Definition of NPR, EVM, and EVM-like criteria for use in linear distortion, nonlinear distortion and noise characterization

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Testing, Modeling and Linearization of Nonlinear RF/microwave Devices and Circuits

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Outline

- Introduction and context
- Definition of EVM
- Effect of additive noise and phase noise
- Effect of linear distortion
- Effect of amplitude and phase imbalance, echoes
- EVM-like criteria
- Effects of nonlinear distortion and NPR
- System characterization and troubleshooting
- Conclusion

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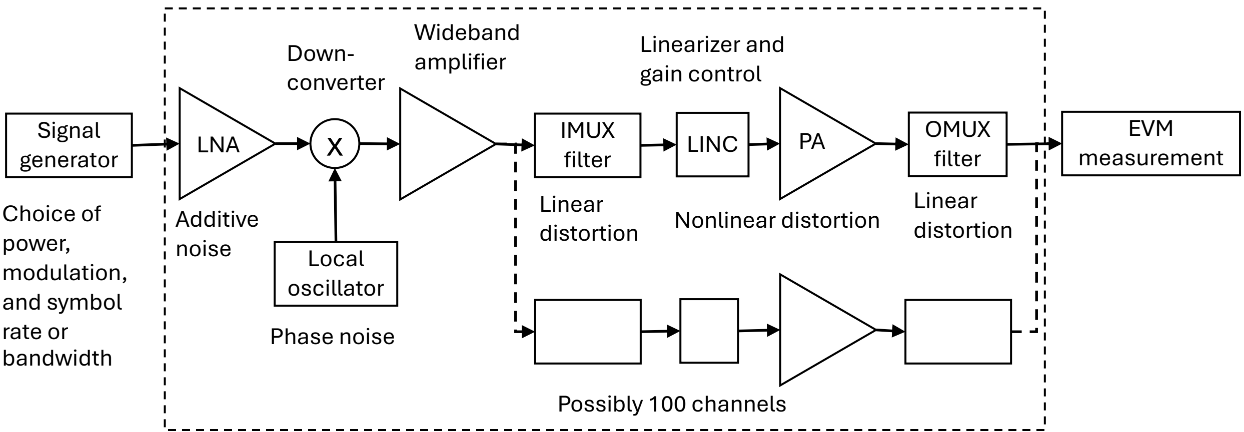
Introduction and context

Why use EVM (Error Vector magnitude) for microwave measurements?

- End-to-end system level measurement
- Used in all modern telecommunication standards to defined worst case effects of noise plus distortion on signals
- Used by customers to accept or reject devices or complete systems
- Single measurement (but for each modulation and each operating condition)
- Easy to measure with commercial equipment

- A lot of information appears on the constellation plot
- EVM-like criterion can be defined for finer measurements of devices and appreciate separately noise, linear distortion, and nonlinear distortion
- NPR (Noise Power Ratio) can be measured in the same way as EVM (on a different signal)

End-to-end measurement of satcom payload



Can this end-to-end measurement detect all problems in the satcom payload?

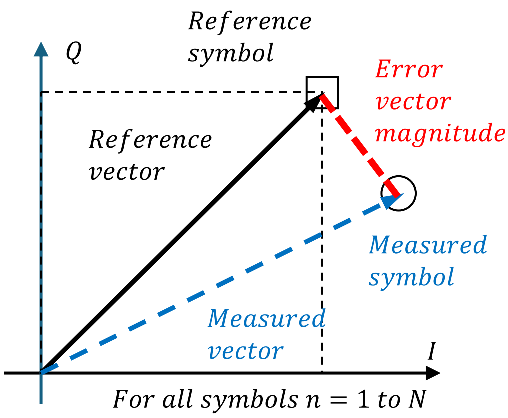
Definition of EVM, NPR, and EVM-like criteria

The initial definition of EVM is based on errors in received (demodulated) IQ symbols compared to ideal symbols

The RMS value of the magnitude of the vector error is computed on all symbols in a frame or in a stream

It is then normalized to the ideal signal magnitude

The received signal must be time synchronized, sampled, phased, and normalized for this measured RMS error to be as small as possible



A. Mashhour and A. Borjak: “A method for computing error vector magnitude in GSM EDGE systems-simulation results”, IEEE COM Letters, vol. 5, No 3, March 2001, pp. 88-91

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Synchronization, sampling, phase alignment and amplitude normalization

- Done by the receiver (either the communication user receiver or the EVM measurement equipment)
- Optimum mathematical method defined extensively by IEEE standard P1765 for calibration of measurement equipment (generators and receivers)
- This method can be used in simulations or computations
- Time synchronization and sampling are based on finding the maximum in time of the cross-correlation between ideal and received signals
- Phase alignment and amplitude normalization are based on the complex value of this cross-correlation

IEEE Std 1765™-2022 “IEEE Recommended Practice for Estimating the Uncertainty in Error Vector Magnitude of Measured Digitally Modulated Signals for Wireless Communications”

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Data-aided or non-data-aided measurements

- Data-aided EVM: the sequence of transmitted symbols is known by the receiver so that it does not need to make assumptions on the received symbols
  - In simulation or laboratory conditions this is easiest to do because the cross-correlation can be computed directly and used for synchronization, sampling, phase alignment, and normalization of he received signal
- Non-data-aided EVM: the sequence of transmitted symbols is not known by the receiver so that it assumes that the ideal symbol nearest to the received symbol has been transmitted
  - The receiver must first synchronize, sample, phase align, and normalize the received signal to get a meaningful value of EVM
  - The non-data-aided EVM is lower than the data-aided EVM because large errors (more than the distance between two ideal symbols) are not considered

Definition of EVM

- The received symbol y_n is compared to the ideal symbol x_n
- We introduce an optimum complex gain and an error
- The optimum complex gain is chosen to minimize the RMS error
- Two possible expressions
  - Used in IEEE standard P1765: $x_n = Gy_n + e_n \qquad e_n = x_n - Gy_n$
  - Used in signal processing for maximum likelihood: $y_n = \gamma x_n + \varepsilon_n \qquad \varepsilon_n = y_n - \gamma x_n$
- In both cases the mean square error is computed on all N symbols:

$$e^2 = \frac{1}{N}\sum_{n=1}^{N}|e_n|^2 = \frac{1}{N}\sum_{n=1}^{N}|x_n - Gy_n|^2$$

$$\varepsilon^2 = \frac{1}{N}\sum_{n=1}^{N}|\varepsilon_n|^2 = \frac{1}{N}\sum_{n=1}^{N}|y_n - \gamma x_n|^2$$

Definition of EVM

- The mean square error is minimum when the N-dimensional vector of errors has zero cross-correlation (at $t = 0$) with the N-dimensional vector of symbols that is multiplied by the gain:

$$\sum_{n=1}^{N} e_n y_n^* = 0 \qquad or \qquad \sum_{n=1}^{N} \varepsilon_n x_n^* = 0$$

- The error vector is uncorrelated with the ideal signal, but it is not independent (because distortions are dependent on the signal) and it is not gaussian (because an uncorrelated gaussian noise would be independent)
- This is obtained when the complex gain is maximum as:

$$G = \frac{\sum_{n=1}^{N} y_n x_n^*}{\sum_{n=1}^{N} y_n y_n^*} \qquad or \qquad \gamma = \frac{\sum_{n=1}^{N} y_n x_n^*}{\sum_{n=1}^{N} x_n x_n^*}$$

- So that:

$$e^2 = \frac{1}{N}\sum_{n=1}^{N}|e_n|^2 = \frac{1}{N}\sum_{n=1}^{N}|x_n|^2 - \frac{|G|^2}{N}\sum_{n=1}^{N}|y_n|^2 \quad or \quad \varepsilon^2 = \frac{1}{N}\sum_{n=1}^{N}|\varepsilon_n|^2 = \frac{1}{N}\sum_{n=1}^{N}|y_n|^2 - \frac{|\gamma|^2}{N}\sum_{n=1}^{N}|x_n|^2$$

Definition of EVM

- The error power is then normalized to the signal power and the square root gives the EVM
- EVM used in IEEE standard P1765:

$$EVM_{P1765} = \sqrt{\frac{e^2}{\frac{1}{N}\sum_{n=1}^{N}|x_n|^2}} = \sqrt{\frac{\sum_{n=1}^{N}|x_n - Gy_n|^2}{\sum_{n=1}^{N}|x_n|^2}} = \sqrt{\frac{\sum_{n=1}^{N}|x_n|^2 - |G|^2\sum_{n=1}^{N}|y_n|^2}{\sum_{n=1}^{N}|x_n|^2}}$$

- EVM used in signal processing for maximum likelihood:

$$EVM_{ML} = \sqrt{\frac{\varepsilon^2}{\frac{1}{N}\sum_{n=1}^{N}|\gamma x_n|^2}} = \sqrt{\frac{\sum_{n=1}^{N}|y_n - \gamma x_n|^2}{\sum_{n=1}^{N}|\gamma x_n|^2}} = \sqrt{\frac{\sum_{n=1}^{N}|y_n|^2 - |\gamma|^2\sum_{n=1}^{N}|x_n|^2}{|\gamma|^2\sum_{n=1}^{N}|x_n|^2}}$$

Definition of EVM

- Using the value of the optimum gains G or γ, we have:

$$EVM_{P1765} = \sqrt{\frac{\sum_{n=1}^{N}|x_n|^2 - \left|\frac{\sum_{n=1}^{N} y_n x_n^*}{\sum_{n=1}^{N} y_n y_n^*}\right|^2 \sum_{n=1}^{N}|y_n|^2}{\sum_{n=1}^{N}|x_n|^2}} = \sqrt{1 - \frac{|\sum_{n=1}^{N} y_n x_n^*|^2}{\sum_{n=1}^{N}|x_n|^2 \sum_{n=1}^{N}|y_n|^2}}$$

$$EVM_{ML} = \sqrt{\frac{\sum_{n=1}^{N}|y_n|^2 - \left|\frac{\sum_{n=1}^{N} y_n x_n^*}{\sum_{n=1}^{N} x_n x_n^*}\right|^2 \sum_{n=1}^{N}|x_n|^2}{\left|\frac{\sum_{n=1}^{N} y_n x_n^*}{\sum_{n=1}^{N} x_n x_n^*}\right|^2 \sum_{n=1}^{N}|x_n|^2}} = \sqrt{\frac{\sum_{n=1}^{N}|y_n|^2 \sum_{n=1}^{N}|x_n|^2}{|\sum_{n=1}^{N} y_n x_n^*|^2} - 1}$$

Definition of EVM

- In both definition, we recognize the square of the normalized cross-correlation or Pearson coefficient, which is the cosine of the angle θ between complex N-dimensional signal vectors $x = [x_1 \cdots x_N]$ and $y = [y_1 \cdots y]$:

$$\cos^2\theta = \frac{\left|\sum_{n=1}^{N} y_n x_n^*\right|^2}{\sum_{n=1}^{N}|x_n|^2 \sum_{n=1}^{N}|y_n|^2}$$

- This angle is between 0 and $\pi/2$ so that:

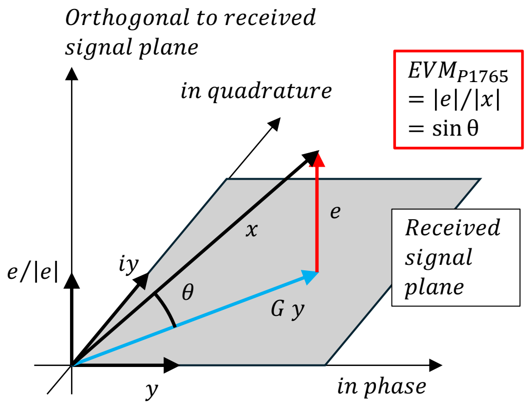
$$EVM_{P1765} = \sqrt{1-\cos^2\theta} = \sin\theta$$

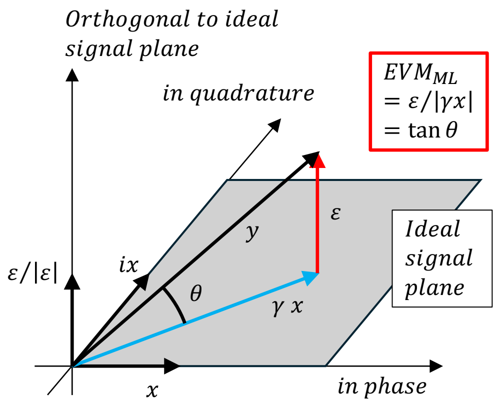
$$EVM_{ML} = \sqrt{1/\cos^2\theta - 1} = \tan\theta$$

- Main difference EVM_{P1765} is limited to 100% while EVM_{ML} can be infinite
- Maximum difference for EVM=17.5% $\quad EVM_{ML} = 0.175 \equiv EVM_{P1765} = 0.172$

Geometrical interpretation

$$\cos^2\theta = \frac{\left|\sum_{n=1}^{N} y_n x_n^*\right|^2}{\sum_{n=1}^{N}|x_n|^2 \sum_{n=1}^{N}|y_n|^2}$$





Orthogonal projections in complex N-dimensional signal vector space

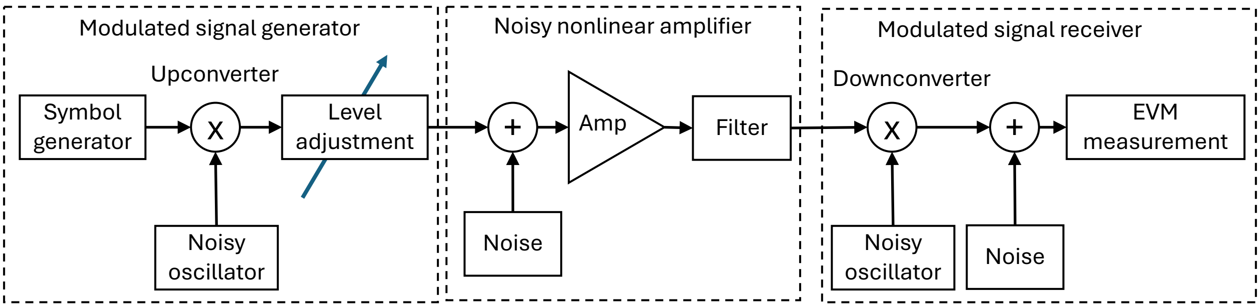
Synchronization and symbol sampling

- To get the minimum value for the error between the ideal and received signals at time 0, we must maximize the normalized cross-correlation at time 0
- We shift in time the signals so that the maximum of the cross-correlation is at time 0
- Rough synchronization with an integer sampling duration shift
- A synchronization much better than 1% of a symbol duration is needed for correct evaluation of EVM
- Generally, a time vernier must be applied as the signal may be sampled at as low as 2 samples per symbol
- This is done in P1765 by applying an optimized phase slope to the Fourier transform of one signal
- Linear distortion (filters, echoes, …) may distort the cross-correlation (smooth, no sharp peak at time 0, many small peaks, …)
- It may be necessary to equalize the received signal
- After synchronization, the signal is sampled at times multiples of the symbol duration to give the sequence of received symbols
- This is performed automatically in a receiver by the carrier and symbol rate phase lock loops

Equalized or non-equalized measurements

- Measurement equipment propose to equalize or not the signal before measuring the EVM
  - Equalized EVM is lower than non-equalized EVM
  - Standards ask for an equalized measurement of EVM if the receiver will include an equalizer
  - Used in modern wideband communication standards if echoes and bandwidth impairments degrade so much the frequency response that the receiver must equalize the signal to function correctly
  - Not to be used if the receiver is not able to equalize the signal
- This is a linear equalization only
- Based on removing some echoes (Rake receiver) or on frequency response equalization for OFDM signals

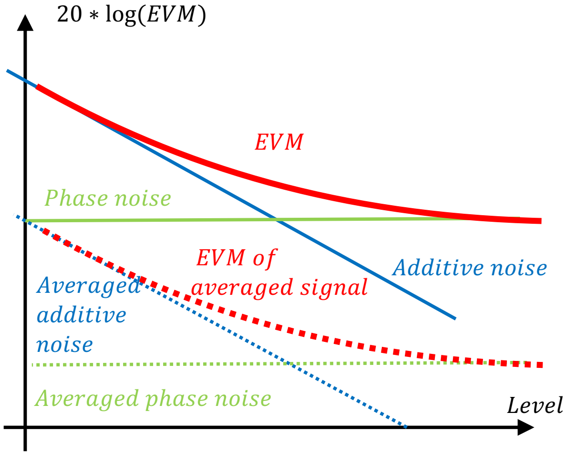
Simple EVM test setup for a nonlinear amplifier



The filter represents all bandwidth limitations and reflections in the DUT and the test setup

Effect of additive noise and phase noise

- Signal level is modified before addition of noise
- Phase noise effect on EVM is independent of the signal level
- Additive noise effect on EVM versus signal level decreases with a slope of 1 dB/dB
- Additive noise and phase noise effect on EVM can be decreased by measuring the EVM on the average of many identical signals (because noises are independent from the signal)
- 100 signals -> 10 dB better
- 10 000 signals -> 20 dB better



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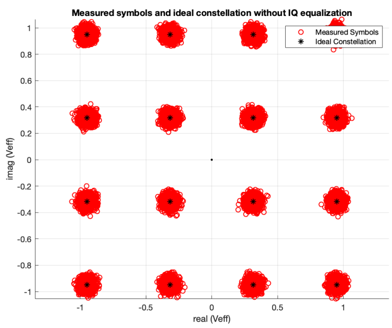
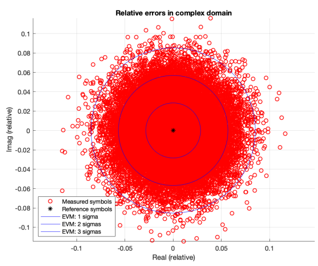
Effect of additive noise on the constellation

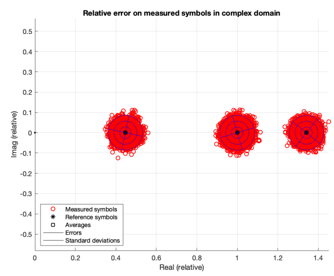




Received symbol – ideal symbol

= translated to the origin

$$y - x$$



Received symbols rotated to the real axis

$$y \cdot x^* / |x|$$

- 16 QAM modulation

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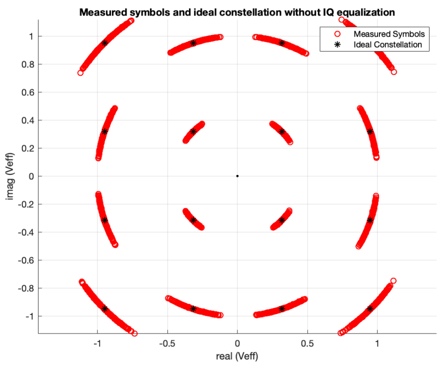
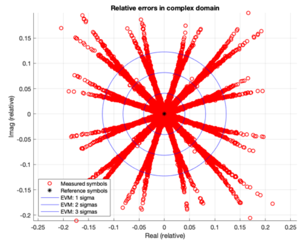
Effect of phase noise on the constellation





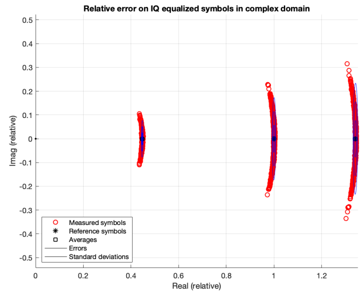
Received symbol – ideal symbol

= Translated to the origin

$$y - x$$

Received symbols rotated to the real axis

$$y \cdot x^*/|x|$$



- 16 QAM modulation

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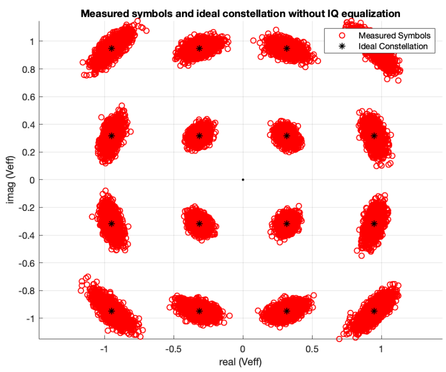
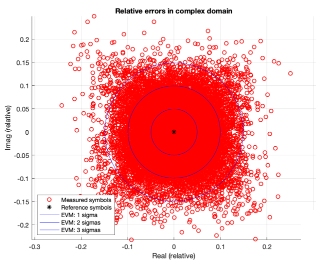
Effect of additive and phase noise





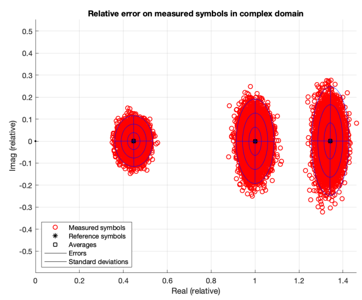
Received symbol – ideal symbol

= translated to the origin

$$y - x$$

Received symbols rotated to the real axis

$$y \cdot x^*/|x|$$



- 16 QAM modulation

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Discrimination between Noise and Distortion in EVM Measurements (1/2)

- Averaging: making 100 or 10 000 measurements for 10 dB or 20 dB gain in signal-to-noise ratio
- Computation of random noise effect (R) and distortion effects(D) from a small number N of measurements (2 to 16)
- Averaged EVM on N measurements

$$\overline{EVM}^2 = \frac{1}{N}\sum_{n=1}^{N} EVM_n^2 = D^2 + R^2$$

- EVM measured on averaged signal $S_{averaged} = \frac{1}{N}\sum_{n=1}^{N} S_n$

$$EVM_{averaged\ signal}^2 = D^2 + R^2/N$$

Discrimination between Noise and Distortion in EVM Measurements (2/2)

- We compute D and R as:

$$D^2 = \left(N \cdot EVM_{averaged\ signal}^2 - \overline{EVM}^2\right)/(N-1)$$

$$R^2 = N \cdot (\overline{EVM}^2 - EVM_{averaged\ signal}^2)/(N-1)$$

- Real and positive values are obtained for D^2 and R^2 if and only if:

$$EVM_{averaged\ signal}^2 \leq \overline{EVM}^2 \leq N \cdot EVM_{averaged\ signal}^2$$

J. Sombrin et al, "Discrimination between Noise and Distortion in EVM Measurements", 102nd ARFTG Conference, Radio & Wireless Week RWW-2024, San Antonio, Texas

Effect of amplitude and phase imbalance

- IQ modulators are used to transfer symbols onto a carrier
- Gain difference between both ways = amplitude imbalance
- Phase difference between both way not equal to 90° = phase imbalance
- Results in distorted constellation
- Can be represented mathematically by the addition of part of the conjugate of the ideal symbols:

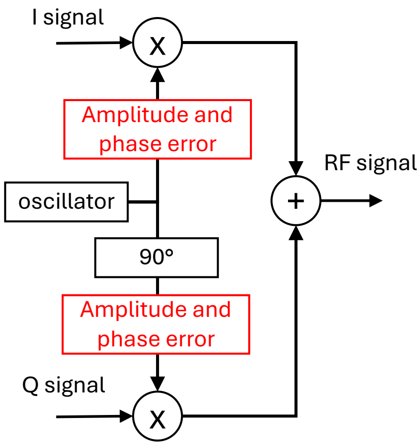
$$Symbol_{distorted} = Symbol_{ideal} + \alpha\, Symbol^{*}_{ideal}$$

- If the ideal constellation is balanced (same average power on I and Q signals), then the cross-correlation of ideal symbols and their conjugate is 0

$$\sum_{n=1}^{N} Symbol_{ideal} \cdot Symbol_{ideal} = 0$$

- Then the EVM can be obtained from the cross-correlation of the received symbols with the conjugate of the ideal symbols

$$EVM = |\alpha|$$



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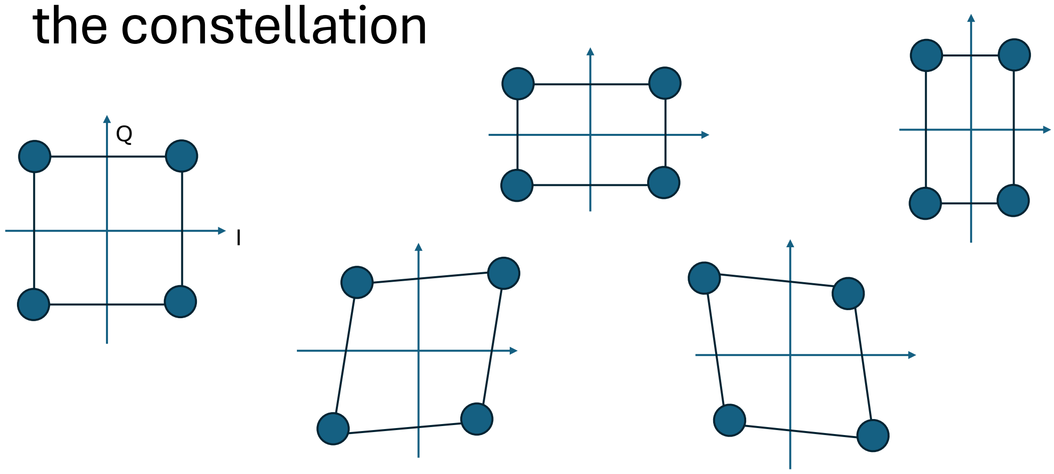
Effect of amplitude and phase imbalance on the constellation



Different constellations obtained with the same magnitude of coefficient α and different phases giving the same $EVM = |\alpha|$

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Correction of amplitude and phase imbalance

- In the receiver, it is possible to correct for amplitude and phase imbalance by adding to the received symbols their conjugate multiplied by a small complex coefficient
- This coefficient can be obtained from the cross-correlation of the received sequence of symbols with
  - The conjugate of the ideal sequence of symbols if data-aided
  - The conjugate of the received sequence of symbols if non-data-aided. This result in a received sequence of symbols having a cross-correlation with its complex conjugate = 0
- Direct computation of an EVM-like criterion
- No lengthy optimization needed for correction

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Effect of echoes

- Echoes or multipath are a kind of linear distortion
- They are present in all mobile communications
- Modern mobile receivers equalize these echoes
- The effect on a modulated single carrier is easy to detect if there is a single main echo
- The cross-correlation of received signal and ideal signal has multiple spurious peaks in addition to the main peak at time 0
- By shifting the signals in time, it is possible to compute directly the amplitude and phase of each peak and compute the EVM due to each echo

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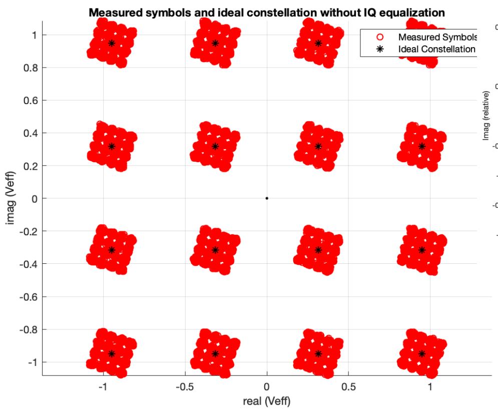
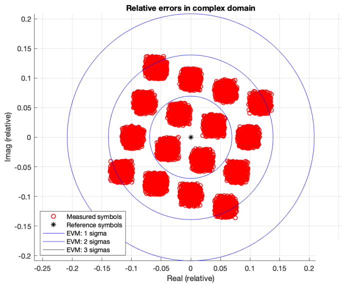
Effect of echoes on a modulated carrier

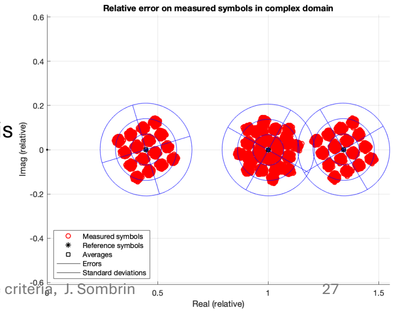




Received symbol – ideal symbol

= translated to the origin

$$y - x$$

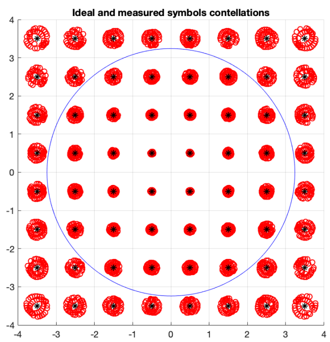


Received symbols rotated to the real axis

$$y \cdot x^* / |x|$$

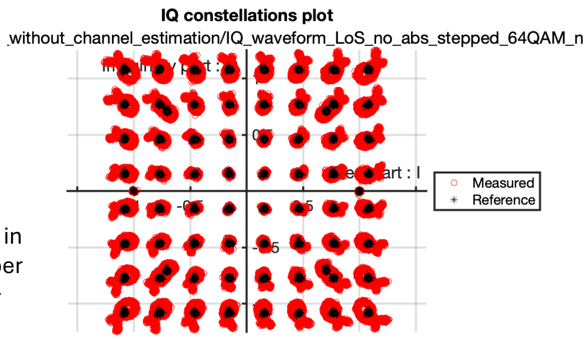
16 QAM modulation

Multicarrier case (5G OFDM 64 QAM signal)



← Simulation with one echo for P1765

Measurement in reverb chamber at ANTENEX→



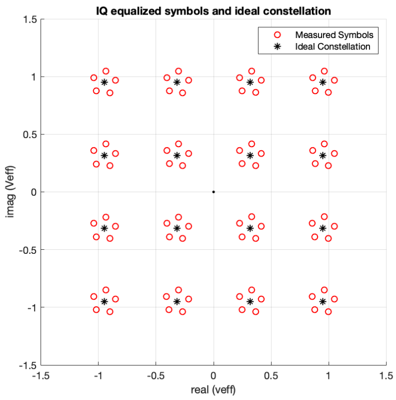
- Non-linear distortions identical on all symbols, do not depend on the symbol amplitude
- Echoes and linear distortions effect higher on higher amplitude symbols

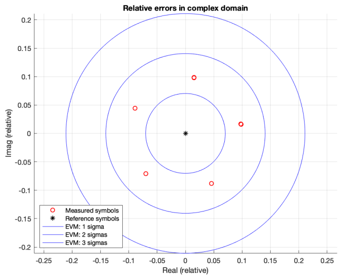
Effect of iterference

Received symbol – ideal symbol

= translated to the origin

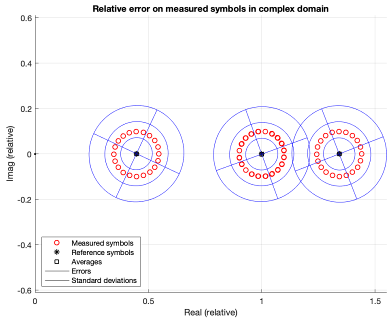
$y - x$





Received symbols rotated to the real axis

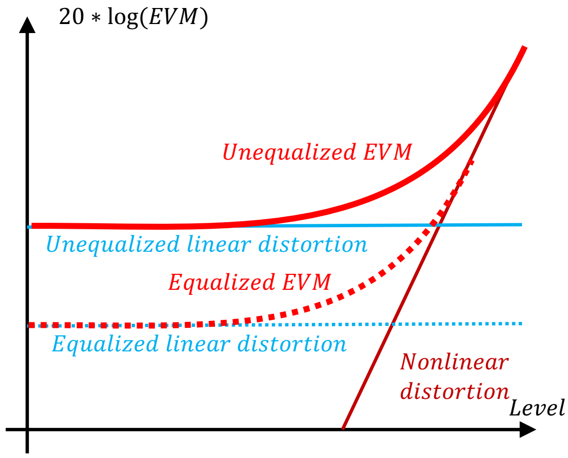
$y \cdot x^*/|x|$



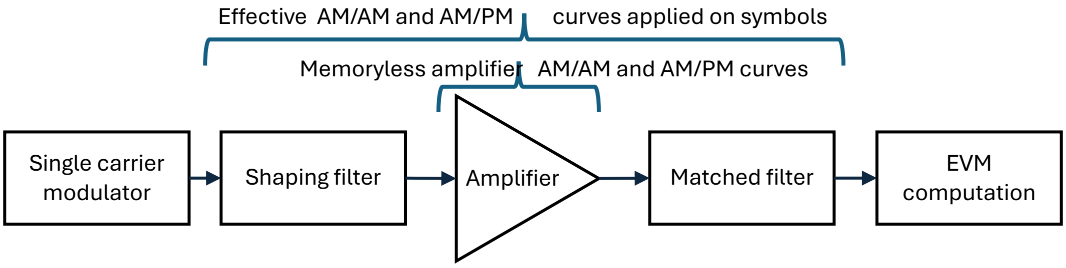
- 16 QAM modulation with CW interference at 1/5 the symbol rate

Effect of linear and nonlinear distortion

- Signal level is modified before nonlinearity
- Linear distortion effect on EVM is independent of the signal level
- Non-linear distortion effect on EVM versus signal level increases with a slope of 2 dB/dB
- Linear distortion can be mostly removed (10 to 20 dB) by equalization
- Distortion cannot be decreased by averaging (because they are deterministic / dependent on the signal)



Nonlinear amplifier effects

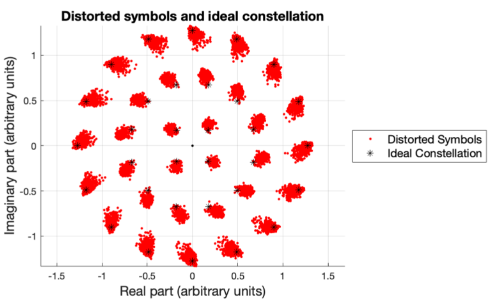


- We consider the effect of AM/AM and AM/PM nonlinear amplifier curves on a noisy signal
- EVM is 0 if there is no amplifier or if it is linear and noiseless
- Amplifier causes EVM through distortion due to AM/AM and AM/PM curves and because the matched filter is no longer optimum to remove inter-symbol interference (ISI)

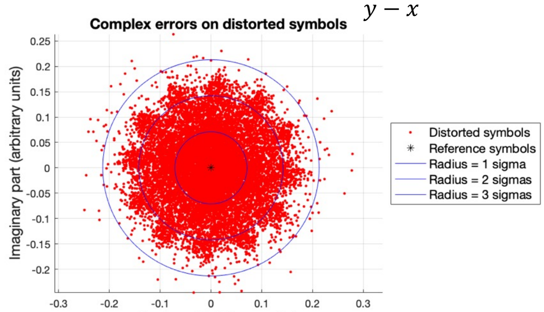
J. Sombrin, "Effective AM/AM and AM/PM curves derived from EVM simulations or measurements on constellations", 99th ARFTG Conference,2022, Denver, Colorado

Noisy and distorted constellation

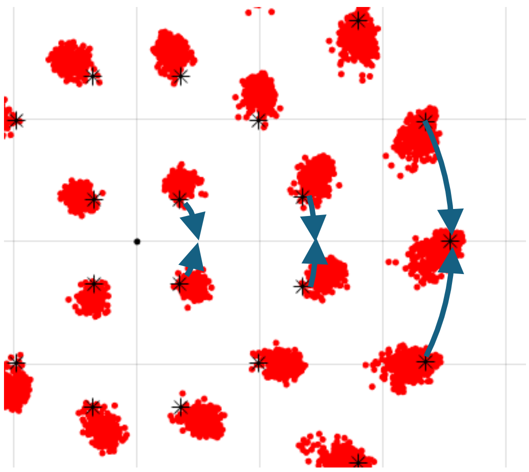
- 32 amplitude and phase modulation
- Error $y - x$ seems to have rotational symmetry



Received symbol – ideal symbol
= translated to the origin
$y - x$



Folding up the constellation



- “Folding fan” like transformation
- Rotation of each ideal symbols to the positive real axis
- Rotation of distorted symbols y by the same angle $-\angle x$ as the ideal symbol x
- $x \Longrightarrow x\dfrac{x^*}{|x|} = |x|$
- $y \Longrightarrow y\dfrac{x^*}{|x|}$

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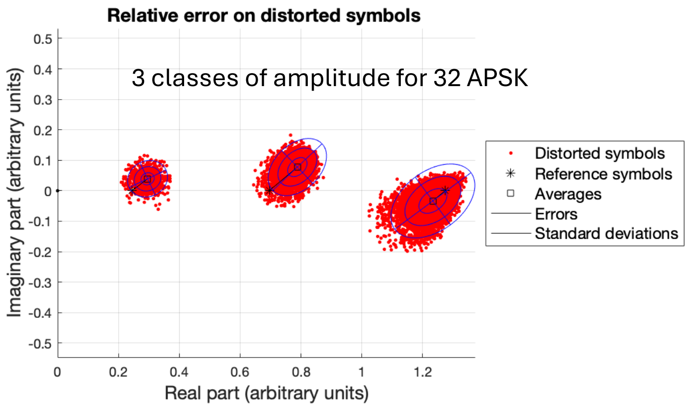
Computation of EVM for each class



- For each class of ideal symbols amplitude, computation of the average value and EVM
- Root mean square is different on x and y axes
- Correlation of x and y coordinates of points
- Gives a tilted ellipsis around each average value
- Noise is not always Gaussian around average values

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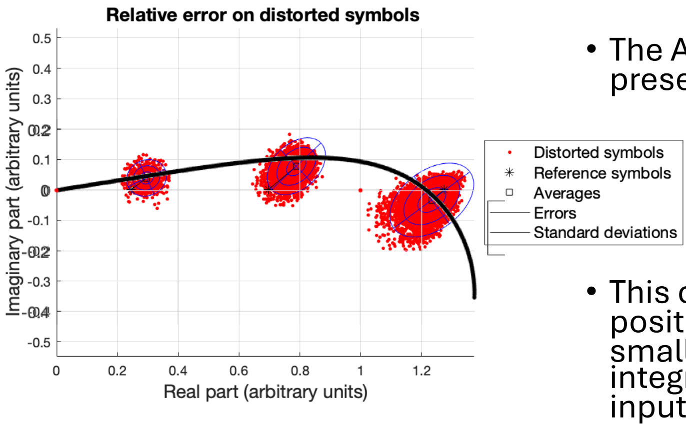
Polar plot of combined AM/AM and AM/PM curve and folded up relative errors



- The AM/AM and AM/PM curves are presented in a polar plot
- This curve fits the average positions of the symbols with a small error due to some integration because of a spread of input amplitudes

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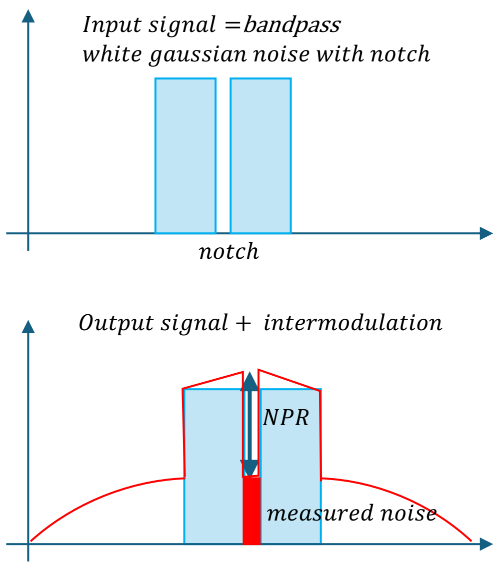
NPR classical definition

- NPR error is measured in a notch at the center of a bandpass limited white gaussian noise
- This means that the cross-correlation between the measured noise (in the notch where there is no ideal signal) and the ideal signal is 0 (product of spectra is 0)
- No synchronization is needed in that case because the cross-correlation is 0 for all time shifts
- By this way, the NPR can be measured only in the notch
- Worst case at the center of bandwidth
- Multiple notches can be used
- The total bandwidth of all notches must be less than 5% otherwise it degrades the measurement



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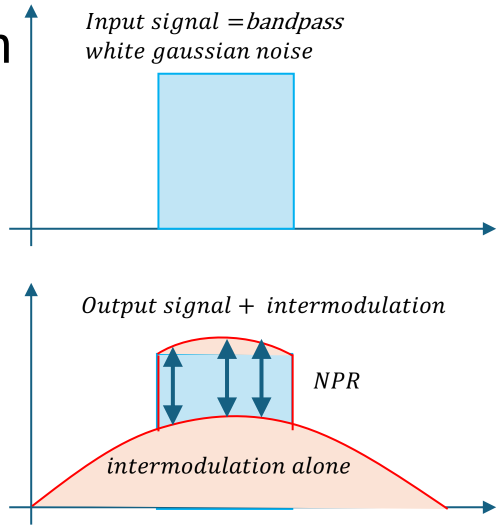
NPR as an EVM-like criterion

- A measurement of NPR anywhere in the bandwidth can be based on cross-correlation like it is in EVM measurement
- Synchronization is necessary
- Equalization can be applied
- Applicable to signals other than a bandpass white gaussian noise:

  OFDM, modulated signals, ...
- NPR versus frequency in the bandwidth
- Average NPR in the bandwidth
- NPR is the worst case of EVM for all types of modulations
- Parseval theorem shows that cross-correlation and power can be computed either in time domain or in frequency domain



Chen, Panton, Gilmore, “Effects of Nonlinear Distortion on CDMA Communication Systems”, IEEE Trans on MTT, Vol. 44, No. 12, December 1996

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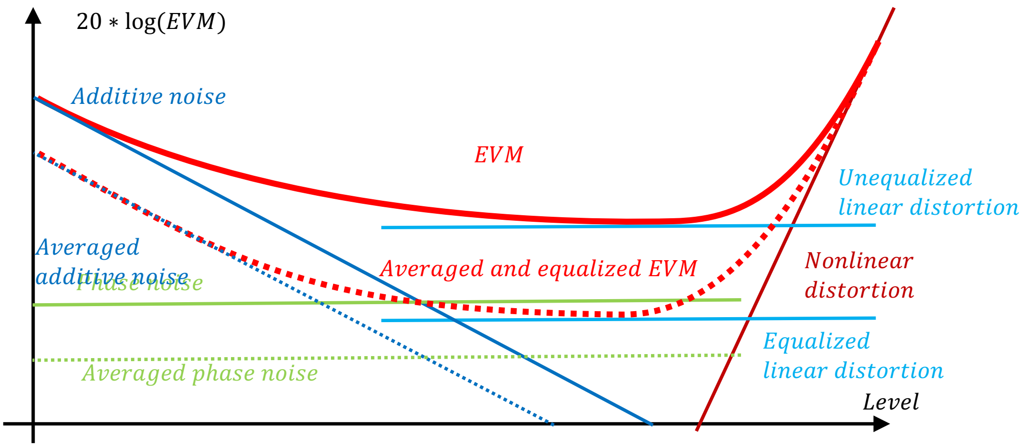
Combination of effects versus signal level



Minimum of measured EVM for a certain signal level

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System characterization and troubleshooting

- A given value of EVM can be obtained through a lot of different impairments in a system
- By itself, it does not prove that the system is nominal
- A trouble shooting process must be defined and followed
- First compute the expected EVM for the system from measurements of each equipment and simulation of the system
- Remove noise system measurements by averaging and evaluate this noise
- Remove linear distortion by equalization and evaluate it
- Extract and evaluate AM/AM and AM/PM effects
- Simulate possible combinations of effects in different devices of the payload if the results of each evaluation is not the expected one

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Conclusion

- A single measurement of EVM will not give access to an evaluation of all possible impairments
- A system simulation predicting the system EVM from equipment performance followed by a measurement in the same conditions may detect changes during assembly and tests thermal vacuum, vibration, …)
- Change in EVM indicate change in the payload performance
- Additional EVM-like measurements may give access to an evaluation of individual impairments and may be sufficient for acceptance in a series of payloads
- EVM gives access to in-band operation only, nothing about out-of-band interference to other systems

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Reference

- IEEE Std 1765™-2022 "IEEE Recommended Practice for Estimating the Uncertainty in Error Vector Magnitude of Measured Digitally Modulated Signals for Wireless Communications"
- M. S. Heutmaker et al, "The error vector and power amplifier distortion", 1997 Wireless Communications Conference, pp. 100-104
- A. A. Saleh, "Frequency-Independent and Frequency-Dependent Nonlinear Models of TWT Amplifiers", IEEE Trans. on Communications, Vol. COM-29, No. 11, November 1981, pp. 1715-1720
- Chen, Panton, Gilmore, "Effects of Nonlinear Distortion on CDMA Communication Systems", IEEE Trans on MTT, Vol. 44, No. 12, December 1996
- A. Mashhour and A. Borjak: "A method for computing error vector magnitude in GSM EDGE systems-simulation results", IEEE COM Letters, vol. 5, No 3, March 2001, pp. 88-91
- J. Sombrin, "Effective AM/AM and AM/PM curves derived from EVM simulations or measurements on constellations", 99th ARFTG Conference,2022, Denver, Colorado
- J. Sombrin et al, "Discrimination between Noise and Distortion in EVM Measurements", 102nd ARFTG Conference, Radio & Wireless Week RWW-2024, San Antonio, Texas
- Sombrin, "On the Formal Identity of EVM and NPR Measurement Methods: Conditions for Identity of Error Vector Magnitude and Noise Power Ratio", European microwave Conference, Manchester, 2011, pp. 337-340
- Freiberger, Enzinger, Vogel, "The Error Power Ratio Estimates EVM for a Wide Class of Impairments: Monte Carlo Simulations", INMIC 2017