# Passive Intermodulation Products Radiated from an Antenna Reflector: Theory and Experiments

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Abstract — This paper presents some measurements made on samples of raw materials used for antenna reflectors on communication satellites. Two theoretical results have been experimented: the first one is the power dependence of the passive intermodulation products versus the power of the carriers; the second one is the direction along which intermodulation products are radiated when the incident carriers come from different

Keywords - passive intermodulation product, antenna, reflector, communication, satellite.

#### I. Introduction

Passive intermodulation products are generated by two or more high power carriers when they go through a common device (filters, waveguides, ...) or illuminate some materials. The power of such products is very low, typically 100 dB to 150 dB under the power of carriers but this low power may be enough to desensitize or jam a receiver. This may prevent the use of an antenna reflector for both transmit and receive on a satellite and mandate the use of two reflectors to the detriment of volume, mass, and cost.

In this paper two theoretical results are exposed along with the corresponding experiments validating these results.

The first theoretical result and experiment show the difference between active and passive intermodulation (IM) products. For easy computation, memoryless nonlinearities are generally modelled by polynomials or by analytical mathematical functions. These functions are continuous, have continuous derivatives of all orders and can be approximated by their Taylor series developments. In small signal conditions, the power of active products depend on carrier power elevated to an exponent equal to the order of the product, e.g. exponent 3 for order 3, exponent 5 for order 5 [1, 2]. This is not the case for passive IM where the power of IM products depends on the carrier power with an exponent that is non-integer and about the same for all orders [3]. A model based on a non-analytical power function has been proposed [4]. The experiment is carried out on different samples of materials, some for reference and some identical to space qualified raw materials used in communication satellites. Two carriers with power up to 500 watts per carrier or three carriers with power up to 155 watts per carrier can be transmitted on the test bench developed for validation.

The second theoretical result and experiment show that IM products generated on a reflector by two incident carriers coming from two different directions are radiated in directions that can be computed from the directions and frequencies of the incident carriers [5]. This may be of consequence for multihorns satellite antennas where products generated by carriers coming from two horns could be radiated to another horn.

#### II. IM POWER DEPENDENCE THEORY

Many passive IM products level dependence in dB on carrier level in dB is given by a linear relationship with a slope generally between 1 and 3 [3-4, 6-9]. This linear relationship can happen for ranges up to 30 dB, from 1 watt to 1 kW as shown in one of the oldest articles on this phenomenon [3].

No analytical function has been found to be able to reproduce correctly this behavior even with a polynomial of

The most evident function that allows to model this behavior is a power function with an exponent equal to the slope in the dB/dB graph. The following function generates odd order IM products with a slope equal to s:

$$f(x) = \alpha_s \operatorname{sign}(x) |x|^s = \alpha_s |x|^{s-1}$$
 (1)

Even order IM products can be obtained by using:

$$f(x) = \beta_s |x|^s \tag{2}$$

The derivatives of order higher than s of these functions are infinite or indefinite at origin. So, the functions have no Taylor series development at origin and cannot be correctly modelled by polynomials of low degree. Approximations can be found but they are of high degree and are valid only on a limited range.

The computation of the IM products is obtained through the Chebyshev transform [1]. This transform is a direct application of Fourier transform of the nonlinear function when the variable x is a pure cosine carrier:

$$x = a\cos(\omega t + \varphi) = a\cos\theta \tag{3}$$

The nonlinearity gives a series of harmonics:

$$f(a\cos\theta) = \frac{1}{2}f_0(a) + \sum_{m=1}^{\infty} f_m(a).\cos(m\theta)$$
 (4)  
The order *m* Chebyshev transform of function *f* is:

$$f_m(a) = \frac{1}{\pi} \int_{-\pi}^{+\pi} f[a.\cos(\theta)] \cos(m\theta) d\theta$$
 (5)

In the case of equation (1) we get only odd order harmonics and odd order transforms for m = 2p + 1:

$$f_{2p+1}(a) = 2 \alpha_s \operatorname{sign}(a) \left(\frac{|a|}{2}\right)^s \frac{\Gamma(s+1)}{\Gamma(\frac{s+3}{2} + p)\Gamma(\frac{s+1}{2} - p)}$$
(6)

This function is of the same type as the initial function and with the same exponent s. The order 1 (p = 0) Chebyshev transform gives the fundamental response and is linked to the complex gain and AM/AM and AM/PM curves:

$$f_1(a) = 2 \alpha_s \operatorname{sign}(a) \left(\frac{|a|}{2}\right)^s \frac{\Gamma(s+1)}{\Gamma\left(\frac{s+3}{2}\right)\Gamma\left(\frac{s+1}{2}\right)}$$
$$= g(a) = \beta_s \operatorname{sign}(a) |a|^s \tag{7}$$

A bandwidth limited signal with 2 equal amplitude carriers can be put into the form:

$$x = \frac{a}{2} \left[ \cos(\omega_1 t + \varphi_1) + \cos(\omega_2 t + \varphi_2) \right]$$

$$= a \cos(\frac{\omega_1 t + \omega_2 t + \varphi_1 + \varphi_2}{2}) \cos(\frac{\omega_1 t - \omega_2 t + \varphi_1 - \varphi_2}{2})$$

$$= a \cos \theta \cos \Omega$$
(8)

Then the IM products of odd orders m=2p+1 at radians frequencies  $(p+1)\omega_1-p\omega_2$  and  $(p+1)\omega_2-p\omega_1$  are obtained with the order m Chebyshev transforms of the function  $g=f_1$ :

$$g_{2p+1}(a) = 2 \beta_s \operatorname{sign}(a) \left(\frac{|a|}{2}\right)^s \frac{\Gamma(s+1)}{\Gamma(\frac{s+3}{2} + p)\Gamma(\frac{s+1}{2} - p)}$$
 (9)

All these IM products have the same slope *s* versus carrier power. In addition, the difference in level between two successive IM orders depends only on the slope and the order, as:

$$\frac{g_{2p-1}(a)}{g_{2p+1}(a)} = \frac{\Gamma(\frac{s+3}{2} + p)\Gamma(\frac{s+1}{2} - p)}{\Gamma(\frac{s+1}{2} + p)\Gamma(\frac{s+3}{2} - p)} = \frac{s+2p+1}{s-2p+1}$$
(10)

As an example, a slope s=2 in equation 10 would result in a level amplitude ratio of 7 or a level difference of 17 dB between orders 3 and 5 (p=2 in equation 10) and an amplitude ratio of 3 or a difference of 9.5 dB between orders 5 and 7 (p=3 in equation 10).

As this level difference becomes verry low for higher values of IM orders, many high order products have measurable levels and can desensitize or jam a receiver at receive frequencies far from the transmit frequencies.

A simulation using the proposed model also shows that some IM 3 levels decrease when carriers are added to a 2-carrier signal. This is not the case in small signal for the polynomial or analytical functions models and for active IM.

# III. IM POWER DEPENDENCE MEASUREMENTS

# A. Test bench description

A 2-carrier test bench has been used to measure IM products of samples of reflector materials and reference materials. A first measurement with a strongly nonlinear material (aluminum honeycomb) allowed us to verify the test bench operation.

The sample is placed at  $45^{\circ}$  with respect to the direction of the incident carriers. The IM product is radiated mainly at  $45^{\circ}$  from the sample and at  $90^{\circ}$  from the incident direction.

The test bench is shown in Fig 1.

Transmit and receive filters are used to combine the carriers and to filter them out of the measured IM. The two transmit filters are respectively in the 11.4 to 11.7 GHz and 12.5 to 12.8 GHz bands, the receive filter in the 13.5 to 14.5 GHz band. The available power is up to 500 watts (2 TWTA in parallel) for each of 2 carriers.

Frequencies  $f_1$  in filter 1 and  $f_2$  in filter 2 are chosen so that at least one IM3 or IM5 or IM7 frequency is in receive filter bandwidth and can be measured.

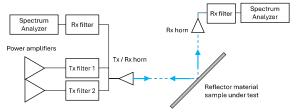


Fig. 1. Reflector material sample in 2-carrier IM test bench.

It is possible to measure the level of IM that is radiated in the direction of the receive horn and the level of the same IM that is reflected in the direction of the transmission horn.

# B. Measured samples

The following material samples were measured:

- Reference aluminum plate.
- Aluminum honeycomb, 3 mm and 7 mm mesh size.
- Plate with magnetic Nickel plating.
- Steel plate.
- Plate with a ferrite layer of < 40 microns grains.
- Plate with a ferrite layer of mixed 40 to 200 microns grains.
- Carbon fiber sheet.

#### C. Measurement results

# 1) Comparison of radiated and reflected IM

The level of the IM product that is radiated in the direction of the receive horn is 10 to 12 dB higher than the level of the IM product that is reflected to the transmit horn for IM order 3 and 7 dB for IM orders 5 and 7.

All following IM levels measurement are made with the receive horn for higher dynamic.

#### 2) Comparison of materials

All homogeneous materials (nickel plated, steel plate, ferrite layers) have given very low or non-measurable IM levels around the noise level of -160 dBm.

Honeycomb materials have given higher IM levels, but they were very difficult to reproduce because of the flexibility of the material and change of shape between measurements.

The carbon fiber sheet gave high and stable levels of IM products. It was used for all measurements of IM products of orders 3, 5 and 7.

#### 3) Measurements on the carbon fiber sheet

The measured IM levels (at 50 dBm per carrier) and IM slopes are given in table 1 for orders 3, 5 and 7.

Table 1. IM levels and slopes for orders 3, 5 and 7.

IM	Level at 50 dBm	Slope of	Slope of
order	per carrier (dBm)	radiated IM	reflected IM
3	-106.9	1.74	1.33
5	-126.3	2.09	1.87
7	-135.2	2.27	2.22

As with other published results the slope of passive IM products is different from the order of the IM [3, 6-9]. Also, the slope increases slightly with the order.

The difference in levels from orders 3 to 5 is 19.4 dB, which could be obtained with a slope of 2.2. The slope of 1.74 should give a difference of 14.6 dB.

The difference in levels between orders 5 to 7 is 8.9 dB, which correspond to a slope of 1.8. The slope of 2.1 should give a difference of 9.9 dB.

The differences in measured levels of successive IM products 3, 5 and 7 correspond to a slope around 2.2 rather than the measured values of 1.74, 2.09 and 2.2.

#### IV. IM RADIATED DIRECTION THEORY

In the previous experiment, both carriers radiate on the sample from the same horn. The IM product is mainly radiated in the same direction as the reflected carriers. This is in accordance with the geometrical (or ray) optics theory of light.

In addition, diffraction due to the dimensions of the horn and sample can be considered by the physical optics.

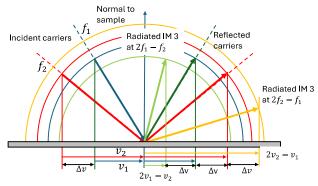
When the carriers come from two different incident directions, the carriers are reflected in two different directions and the IM products are radiated in directions that are also different from that of the carriers.

For each carrier, the length of the wavevector (the wavenumber) and the projection of the wavevector on the reflection surface are conserved. This can be simplified in the computation of incident and reflected angles.

The computation of the IM radiated directions is based on the same method.

We apply it here on the IM3 product at frequency  $2f_1 - f_2$ . The wavenumber of the IM product (the length of its wavevector) is proportional to its frequency  $2f_1 - f_2$ .

The projection of its wavevector on the sample surface is given by applying the same equation to the projections of the wavevectors:  $2v_1 - v_2$  (see Fig. 2).



Projections of wavevectors on sample surface and their differences  $\Delta v$ 

Fig.2 Reflected carriers and radiated IM 3 products for different directions and frequencies of incident carriers. The radii of the circles are proportional to the wavenumbers of the carriers and IM 3 products. The projections of the wavevectors and their differences are given under the sample surface.

The same computation can be done for all IM products. The spread between the projections of the wavevectors is analog to

the spread of the frequencies (and phases) of IM products when replacing  $\Delta f$  by  $\Delta v$ , see Fig 3.

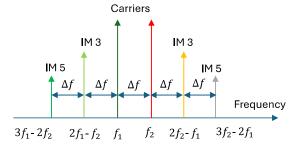


Fig.3 Frequencies of carriers and IM products.

This gives a spread of IM products with different angles between the first radiated IM products (IM3) and the reflected carriers and between successive radiated IM products.

Some IM products can even be radiated in the direction of the incident carriers (and the transmit horns).

If an IM product wavenumber is smaller than the computed projection of its wavevector on the sample, then the corresponding IM product cannot be radiated. This is analog to the total internal refraction of an incident wave when the wavenumber in the second medium is smaller than the projection of the wavevector on the boundary between the two media.

This theoretical result has been confirmed by an electromagnetic software computation of diagrams of radiated IM. The peak of each IM diagram for orders 3, 5 and 7 is at the angle value given by the ray optics computation. This incited us to measure radiated IM products in configurations where the IM product can be received in a direction different from that of the carriers and thus improve the dynamic of the test bench.

# V. IM RADIATED DIRECTION MEASUREMENT

#### A. Test bench description

To measure the direction of the radiated IM, the test bench is reconfigured: the Rx/Tx horn is replaced by two transmit horns, the first one at 45° to the sample surface (as in section III.A) and the second one at 15° from the first (and 60° from the sample surface).

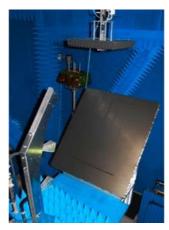
The receive horn can be moved in an arc of a circle from 45° to 90° to the sample surface, see Fig. 4. Absorbing material is used to minimize unwanted reflections and improve isolation between horns.

# B. Configurations

Two configurations have been measured:

- Two carriers: one carrier on each transmit horn (up to 500 watts per carrier).
- Three carriers: two carriers on 45° transmit horn and one carrier on 60° transmit horn (up to 155 watts per carrier).

The same material samples as in III B have been used and measurements are given for the carbon fiber sheet.



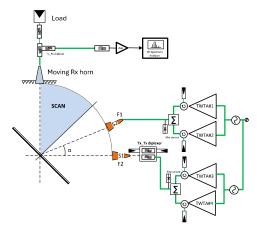


Fig.4 Picture and schematics of test bench with two transmit horns at  $45^{\circ}$  and  $60^{\circ}$  and one mobile receive horn from  $45^{\circ}$  to  $90^{\circ}$ 

#### C. Measurement results

The receive horn is moved at the position that gives the highest level for the wanted IM product. The higher measured power increases the dynamic of the test bench.

The angle where the maximum is obtained is less than 2 or 3° from the one obtained through geometrical optics computation. The maximum is flat, and some minima appears around the main lobe because of diffraction from the limited area of sample irradiated by the horns (and maybe grating lobes from 2 hot spots at around 30 cm distance).

When the position of the receive horn is different from the directions of reflected carriers, the measured carrier power is lower: this also improves the dynamic of the test bench.

The first measurement has been made with two carriers in the same frequency configuration as in the section III (single transmit horn) and gave 3.5 dB higher level for the IM 3 product. However, IM 5 and IM 7 products had lower levels (respectively 10 dB and 5 dB). The slopes are more coherent, 2.7 for IM 3 and IM 7, 3.3 for IM5).

When adding a third carrier, the level of the IM 3 product generated from the first 2 carriers decreases by 6 dB and the level of the IM 5 by around 14 dB (noisy measurement).

The level of IM 7 decreases by 3 dB (noisy) when compared to a 2-carrier measurement (both carriers transmitted from the same horn).

These level changes are also obtained through a nonlinear simulation using the non-analytic power law model with the measured slope as exponent. They are not a result form using 2 or more transmit horns.

# VI. CONCLUSION

The experiments presented in this paper validate two main theoretical results:

• The levels of IM products are correctly obtained in different configurations of carriers and power by using the non-analytical model based on a power law nonlinearity. This ensures that the 2-carrier IM test for reflector materials can be relaxed if the slope is less than 3 and if the reflector will be used for multi-carrier operations [10].

• The main directions in which IM products are radiated when the carriers are transmitted from one or more horns on a reflector are correctly obtained by using a nonlinear complement to geometrical optics (ray optics). The wavenumber of an IM product and the projection of its wavevector on the sample surface follow the same equation as its frequency (when replacing the frequencies of carriers by their wavenumbers or projection of their wavevectors).

#### ACKNOWLEDGMENT

The research presented in this paper has been financed by CNES (French Space Agency) under its R&D program R-S13/TC0007-051.

Test bench has been manufactured and measurements have been performed by TAS (Thales Alenia Space) Antenna department in Toulouse.

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