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# A MAXIMUM LIKELIHOOD-BASED UNSCENTED KALMAN FILTER FOR MULTIPATH MITIGATION IN A MULTI-CORRELATOR BASED GNSS RECEIVER

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#### **ABSTRACT**

In complex environments, the presence or absence of multipath signals not only depends on the relative motion between the GNSS receiver and navigation satellites, but also on the environment where the receiver is located. Thus it is difficult to use a specific propagation model to accurately capture the dynamics of multipath signal parameters when the GNSS receiver is moving in urban canyons or other severe obstructions. This paper introduces a statistical model for the line-of-sight and multipath signals received by a GNSS receiver. A multi-correlator based GNSS receiver is also exploited with the advantage to fully characterizing the impact of multipath signals on the correlation function by providing samples of the whole correlation function. Finally, a maximum likelihood-based unscented Kalman filter is investigated to estimate the line-of-sight and multipath signal parameters. Numerical simulations clearly validate the effectiveness of the proposed approach.

*Index Terms*— Global navigation satellite systems, multipath mitigation, multi-correlator, maximum likelihood principle, unscented Kalman filter.

### 1. INTRODUCTION

With the new application requirements for global navigation satellite systems (GNSS) in complex environments, such as in urban canyons, one of the largest challenges is to address the impact of multipath (MP) interferences on positioning errors. MP interferences are mainly due to the fact that a signal transmitted by a navigation satellite is very likely to be reflected or diffracted and can follow different paths before arriving at the GNSS receiver [1]. When the relative time delay of an MP is short, the correlator outputs in the receiver are distorted as the line-of-sight (LOS) signal entering the receiver is affected by MP signals. This results in tracking errors which introduce biases in pseudo-range, carrier phase and Doppler frequency measurements and severely impair the positioning solution based on GNSS [2]. Thus mitigating MP interferences inside the GNSS receiver is a critical issue to obtain accurate positioning in a complex environment.

MP mitigation techniques inside the GNSS receiver are often based on statistical estimation methods trying to estimate the parameters of the LOS and MP signals. Several estimation methods proposed in recent years are based on the maximum likelihood principle [3–5], such as the vision correlator (VC) [6] and the fast iterative maximum likelihood algorithm (FIMLA) [7]. However, the maximum likelihood-based approaches assume that the signal parameters are not time varying and do not exploit any dynamic information for

the signal parameters. On the contrary, some alternative approaches exploit time propagation models associated with the unknown parameters of LOS and MP signals. The objective is then to estimate recursively the posterior probability density function of the unknown parameters associated with the LOS and MP signals. Considering that the GNSS measurements are related to the unknown parameters by highly nonlinear equations, the use of particle filters (PF) has been addressed in the literature [8-10]. Although many approaches have been suggested to improve the efficiency of these filters [11-15], a high computation still limits the real-time application of PF-based MP mitigation approaches. In addition, the presence or absence of MP signals not only depends on the relative position of the receiver and GNSS satellites, but also on the environment where the receiver is located. Thus it is difficult to use a specific propagation model for the MP signal parameters when the receiver is moving. This paper introduces a statistical model for the LOS and MP signals and investigates a maximum likelihood-based unscented Kalman filter (UKF) for estimating the parameters of this model.

The paper is organized as follows: the mathematical models for the LOS and MP signal parameters in a multi-correlator based receiver are presented in Section 2. Section 3 studies the proposed maximum likelihood-based UKF for MP mitigation in a GNSS receiver. The performance of the resulting algorithm is evaluated in Section 4. Conclusions are finally reported in Section 5.

### 2. SIGNAL MODEL

## 2.1. Measurement model for a multi-correlator based receiver

In a pilot channel, the received complex baseband signal associated with GNSS satellites affected by M MP signals, sampled at time instants  $nT_s$  where  $n=1,\ldots$ , can be written as follows [2]

$$r(nT_s) = \sum_{m=0}^{M} a_m c(nT_s - \tau_m) \exp(j\varphi_m) + e(nT_s)$$
 (1)

with

$$\frac{\mathrm{d}\varphi_m}{\mathrm{d}t} = 2\pi f_m^d$$

where  $m=0,\ldots,M$  and M is the number of non-light-of-sight (NLOS) signals. Here the subscript m=0 denotes the LOS signal,  $a_m$  is the signal amplitude associated with the mth NLOS signal, c(t) is the pseudo-random noise (PRN) code associated with the given GNSS signal,  $\tau_m$ ,  $\varphi_m$  and  $f_m^d$  are the code delay, carrier phase and Doppler frequency associated with the mth NLOS signal and  $e(nT_s)$  is a zero mean additive Gaussian white noise.

In order to fully characterize the impact of MP interferences on the correlation function, a multi-correlator based receiver [16], which allows a complete sampling of the whole useful part of the correlation function, is considered in this work. The architecture of such a receiver is illustrated in Fig. 1. First, the received signal associated with the given satellite is decomposed into its in-phase (I) and quadrature (Q) components after being multiplied by the in-phase and quadrature local generated carriers. Then each component is correlated with 2J+1 replicas of the PRN code for obtaining a multi-correlator structure. Thus the jth in-phase output at time k, denoted as  $I_{j,k}$  resulting from an integration over an interval  $T_a$ , can be defined as

$$I_{j,k} = \sum_{m=0}^{M} a_{m,k} R \left( \Delta \tilde{\tau}_{m,k} + \theta_j \right) \operatorname{sinc} \left( \pi \Delta \tilde{f}_{m,k}^d T_a \right)$$

$$\operatorname{cos} \left( \Delta \tilde{f}_{m,k} \right) + n_{J,k}$$
(2)

where  $T_a=N_sT_s$  and  $N_s$  is the number of GNSS signal samples in an integration interval  $T_a$ , sinc  $(\cdot)$  is the cardinal sine function,  $\theta_j=j\Delta\theta$  is the jth correlator delay expressed in chips where  $\Delta\theta=\theta_{j+1}-\theta_j>0$   $(j=-J,\ldots,0,\ldots,J)$  is a correlation spacing between adjacent correlators,  $R(\cdot)$  is the auto-correlation function of the PRN code,  $\Delta\tilde{\tau}_{m,k},\Delta\tilde{\varphi}_{m,k}$  and  $\Delta\tilde{f}_{m,k}^d$  denote differences between the code delay, carrier phase and Doppler frequency for the mth signal path  $(\tau_{m,k},\varphi_{m,k},f_{m,k}^d)$  and those of local generated replicas  $(\tilde{\tau}_k,\tilde{\varphi}_k,\tilde{f}_k^d),n_{I_j,k}$  is a zero mean Gaussian white noise associated with the jth in-phase correlation function samples. Note that the jth quadrature integration outputs can be obtained similarly by changing  $I_{j,k}$  to  $Q_{j,k}$ ,  $\cos(\cdot)$  to  $\sin(\cdot)$  and  $n_{I_j,k}$  by  $n_{Q_j,k}$  in (2). Accordingly, the measurement equation at time k can be defined as

$$\boldsymbol{z}_k = \mathbf{h}(\boldsymbol{x}_k) + \boldsymbol{n}_k \tag{3}$$

with

$$\begin{aligned} \boldsymbol{z}_k &= (I_{-J,k}, \dots, I_{J,k}, Q_{-J,k}, \dots, Q_{J,k})^{\mathrm{T}} \\ \boldsymbol{x}_k &= (\boldsymbol{x}_{0,k}, \dots, \boldsymbol{x}_{M,k})^{\mathrm{T}} \\ \boldsymbol{n}_k &= \begin{pmatrix} n_{I-J}, k, \dots, n_{IJ,k}, n_{Q-J,k}, \dots, n_{QJ,k} \end{pmatrix}^{\mathrm{T}} \end{aligned}$$

where  $k=1,\ldots,K$  denotes the kth time instant of the correlator integration output,  $\mathbf{h}(\cdot)$  is a non-linear function resulting from (2),  $\mathbf{x}_{0,k} = \left(a_{0,k},\tau_{0,k},\varphi_{0,k},f_{0,k}^d,\xi_{0,k}^d\right)^{\mathrm{T}}$  and  $\mathbf{x}_{m,k} = \left(a_{m,k},\tau_{m,k},\varphi_{m,k},f_{m,k}^d\right)^{\mathrm{T}}$  ( $m=1,\ldots,M$ ) are the LOS and MP signal parameter vectors, respectively. Here  $\xi_{0,k}^d$  denotes a drift associated with the carrier Doppler frequency  $f_{0,k}^d$ . Note that the noise terms  $n_{I_j,k}$  and  $n_{Q_j,k}$  for  $j=-J,\ldots,J$  of the in-phase and quadrature phase components are correlated with a covariance matrix denoted as

$$R_k = \sigma^2 \begin{pmatrix} R_I & \mathbf{0} \\ \mathbf{0} & R_O \end{pmatrix} \tag{4}$$

where the matrices  $R_I$  and  $R_Q$  depend on the correlation spacing  $\Delta\theta$  [17] and  $\sigma^2 = N_0/2T_a$  is the noise variance depending on the signal power spectral density  $N_0$  and on the integration time  $T_a$ .

### 2.2. Propagation model for LOS signal parameters

When the GNSS signal has been locked inside the receiver, a discrete-time state model, which describes the time propagation of the LOS parameter vector  $\boldsymbol{x}_{0,k}$  at time k, can be formulated as

$$x_{0,k} = F_{k|k-1}x_{0,k-1} + \Gamma_{k-1}\omega_{k-1}$$
 (5)

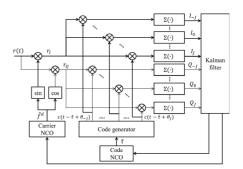


Fig. 1: GNSS baseband signal processing channel with a multi-correlator [11].

where  $\omega_{k-1} = (\omega_{a,k-1}, \omega_{\tau,k-1}, \omega_{\varphi,k-1}, \omega_{f,k-1}, \omega_{\xi,k-1})^{\mathrm{T}}$  and where the definitions of the matrices  $F_{k|k-1}$  and  $\Gamma_{k-1}$  can be found in [18].

#### 2.3. Likelihood model for MP signal parameters

As mentioned above, it is difficult to use a specific propagation model for the MP signal parameters when the receiver is moving in complex environments. However, a likelihood function for the measurements provided by the bank of correlators can be defined to construct an estimator of the unknown MP signal parameters. According to (3), a function of the MP parameter vectors  $(\boldsymbol{x}_{1,k},\ldots,\boldsymbol{x}_{M,k})$  and the correlation function samples at time k can be written as

$$\boldsymbol{z}_{k} = \mathbf{h}_{0} \left( \boldsymbol{x}_{0,k} \right) + \mathbf{h}_{1} \left( \boldsymbol{x}_{1,k}, \dots \boldsymbol{x}_{M,k} \right) + \boldsymbol{n}_{k}. \tag{6}$$

Thus the likelihood function of the MP signal parameters defined from the correlation function samples can be defined as  $p(\mathbf{z}_k'|\mathbf{z}_{1,k},\ldots,\mathbf{z}_{M,k})$  where  $\mathbf{z}_k'=\mathbf{z}_k-\mathbf{h}_0(\mathbf{z}_{0,k})$ . Note that the LOS parameter vector  $\mathbf{z}_{0,k}$  is first assumed to be known (the unknown case will be discussed in the next section).

# 3. THE MAXIMUM LIKELIHOOD-BASED UKF FOR MP MITIGATION

### 3.1. Problem formulation

In the presence of MP interferences, the MP mitigation problem can be formulated as how to accurately estimate the LOS signal parameters when the correlation function in the receiver is distorted by the MP signals. According to section 2.1, the received signal parameter vector  $\boldsymbol{x}_k$  can be partitioned into a LOS parameter vector  $\boldsymbol{x}_{0,k}$  and MP parameter vectors  $(\boldsymbol{x}_{1,k},\ldots,\boldsymbol{x}_{M,k})$ . We assume that the parameter vectors of the LOS and MP signals are pairwise independent. Using Bayes theorem, the posterior probability density function (pdf) of the parameter vector  $\boldsymbol{x}_k$  is defined as

$$p\left(\boldsymbol{x}_{k}|\boldsymbol{z}_{1:k}\right) = p\left(\boldsymbol{x}_{0,k}, \dots, \boldsymbol{x}_{M,k}|\boldsymbol{z}_{1:k}\right)$$

$$\propto p\left(\boldsymbol{z}_{k}|\boldsymbol{x}_{0,k}, \dots, \boldsymbol{x}_{M,k}, \boldsymbol{z}_{1:k-1}\right) p\left(\boldsymbol{x}_{0,k}, \dots, \boldsymbol{x}_{M,k}|\boldsymbol{z}_{1:k-1}\right)$$

$$= p\left(\boldsymbol{z}_{k}|\boldsymbol{x}_{0,k}, \dots, \boldsymbol{x}_{M,k}, \boldsymbol{z}_{1:k-1}\right) p\left(\boldsymbol{x}_{0,k}|\boldsymbol{z}_{1:k-1}\right)$$

$$p\left(\boldsymbol{x}_{1,k}, \dots, \boldsymbol{x}_{M,k}|\boldsymbol{z}_{1:k-1}\right).$$
(7)

According to section 2.3, the pdf  $p(x_{1,k},\ldots,x_{M,k}|z_{1:k-1})$  associated with the MP parameter vectors is assumed to be constant (unin-

formative prior). As a consequence, (7) can be rewritten as

$$p\left(\boldsymbol{x}_{k}|\boldsymbol{z}_{1:k}\right) \propto p\left(\boldsymbol{z}_{k}|\boldsymbol{x}_{0,k},\ldots,\boldsymbol{x}_{M,k},\boldsymbol{z}_{1:k-1}\right)p\left(\boldsymbol{x}_{0,k}|\boldsymbol{z}_{1:k-1}\right). \tag{8}$$

Thus the estimation of the received signal parameter vector  $\boldsymbol{x}_k$  from (8) is not straightforward. As an alternative, this work introduces a method for computing the Bayesian estimators of the signal parameter vector  $\boldsymbol{x}_k$ , as explained below.

Step 1. Posterior pdf of  $x_k$  for a given LOS parameter vector Assume that the LOS parameter vector  $x_{0,k}$  is known at time k. The following results can be obtained

$$p\left(\boldsymbol{x}_{k}|\boldsymbol{z}_{1:k}\right) = p\left(\boldsymbol{x}_{0,k}, \boldsymbol{x}_{1,k}, \dots, \boldsymbol{x}_{M,k}|\boldsymbol{z}_{1:k}\right)$$

$$\propto p\left(\boldsymbol{x}_{1,k}, \dots, \boldsymbol{x}_{M,k}|\boldsymbol{x}_{0,k}, \boldsymbol{z}_{1:k}\right)$$
(9)

where  $p\left(\boldsymbol{x}_{1,k},\ldots,\boldsymbol{x}_{M,k}|\boldsymbol{x}_{0,k},\boldsymbol{z}_{1:k}\right)$  is the posterior pdf of the MP parameter vectors  $(\boldsymbol{x}_{1,k},\ldots,\boldsymbol{x}_{M,k})$ , under the assumption of a known LOS parameter vector. Since the prior pdf associated with the MP parameter vectors is assumed to be constant, the posterior pdf of  $(\boldsymbol{x}_{1,k},\ldots,\boldsymbol{x}_{M,k})$  is defined as

$$p\left(\boldsymbol{x}_{1,k},\ldots,\boldsymbol{x}_{M,k}|\boldsymbol{x}_{0,k},\boldsymbol{z}_{1:k}\right) \propto p\left(\boldsymbol{z}_{1:k}|\boldsymbol{x}_{0,k},\boldsymbol{x}_{1,k},\ldots,\boldsymbol{x}_{M,k}\right)$$
(10)

where  $p(z_{1:k}|x_{0,k},x_{1,k},...,x_{M,k})$  is the likelihood function of the measurements  $z_{1:k}$  at time k. Thus the estimation of  $x_k$  from the measurements  $z_{1:k}$  for a given LOS parameter vector can be converted into a maximum likelihood estimator (MLE).

Step 2. Posterior pdf of  $x_k$  for given MP parameter vectors Assume now that the MP parameter vectors  $(x_{1,k},\ldots,x_{M,k})$  are known at time k. The following results can be obtained

$$p(\mathbf{x}_{k}|\mathbf{z}_{1:k}) = p(\mathbf{x}_{0,k}, \mathbf{x}_{1,k}, \dots, \mathbf{x}_{M,k}|\mathbf{z}_{1:k})$$

$$\propto p(\mathbf{x}_{0,k}|\mathbf{x}_{1,k}, \dots, \mathbf{x}_{M,k}, \mathbf{z}_{1:k})$$
(11)

where  $p\left(\boldsymbol{x}_{0,k}|\boldsymbol{x}_{1,k},\ldots,\boldsymbol{x}_{M,k},\boldsymbol{z}_{1:k}\right)$  is the posterior pdf of the LOS parameter vector  $\boldsymbol{x}_{0,k}$ , under the assumption of known MP parameter vectors

The proposed method for estimating  $x_k$  consists of sequentially implementing **Step 1** and **2** for k = 1, ..., K, as detailed in the next sections.

### 3.2. Step 1: MP Parameter Estimation Based on the MLE

An estimator of the MP parameter vectors can be obtained by maximizing the likelihood function defined in (10) with respect to the MP parameter vectors  $(\boldsymbol{x}_{1,k},\ldots,\boldsymbol{x}_{M,k})$ . However, it requires the value of the LOS parameter vector  $x_{0,k}$  which is not straightforward to obtain at time k. Assuming that the posterior pdf  $p\left(\tilde{\boldsymbol{x}}_{0,k-1}|\boldsymbol{z}_{1:k-1}\right)$  at time k-1 is available, the conditional pdf of the LOS parameter vector  $\boldsymbol{x}_{0,k}$ , conditionally upon the k-1 first measurements  $\boldsymbol{z}_{1:k-1}$ , can be obtained by using the time propagation model of the LOS signal parameters as presented in (5), i.e.,  $\boldsymbol{x}_{0,k} \sim \mathcal{N}\left(\hat{\boldsymbol{x}}_{0,k|k-1}, P_{k|k-1}\right)$ where  $\mathcal{N}\left(\hat{m{x}}_{0,k|k-1},m{P}_{k|k-1}
ight)$  denotes the Gaussian distribution with mean vector  $\hat{x}_{0,k|k-1}$  and covariance matrix  $P_{k|k-1}$  [19]. More precisely, according to the principle of the unscented transformation (UT), we generate a set of sigma-points that are deterministically calculated using the mean and covariance matrix of the random vector of interest [20]. In this work, we propose to generate a set of sigma points  $\left\{oldsymbol{\chi}_{i,0}
ight\}_{i=0}^{2L}$  according to the conditional pdf  $p(x_{0,k}|z_{1:k-1})$  at time k, i.e.,

$$\chi_{i,0} = \hat{x}_{0,k|k-1}, i = 0$$

$$\chi_{i,0} = \hat{x}_{0,k|k-1} + \left(\sqrt{(L+\lambda)P_{k|k-1}}\right)_i, i = 1,\dots, L$$

$$\chi_{i,0} = \hat{x}_{0,k|k-1} - \left(\sqrt{(L+\lambda)P_{k|k-1}}\right)_i, i = L+1,\dots, 2L$$
(12)

where L is the dimension of the state vector  $\hat{\boldsymbol{x}}_{0,k|k-1}$  and  $\lambda$  is a scaling parameter. The definitions of the weights  $w_i^s$  and  $w_i^c$  associated with the sigma point  $\boldsymbol{\chi}_{i,0}$  and its covariance matrix can be found in [21]. The vector  $\boldsymbol{x}_{0,k}$  in **Step 1** can be approximated by using the sigma points of the LOS parameter vector associated with the conditional pdf  $p\left(\boldsymbol{x}_{0,k}|\boldsymbol{z}_{1:k-1}\right)$  at time k, i.e.,  $\boldsymbol{x}_{0,k}\approx\boldsymbol{\chi}_{i,0}$  where  $i=0,\ldots,2L$ . As a consequence, the likelihood functions of the MP parameter vectors for the ith sigma point  $\boldsymbol{\chi}_{i,0}$  at time k can be written as

$$p\left(\hat{\boldsymbol{z}}_{i,k}'|\boldsymbol{x}_{1,k},\ldots,\boldsymbol{x}_{M,k}\right) = \mathcal{N}\left(\mathbf{h}_{1}(\boldsymbol{x}_{1,k},\ldots\boldsymbol{x}_{M,k}),\boldsymbol{R}_{k}\right) \quad (13)$$

with

$$\hat{oldsymbol{z}}_{i,k}' = oldsymbol{z}_k - \mathbf{h}_0 \left( oldsymbol{\chi}_{i,0} 
ight)$$

and  $i=0,\ldots,2L$ . Thus the estimation of the MP parameter vectors associated with the ith sigma point  $\chi_{i,0}$  can be obtained by maximizing the likelihood function in (13). This maximization is not straightforward since all terms in  $\mathbf{h}_1(\cdot)$  are related to the unknown MP signal parameters. Thus we propose an interval grid search based on the maximum likelihood principle to perform the estimation of MP parameters [5, 17]. Finally, the estimator of the mth MP parameter vector can be expressed as follows

$$\hat{\boldsymbol{x}}_{m,k} = \sum_{i=0}^{2L} w_i^s \hat{\boldsymbol{x}}_{m,k}^i$$
 (14)

where  $m=1,\ldots,M$ ,  $\hat{\boldsymbol{x}}_{m,k}^{i}$  is the maximum likelihood estimator of the mth MP parameter vector associated with the ith sigma point.

### 3.3. Step 2: LOS Parameter Estimation Based on the UKF

According to (11), the posterior pdf of the LOS parameter vector  $x_{0,k}$  can be obtained as a function of the MP parameter vectors. Considering that the MLE of the MP parameter vectors at time k, denoted as  $(\hat{x}_{1,k},\ldots,\hat{x}_{M,k})$ , have been obtained in (14), the vectors  $(x_{1,k},\ldots,x_{M,k})$  required in **Step 2** can be approximated by these estimators. The measurement equations (3) being highly nonlinear, we consider a UKF based on a UT technique that provides an efficient and low-cost estimator for highly non-linear equations.

The generated sigma points  $\chi_{i,0}$  in (12) are transformed to obtain the *i*th predicted measurement when the MLE of the MP parameter vectors  $(\hat{x}_{1,k}, \ldots, \hat{x}_{M,k})$  is given [21]

$$\hat{z}_{i,k|k-1} = \mathbf{h}(\chi_{i,0}, \hat{x}_{1,k}, \dots, \hat{x}_{M,k})$$
 (15)

where  $i=0,\dots,2L$ . Accordingly, the mean and covariance of the predicted measurement  $\hat{z}_{k|k-1}$  can be obtained as

$$\hat{\mathbf{z}}_{k|k-1} = \sum_{i=0}^{2L} w_i^s \hat{\mathbf{z}}_{i,k|k-1} \tag{16}$$

$$\mathbf{P}_{k|k-1}^{\hat{\mathbf{z}}\hat{\mathbf{z}}} = \sum_{i=0}^{2L} w_i^c \left( \hat{\mathbf{z}}_{i,k|k-1} - \hat{\mathbf{z}}_{k|k-1} \right) \left( \hat{\mathbf{z}}_{i,k|k-1} - \hat{\mathbf{z}}_{k|k-1} \right)^{\mathrm{T}} + \mathbf{R}_k.$$
(17)

The cross covariance matrix between the predicted LOS parameter vector  $\hat{x}_{0,k|k-1}$  and the predicted measurement  $\hat{z}_{k|k-1}$  is

$$\boldsymbol{P}_{k|k-1}^{\hat{\boldsymbol{x}}_0\hat{\boldsymbol{z}}} = \sum_{i=0}^{2L} w_i^c \left( \boldsymbol{\chi}_{0,i} - \hat{\boldsymbol{x}}_{0,k|k-1} \right) \left( \hat{\boldsymbol{z}}_{i,k|k-1} - \hat{\boldsymbol{z}}_{k|k-1} \right)^{\mathrm{T}}.$$
(18)

The conventional Kalman filter gain is calculated as follows

$$\boldsymbol{K}_{k} = \boldsymbol{P}_{k|k-1}^{\hat{\boldsymbol{x}}_{0}\hat{\boldsymbol{z}}} \left(\boldsymbol{P}_{k|k-1}^{\hat{\boldsymbol{z}}\hat{\boldsymbol{z}}}\right)^{-1}. \tag{19}$$

As a consequence, the maximum a posteriori estimator of the LOS parameter vector and its covariance matrix are defined as

$$\hat{x}_{0,k|k} = \hat{x}_{0,k|k-1} + K_k \left( z_k - \hat{z}_{k|k-1} \right)$$
 (20)

$$\boldsymbol{P}_{k|k} = \boldsymbol{P}_{k|k-1} - \boldsymbol{K}_k \boldsymbol{P}_{k|k-1}^{\hat{\boldsymbol{z}}\hat{\boldsymbol{z}}} \boldsymbol{K}_k^{\mathrm{T}}.$$
 (21)

### 4. SIMULATION RESULTS

In order to validate the proposed MP mitigation approach, we have first simulated a GPS L1 C/A signal assuming a scenario composed of a LOS signal and one reflected MP signal, i.e., M=1. This assumption is realistic in many practical scenarios due to the fact that two reflected signals very close in time can be considered as only one perturbation [22]. The GPS L1 C/A signal is based on a pseudo random sequence that is used to spread the data signal around the carrier frequency  $f_{ca} = 1575.42$  MHz. The PRN code rate is  $f_{co} =$  $1/T_{co} = 1.023$  MHz and the length of the PRN code is N = 1023, resulting in a code period of 1 ms. In the following simulation, the carrier-to-noise density ratio of the LOS signal is 42 dB-Hz and a power signal-to-multipath ratio of 6 dB is considered. The sampling frequency of the baseband signal entering the digital receiver is set to  $f_s = 10.23$  MHz, providing 10230 data samples during the code period. The noises associated with the amplitude, code delay, and carrier phase of the LOS signal have variances  $\omega_a = 0.00001$ ,  $\omega_\tau =$ 0.1 and  $\omega_{\varphi}=0.1$ , respectively. The run time for all simulations is 2 s and the filter rate is equal to the correlator integration time, i.e.,  $T_a = 20$  ms. Two scenarios has been generated according to the GNSS signal model (1) as follows

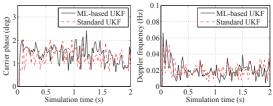
- Scenario 1 Only a LOS signal is processed inside the receiver, i.e., M = 0.
- Scenario 2 An MP signal appears during the simulation time interval (0.8s, 1.6s). The MP relative code delay with respect to the LOS signal is set to a random value uniformly distributed over the interval (0, 0.2), i.e.,  $(\tau_1 - \tau_0) \sim U(0, 0.2)$ . Moreover, the LOS and MP signals are in-phase, i.e.,  $\varphi_1=\varphi_0$  and their carrier Doppler frequencies are equal, i.e.,  $f_1^d=f_0^d$ .

This section compares the performance of the proposed approach for the estimation of the LOS parameters (denoted as ML-based UKF) with that obtained using the standard UKF. Note that  $N_m$ Monte Carlo simulations have been run for any scenario. The root mean square error (RMSE) of the estimates is defined as

$$RMSE = \sqrt{\frac{1}{N_m} \sum_{i=1}^{N_m} \left(\hat{\boldsymbol{x}}_{0,k}^{(i)} - \boldsymbol{x}_{0,k}\right)^2}$$
 (22)

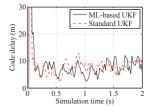
where  $\hat{x}_{0,k}^{(i)}$  is the ith estimate. Fig. 2 shows the RMSEs of the estimated code delay, carrier phase and Doppler frequency of the LOS signal with the different approaches for Scenario 1. Since there is not MP, the RMSEs of the two approaches are very similar. Thus the proposed ML-based UKF is able to effectively track the LOS signal parameters and provides the same performance as a standard UKF in the absence of MP interferences.

In Scenario 2, MP interferences appear or disappear randomly and the MP code delay changes quickly with time. This situation often happens when the receiver is located in severe obstruction environments. Since the carrier phases and Doppler frequencies of the LOS and MP signals are equal in this scenario, the estimations of the carrier phase and Doppler frequency of the LOS signal are not impacted by the MP interference (for both approaches). However, since the correlation function that is sampled at the multi-correlator outputs is distorted in the presence of the MP signal, the estimation of



(a) RMSE of carrier phase

(b) RMSE of Doppler frequency



(c) RMSE of code delay

Fig. 2: RMSEs of LOS parameter estimators in Scenario 1.

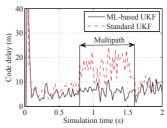


Fig. 3: RMSE of LOS signal code delay in Scenario 2.

the LOS signal code delay obtained with the standard UKF degraded in this scenario. Fig. 3 shows that the RMSE of the estimated LOS code delay using the standard UKF filter fluctuates strongly since the MP relative code delay takes a random value in this scenario. On the contrary, since the impact of the MP interference on the correlation function has been mitigated by the MLE (which estimates the MP signal parameters), the proposed ML-based UKF improves the accuracy of the code delay estimator. Our results prove that the proposed approach is more robust than the standard UKF due to the fact that it corrects the presence of abrupt changes corrupting the LOS signal acquired by the receiver in the presence of MP interferences.

### 5. CONCLUSIONS

This paper studied a maximum likelihood-based UKF to estimate the LOS signal parameters in the presence of MP interferences. An MLE was implemented to estimate the MP signal parameters by using the sigma points of the LOS signal parameters, and a UKF method was developed to estimate the LOS signal parameters from estimators of the MP signal parameters. A simulation study was conducted in order to compare the performance of the proposed approach with the standard UKF. In the absence of MP interferences, the performance of the proposed approach is equivalent to that of the standard UKF. On the contrary, in the presence of MP interferences, the estimation accuracy for the LOS signal parameters, especially for the code delay, can be improved by the proposed approach. Future work includes the consideration of optimization algorithms for determining the code delays of MP signals, reducing the approximation error resulting from the proposed interval grid search method.

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