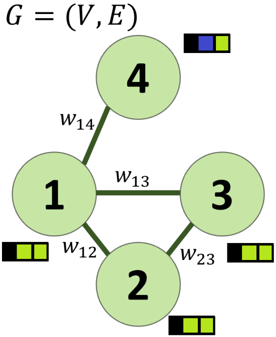


$*$ Attributed graphs are the interest of the current work.

Attributed Graph

Let $G = (V, E, \mathbf{X})$ be an attributed graph, where

- V: set of nodes, and $|V| = N$
- E: set of edges
- $\mathbf{X} \in \mathbb{R}^{N \times d}$: attributes, $\mathbf{X}_u \in \mathbb{R}^d$
- d: number of node features



→ Weighted Adjacency Matrix $\mathbf{W} \in \mathbb{R}^{N \times N}$

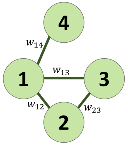
$$\mathbf{W}_{u,v} = \begin{cases} \exp\left(\frac{-\|\mathbf{X}_u - \mathbf{X}_v\|^2}{2\sigma^2}\right), & \text{if } (u,v) \in E \\ 0, & \text{otherwise} \end{cases}$$

Attributed Graph

→ Degree matrix: $\mathbf{D}_{u,u} = \sum_{v=1}^{N} \mathbf{W}_{u,v}$

→ Laplacian Matrix: $\boxed{\mathbf{L} = \mathbf{D} - \mathbf{W}}$

$G = (V, E)$

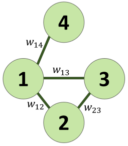


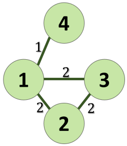
Adjacency matrix (**W**)	Degree matrix (**D**)	Laplacian matrix (**L = D-W**)
$\begin{pmatrix} 0 & w_{1,2} & w_{1,3} & w_{1,4} \\ w_{2,1} & 0 & w_{2,3} & 0 \\ w_{3,1} & w_{3,2} & 0 & 0 \\ w_{4,1} & 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} d_1 & 0 & 0 & 0 \\ 0 & d_2 & 0 & 0 \\ 0 & 0 & d_3 & 0 \\ 0 & 0 & 0 & d_4 \end{pmatrix}$	$\begin{pmatrix} d_1 & -w_{1,2} & -w_{1,3} & -w_{1,4} \\ -w_{2,1} & d_2 & -w_{2,3} & 0 \\ -w_{3,1} & -w_{3,2} & d_3 & 0 \\ -w_{4,1} & 0 & 0 & d_4 \end{pmatrix}$

Attributed Graph

→ Degree matrix: $\mathbf{D}_{u,u} = \sum_{v=1}^{N} \mathbf{W}_{u,v}$

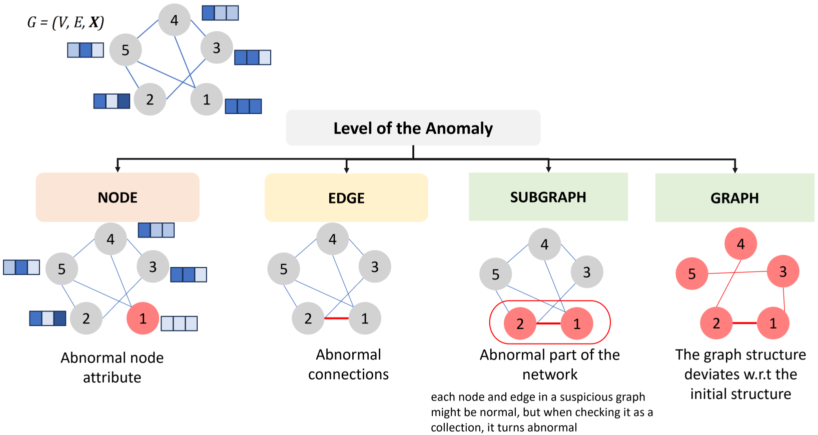
→ Laplacian Matrix: $\boxed{\mathbf{L} = \mathbf{D} - \mathbf{W}}$

$G = (V, E)$	Adjacency matrix (**W**)	Degree matrix (**D**)	Laplacian matrix (**L = D-W**)
	$\begin{pmatrix} 0 & w_{1,2} & w_{1,3} & w_{1,4} \\ w_{2,1} & 0 & w_{2,3} & 0 \\ w_{3,1} & w_{3,2} & 0 & 0 \\ w_{4,1} & 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} d_1 & 0 & 0 & 0 \\ 0 & d_2 & 0 & 0 \\ 0 & 0 & d_3 & 0 \\ 0 & 0 & 0 & d_4 \end{pmatrix}$	$\begin{pmatrix} d_1 & -w_{1,2} & -w_{1,3} & -w_{1,4} \\ -w_{2,1} & d_2 & -w_{2,3} & 0 \\ -w_{3,1} & -w_{3,2} & d_3 & 0 \\ -w_{4,1} & 0 & 0 & d_4 \end{pmatrix}$

$G = (V, E)$	Adjacency matrix (**W**)	Degree matrix (**D**)	Laplacian matrix (**L = D-W**)
	$\begin{pmatrix} 0 & 2 & 2 & 1 \\ 2 & 0 & 2 & 0 \\ 2 & 2 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 5 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 5 & -2 & -2 & -1 \\ -2 & 4 & -2 & 0 \\ -2 & -2 & 4 & 0 \\ -1 & 0 & 0 & 1 \end{pmatrix}$

→ The **Laplacian** matrix captures the local geometric structure of the graph.

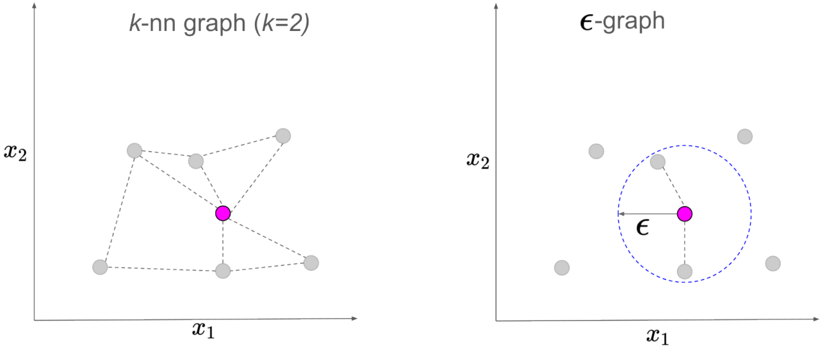
Anomaly Detection on Graphs



From data to graphs: Similarity graph

Given a set of points $x_1, x_2, ..., x_N$

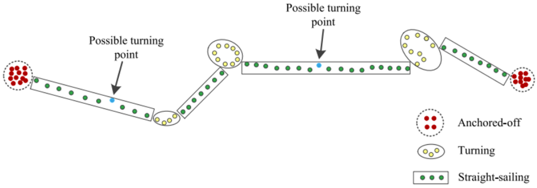
- Set the nodes of the graph
- Compute a similarity distance $dist(x_i, x_j)$ (e.g. euclidean)
- Set the edges using the distance and following either k-nn graph/ϵ-graph



k-nearest neighbor graph vs ϵ-graph

State-of-the-art for Graph-based AD

- **Spatial Patterns**[1,2]
AIS data points (center of clusters) are defined as the nodes.



- **Spatial Patterns + Semantic Knowledge**[3,4]
Other information is added to build the graph: ship direction (COG), speed over ground (SOG), angle.

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(1) Pallotta et al. (2013). Vessel pattern knowledge discovery from AIS data: A framework for anomaly detection and route prediction.. Entropy

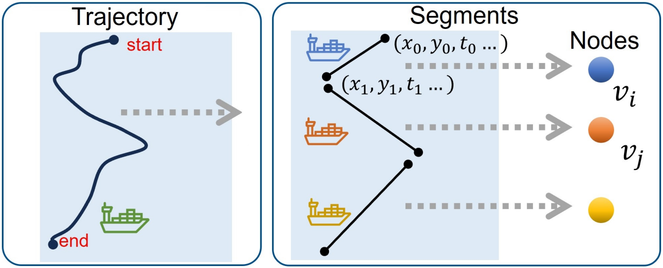
(2) Shi et al. (2022). Abnormal ship behavior detection based on AIS data. MDPI Applied Sciences

(3) Yan et al. (2020). Exploring AIS data for intelligent maritime routes extraction, Applied Ocean Research.

(4) Singh et al. (2022). Leveraging graph and deep learning uncertainties to detect anomalous trajectories. IEEE Transactions on Intelligent Transportation Systems.

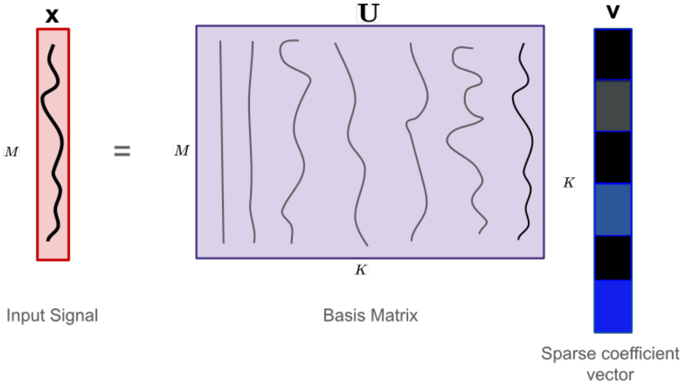
Graph on Trajectories

- **Vessels**: Let $V = \{v_1, .., v_N\}$ be a set of N the vessels.
- **AIS data**: Each vessel v_i can have d features $\mathbf{x}_i \in \mathbb{R}^d$.
- **Graph**: $G = (V, E, \mathbf{X})$ is composed of:
  - Nodes $V = \{v_1, ..., v_N\}$ represent vessels.
  - Node Attributes $\mathbf{X} \in \mathbb{R}^{N \times d}$ are the AIS data.
  - Edges E are set via the similarity/distance between nodes.



Towards the Regularization-based Approach

Sparse Coding

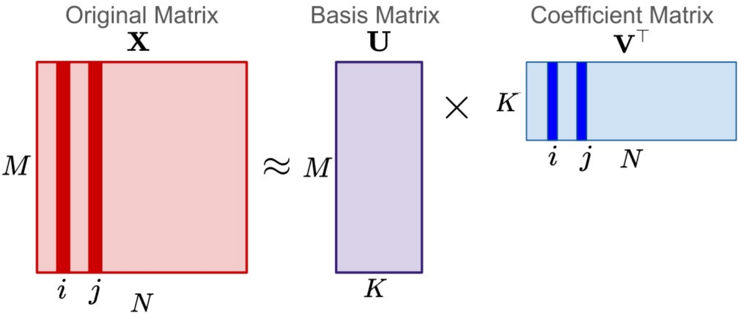


Sparse Coding Goal: to find the sparse coefficient vector $\mathbf{v}$ that represents $\mathbf{x}$ in the basis $\mathbf{U}$ via

$$\min_{\mathbf{v}} \frac{1}{2} \left\| \mathbf{x} - \mathbf{U}\mathbf{v} \right\|^2 + \lambda \left\| \mathbf{v} \right\|_1 . \tag{1}$$

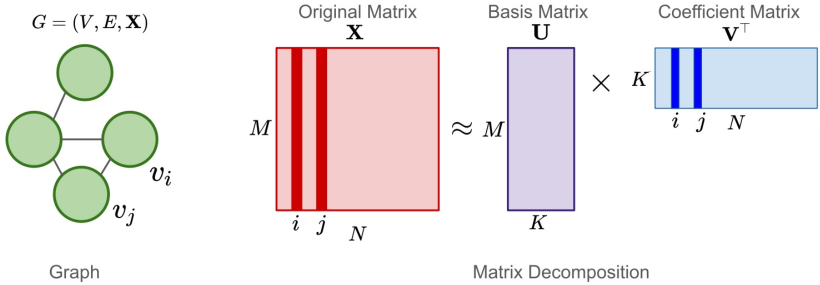
Towards the Regularization-based Approach

Matrix Decomposition



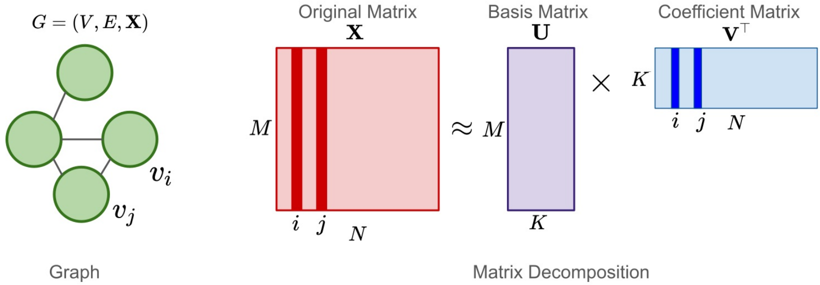
Matrix Decomposition

Graph Laplacian Restriction



*Hu, W. et al. (2021). Graph signal processing for geometric data and beyond: Theory and applications. IEEE Transactions on Multimedia, 24, 3961-3977.

Graph Laplacian Restriction



The operator is written as $(\mathbf{L} = \mathbf{D} - \mathbf{W})$:

$$\mathcal{R} = \sum_{i,j \in E} W_{i,j} \|\mathbf{v}_i - \mathbf{v}_j\|^2 = \mathbf{V}^\top \mathbf{L} \mathbf{V} \tag{2}$$

* Hu, W. et al. (2021). Graph signal processing for geometric data and beyond: Theory and applications. IEEE Transactions on Multimedia, 24, 3961-3977.

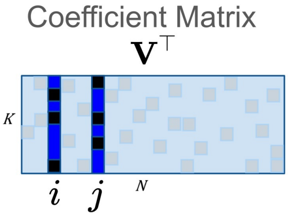
Graph-based Regularization Optimization

The regularization problem considering the sparsity and the graph connectivity is expressed as:

$$\min_{\mathbf{V}} \underbrace{\frac{1}{2}\|\mathbf{X}-\mathbf{U}\mathbf{V}\|_F^2}_{\text{fidelity term}} + \underbrace{\lambda_s \sum_{i=1}^{N}\|\mathbf{v}_i\|_1}_{\text{sparsity term}} + \underbrace{\frac{\lambda_g}{2}\mathrm{Tr}(\mathbf{V}\mathbf{L}\mathbf{V}^T)}_{\text{graph term}} \tag{3}$$

where

- λ_s: regularization for the sparsity term
- λ_g: regularization for the graph term
- $\mathbf{X} \in \mathbb{R}^{M\times N}$: the data features
- $\mathbf{U} \in \mathbb{R}^{M\times K}$ is a (known) dictionary
- $\mathbf{V} \in \mathbb{R}^{K\times N}$ coefficient matrix



Proposed Method

Objective function. The problem can be divided and solved for each sample $\mathbf{x}_i$ as:

$$\boxed{\min_{\mathbf{v}_i} \frac{1}{2} \left\| \mathbf{x}_i - \mathbf{U}\mathbf{v}_i \right\|^2 + \lambda_s \left\| \mathbf{v}_i \right\|_1 + \frac{\lambda_g}{2} \sum_{j=1}^{N} \left\| \mathbf{v}_i - \mathbf{v}_j \right\|^2 W_{i,j},} \qquad (4)$$

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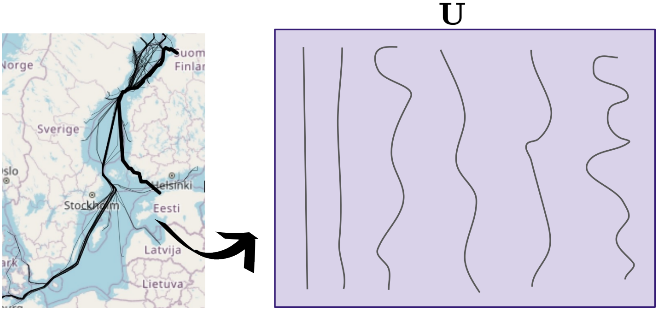
→ The dictionary is built from a set of **segments** of the data



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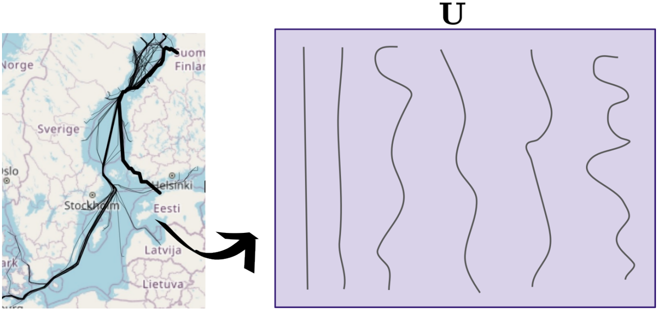


→ The problem (4) can be efficiently solved via the alternating direction method of multipliers (**ADMM**) method!

ADMM Recipe

Algorithm ADMM: For $i = 1, ..., N$:

1. Set $\mathbf{z} = \mathbf{v}_i$ (segment coefficient vector).
2. Update $\mathbf{z}$:

$$\mathbf{z}^{(t+1)} = \Big(\mathbf{U}^T\mathbf{U} + (\mu + \lambda_g \sum_{j=1}^{N} w_{i,j})\mathbf{I}_R\Big)^{-1}\big(\mathbf{U}^T\mathbf{x}_i + \mu\mathbf{q}^{(t)} - \mathbf{m}^{(t)} + \lambda_g \sum_{j=1}^{N} w_{i,j}\mathbf{v}_j\big) \tag{5}$$

3. Update $\mathbf{q}$ (for the sparsity constraint):

$$\mathbf{q}^{(t+1)} = \mathrm{soft}\Big(\mathbf{z}^{(t+1)} + \frac{\mathbf{m}^{(t)}}{\mu}, \frac{\lambda_s}{\mu}\Big) \tag{6}$$

4. Update $\mathbf{m}$ (dual multiplier):

$$\mathbf{m}^{(t+1)} = \mathbf{m}^{(t)} + \mu\big(\mathbf{z}^{(t+1)} - \mathbf{q}^{(t+1)}\big). \tag{7}$$

Anomaly detection using reconstruction scores

The reconstruction error is computed as

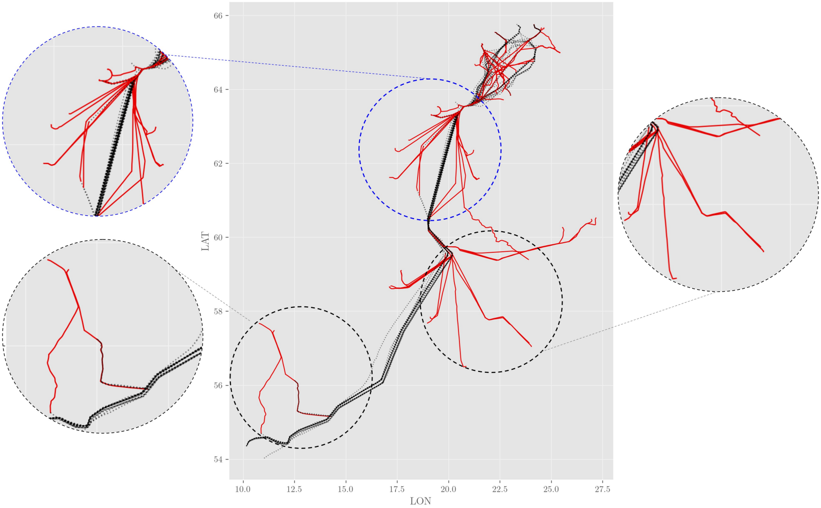
$$error_i = \left\| \mathbf{x}_i - \sum_{k=1}^{K} \mathbf{u}_k \hat{\mathbf{v}}_{i,k} \right\|^2 \tag{8}$$

where $\hat{\mathbf{v}}_i$ is the obtained coefficient vector in the optimization.

$\rightarrow$ **Remark:** High reconstruction scores indicate not smooth representations in the graph, then, potential anomalies

Numerical Experiments

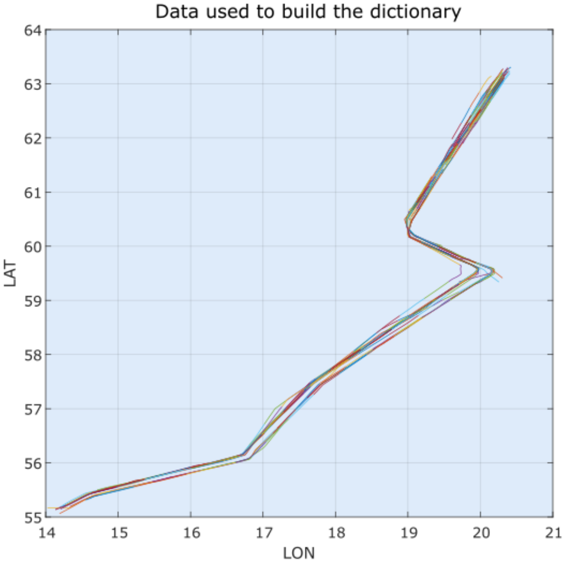
AIS Dataset (Baltic sea at Winter)



- Set 61 trajectories.
- Segments: 1446 **sub-trajectories** by a sliding window of 20 AIS points.

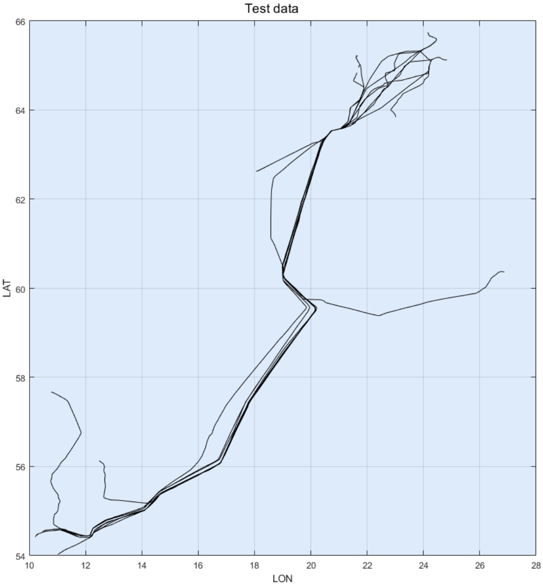
Setup

→ The dataset is split into two subsets: 80% for training and 20% for testing.

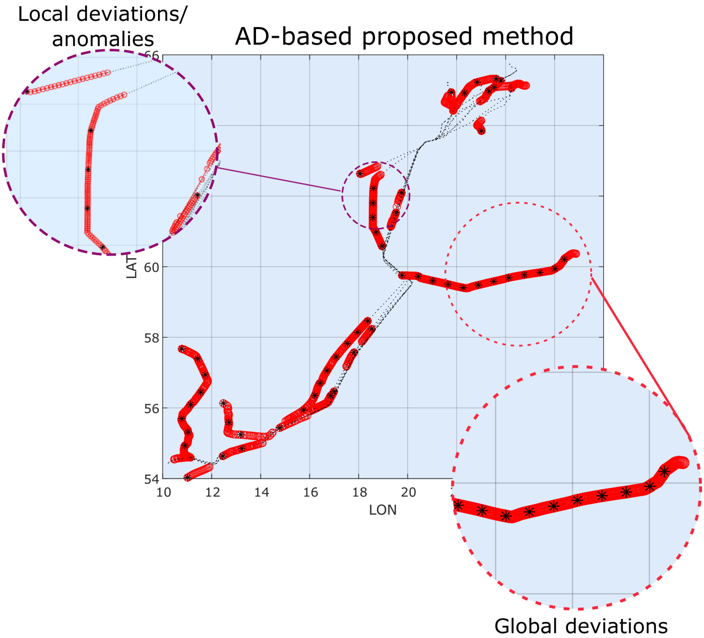


Setup

Test data



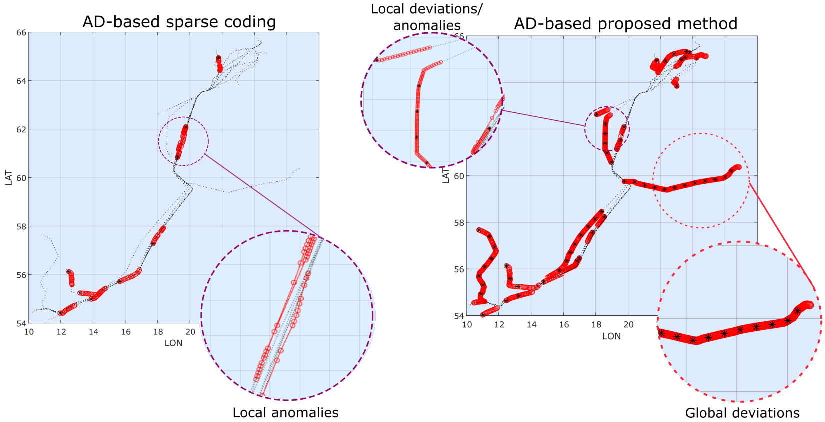
AD Results



Local deviations/ anomalies
AD-based proposed method
LAT
LON
54
56
58
60
66
10
12
14
16
18
20
Global deviations

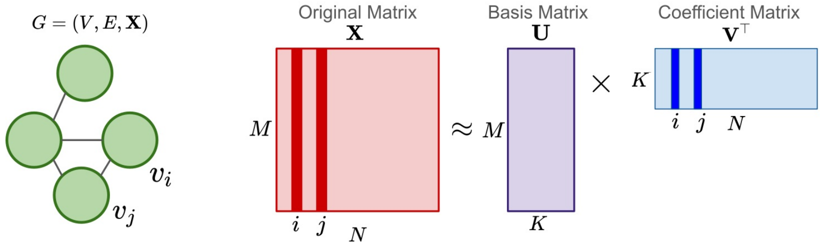
Comparison AD Results

→ Anomaly detection results using the reconstructions methods.



To take away

- Signals/images can be embedded into graphs **to exploit their correlations**.
- The **graph structure** imposed as a regularization term promotes the reconstruction of signals given the chosen structure of the graph, making it a versatile tool for modeling different problems.



Graph

Matrix Decomposition

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- **Anomaly detection on graphs.**

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- **Anomaly detection on trajectories.**

• Laxhammar, R., et Falkman, G. Online learning and sequential anomaly detection in trajectories. IEEE transactions on pattern analysis and machine intelligence, 36(6), 1158-1173, 2013.

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Thanks for your attention!

Contact Information:

Atoms in the dictionary