

An adaptive detection of Spread Targets in Locally Gaussian Ground Clutter using a Long integration time

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Abstract

This paper deals with the problem of detecting a collision target in ground clutter, using a long integration time. A single reception channel being available, classical space time adaptive processing (STAP) cannot be used. After range processing, ground clutter can be modeled as a known interference subspace in the Doppler domain depending on its radial and orthoradial speeds. We exploit this a priori knowledge to perform an adaptive detection of a collision target supposed to lie in a known and different subspace. A GLRT detector is first derived for known clutter covariance matrix. Then, the unknown covariance matrix is adaptively estimated from the projection of the data onto the modeled clutter subspace, and is plugged in the GLRT to form a sub-optimal detector. The proposed scheme can be viewed as a synthetic STAP, for which the space domain is replaced by a clutter orthoradial information and longer integration time.

1 Introduction

Moving target indicator (MTI) are commonly used in airborne radars to detect and track moving targets. Due to the platform motion, ground clutter returns spread over range and Doppler. Thus the target detection is heavily degraded in the clutter region. A well known solution to cope with this problem is to combine the use of an antenna array and STAP technique to reject clutter and detect the target [4], which can be extremely costly. Consequently, this strategy is limited to specific applications. In the radar literature, clutter returns are frequently modeled as Gaussian disturbance [2]. More recently, SIRV clutter models [7] have been proposed to take into account non-Gaussian clutter disturbance: the clutter vector is then the product of two components, a Gaussian vector (speckle) of known covariance matrix, and a random positive amplitude (texture) of known prior distribution. Equivalently, when textures are considered unknown deterministic parameters, locally Gaussian models considered in this paper are invoked to describe ground clutter. In practice, the clutter covariance matrix may not be available, and must be estimated using secondary data [4] to form an adaptive detector. In [3], a

long integration time has been used to separate a collision point target from ground clutter for short and middle range applications. For this, ground clutter has been structured as a deterministic known interference subspace containing quadratic Doppler phase exponentials, depending on both clutter radial and orthoradial speeds. Conversely, the collision point target has been modeled as a deterministic signal known to lie in a subspace of linear Doppler phase exponentials. The optimal detector is then a subspace matched detector [8]. We propose here to use a GLRT detector designed for range and Doppler spread target in locally Gaussian clutter whose covariance matrix is known a priori. The covariance matrix is then estimated from the projection of the data onto the modeled clutter subspace, and inserted in the GLRT to form an adaptive suboptimal detection scheme. This paper is organized as follows. Section 2 formulates the problem and describes the clutter and signal models investigated in this study. In section 3, the GLRT for range and Doppler spread targets in locally Gaussian clutter of known covariance matrix will be derived. The estimation of the clutter covariance matrix is addressed using the modeled clutter subspace in the Doppler domain. In section 4, the theoretical performances of the proposed detector are discussed. Simulation results based on real clutter data are also presented. Section 5 draws conclusions and perspectives.

2 Data Model and Assumptions

We consider an airborne radar embedded on an aircraft flying at constant speed v_a . A possible collision target at initial range R_0 with constant velocity v_t is heading toward the aircraft up to an impact point, defined by the intersection of both aircraft and target directions. Ground clutter is also present at range R_0 and will compete with the target, as shown in Fig.1. Due to long integration time T_{int} , we use a second order Taylor expansion of the target-radar distance [6]

$$R(t) = R_0 + v_r t + \frac{1}{2} \left(a_r + \frac{v_{\perp}^2}{R_0} \right) t^2 \quad (1)$$

where v_r and a_r are the relative, radial velocity and acceleration, and v_{\perp} is the orthoradial velocity. We assume a constant radial velocity such that $a_r = 0$.

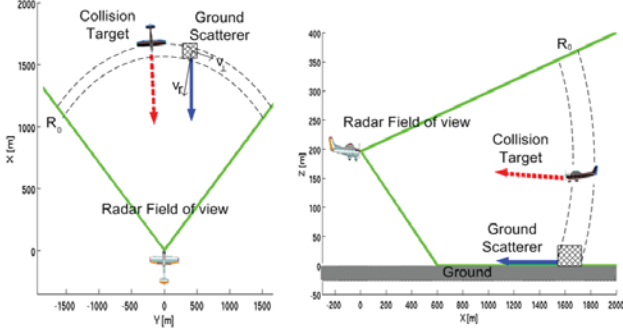


Figure 1: Collision Target/Ground Scatterer Trajectories.

2.1 Data Model

After range processing, we assume that the target's signal is present in L adjacent cells and is in the Doppler clutter region, for instance due to Doppler aliasing [6]. The detection problem can thus be formulated as follows:

$$\begin{aligned} H_0 : \mathbf{z}_r &= \mathbf{c}_r + \mathbf{n}_r \\ H_1 : \mathbf{z}_r &= \mathbf{x}_r + \mathbf{c}_r + \mathbf{n}_r \end{aligned} \quad (2)$$

where \mathbf{z}_r is the observations vector at range cell r , \mathbf{x}_r is the target signal described in section 2.2, \mathbf{c}_r is the clutter signal described in section 2.3, and \mathbf{n}_r is the additive white Gaussian noise of power σ^2 . The observations are supposed to be independent from one range cell to another. Hypothesis H_0 corresponds to the case where only clutter is present whereas hypothesis H_1 corresponds to the case where clutter and target are present.

2.2 Collision Target Subspace

We consider the target to be a so-called collision target, i.e., its relative speed to the radar v_{rel} is only a radial velocity [3]. In other words, with constant velocity assumptions, its direction of arrival (DOA) and Doppler frequency are constant over time. Moreover, due to long integration time T_{int} and high Doppler resolution $\Delta f = 1/T_{\text{int}}$, the target's scatterers are present in $p > 2$ adjacent Doppler bins: the target is thus Doppler spread. As a consequence, at range cell r , the radar signal backscattered by the target can be written in the following compact form

$$\mathbf{x}_r = \mathbf{H}_r \mathbf{A}_r \quad (3)$$

where \mathbf{A}_r is the unknown deterministic $p \times 1$ target amplitudes vector. The $N \times p$ target steering matrix \mathbf{H}_r is supposed to be known and can be written as $\mathbf{H}_r = [\mathbf{h}_{r,1} | \mathbf{h}_{r,2} | \dots | \mathbf{h}_{r,p}]$ with

$$h_{r,k}(n) = \exp(j2\pi f_{r,k} n) = \exp\left(j \frac{4\pi v_{\text{rel},k}}{\lambda} n\right). \quad (4)$$

Note that $f_{r,k}$ denotes the adjacent Doppler frequency of a target scatterer, so that $f_{r,k+1} = f_{r,k} + \Delta f$. More precisely, \mathbf{H}_r is not range dependent for a collision target. Note that this model can account for small and unknown Doppler variations as well and is not restricted to collision target only.

2.3 Locally Gaussian Clutter

Ground clutter is assumed to be a locally Gaussian process: at range cell r , the clutter vector can be written as the product of two components, a Gaussian vector (speckle) of covariance matrix \mathbf{M}_r and a deterministic positive scale parameter (texture) τ_r [7]

$$\mathbf{c}_r = \sqrt{\tau_r} \mathbf{s}_r, \text{ with } \mathbf{s}_r \sim \mathcal{CN}(0, \mathbf{M}_r). \quad (5)$$

The multivariate distribution of the clutter vector is given conditionally to the texture by:

$$p(\mathbf{c}_r | \tau_r) = \frac{1}{(\pi \tau_r)^N |\mathbf{M}_r|} \exp\left(-\frac{\mathbf{c}_r^H \mathbf{M}_r^{-1} \mathbf{c}_r}{\tau_r}\right). \quad (6)$$

The clutter covariance matrix \mathbf{M}_r is supposed to be known a priori in this paper.

3 Target Detection in Ground Clutter

We propose to use a GLRT detector derived for known clutter covariance matrix. The unknown clutter covariance matrix is then estimated adaptively from the data thanks to a given structured clutter subspace, and inserted in the GLRT to form the proposed suboptimal detection scheme.

3.1 GLRT Derivation

We suppose in this work that thermal noise can be neglected with respect to ground clutter, and that the observations are independent from one range cell to another. Following the steps of [1], we derive the GLRT for the general model (2) and report the main results here. We form the likelihood ratio of the data under H_0 and H_1 , and we replace, for each range cell, the unknown deterministic parameters, namely the textures τ_r under hypotheses H_0 and H_1 , and the target amplitudes \mathbf{A}_r , by their maximum likelihood (ML) estimates. As the target steering matrix and clutter covariance are supposed to be known, the ML estimates for the target amplitudes are given by:

$$\hat{\mathbf{A}}_r = (\mathbf{H}_r^H \mathbf{M}_r^{-1} \mathbf{H}_r)^{-1} \mathbf{H}_r^H \mathbf{M}_r^{-1} \mathbf{z}_r. \quad (7)$$

The ML estimates of the texture under H_0 and H_1 are respectively given by

$$\hat{\tau}_{r|H_0} = \frac{\mathbf{z}_r^H \mathbf{M}_r^{-1} \mathbf{z}_r}{N}, \quad \hat{\tau}_{r|H_1} = \frac{\mathbf{z}_r^H (\mathbf{M}_r^{-1} - \mathbf{Q}_r) \mathbf{z}_r}{N} \quad (8)$$

where the matrix \mathbf{Q}_r is the projection matrix:

$$\mathbf{Q}_r = \mathbf{M}_r^{-1} \mathbf{H}_r (\mathbf{H}_r^H \mathbf{M}_r^{-1} \mathbf{H}_r)^{-1} \mathbf{H}_r \mathbf{M}_r^{-1}. \quad (9)$$

The generalized log-likelihood ratio is then given by the sum of the individual log-likelihood ratio of each range cell:

$$\ln \Lambda_G = N \sum_{r=1}^L \ln \left[\frac{\mathbf{z}_r^H \mathbf{M}_r^{-1} \mathbf{z}_r}{\mathbf{z}_r^H (\mathbf{M}_r^{-1} - \mathbf{Q}_r) \mathbf{z}_r} \right]. \quad (10)$$

In practice, the matrices \mathbf{M}_r may not be available and must be estimated to form an adaptive detector [2]. To this end, we now propose to model ground clutter returns similarly to SAR processing by a sum of P local scatterers whose quadratic Doppler phase shifts are known a priori.

3.2 A Structured Clutter Subspace

Ground clutter can be decomposed by a multitude of local scatterers whose relative velocity to the aircraft only depends on the aircraft velocity and aspect angle α :

$$v_r = v_a \cos \alpha, \quad v_\perp = v_a \sin \alpha.$$

Similarly to SAR processing [6], the radar response of a ground scatterer at the aspect angle α , range R_0 , and time instant n , consists of a complex amplitude $B_{r,p}$ and a quadratic Doppler phase shift given by:

$$\phi(n, \alpha) = \frac{-4\pi v_a \cos(\alpha)}{\lambda} n + \frac{2\pi v_a \sin^2(\alpha)}{\lambda R_0} n^2 \quad (11)$$

with λ the radar wavelength, and $f_D(\alpha) = \frac{-2v_a \cos(\alpha)}{\lambda}$ and $B_D(\alpha) = \frac{2v_a \sin^2(\alpha)}{\lambda R_0} T_{\text{int}}$ are respectively the ground scatterer initial Doppler frequency and Doppler bandwidth. Equivalently, one can say that each ground scatterer migrates linearly in Doppler with a known bandwidth depending on the aircraft speed v_a and its initial Doppler frequency. Moreover, we suppose that the minimum clutter Doppler bandwidth is greater than the target Doppler spread. One can then represent the clutter signal with the following compact form

$$\mathbf{c}_r = \mathbf{S}_r \mathbf{B}_r \quad (12)$$

where \mathbf{S}_r is an $N \times P$ matrix containing the exponential of the quadratic Doppler phase $\phi(n, \alpha)$ at range cell r and \mathbf{B}_r is a $P \times 1$ vector containing the complex amplitudes of the clutter sources. \mathbf{B}_r can be modeled as a zero-mean Gaussian vector and the disturbance covariance matrix is:

$$\mathbf{C} = E[\mathbf{c}_r \mathbf{c}_r^H] = \mathbf{S}_r \mathbf{\Lambda}_r \mathbf{S}_r^H + \sigma^2 \mathbf{I}_N. \quad (13)$$

where $\mathbf{\Lambda}_r = E[\mathbf{B}_r \mathbf{B}_r^H]$ is a diagonal matrix of clutter powers, and $\sigma^2 \mathbf{I}_N$ accounts for the thermal noise. The inverse covariance matrix is written using the classic inversion lemma

$$\mathbf{C}^{-1} = \frac{\mathbf{I}_N - \mathbf{S}_r (\sigma^2 \mathbf{\Lambda}_r^{-1} + \mathbf{S}_r^H \mathbf{S}_r)^{-1} \mathbf{S}_r^H}{\sigma^2}. \quad (14)$$

We assumed that the clutter is dominant against thermal noise, so that the inverse covariance matrix can be approximated as follows

$$\mathbf{C}^{-1} \approx \frac{\mathbf{I}_N - \mathbf{S}_r (\mathbf{S}_r^H \mathbf{S}_r)^{-1} \mathbf{S}_r^H}{\sigma^2} = \frac{1}{\sigma^2} \mathbf{P}_{\mathbf{S}_r}^\perp \quad (15)$$

where $\mathbf{P}_{\mathbf{S}_r}^\perp$ is the matrix projection onto the subspace orthogonal to the columns of \mathbf{S}_r . Besides, detectors defined in (10) are independent of scale parameters [5], namely σ^2 . The clutter covariance can thus be replaced by the orthogonal projection in the detector. In fact, this approximation is equivalent to calculate the GLRT for deterministic and known interferences:

$$\ln \Lambda_D = N \sum_{r=1}^L \ln \left[1 + \frac{\tilde{\mathbf{z}}_r^H \mathbf{P}_{\tilde{\mathbf{H}}_r} \tilde{\mathbf{z}}_r}{\tilde{\mathbf{z}}_r^H \mathbf{P}_{\tilde{\mathbf{H}}_r}^\perp \tilde{\mathbf{z}}_r} \right] \quad (16)$$

where $\tilde{\mathbf{z}}_r = \mathbf{P}_{\mathbf{S}_r}^\perp \mathbf{z}_r$ and $\tilde{\mathbf{H}}_r = \mathbf{P}_{\mathbf{S}_r}^\perp \mathbf{H}_r$ are the projected data and steering matrix.

3.3 Covariance Matrix Estimation

In practice, the clutter Range/Doppler region can be predicted by knowing the aircraft velocity and the radar field of view, or determined by forming the range/Doppler map of the scene [6]. For each range cell r , we form an over-sampled clutter matrix \mathbf{S}_r in Doppler frequency/aspect angle $f_D(\alpha)$ dimension, and then perform its singular value decomposition (SVD), $\mathbf{S}_r = \mathbf{U}_r \mathbf{O}_r \mathbf{V}_r^H$, to determine an orthonormal basis \mathbf{U}_r of the modeled clutter subspace. The next step is to estimate the clutter powers associated with the columns of \mathbf{U}_r from the data following [3]. More precisely, an oblique projection is used to remove the target

$$\hat{\mathbf{B}}_r = (\mathbf{U}_r^H \mathbf{P}_{\tilde{\mathbf{H}}_r}^\perp \mathbf{U}_r)^{-1} \mathbf{U}_r^H \mathbf{P}_{\tilde{\mathbf{H}}_r}^\perp \mathbf{z}_r \quad (17)$$

As in reduced rank processing [4], only the K most significant clutter amplitudes and the corresponding columns of \mathbf{U}_r are selected to form the orthogonal projection

$$\mathbf{P}_{\mathbf{S}_r}^\perp \approx \mathbf{I}_N - \tilde{\mathbf{U}}_r \tilde{\mathbf{U}}_r^H \quad (18)$$

which is finally plugged in (16) to form the proposed detector. This suboptimal procedure approximates the clutter covariance matrix without resorting to secondary data, thanks to the assumed structured clutter subspace. It can be viewed as a synthetic STAP, for which the spatial domain is replaced by the clutter orthoradial speed information via longer integration time. Note that the detector (10) can be used inserting Eq.(14-17) as well. For a single reception channel, target and clutter belong to the same subspace and cannot be separated in DOA. Only classical detectors based on power contrast between range/Doppler cells and their direct environment are available. However, these detectors show poor performance in the clutter region [9]. Thanks to long integration time, the target and ground clutter belong to two different subspaces. Thus one can estimate the clutter covariance matrix from the data and the modeled clutter subspace.

4 Performances and Simulation Results

We study in this section the performances of the proposed detector for synthetic signals. They depend on the rank p of the target subspace and on the rank K of the clutter subspace. We then compare it to the detector of (10) for known covariance matrix.

4.1 Performances

We define the detection probability of false alarm so that the log-likelihood ratio is higher than a threshold η under H_0

$$P_{\text{fa}} = Pr\{\ln \Lambda_D |_{H_0} > \eta\}. \quad (19)$$

The proposed detector (16) satisfies the Constant False Alarm Rate (CFAR) property. Its performances are well known for a single range cell [8]. Under hypothesis H_0 , the following result is obtained:

$$\frac{\tilde{\mathbf{z}}^H \mathbf{P}_{\tilde{\mathbf{H}}} \tilde{\mathbf{z}}}{\tilde{\mathbf{z}}^H \mathbf{P}_{\tilde{\mathbf{H}}}^\perp \tilde{\mathbf{z}}} \sim \frac{\chi^2(2p)}{\chi^2(2(\tilde{N} - p))} \sim \frac{p}{\tilde{N} - p} F_{2p, 2(\tilde{N} - p)} \quad (20)$$

where $F_{2p,2(\tilde{N}-p)}$ is the well-known F distribution, p is the rank of the target subspace and $\tilde{N} = N - K$ the dimension of the subspace orthogonal to clutter subspace. The detection performance depends on the separability between clutter and target subspaces [8]. Intuitively, it is a function of the ratio between the reduced-rank K and the dimension of \mathbf{U}_r . If the entire clutter subspace \mathbf{U}_r is taken into account in the projection, the target will be suppressed from the signal along with clutter. However if K is too small, clutter returns won't be reduced enough to exhibit the target signal. The choice of K is thus critical to maximize the signal-to-clutter ratio (SCR) for detection. In the general case of $L > 1$ range cells, there is no known closed-form expressions for P_{fa} and for the detection probability P_d . Therefore, we need to resort to Monte Carlo trials to determine the $(P_d\text{-SCR})$ curves for fixed P_{fa} .

4.2 Simulation Results

We consider a synthetic target distributed over $L = 4$ range cells, for a range resolution of 5m. The integration time is 2.8s, the aircraft speed is 65m/s and the observed ranges are centered around 1880m. The radar is looking forward with an aperture of 20° . Migration is compensated for the main lobe clutter. For each range cell, the target scatterers are located in ground clutter region at the same $p = 20$ adjacent Doppler frequencies. Real ground clutter data are projected onto the modeled clutter subspace (17) to obtain the clutter powers needed for the simulation. The rank K is fixed to suppress clutter sources of power greater than the average clutter power. This approach is equivalent to selecting approximately 45% of the clutter subspace columns, and seems to be a good compromise between clutter suppression and signal restoration.

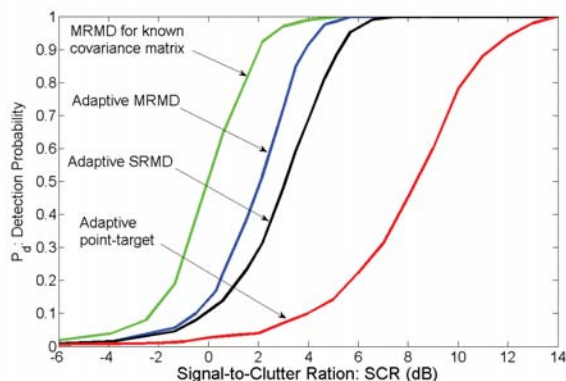


Figure 2: $(P_d\text{-SCR})$ curves for $L = 4, p = 20, P_{fa} = 10^{-3}$

We now compare the performances of the adaptive multi-range/multi-Doppler (MRMD) detector and its single-range (SRMD) version defined in (16) with the detector defined in (10) for a known covariance matrix and a cell-averaging point-target detector. Figure 2 shows the $(P_d\text{-SCR})$ curves of the 4 detectors for $P_{fa} = 10^{-3}$. Accordingly, the point-target detector shows poor performances with SCR smaller than 13dB. The adaptive MRMD detector is equivalent to

summing incoherently the $L = 4$ single-range detectors, with a gain of $1.3dB$ at $P_d = 0.9$. Note that the processing loss for adaptation is less than $2dB$.

5 Conclusion

We proposed in this paper an adaptive detector for detecting a collision target against ground clutter, using a long integration time. The detector results from a GLRT for range/Doppler spread target in locally Gaussian clutter derived for known clutter covariance. The unknown covariance matrix is then estimated and replaced in the GLRT to form the adaptive detector. To this end, the ground clutter is modeled as a known interference subspace in Doppler domain related to clutter radial and orthoradial velocities. Data are projected onto the clutter subspace. The principal components are selected to approximate the clutter covariance matrix. The proposed scheme can be viewed as a synthetic STAP, for which the space domain is replaced by clutter orthoradial information and longer integration time. Perspectives include the study of the influence of the reduced-rank K on SCR.

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