ESTIMATING INSTRUMENT SPECTRAL RESPONSE FUNCTIONS USING SPARSE APPROXIMATION

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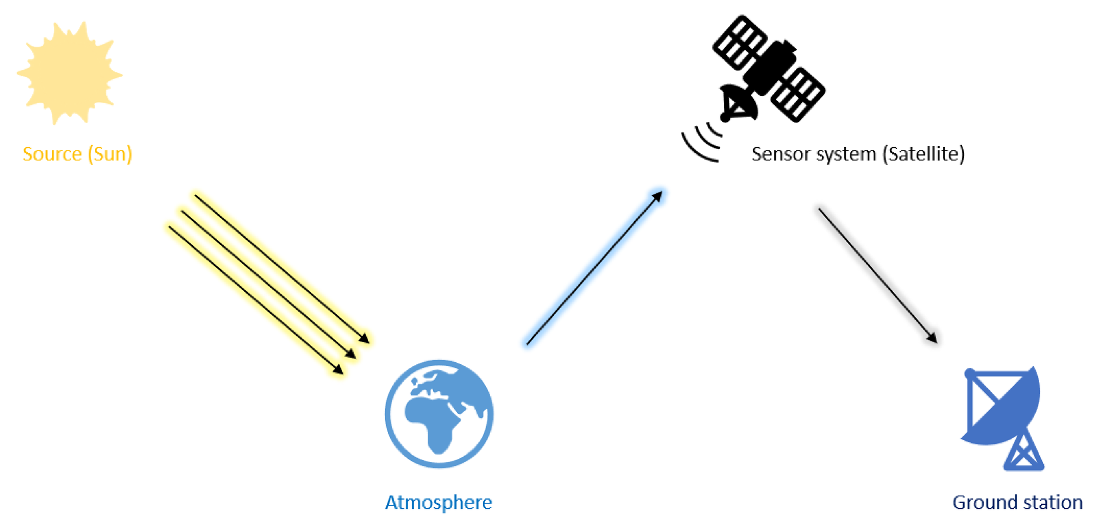
Introduction

Context



- Determination of the concentration of different gases in an atmosphere column
- Use of high-resolution spectrometers

Objectives

- Characterize the instrument model
- Estimate as accurately as possible the Instrument Spectral Response Functions (ISRFs)

ISRF estimation

Observation model

$$S_{\text{meas}}(\lambda_l) = (S_{\text{th}} * I_l)(\lambda_l) \approx \sum_{n=1}^{N} S_{\text{th}}(\lambda_n^*) I_l(\lambda_l - \lambda_n^*) \qquad (1)$$

where $I_l(\lambda_l)$, $S_{\text{meas}}(\lambda_l)$ and $S_{\text{th}}(\lambda_n^*)$ are the ISRF associated with the l-th pixel, the measured spectrum at the wavelength λ_l and the theoretical spectrum defined at the wavelengths λ_n^*.

Hypothesis: I_l is assumed to be constant on a sliding window, i.e., for L measurements associated with the wavelengths $\lambda_{l-\frac{L}{2}}$, ..., $\lambda_{l+\frac{L}{2}}$.

Parametric modeling [1]

Gaussian parameterization

$$I_l(\lambda, \boldsymbol{\beta}_{\text{G}}) = A_{\text{G}} \exp\left(-\frac{(\lambda - \lambda_l - \mu_{\text{G}})^2}{2\sigma^2}\right), \text{ where the vector } \boldsymbol{\beta}_{\text{G}} = (A_{\text{G}}, \mu_{\text{G}}, \sigma^2)^T \text{ is estimated.} \qquad (2)$$

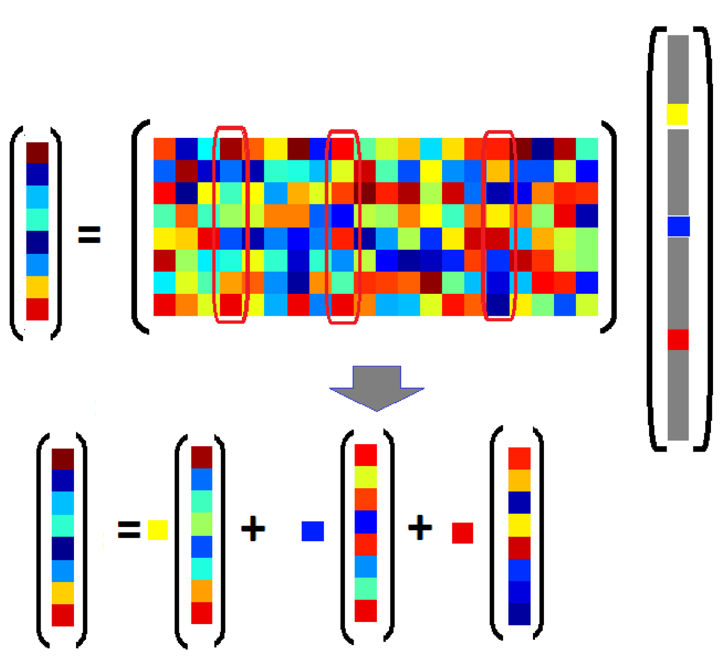
Super-Gaussian parameterization

$$I_l(\lambda, \boldsymbol{\beta}_{\text{SG}}) = A_{\text{SG}} \exp\left(-\left|\frac{\lambda - \lambda_l - \mu_{\text{SG}}}{w}\right|^k\right), \text{ where the vector } \boldsymbol{\beta}_{\text{SG}} = (A_{\text{SG}}, \mu_{\text{SG}}, w, k)^T \text{ is estimated.} \qquad (3)$$

Minimization of the residuals using the Least Square method

$$R_l(\boldsymbol{\beta}) = \sum_{l'=l-\frac{L}{2}}^{l-\frac{L}{2}} \left[S_{\text{meas}}(\lambda_{l'}) - \sum_{n=1}^{N} S_{\text{th}}(\lambda_n^*) I_l(\lambda_{l'} - \lambda_n^*; \boldsymbol{\beta})\right]^2 \qquad (4)$$

Sparse representation



Sparse approximation

- Dictionary $\boldsymbol{\Phi} \in \mathbb{R}^{N \times N_{\text{D}}}$ created using an SVD
- $\boldsymbol{I}_l$ modeled using K atoms from the dictionary $\boldsymbol{\Phi}$

$$\boldsymbol{I}_l \approx \boldsymbol{I}_l^K = \sum_{k=1}^{K} \alpha_k \boldsymbol{\Phi}_{\gamma_k} \qquad (5)$$

- Sparse approximation problem [2]

$$\arg\min_{\boldsymbol{\alpha}} L(\boldsymbol{\alpha}, \mu) = \arg\min_{\boldsymbol{\alpha}} \left\{ ||\boldsymbol{I_l} - \boldsymbol{\Phi}\boldsymbol{\alpha}||_2^2 + \mu||\boldsymbol{\alpha}||_0 \right\} \qquad (6)$$

- Use of the Orthogonal Matching Pursuit algorithm

Solving the inverse problem

$$S_{\text{meas}}(\lambda_l) \approx \sum_{n=1}^{N} S_{\text{th}}(\lambda_n^*) \sum_{k=1}^{K} \alpha_k \boldsymbol{\Phi}_{\gamma_k}(\lambda_l - \lambda_n^*) = \sum_{k=1}^{K} \alpha_k \boldsymbol{\Psi}_{\gamma_k}(\lambda_l) \qquad (7)$$

Key results

Dataset and simulation scenarios

CNES simulations for the MicroCarb mission

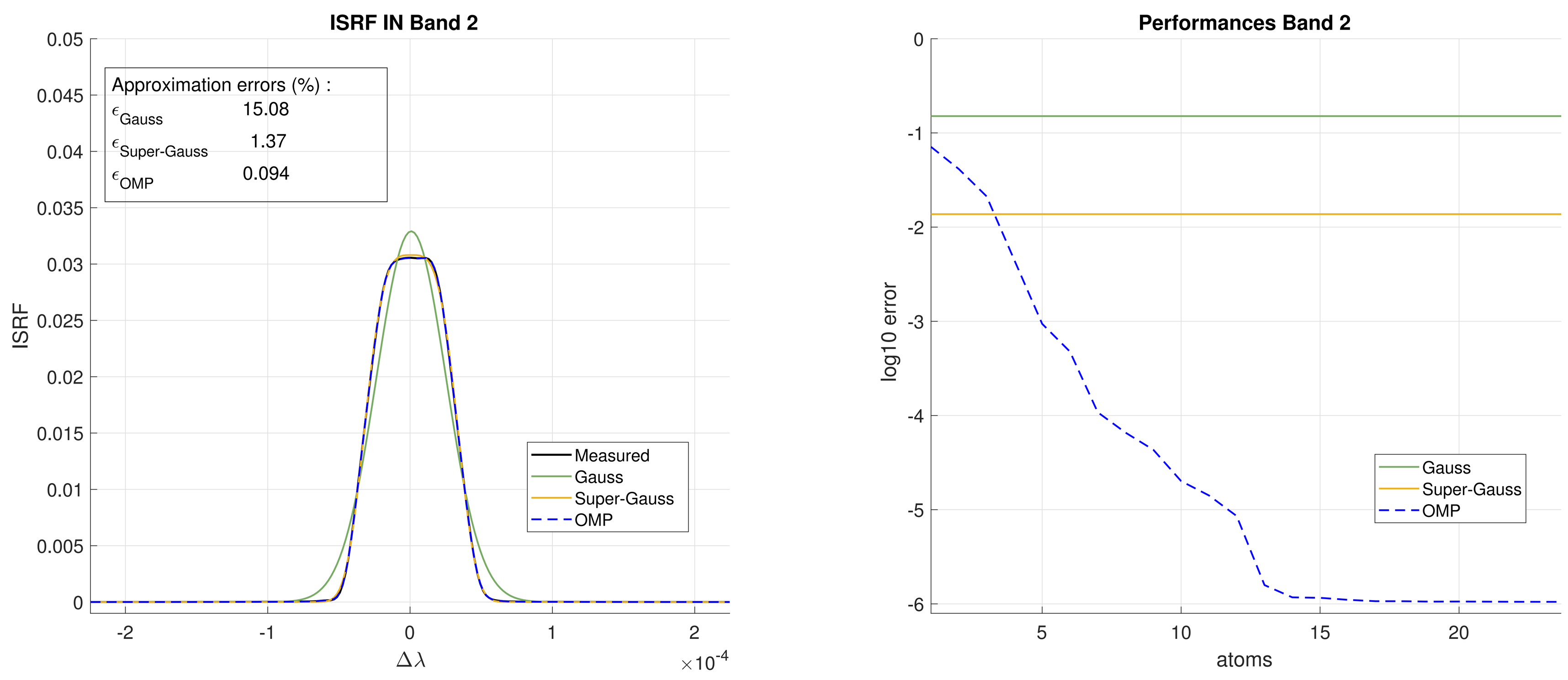
- Theoretical spectra obtained with the 4A/OP radiative transfer software
- Measured spectra computed by convolution

Estimation methods applied to band 2 associated with CO_2

- Minimization of $L(\boldsymbol{\alpha}, \mu)$ using the simplex method (MATLAB function *fminsearch*)
- Dictionary built using the SVD of a matrix of 1024 examples of ISRFs for each spectral band
- Performance evaluated using the normalized absolute error: $\epsilon_l = \frac{\sum_n^N |I(\lambda_l - \lambda_n^*) - \hat{I}(\lambda_l - \lambda_n^*)|}{\sum_n^N I(\lambda_l - \lambda_n^*)}$

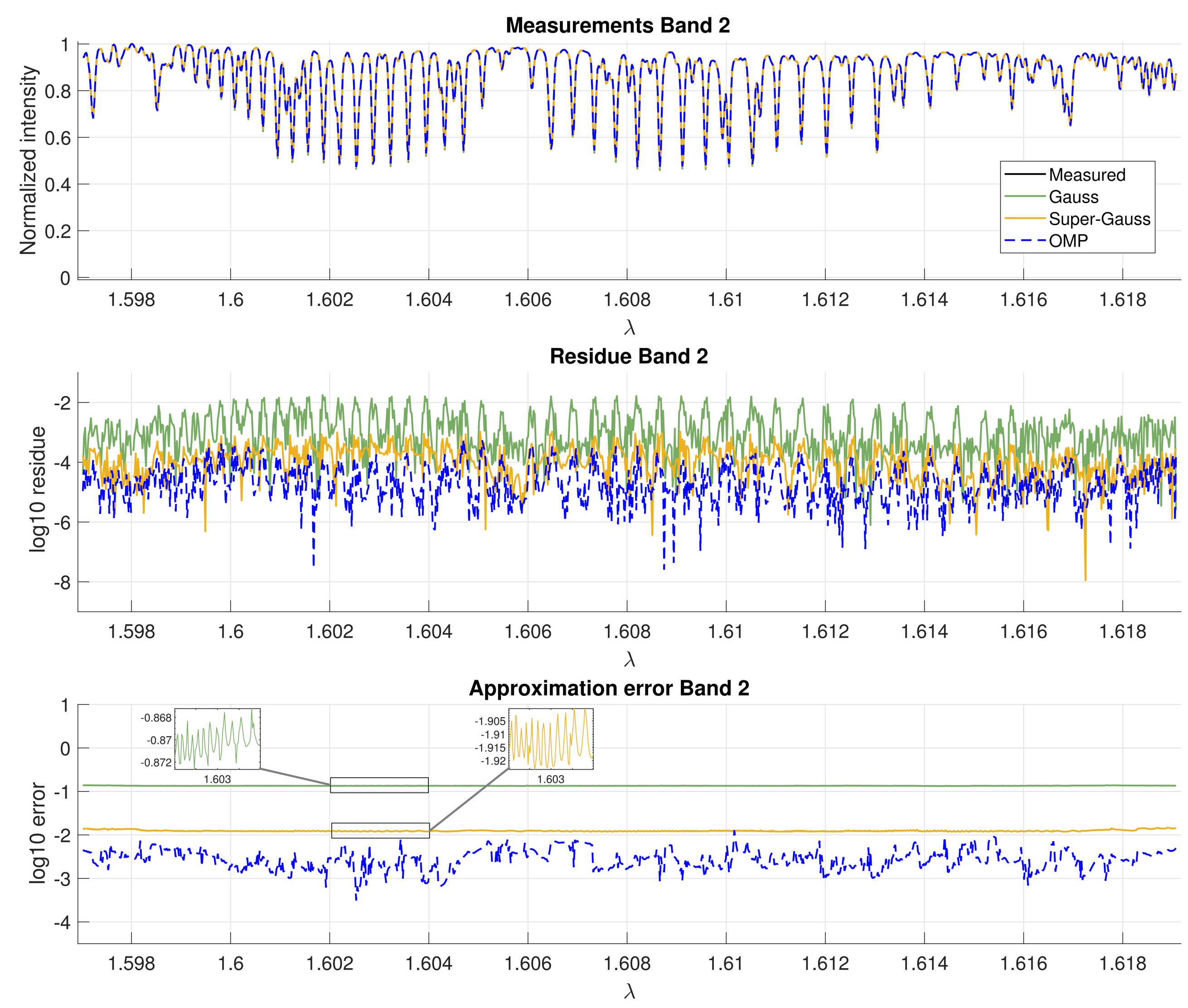
Approximation problem

Approximating ISRFs ($N_{\text{D}} = 25$, $K = 5$ atoms)



Solving the inverse problem

Reconstruction of the measured spectrum ($N_{\text{D}} = 25$, $K = 5$ atomes)



Conclusion & Prospects

- A new method for estimating spectral responses of spectrometers
- Analysis of other dictionary learning methods
- Take into account spectrum errors (scattered or stray light) that can degrade the estimation.
- ISRF denoising problem

References

[1] S. Beirle et al., "Parameterizing the instrumental spectral response function and its changes by a super-Gaussian and its derivatives," *Atmos. Meas. Tech.*, vol. 10, no. 2, pp. 581-598, 2017.
[2] Z. Zhang et al., "A Survey of Sparse Representation: Algorithms and Applications," *IEEE Access*, vol. 3, pp. 490-530, 2015.