# Memristors as Non-Linear Behavioral Models for Passive Inter-Modulation Simulation

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***Abstract*—We propose to use memristors as memory non-linear circuits to build behavioral models useful in the simulation of passive inter-modulation in RF and microwave devices such as filters, antennas and in general connections.**

***Keywords*—memristor; behavioral modelling; non-linear; passive inter-modulation; filters; antenna.**

## I. Introduction

Leon Chua introduced memristors in 1971 [1] as an ideal two-terminal circuit element in complement to the already known three basic circuit elements: resistor, inductor and capacitor (RLC).

Responses of devices and systems using memristors were published in 1976 [2].

Hewlett-Packard proposed some realizations in 1998 [3]. They attracted great interest particularly as memory elements for future computers.

The properties of some metal oxides explain the physical behavior of HP memristors and could also explain some passive inter-modulation effects.

In this paper, we use the ideal memristor circuit element as a behavioral model for passive non-linearity in filters, antennas and connections.

### A. Memristor theory

The memristor permits us to express a new relationship between the flux $\varphi$ and the charge $q$. This fourth two-terminal circuit element is defined by a curve relating flux to charge. The voltage across a charge-controlled memristor is given by:

$$v(t) = M(q(t)) \cdot i(t) \quad (1)$$

$$M(q) = d\varphi(q)/dq \quad (2)$$

The value $M$ is a resistance. If its value is constant, independent of $q$, then the memristor is equivalent to a fixed resistor. If $M$ varies with $q$, it cannot be replaced by a resistor. It will then vary as a function of the integral of the current in time, from $t = -\infty$ to the present. Its resistance depends on the past of the component; this justifies the name memristor as a contraction of memory resistor.

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A memristor is a passive component if the value of $M(q)$ is non-negative: $M(q) \geq 0$.

The completion of the relations between voltage, current, charge and flux by the memristor is shown on the following graph.

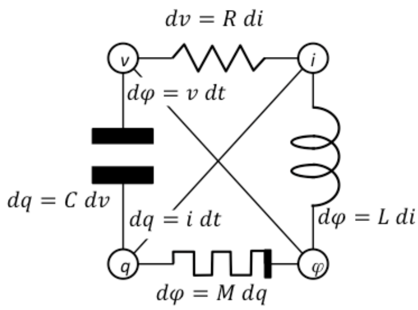


Fig. 1. Graph of relations between voltage, current, charge and flux

Curves of $M(q)$ as a function of $q$ are generally said to be non-linear curves but it is necessary only for theses curves to be non-constant for the memristor to be different from a resistor.

However $\phi(q)$ or $q(\phi)$ curves must be non-linear for their derivatives to be non-constant.

For passive memristors, their slopes must be non-negative. They can always be inverted and either curve can be used.

### B. First uses of memristors as behavioral models

In his first paper, Leon Chua proposed to use memristors as behavioral models for "amorphous Ovonic threshold switches" and for "electrolytic E-cells".

In both cases, the $\phi - q$ curves were piecewise linear curves.

This memristor behaves as a resistor of which the resistance value $M$ changes, depending on the charge that passed through the memristor in the past, see Fig. 2. Applications to signal processing were also proposed.

Leon Chua also built memristors from operational amplifiers and common RLC circuits.

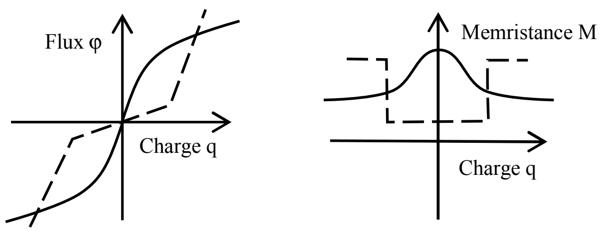


Fig. 2. Typical flux versus charge curves and corresponding memristance versus charge curves, continuous (solid curve) or piecewise linear (dash).

In a 1976 paper [2], Leon Chua and Sung Kang published $i-v$ response curves of memristors for different frequencies. They called them "pinched hysteresis" curves; see Fig. 3.

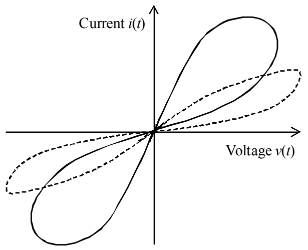


Fig. 3. $i-v$ reponse curve of a memristor for 2 frequencies

### C. *Hewlett-Packard memristors*

Stanley Williams and his team in Hewlett-Packard were working on dense computer memories. Each memory point is made of the crossing of two wires separated by a layer of Titanium dioxide $TiO_2$ and $TiO_{2-x}$. The $i-v$ "bow-tie" response of these memory points was comparable to the "pinched hysteresis" curves in Chua's article.

This led the HP team to identify the circuit as a memristor in published results in 2008 [3, 4].

## II. MODELS FOR PASSIVE NON-LINEARITY

Many models have been proposed for passive non-linear contacts or impedances in RF transmission lines, connections, filters and antennas. One of them is based on oxidized metals exhibiting non-linear semi-conductor contacts that depend on the pressure applied to the contact. Others depend on temperature or magnetic hysteresis.

The explanation of the memory and non-linearity of a contact made of $TiO_2$ by a memristor model was quite interesting as a possible cause for some passive RF inter-modulation products.

$TiO_2$ does not seem to work at microwave frequencies but other oxides have wider bandwidth. Vanadium dioxide ($VO_2$) is non-linear at RF, THz and optic frequencies [5].

Even if the physical explanation is different and will certainly depend on the materials used in the contacts, the use of a memristor as a behavioral model is interesting as it is a much simpler behavioral model than existing physical models.

Some physical models of passive non-linearity are based on semiconductors and capacitors together with RF couplings and transmission lines, see Fig. 4 [6]. Other models take into account thermal effects [7] or hysteresis [8].

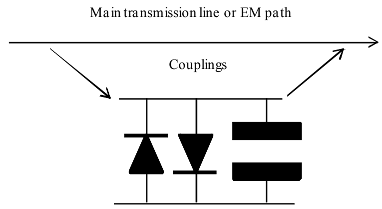


Fig. 4. Schematic of classical physical model of metal contacts

All the values of elements in the model are fitted from measurements of harmonics or inter-modulation products at the output of the device when the input consists of one or more sinusoidal carriers.

Inter-modulation products are generally very low, around 150 dB under the carrier power as the device is mainly passive and linear. Couplings of the non-linear elements to the main power path are adjusted to obtain the required output power.

The statistical combination of the output of many elements like this must be taken into account for a given contact, e.g. a flange contact, a connector or a bolt. A simpler behavioral model will permit us to reduce the simulation time for the great number of non-linear circuits that is necessary for a statistical approach.

Figure 5 shows a possible schematic of one non-linear element where the diodes and capacitor in Fig.4 have been replaced by a memristor.

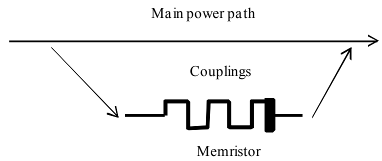


Fig. 5. Schematic of memristor model of metal contacts

Figure 6 shows the behavioral model of the memristors used for simulations. Integration can be performed either in the time domain or in the frequency domain by multiplyng the spectrum by $1/j\omega$. The integrator is mainly a low-pass filter.

Computation of the non-linear gain and multiplication by input signal is performed in time domain.

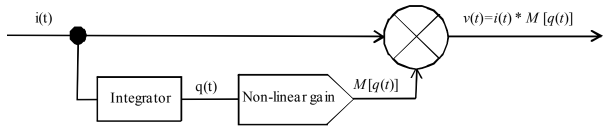


Fig. 6. Proposed behavioral model using a memristor

Replacing the integrator by a more general filter, filtering the gain and including filters in the RF path gives us a more general behavioral model. We obtain a schematic (Fig.7) similar to a typical first order dynamic Volterra model with a high frequency chain and a low frequency non-linear chain.

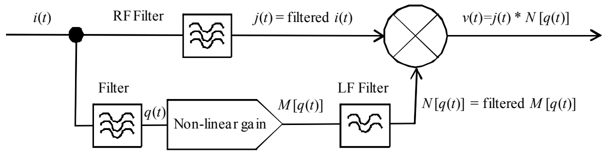


Fig. 7. Extended memristor behavioral model

Depending on the filters used, we are able to represent memristors but also thermal effects that have been proposed as models [7] or hysteresis effects measured in [8].

## III. NON-ANALYTICAL MEMRISTOR

We represent the non-linear flux versus charge in the memristor by non-analytical curves that were introduced in [9]. The memristance $M(q)$ or the gain in the behavioral model is then a function of the modulus of the input. One of the simplest models is obtained by elevating this modulus to a non-integer power. An exponent of 0.6 has been used in [9] to fit measurements given in [10]. Passive inter-modulation (PIM) products power expressed in dBm versus input power in dBm increase with a slope of 1.6 dB/dB.

The non-linear flux and memristance equations versus charge are the following ones:

$$\varphi(q) = \left(1 + \alpha \cdot |q|^{0.6}\right) \cdot q \tag{3}$$

$$M(q) = 1 + 1.6 \cdot \alpha \cdot |q|^{0.6} \tag{4}$$

The coefficient $\alpha$ is quite small (around $10^{-6}$) and its value is adjusted to obtain the measured value of PIM power, around -100 to -150 dBc (dB with respect to carrier power).

Figure 8 shows the non-linear part only of flux and memristance curves versus charge. Both are continuous and they are analytic everywhere except at origin. Here, only the second and higher derivatives of flux are discontinuous. The first derivative (memristance) is continuous. Memristance is non-negative. Its non-linear part is zero at origin only and strictly positive otherwise.

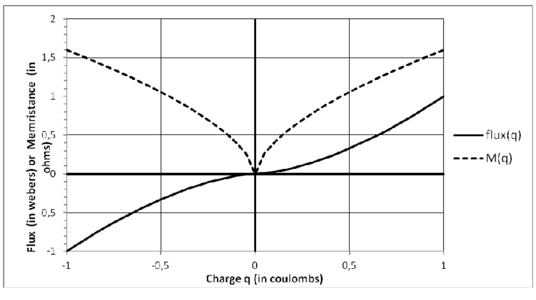


Fig. 8. Typical non-linear non-analytic part of flux versus charge curve used for passive inter-modulation model and giving a 1.6 dB/dB slope and corresponding memristance versus charge

Either the charge at time origin or the origin of current integration can be independently chosen. The equations linking current and voltage are the following:

$$q(t) = \int_{\tau_0}^{t} i(\tau)d\tau = q_0 + \int_0^t i(\tau)d\tau \tag{5}$$

$$v(t) = \left(1 + 1.6 \cdot \alpha \cdot \left| q_0 + \int_0^t i(\tau)d\tau \right|^{0.6}\right) \cdot i(t) \tag{6}$$

Using a random time origin (e.g. a random phase for the current) or a random integration constant, we get different shapes for the memristance and the voltage versus current response. This will change the non-linear response and the PIM products power. This may explain some of the high variability of PIM measurements.

The flux-charge curve is odd-symmetric with respect to origin. However it is applied to the integral of current, which is not symmetric because of the integration constant so that the model may generate odd and even products and harmonics.

## IV. RESULTS OF SIMULATION AND VALIDATION

### *A. Response to a sinusoidal input*

Figure 9 shows a cosine input current and the corresponding charge, non-analytical memristance and voltage for some values of integration constant and memristance linear and non-linear coefficients.

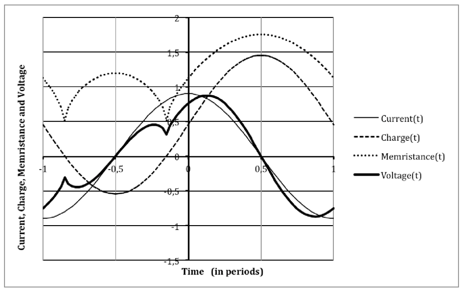


Fig. 9. Exemple of input current, charge, memristance and voltage

Figure 10 (a) shows the corresponding voltage versus current response. The bow-tie curve presents singularities at the points where the charge is 0 (the integration origin). Then, the memristance is minimum and has singular points.

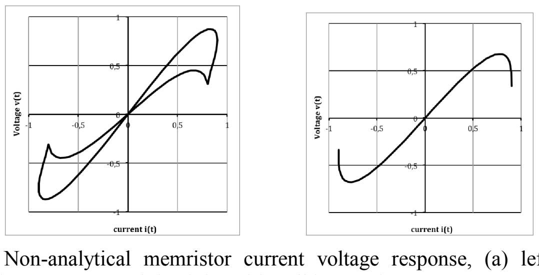


Fig. 10. Non-analytical memristor current voltage response, (a) left, with integration constant; and (b) right, with null integration constant

### *B. Response with null integration constant*

When using a null integration constant (or integrating from 0), this singular point is at time origin. Then, the memristance curve has the same symmetry as the current and both up-going and down-going branches of the curve are identical. The bow-tie is reduced to a curve that retraces its steps after cusps at extreme values of input current, see Fig. 10 (b).

A realizable low-pass filter in place of the integrator gives us the bow-tie curves of Fig. 10(a), even for null integration constant, at frequencies where the phase is different from 90°. It gives us a curve like Fig. 10(b) if the phase is exactly 90°.

### *C. Harmonics in response*

Figure 11 shows the spectrum, obtained through a Fourier transform, of the response in Fig. 9 for a given integration constant. Harmonics exhibit a global but not always monotonic decrease that is generally obtained in experiments. Even harmonics are due to the integration constant and will cancel in average when taking into account a random distribution of integration constants.

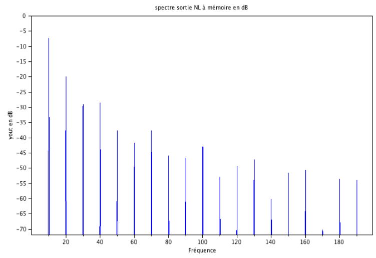


Fig. 11. Spectrum of the first 20 harmonics of voltage obtained in Fig. 9 and 10(a) for a non-analytical memristor with non-null integration constant

### *D. Inter-Modulation products*

For a 2-carrier input signal, the memristor will generate inter-modulation products around carriers and their harmonics.

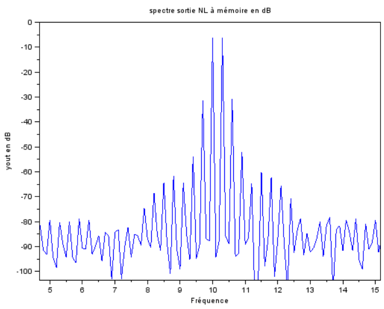


Fig. 12. Spectrum of inter-modulation generated by a non-analytical memristor model for 2 carriers at 10 and 10.3 GHz.

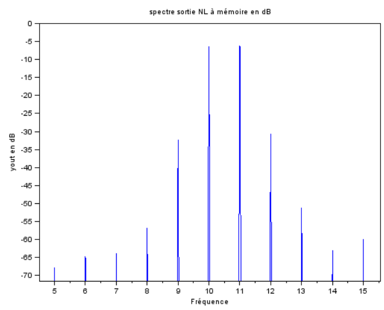


Fig. 13. Spectrum of inter-modulation generated by a non-analytical memristor model for 2 carriers at 10 and 11 GHz.

Figure 12 shows a zoom of the spectrum around the first harmonic or fundamental signal for 2 carriers at 10 and 10.3 GHz.

Because of the closeness of both frequencies, the effect of memory is small; the levels of left and right products are nearly the same.

The classical effect of memory is more important for a larger frequency bandwidth. Figure 13 shows the output spectrum for a 10% bandwidth input signal. Third order PIM levels differ by 1.5 dB and fifth order by 6 dB.

### *E. Experimental validation*

Simulations results obtained up to now are in good agreement with published experimental results. We keep the non-integer slopes in dB/dB for PIM product levels as a function of input carrier level that have been reported by authors [10] and simulated in [9].

More focused experimental validation is underway and will be presented at the conference.

## V. Conclusion

Simulation results obtained with memristor behavioral models of passive non-linearity are in good agreement with published experimental results.

In addition, a generalized behavioral model is easily applicable to other physical effects (thermal and hysteresis) and the memristor could be a physical model for oxidized contacts.

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