P and T Wave Analysis in ECG signals using Bayesian methods

Chao Lin

PhD advisors: Prof. Corinne Mailhes and Prof. Jean-Yves Tourneret

PhD Defense Presentation, July 2012, Toulouse France

Outline

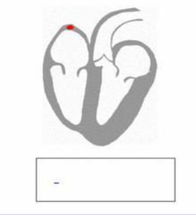
1. **Introduction to cardiac electrophysiology**
2. **Window based Bayesian analysis of P and T waves**
   - Window based Bayesian model and a PCGS
   - Modified Bayesian model and a block Gibbs sampler
3. **Beat-to-beat Bayesian analysis of P and T waves**
   - Beat-to-beat Bayesian model and a block Gibbs sampler
   - Particle filters for beat-to-beat P and T wave analysis
4. **Application in clinical research: TWA detection**
   - TWA detection in surface ECG
   - Endocardial TWA detection
5. **Conclusion and future works**

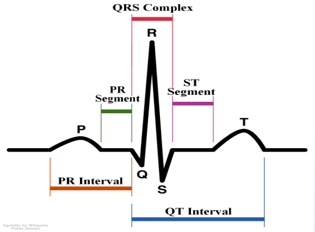
Outline

1. **Introduction to cardiac electrophysiology**
2. **Window based Bayesian analysis of P and T waves**
   - Window based Bayesian model and a PCGS
   - Modified Bayesian model and a block Gibbs sampler
3. **Beat-to-beat Bayesian analysis of P and T waves**
   - Beat-to-beat Bayesian model and a block Gibbs sampler
   - Particle filters for beat-to-beat P and T wave analysis
4. **Application in clinical research: TWA detection**
   - TWA detection in surface ECG
   - Endocardial TWA detection
5. **Conclusion and future works**

Electrocardiogram (ECG)

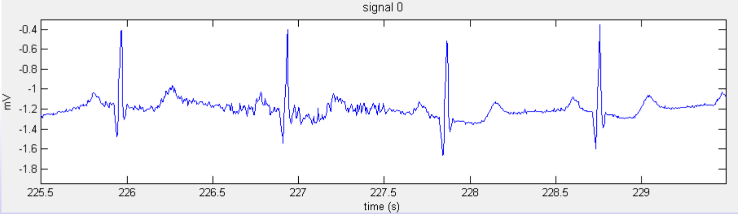
- A recording of the electrical activity of the heart over time
- 3 distinct waves are produced during cardiac cycle
  - P wave caused by atrial depolarization
  - QRS complex caused by ventricular depolarization
  - T wave results from ventricular repolarization and relax
- Wave shapes and interval durations indicate clinically useful information





ECG delineation

- Delineation: determination of peaks and boundaries of the waves
- P and T wave delineation—a challenging problem
  - Low slope and low magnitude
  - Presence of noise, interference and baseline fluctuation
  - Lack of universal delineation rule
  - Waveform estimation

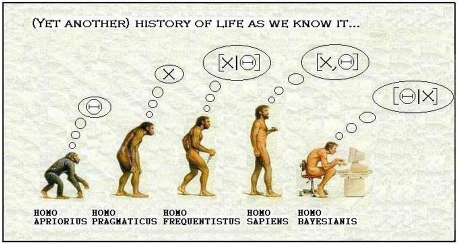


Literature review

- Filtering techniques: nested median filtering, adaptive filtering, low-pass differentiation (LPD)
- Basis expansions: Fourier transform, discrete cosine transform, wavelet transform (WT)
- Classification and pattern recognition: fuzzy theory, hidden Markov models, pattern grammar (PG)
- Bayesian inference: extended Kalman filter (EKF)

LPD: **P. Laguna et al.**, New algorithm for QT interval analysis in 24 hour Hotler ECG: Performance and applications. *Med. Biological Eng. and Comput.*, 1990
WT: **L. Senhadji et al.**, Comparing wavelet transforms for recognizing cardiac patterns. *IEEE Eng. in Medicine and Biology*, 1995
J. P. Martínez et al., A Wavelet-based ECG delineator: Evaluation on standard databases. *IEEE Trans. Biomed. Eng.*, 2004
PG: **P. Trahanias et al.**, Syntactic Pattern Recognition of the ECG. *IEEE Trans. Pattern Anal. Mach. Intell.*, 1990
EKF: **O. Sayadi et al.**, A model-based Bayesian framework for ECG beat segmentation. *J. Physiol. Meas.*, 2009

Why using a Bayesian approach?



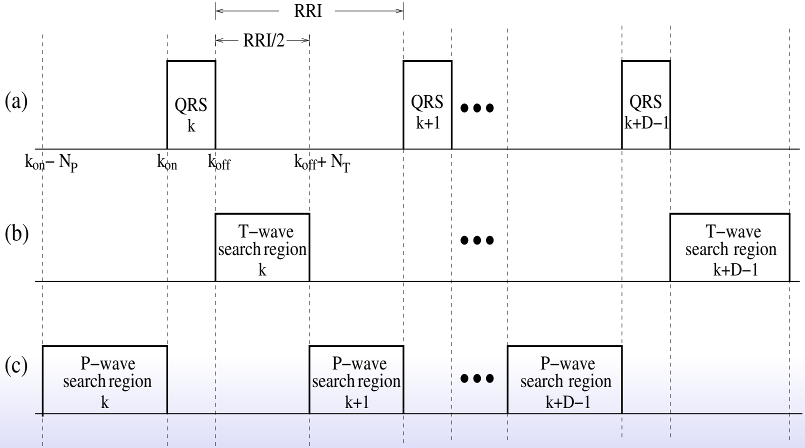
Bayesian models are well suited to the ECG processing:

- Natural way to express what is known and unknown in a probabilistic sense and "get it into the problem"
- Allowing to evaluate which one of many alternatives is most likely the source of the observations

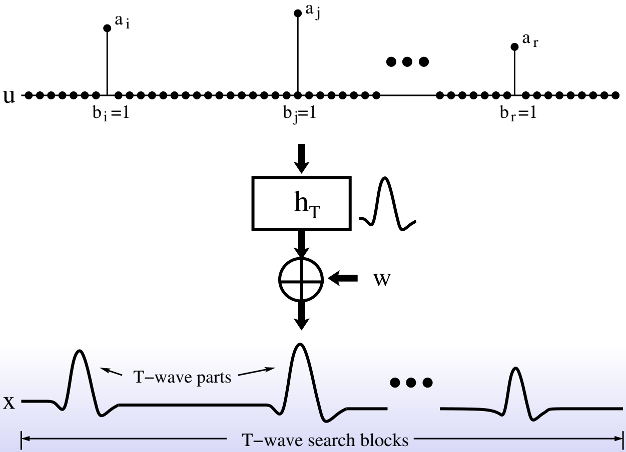
Outline

Outline

Construction of P and T wave blocks



Modeling of T wave parts within the *D*-beat window



Signal model for T wave search blocks

Deconvolution model

$$x_k = \sum_{l=-L}^{L} h_l u_{k-l} + w_k, \ \ k \in \{1, \ldots, K\}$$

- $\mathbf{u} = (u_1 \cdots u_M)^T$: unknown "impulse" sequence
- $\mathbf{h} = (h_{-L} \cdots h_L)^T$: unknown T waveform
- $K = M + 2L$: the processing window length
- $u_k = b_k a_k$: u_k can be further decomposed by using a binary indicator $b_k \in \{0, 1\}$ representing the T wave locations multiplied by weights a_k representing the T wave amplitudes.
- w_k: white Gaussian noise

Signal model for T wave search blocks

Vector representation of T wave components

$$\mathbf{x} = \mathbf{FBa} + \mathbf{w} \tag{1}$$

- $\mathbf{x} = (x_1 \cdots x_K)^T$ denotes the T wave search block portion
- $\mathbf{a} = (a_1 \cdots a_M)^T$ denotes the T wave amplitude vector
- $\mathbf{B} = \mathrm{diag}(\mathbf{b})$ denotes the $M \times M$ diagonal matrix whose diagonal elements are the components of $\mathbf{b} = (b_1 \cdots b_M)^T$
- $\mathbf{F}$ is the $K \times M$ Toeplitz with first row $(\mathbf{h}_{0:-L}\ \mathbf{0})$ and first column $(\mathbf{h}_{0:L}^T\ \mathbf{0}^T)^T$
- $\mathbf{w} = (w_1 \cdots w_K)^T$ denotes the noise vector

Model parameters

Bayesian estimation relies on the posterior distribution

$$p(\boldsymbol{\theta}|\mathbf{x}) \propto p(\mathbf{x}|\boldsymbol{\theta})p(\boldsymbol{\theta})$$

- $\propto$ means "proportional to"
- $\boldsymbol{\theta} = (\mathbf{b}^T \ \mathbf{a}^T \ \mathbf{h}^T \ \sigma_w^2)^T$ are the unknown parameters resulting from (1)

Likelihood function

$$p(\mathbf{x}|\boldsymbol{\theta}) = \frac{1}{(2\pi)^{\frac{K}{2}} \sigma_w^K} \exp\left(-\frac{1}{2\sigma_w^2} \|\mathbf{x} - \mathbf{FBa}\|^2\right)$$

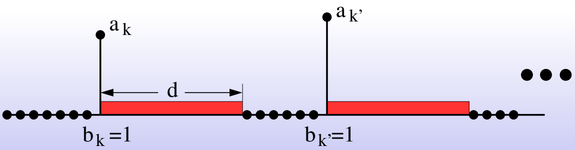
- where $\|\cdot\|$ is the ℓ_2 norm, i.e., $\|\mathbf{x}\|^2 = \mathbf{x}^T\mathbf{x}$

Prior distributions

T wave indicator prior: minimum-distance prior

$$p(\mathbf{b}) \propto \left[\prod_{k=1}^{K} p(b_k)\right] I_C(\mathbf{b}) = \lambda^{\|\mathbf{b}\|^2}(1-\lambda)^{K-\|\mathbf{b}\|^2} I_C(\mathbf{b})$$

- binary T wave indicator b_k is modeled as a Bernoulli sequence
- $\mathbf{b}$ cannot have two elements $b_k = 1$ and $b_{k'} = 1$ closer than a minimum-distance d
- $I_C(\mathbf{b}) = 1$ if $\mathbf{b} \in C$ and $I_C(\mathbf{b}) = 0$ if $\mathbf{b} \notin C$



Prior distributions

T wave amplitude prior

$$p(a_k | b_k = 1) = \mathcal{N}(a_k; 0, \sigma_a^2)$$

- a_k are only defined at time instants k where $b_k = 1$,
- $u_k = b_k a_k$ is a Bernoulli-Gaussian sequence with minimum-distance constraints.

J. Idier and Y. Goussard, Stack algorithm for recursive deconvolution of Bernoulli-Gaussian processes, *IEEE Trans. Geosci. Remote Sens.*, 1990

C. Soussen, J. Idier, D. Brie and J. Duan, From Bernoulli-Gaussian deconvolution to sparse signal restoration, *IEEE Trans. Signal Processing*, 2011

G. Kail, J.-Y. Tourneret, F. Hlawatsch and N. Dobigeon, Blind deconvolution of sparse pulse sequences under a minimum distance constraint: A partially collapsed Gibbs sampler method, *IEEE Trans. Signal Processing*, 2012

...

Posterior distribution

T waveform coefficients prior

$$p\left(\mathbf{h}\right) = \mathcal{N}\left(\mathbf{0}, \sigma_h^2 \mathbf{I}_{2L+1}\right)$$

Noise variance prior

$$p(\sigma_w^2) = \mathcal{IG}\left(\xi, \eta\right) = \frac{\eta^{\xi}}{\Gamma(\xi)} \frac{1}{(\sigma_w^2)^{\xi+1}} \exp\left(-\frac{\eta}{\sigma_w^2}\right) I_{\mathbb{R}^+}(\sigma_w^2)$$

Posterior distribution

$$p\left(\boldsymbol{\theta}|\mathbf{x}\right) \propto p\left(\mathbf{x}|\boldsymbol{\theta}\right) p\left(\mathbf{a}|\mathbf{b}\right) p\left(\mathbf{b}\right) p\left(\mathbf{h}\right) p\left(\sigma_w^2\right)$$

Complex distribution

Partially collapsed Gibbs sampler

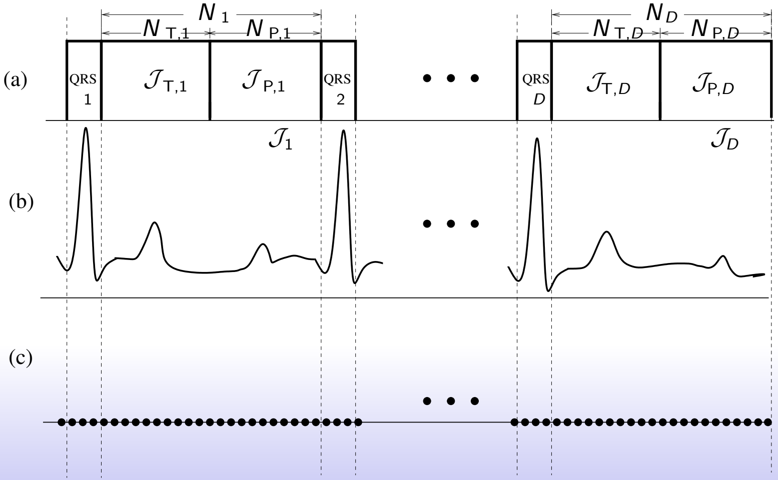
- Set $k = 1$
- **While** $k \leq K$
  - Sample the T wave indicator b_k
  - **If** $b_k = 1$

    Sample the T wave amplitudes a_k
    Set the right-hand neighborhood $\mathbf{b}_{J_d(k)\setminus k} = \mathbf{0}$
    Set $k = k + d - 1$
  - Set $k = k + 1$
- Sample the T waveform coefficients $\mathbf{h}$
- Sample the noise variance σ_w^2

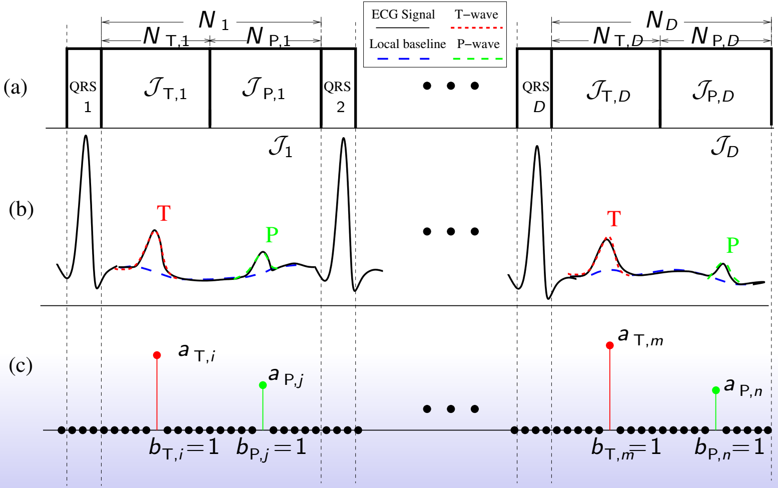
C. Lin et al., P and T wave delineation in ECG signals using a Bayesian approach and a partially collapsed Gibbs sampler, *IEEE Trans. Biomed. Eng.*, 2010

Outline

Modified signal model for the non-QRS intervals within a D-beat window



Modified signal model for the non-QRS intervals within a D-beat window



Modified signal model for the non-QRS intervals within a D-beat window

non-QRS signal components within a D-beat window

$$x_k = \sum_{l=-L}^{L} h_{\mathrm{T},l} u_{\mathrm{T},k-l} + \sum_{l=-L}^{L} h_{\mathrm{P},l} u_{\mathrm{P},k-l} + c_k + w_k \,, \quad k \in \mathcal{J}$$

- $u_{\mathrm{T},k} = b_{\mathrm{T},k} a_{\mathrm{T},k}$: unknown "impulse" sequence indicating T wave locations and amplitudes,
- $u_{\mathrm{P},k} = b_{\mathrm{P},k} a_{\mathrm{P},k}$: unknown "impulse" sequence indicating P wave locations and amplitudes,
- $\mathbf{h}_{\mathrm{T}} = (h_{\mathrm{T},-L} \cdots h_{\mathrm{T},L})^T$: unknown T waveform,
- $\mathbf{h}_{\mathrm{P}} = (h_{\mathrm{P},-L} \cdots h_{\mathrm{P},L})^T$: unknown P waveform,
- c_k: baseline sequence, w_k: white Gaussian noise

Modified signal model for the non-QRS intervals within a D-beat window

- Representation of the P and T waveforms by a Hermite basis expansion

$$\mathbf{h}_{\mathrm{T}} = \mathbf{H}\boldsymbol{\alpha}_{\mathrm{T}}\,, \quad \mathbf{h}_{\mathrm{P}} = \mathbf{H}\boldsymbol{\alpha}_{\mathrm{P}}\,,$$

  - $\mathbf{H}$ is a $(2L+1) \times G$ matrix whose columns are the first G Hermite functions with $G \leq (2L+1)$
  - $\boldsymbol{\alpha}_{\mathrm{T}}$ and $\boldsymbol{\alpha}_{\mathrm{P}}$ are unknown coefficient vectors of length G

- Modeling of the local baseline within the n-th non-QRS interval by a 4*th*-degree polynomial

$$\mathbf{c}_n = \mathbf{M}_n \boldsymbol{\gamma}_n\,,$$

  - $\mathbf{M}_n$ is the known $N_n \times 5$ Vandermonde matrix
  - $\boldsymbol{\gamma}_n = (\gamma_{n,1} \cdots \gamma_{n,5})^T$ is the unknown coefficient vector

Modified signal model for the non-QRS intervals within a D-beat window

vector representation of the non-QRS components

$$\mathbf{x} = \mathbf{F}_{\mathrm{T}}\mathbf{B}_{\mathrm{T}}\mathbf{a}_{\mathrm{T}} + \mathbf{F}_{\mathrm{P}}\mathbf{B}_{\mathrm{P}}\mathbf{a}_{\mathrm{P}} + \mathbf{M}\boldsymbol{\gamma} + \mathbf{w}\,, \tag{2}$$

- $\mathbf{b}_{\mathrm{T}}$, $\mathbf{b}_{\mathrm{P}}$, $\mathbf{a}_{\mathrm{T}}$, and $\mathbf{a}_{\mathrm{P}}$ denote the $M \times 1$ vectors corresponding to $b_{\mathrm{T},k}$, $b_{\mathrm{P},k}$, $a_{\mathrm{T},k}$, and $a_{\mathrm{P},k}$, respectively.
- $\mathbf{B}_{\mathrm{T}} \triangleq \mathrm{diag}(\mathbf{b}_{\mathrm{T}})$ and $\mathbf{B}_{\mathrm{P}} \triangleq \mathrm{diag}(\mathbf{b}_{\mathrm{P}})$,
- $\mathbf{F}_{\mathrm{T}}$ and $\mathbf{F}_{\mathrm{P}}$ are the $K \times M$ Toeplitz matrices with first row $\left(\mathbf{h}_1^T\boldsymbol{\alpha}_{\mathrm{T}} \;\; \mathbf{0}_{M-1}^T\right)$ and $\left(\mathbf{h}_1^T\boldsymbol{\alpha}_{\mathrm{P}} \;\; \mathbf{0}_{M-1}^T\right)$, respectively.
- $\mathbf{M}$, and $\boldsymbol{\gamma}$ are obtained by concatenating the $\mathbf{M}_n$ and $\boldsymbol{\gamma}_n$, for $n = 1, \ldots, D$.

Modified window based Bayesian model

T wave indicator prior: block constraint

$$p(\mathbf{b}_{\mathcal{J}_{\mathrm{T},n}}) = \begin{cases} p_0 & \text{if } \|\mathbf{b}_{\mathcal{J}_{\mathrm{T},n}}\| = 0 \\ p_1 & \text{if } \|\mathbf{b}_{\mathcal{J}_{\mathrm{T},n}}\| = 1 \\ 0 & \text{otherwise,} \end{cases}$$

Assuming independence between consecutive non-QRS intervals, the prior of $\mathbf{b}_{\mathrm{T}}$ is given by

$$p(\mathbf{b}_{\mathrm{T}}) = \prod_{n=1}^{D} p(\mathbf{b}_{\mathcal{J}_{\mathrm{T},n}}).$$

The priors of other parameters are defined similarly to the window based Bayesian model.

Block Gibbs sampler (BGS)

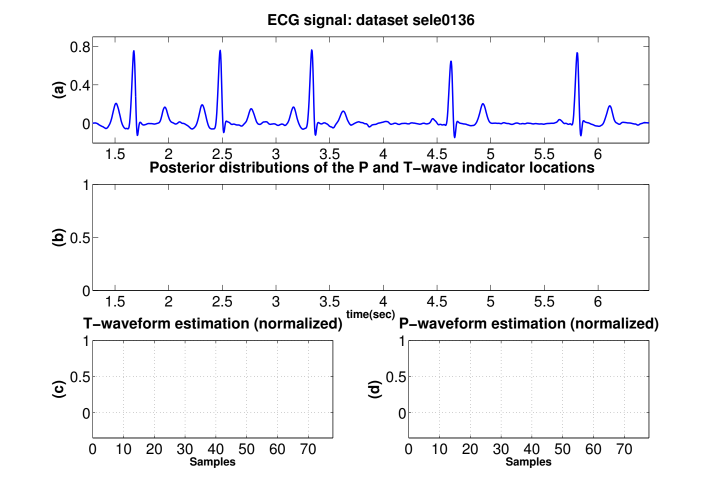
- In a D-beat processing window, for each non-QRS interval:
  - Sample the T indicator block $\mathbf{b}_{\mathcal{J}_{\mathrm{T},n}}$
  - For the k where $b_{\mathrm{T},k} = 1$, sample the T amplitudes $a_{\mathrm{T},k}$
  - Sample the P indicator block $\mathbf{b}_{\mathcal{J}_{\mathrm{P},n}}$
  - For the k where $b_{\mathrm{P},k} = 1$, sample the P amplitudes $a_{\mathrm{P},k}$
- Sample P and T waveform coefficients $\boldsymbol{\alpha}_{\mathrm{T}}$ and $\boldsymbol{\alpha}_{\mathrm{P}}$
- Sample baseline coefficients $\boldsymbol{\gamma}$
- Sample noise variance σ_w^2

C. Lin et al., P and T wave delineation and waveform estimation in ECG signals using a block Gibbs sampler, *IEEE ICASSP*, 2011

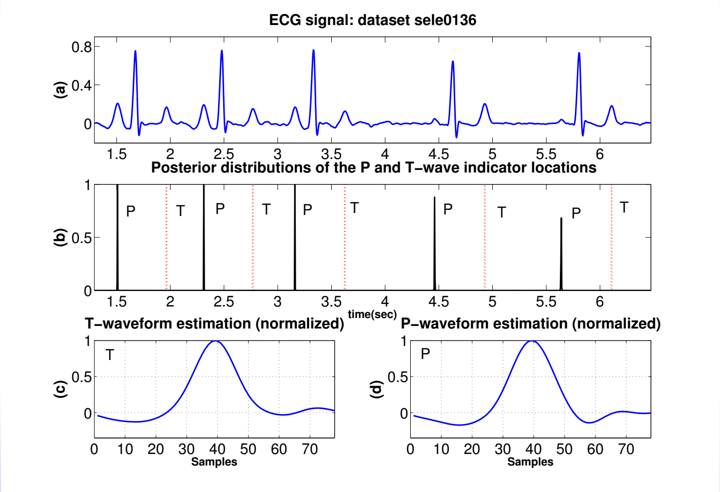
Simulation parameters

- Preprocessing: QRS complexes detection using the algorithm of **Pan et al.** (*IEEE Trans. Biomed. Eng.*, 1985)
- Processing window length: $D = 10$
- The waveform amplitude are normalized to avoid scale ambiguity
- Time-shift ambiguity is addressed by using deterministic shifts after sampling waveform coefficients
- For each estimation, the 40 first iterations are disregarded (burn-in period) and 60 iterations are used to compute the estimates
- Real ECG datasets from the QT database
- Computation time: 8 seconds to run 100 iterations on a 10-beat ECG block (Matlab implementation).

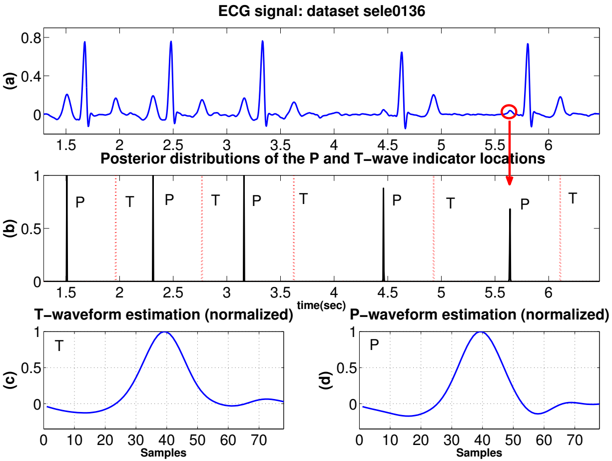
Typical examples



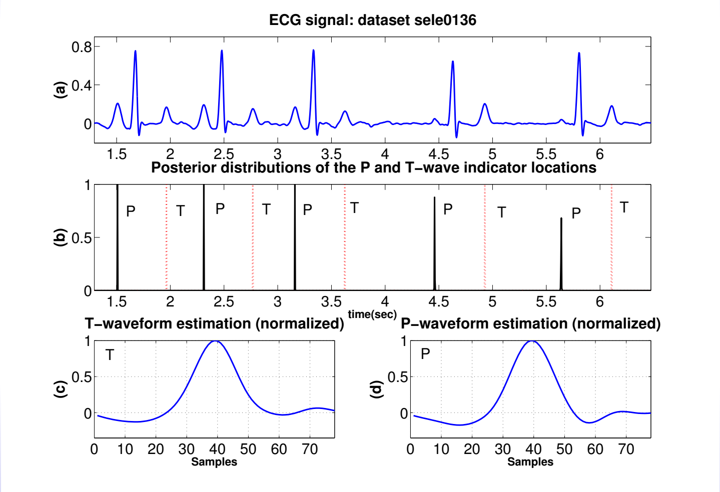
Typical examples



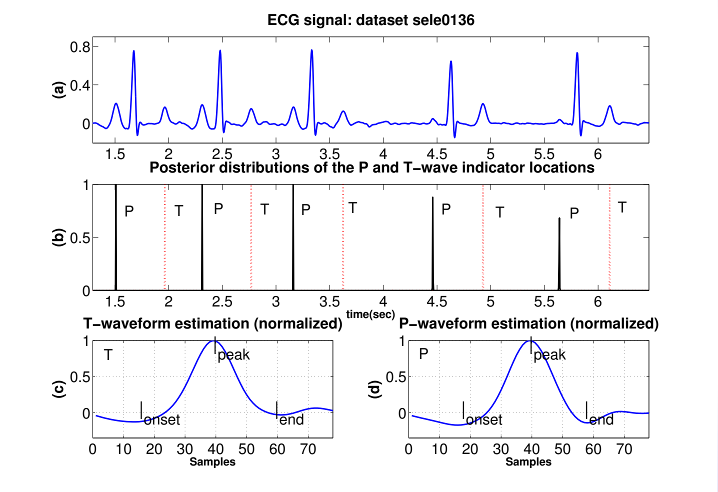
Typical examples



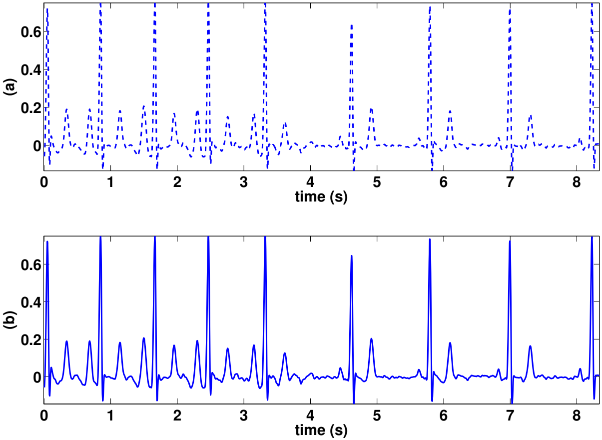
Typical examples



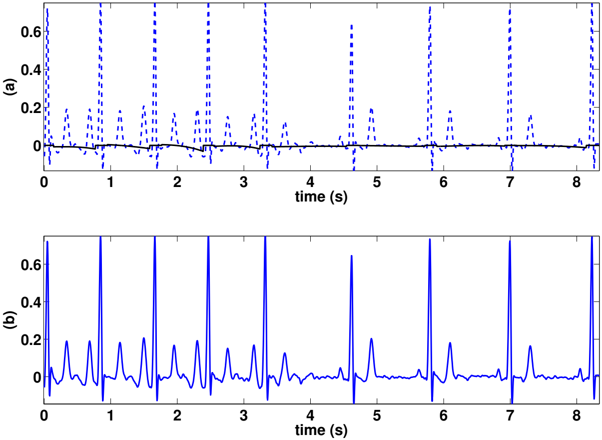
Typical examples



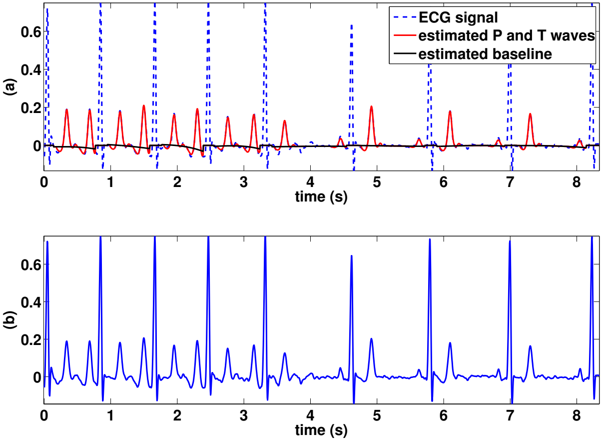
Typical examples



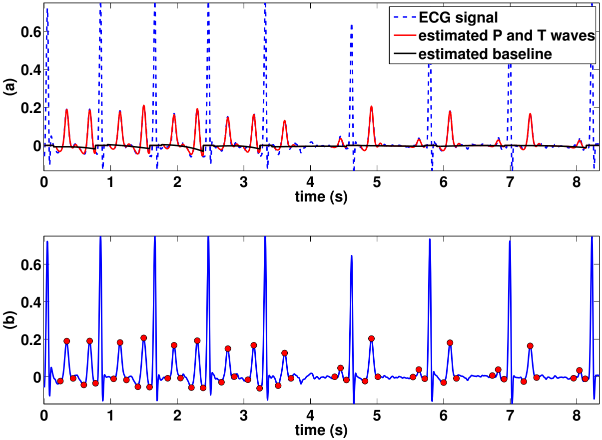
Typical examples



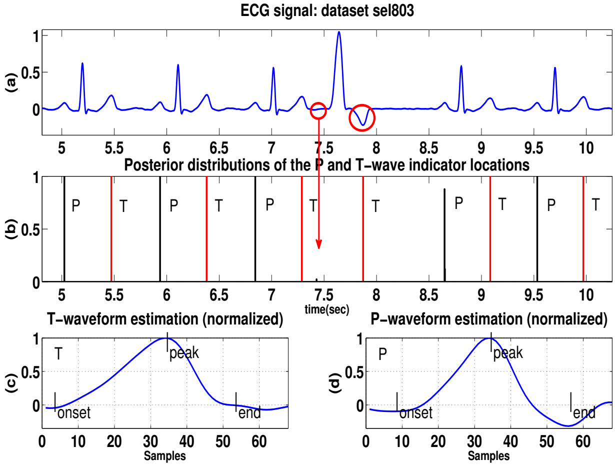
Typical examples



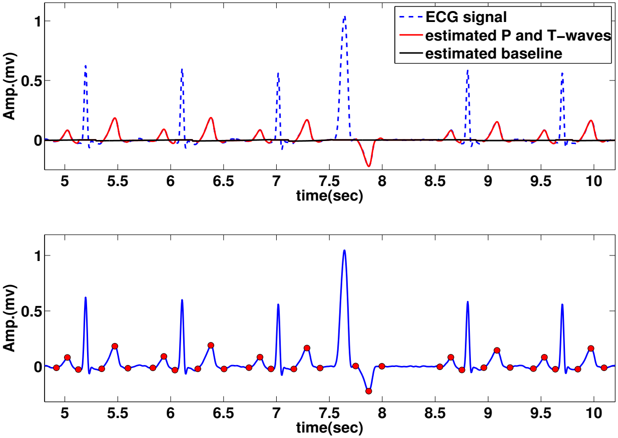
Typical examples



Premature ventricular contraction ECG



Premature ventricular contraction ECG



Evaluation on QTDB

Parameter		Window based Block GS	LPD	WT
$\mathbf{b_P}$:	Se[1] (%)	99.60	97.70	98.87
$\mathbf{b_P}$:	P^{+}[2] (%)	98.04	91.17	91.03
Onset-P:	$\mu \pm \sigma$ (ms)	1.7 ±10.8	14.0 ±13.3	2.0 ±14.8
Peak-P:	$\mu \pm \sigma$ (ms)	2.7 ±8.1	4.8 ±10.6	3.6 ±13.2
End-P:	$\mu \pm \sigma$ (ms)	2.5 ± 11.2	−0.1 ±12.3	1.9 ±12.8
$\mathbf{b_T}$:	Se (%)	100	99.00	99.77
$\mathbf{b_T}$:	P^{+} (%)	99.15	97.74	97.79
Onset-T:	$\mu \pm \sigma$ (ms)	5.7 ±16.5	N/A	N/A
Peak-T:	$\mu \pm \sigma$ (ms)	0.7 ±9.6	−7.2 ±14.3	1.2 ±13.9
End-T:	$\mu \pm \sigma$ (ms)	2.7 ± 13.5	13.5 ± 27.0	−1.6 ±18.1

[1] $\mathrm{Se} \triangleq N_{\mathrm{TP}}/(N_{\mathrm{TP}} + N_{\mathrm{FN}})$, N_{TP} is the number of true positive detections, N_{FN} is the number of false negative detections

[2] $P^{+} \triangleq N_{\mathrm{TP}}/(N_{\mathrm{TP}} + N_{\mathrm{FP}})$, N_{FP} stands for the number of false positive

Contributions and issue

Contributions

- Window based Bayesian models for simultaneous P and T wave delineation and waveform estimation
- A PCGS and a block GS to resolve the unknown parameters of the Bayesian models
- Promising delineation results on QTDB database

Unresolved issue

- ECG waveforms are homogeneous from their neighbor beats but not exactly the same
- Multi-beat processing scheme is not suitable for real-time applications

Contributions and issue

Contributions

- Window based Bayesian models for simultaneous P and T wave delineation and waveform estimation
- A PCGS and a block GS to resolve the unknown parameters of the Bayesian models
- Promising delineation results on QTDB database

Unresolved issue

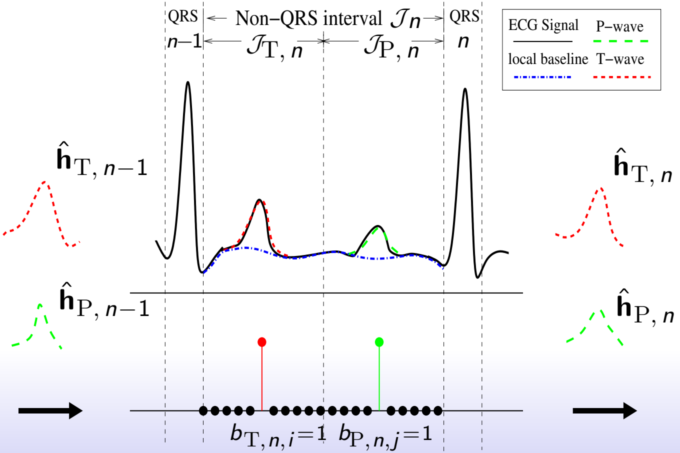
- ECG waveforms are homogeneous from their neighbor beats but not exactly the same
- Multi-beat processing scheme is not suitable for real-time applications

Solution: Beat-to-beat analysis / sequential analysis

Outline

Outline

Signal model for one non-QRS interval



Signal model for one non-QRS interval

Vector representation of the *n*th non-QRS component

$$\mathbf{x}_n = \mathbf{B}_{\mathrm{T},n}\mathbf{H}\boldsymbol{\alpha}_{\mathrm{T},n} + \mathbf{B}_{\mathrm{P},n}\mathbf{H}\boldsymbol{\alpha}_{\mathrm{P},n} + \mathbf{M}\boldsymbol{\gamma}_n + \mathbf{w}_n \tag{3}$$

- $\mathbf{x}_n = (x_{n,1} \cdots x_{n,N_n})^T$ denotes the signal portion within the *n*th non-QRS interval
- $\mathbf{B}_{\mathrm{T},n}$ is the $N_n \times (2L+1)$ Toeplitz matrix with first row $(b_{n,L+1} \cdots b_{n,1}\ 0 \cdots 0)$ and first column $(b_{n,L+1} \cdots b_{n,N_{\mathrm{T},n}}\ 0 \cdots 0)^T$
- $\mathbf{B}_{\mathrm{P},n}$ is the $N_n \times (2L+1)$ Toeplitz matrix with last row $(0 \cdots 0\ b_{n,N_n} \cdots b_{n,N_n-L})$ and last column $(0 \cdots 0\ b_{n,N_{\mathrm{T},n}+1} \cdots b_{n,N_n-L})^T$
- $\mathbf{w}_n = (w_{n,1} \cdots w_{n,N_n})^T$ denotes a white Gaussian noise with a unknown variance $\sigma^2_{w,n}$

Beat-to-beat Bayesian model

Modified T waveform prior

$$p(\boldsymbol{\alpha}_{\mathrm{T},n}|\mathbf{b}_{\mathrm{T},n}, \hat{\boldsymbol{\alpha}}_{\mathrm{T},n-1}) = \begin{cases} \delta(\boldsymbol{\alpha}_{\mathrm{T},n} - \hat{\boldsymbol{\alpha}}_{\mathrm{T},n-1}) & \text{if } \|\mathbf{b}_{\mathrm{T},n}\| = 0 \\ \mathcal{N}(\hat{\boldsymbol{\alpha}}_{\mathrm{T},n-1}, \sigma_\alpha^2 \mathbf{I}_G) & \text{if } \|\mathbf{b}_{\mathrm{T},n}\| = 1 \end{cases}$$

- $\hat{\boldsymbol{\alpha}}_{\mathrm{T},n-1}$ is the estimate of the T waveform coefficient vector associated with the previous non-QRS interval $\mathcal{J}_{n-1}$
- $\mathbf{I}_G$ is the identity matrix of size $G \times G$

The priors of other parameters are defined similarly to the modified window based Bayesian model.

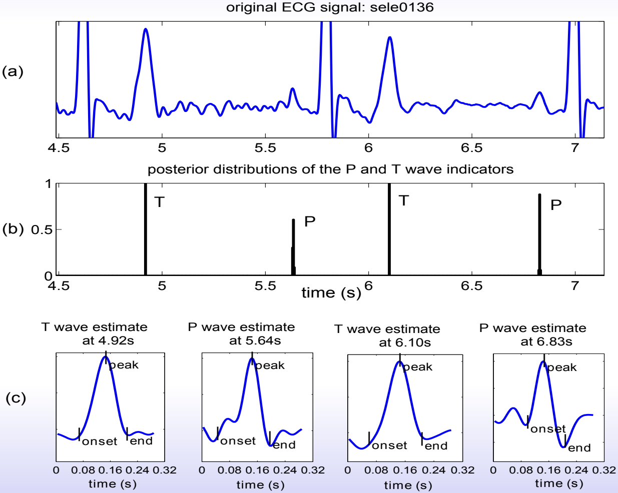
Beat-to-beat block Gibbs sampler

The block Gibbs sampler for the nth non-QRS interval $\mathcal{J}_n$:

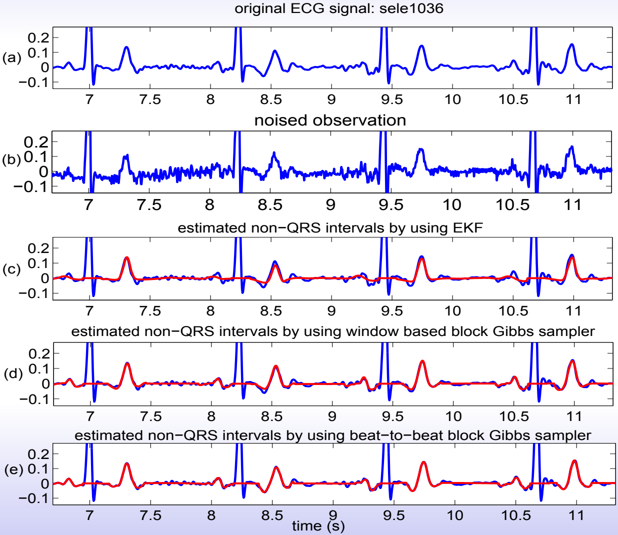
- Sample the T wave indicator block $\mathbf{b}_{\mathrm{T},n}$
- Sample the T waveform coefficients $\boldsymbol{\alpha}_{\mathrm{T},n}$
- Sample the P wave indicator block $\mathbf{b}_{\mathrm{P},n}$
- Sample the P waveform coefficients $\boldsymbol{\alpha}_{\mathrm{P},n}$
- Sample the baseline coefficients $\boldsymbol{\gamma}_n$
- Sample the noise variance σ_w^2

C. Lin et al., Endocardial T wave alternans detection using a beat-to-beat Bayesian approach and a block Gibbs sampler, *IEEE Trans. Biomed. Eng.*, 2012, to be submitted

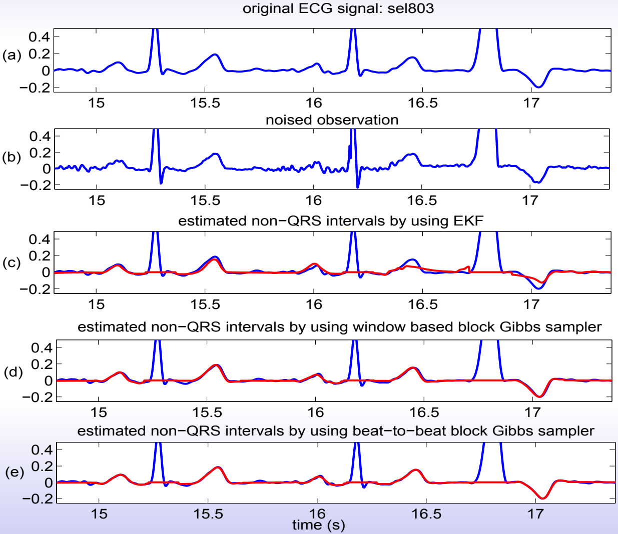
Typical example



Qualitative comparisons



Qualitative comparisons



Outline

A marginalized particle filter

Measurement equation of the nth T wave interval

$$\mathbf{x}_{\mathrm{T},n} = \mathbf{B}_{\mathrm{T},n}\mathbf{H}\boldsymbol{\alpha}_{\mathrm{T},n} + \mathbf{w}_n$$

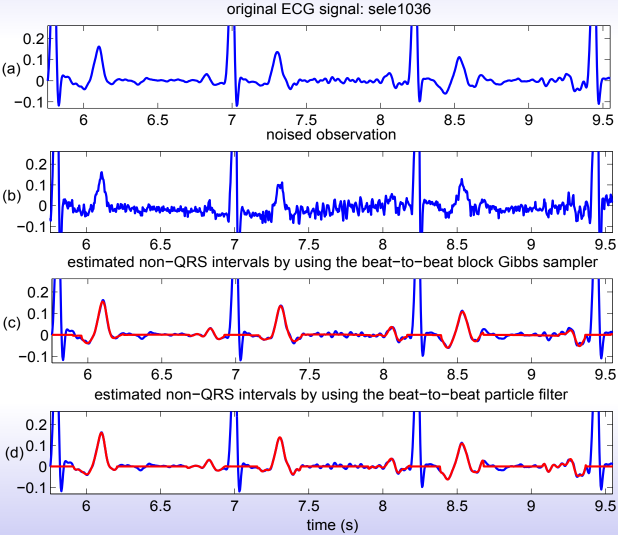
- $\mathbf{x}_{\mathrm{T},n} = (x_{n,1} \cdots x_{n,N_{\mathrm{T},n}})^T$ denotes the T wave interval within the nth non QRS interval

Marginalization of the state variables

$$p(\mathbf{b}_{0:n}, \boldsymbol{\alpha}_n | \mathbf{x}_{1:n}) = \underbrace{p(\boldsymbol{\alpha}_n | \mathbf{b}_{0:n}, \mathbf{x}_{1:n})}_{\text{Optimal KF}} \underbrace{p(\mathbf{b}_{0:n} | \mathbf{x}_{1:n})}_{\text{PF}}$$

F. Gustafsson et al., Marginalized Particle Filters for Mixed Linear/Nonlinear State-space Models, *IEEE Trans. Signal processing*, 2005
C. Lin et al., Beat-to-beat P and T wave delineation in ECG signals using a marginalized particle filter, *EUSIPCO*, 2012

Qualitative comparisons



Quantitative comparison on QTDB

Parameter		Beat-to-beat MPF	Beat-to-beat Block GS	Window based Block GS
$\mathbf{b_P}$:	Se (%)	99.95	99.93	99.60
$\mathbf{b_P}$:	P^+ (%)	99.23	99.10	98.04
Onset-P:	$\mu \pm \sigma$ (ms)	1.1 $\pm$8.3	3.4 $\pm$14.2	1.7 $\pm$10.8
Peak-P:	$\mu \pm \sigma$ (ms)	1.2 $\pm$5.3	1.1 $\pm$5.3	2.7 $\pm$8.1
End-P:	$\mu \pm \sigma$ (ms)	1.7 $\pm$ 9.8	-3.1 $\pm$9.8	2.5 $\pm$11.2
$\mathbf{b_T}$:	Se (%)	100	100	100
$\mathbf{b_T}$:	P^+ (%)	99.20	99.30	99.15
Onset-T:	$\mu \pm \sigma$ (ms)	5.5 $\pm$16.3	6.8 $\pm$ 19.3	5.7 $\pm$ 16.5
Peak-T:	$\mu \pm \sigma$ (ms)	-0.4 $\pm$4.8	-0.8 $\pm$14.0	0.7 $\pm$9.6
End-T:	$\mu \pm \sigma$ (ms)	-1.8 $\pm$ 14.2	-3.8 $\pm$ 14.0	2.7 $\pm$13.5

Contributions and applications

Contributions

- A beat-to-beat Bayesian approach which leads to smaller memory requirements and a lower computational complexity compared to window based approaches
- Ideally suited for real-time ECG monitoring and for on-line pathology analysis
- A dynamic model which exploits the sequential nature of the ECG
- A marginalized particle filter which considers all the available beats in the waveform estimation

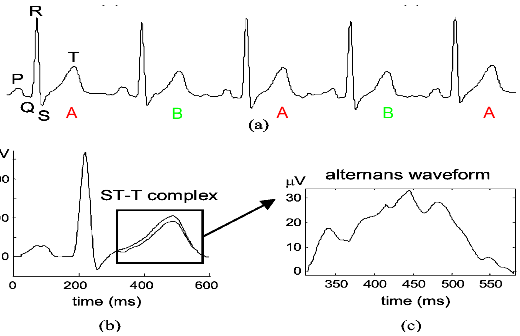
Applications

- ECG interval analysis
- Pathology analysis: T wave alternans detection

Outline

Outline

T-wave alternans (TWA) detection



- TWA: a consistent fluctuation in the T waves on an every-other-beat basis (A-B-A-B-…)
- A challenging problem: non-visible (microvolt-level) TWA detection

T-wave alternans (TWA) detection

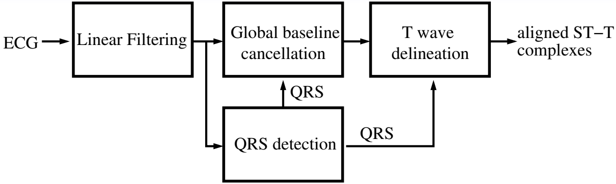


Figure: General TWA preprocessing stage

- Residual local baseline problematic for TWA detection
- T-wave delineator must show inter-beat stability in the fiducial point determination
- TWA waveform analysis

T-wave alternans (TWA) detection

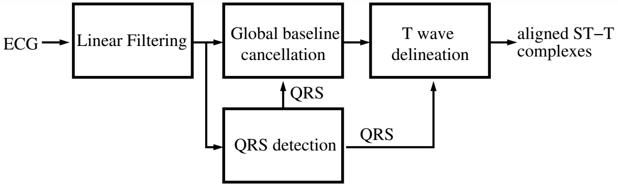
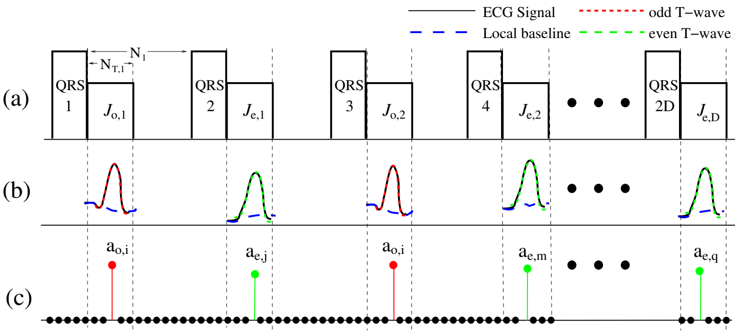


Figure: General TWA preprocessing stage

- Residual local baseline problematic for TWA detection
- T-wave delineator must show inter-beat stability in the fiducial point determination
- TWA waveform analysis

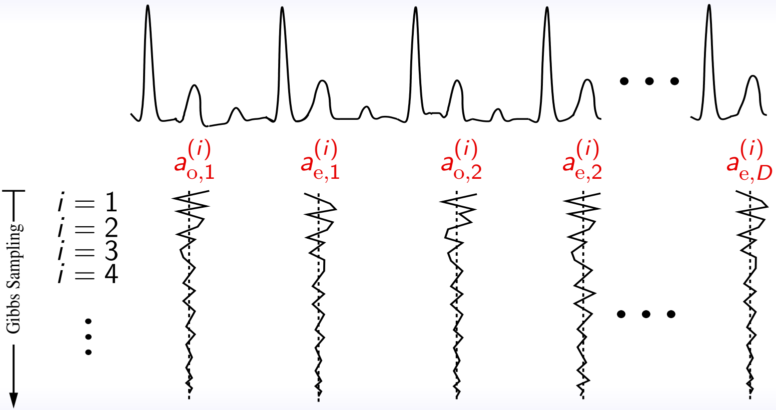
The proposed Bayesian approaches serve as a preprocessing step for TWA detection

Window based Bayesian model for TWA detection in surface ECG



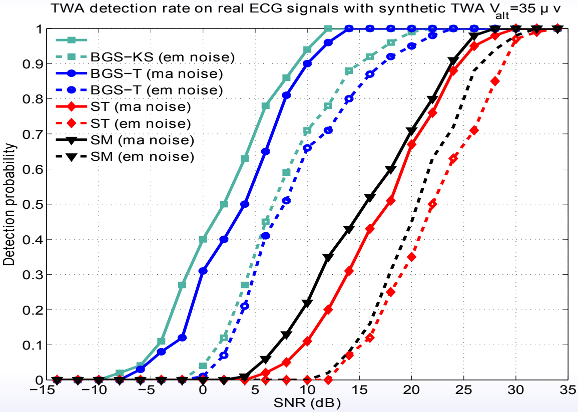
C. Lin et al., T-wave Alternans Detection Using a Bayesian Approach and a Gibbs Sampler, *IEEE EMBC*, 2011

Block Gibbs sampler



Multiple test statistics resulting from the Gibbs sampling can be used to derive reliability information.

Detection performance comparison



ST: **T. Srikanth et al.**, Presence of T wave alternans in the statistical context, 2002

SM: **J. M. Smith et al.**, Electrical alternans and cardiac electrical instability, 1994

Outline

ETWAS project

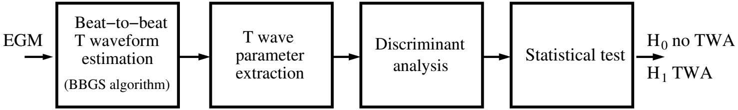
Endocardial T-wave Alternans Study (ETWAS) project

- Collaboration with **St. Jude Medical** and **Rangueil Hospital**
- To assess the feasibility of TWA detection in intracardiac electrograms (EGMs) stored in implantable cardioverter defibrillators
- Pre-onset episode signals and control reference signals are available

Endocardial TWA detection limitations:

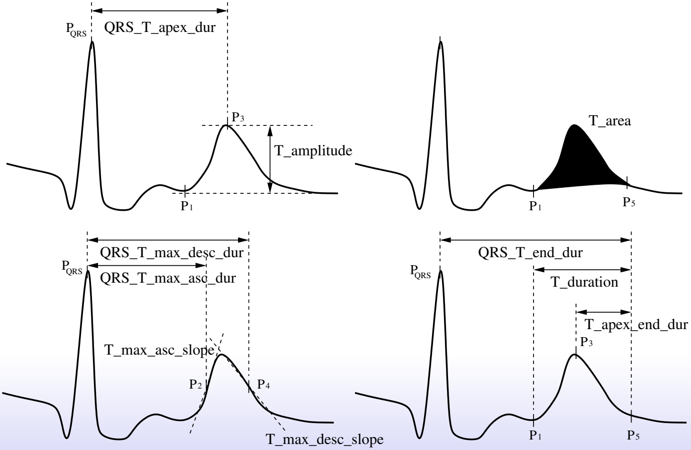
- Very short periods of recordings available (usually 10 to 30 beats)
- Other patterns (A-B-C-A-B-C-$\cdots$) rather than A-B-A-B-$\cdots$
- Perspective defibrillator implementation: real-time processing

Endocardial TWA detection using the beat-to-beat Bayesian approach



- Beat-to-beat block Gibbs sampler to estimate the T waveforms
- 10 T wave parameters defined by a cardiologist
- Discriminant analysis to reduce the dimensionality (Fisher score)
- Univariate and multivariate statistical tests (*t*-test, KS-test, Wilcoxon-test)

T wave parameters



Multivariate TWA detection

$10 \times D$ difference matrix For a $D+1$ beat EGM signal portion

$$\boldsymbol{\Delta} = \{\delta_{p,n}\}_{p=1,\cdots,10,\, n=1,\cdots,D}$$

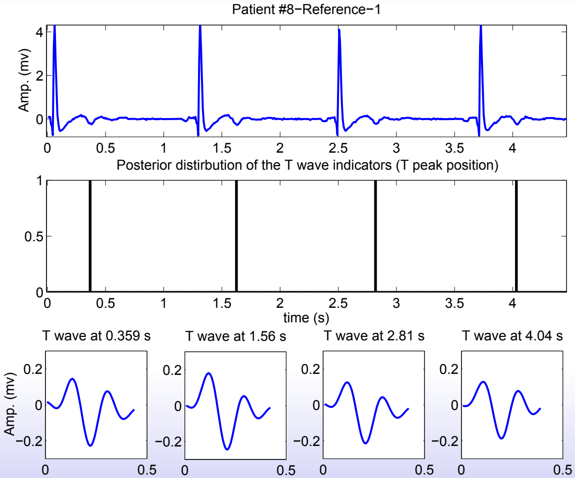
- $\delta_{p,n}$ represents the absolute difference of the pth parameter between beats n and $n+1$

TWA detection is formulated as a multivariate two-class problem:

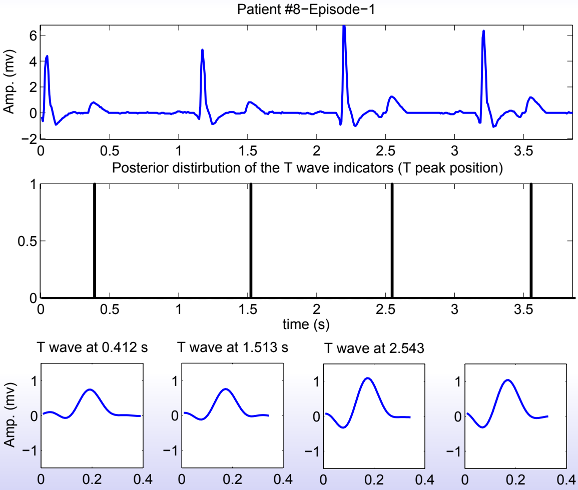
$$\mathcal{H}_0 : \text{No significant beat-to-beat wave parameter variation.}$$
$$\mathcal{H}_1 : \text{Significant beat-to-beat wave parameter variation.}$$

C. Lin et al., Endocardial T wave alternans detection using a beat-to-beat Bayesian approach and a block Gibbs sampler, *IEEE Trans. Biomed. Eng.*, 2012, to be submitted

One reference EGM portion



One episode EGM portion



Fisher score of T wave parameters

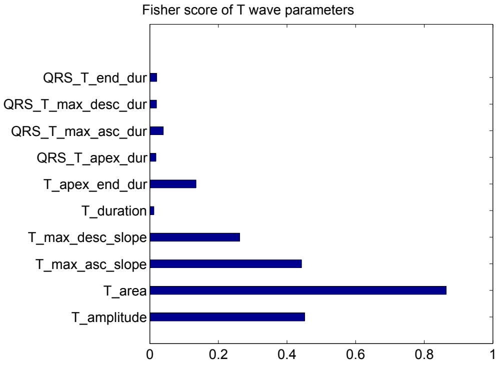


Figure: Fisher score of beat-to-beat parameter variations between reference and episode signals of patient ♯8

Beat-to-beat variation box-and-whisker diagram

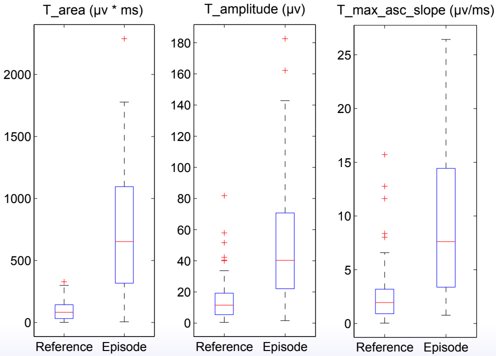


Figure: Beat-to-beat variation box-and-whisker diagram of the three most discriminant parameters of patient ♯8

Statistical test results

Table: Statistical test results on reference and episode signals of patient ♯8.

Parameter	normalized Fisher score	cumulative Fisher score	t-test	KS-test	Wilcoxon-test	Multivariate t-test
T_area	0.3814	0.3814	H_1	H_1	H_1	H_1
T_amplitude	0.1995	0.5809	H_1	H_1	H_1	
T_max_asc_slope	0.1953	0.7763	H_1	H_1	H_1	
T_max_desc_slope	0.1158	0.8920	H_1	H_1	H_1	
T_apex_end_dur	0.0596	0.9516	H_1	H_1	H_1	
QRS_T_max_desc_dur	0.0174	0.9690	H_1	H_1	H_1	
QRS_T_end_dur	0.0091	0.9781	H_1	H_1	H_1	
QRS_T_max_asc_dur	0.0087	0.9868	H_0	H_1	H_1	
QRS_T_apex_dur	0.0079	0.9947	H_0	H_1	H_1	
T_duration	0.0053	1.0000	H_0	H_1	H_1	

Outline

Conclusions

- Bayesian models based on a multiple-beat processing window which simultaneously solves the P and T wave delineation and the waveform estimation problems
  - A PCGS with minimum distance constraint
  - A block GS with block constraint
- Bayesian model that enables P and T wave delineations and waveform estimation on a beat-to-beat basis
  - A beat-to-beat block GS
  - Dynamical model issued from the same Bayesian framework and particle filters
- Applications of the different Bayesian models to T wave alternans detection
  - TWA detection in surface ECG signals by using the window based Bayesian model
  - Endocardial TWA detection in ICD stored by using the beat-to-beat Bayesian approach

Perspectives

- Preprocessing tools for other P and T wave pathology analysis problems
    - Arrhythmia detection
    - P wave morphology classification
- Extension to multi-lead surface ECG recordings
    - Bayesian model for multi-lead ECG
    - Data fusion

Thank you for your attention!

Low-pass differentiation (LPD)

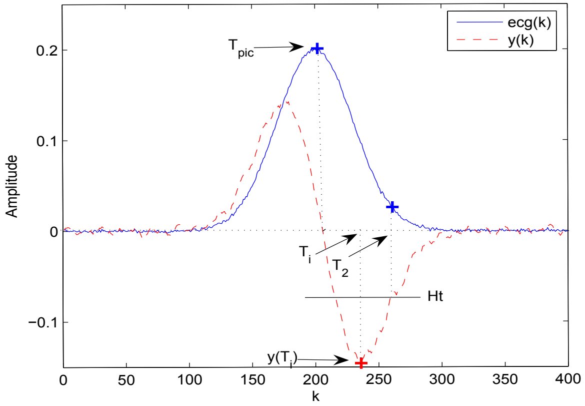
ECG → low-pass filter $G_1(z)$ → derivative filter $G_2(z)$ → wave delineation

$$G_1(z) = \frac{1 - z^{-8}}{1 - z^{-1}}, \quad G_2(z) = 1 - z^{-6}$$

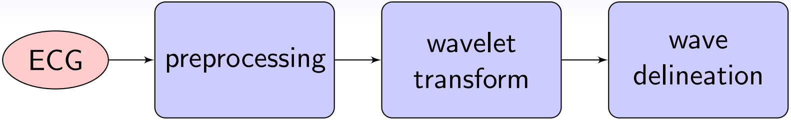
- Advantages: simple to implement, robust to waveform variations
- Drawbacks: sensitive to noise, arbitrary thresholds

P. Laguna et al., New algorithm for QT interval analysis in 24 hour Hotler ECG: Performance and applications. *Med. Biological Eng. and Comput.*, 1990

Low-pass differentiation (LPD)



Wavelet transform (WT)



WT of a signal $x(t)$:

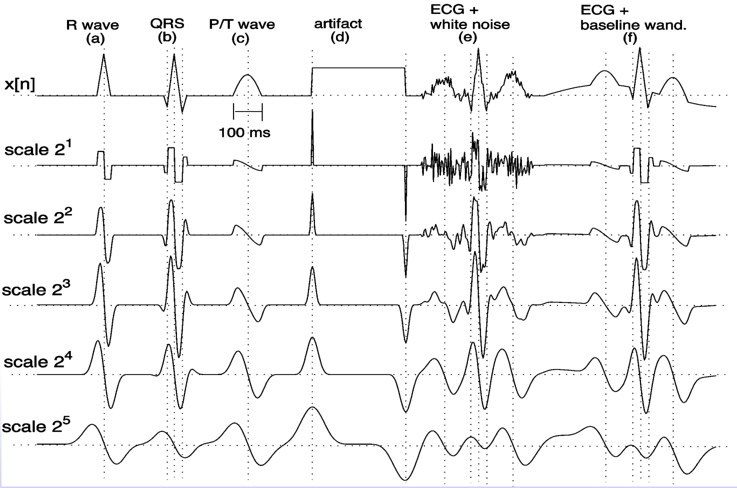
$$W_a x(b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{+\infty} x(t)\, \psi\left(\frac{t-b}{a}\right) \mathrm{d}t,\ \ a > 0$$

Discretization of the dilatation factor $a = 2^k$ and the translation parameter $b = 2^k l$ to form a discrete wavelet transform (DWT):

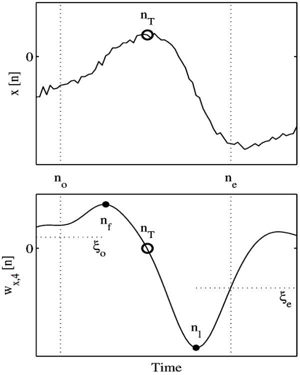
$$\psi_{k,l}(t) = 2^{-k/2}\psi(2^{-k}t - l),\ \ k, l \in \mathbf{Z}^+$$

J. P. Martínez, et al., A Wavelet-based ECG delineator: Evaluation on standard databases. *IEEE Trans. Biomed. Eng.*, 2004

Wavelet transform (WT)

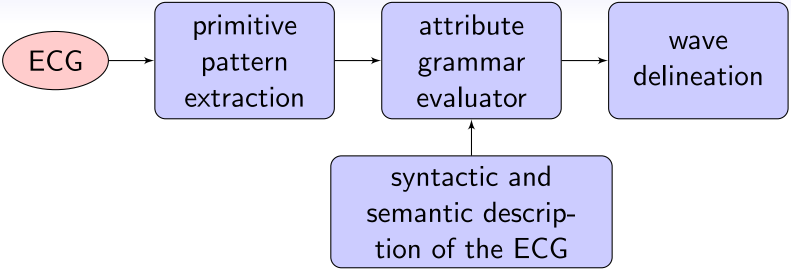


Wavelet transform (WT)



- Advantages:
  - suitable to locate different waves with typical frequency characteristics
- Drawbacks:
  - require *a priori* information on the waveform and width
  - rigid arbitrary thresholds to determine the significance of the wave components

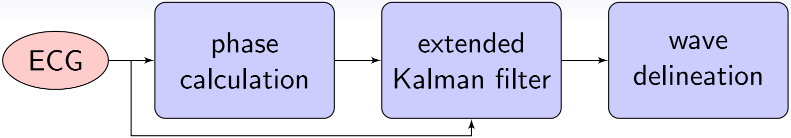
Pattern Recognition



- Advantages: syntactic approach, simple to implement
- Drawbacks: insufficient delineation accuracy, sensitive to noise

P. Trahanias et al., Syntactic Pattern Recognition of the ECG. *IEEE Trans. Pattern Anal. Mach. Intell.*, 1990

Extended Kalman filter

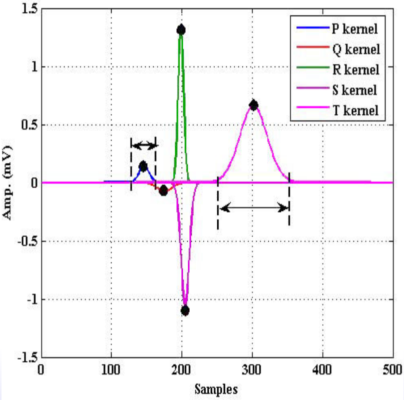


A dynamic Gaussian mixture model to fit ECG:

$$\begin{cases} \theta_{k+1} = \theta_k + \omega\delta \\ z_{k+1} = -\displaystyle\sum_{j \in P,Q,R,S,T} \frac{\alpha_j \omega \delta}{b_j^2} \Delta\theta_j \exp\left(-\frac{\Delta\theta_j^2}{2b_j^2}\right) + z_k + \eta_k \end{cases}$$

O. Sayadi et al., A model-based Bayesian framework for ECG beat segmentation. *J. Physiol. Meas.*, 2009

Extended Kalman filter



- Advantages:
  - sequential Bayesian approach
  - light computational load
- Drawbacks:
  - number of Gaussian kernels known *a priori*
  - difficulties on handling abnormal rhythms

PCGS principle

The PCGS is an extension of the Gibbs sampler.

- **Marginalization**: marginalize some subsets of $\boldsymbol{\theta}$ out of some steps of the sampler
- **Trimming**: discard a subset of the components that were to be sampled in one or more steps of a Gibbs sampler
- **Permutation**: reorder Gibbs sampling steps into different permutations

The PCGS is flexible regarding the choice of the sampling distributions, especially when there are strong dependencies among certain subsets of $\boldsymbol{\theta}$.

D. A. Van Dyk and T. Park, Partially collapsed Gibbs samplers: Theory and methods, *J. Acoust. Soc. Amer.*, 2008

Time-shift and scale ambiguities

Issue: No unique solution for a convolution model

- Scale ambiguity: $\mathbf{h} \star \mathbf{u} = (a\mathbf{h}) \star (\mathbf{u}/a), \ \forall a \neq 0,$
- Time-shift ambiguity: $\mathbf{h} \star \mathbf{u} = (d_\tau \star \mathbf{h}) \star (d_{-\tau} \star \mathbf{u}), \ \forall \tau \in \mathbb{Z}.$

Solution: Hybrid Gibbs sampling

- Metropolis-Hastings within Gibbs after sampling waveform coefficients,
- Deterministic shifts after sampling waveform coefficients:
  - Time-shifts to have $h'_0 = \max |\mathbf{h}|$,
  - Scale-shifts to have $h'_0 = 1$,

C. Labat et al., Sparse blind deconvolution accounting for time-shift ambiguity, *ICASSP*, 2006

Delineation criteria

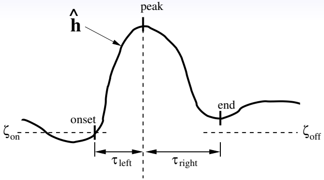
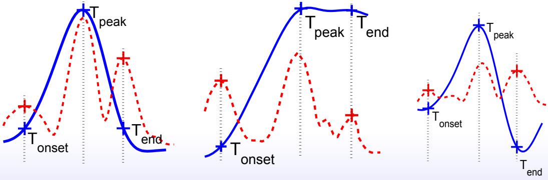


Figure: Wave delineation based on the waveform



(a) (b) (c)

Figure: Wave delineation based on the waveform curvature

Boundary issue between intervals

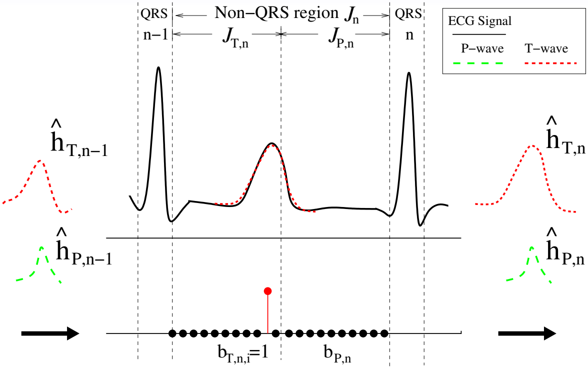


Figure: An example of the boundary problem with PVC signal

Spectral methods for TWA detection

Consider an aligned ST-T complexes matrix of a $2D$-beat window:

$$T = \begin{pmatrix} T_1(1) & T_1(2) & \dots & T_1(N) \\ \vdots & \vdots & \ddots & \vdots \\ T_{2D}(1) & T_{2D}(2) & \dots & T_{2D}(N) \end{pmatrix}$$

Spectral analysis by using periodogram:

$$\widehat{S}_n(f) = \frac{1}{2D} \left|\mathrm{TF}(T_k(n))\right|^2, \; k = 1, \dots, D$$

$$\frac{\frac{1}{N}\sum_{n=1}^{N} \widehat{S}_n(0.5) - \mu}{\sigma} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \gamma$$

Drawbacks: large window size ($2D \geq 128$), sensitive to noise

Statistical test for TWA detection

T-wave amplitudes are estimated as follows:

$$
\begin{aligned}
a_i &= \max(T_i(1), T_i(2), \ldots, T_i(N)) \\
\mu_{\text{odd}} &= \frac{1}{D}\sum_{n=1}^{D} a_i,\ \ i = 1, 3, \ldots, 2D-1 \\
\mu_{\text{even}} &= \frac{1}{D}\sum_{n=1}^{D} a_i,\ \ i = 2, 4, \ldots, 2D
\end{aligned}
$$

The statistical test can be formalized as:

$$
\mathcal{H}_0 : \mu_{\text{odd}} = \mu_{\text{even}}, \quad \mathcal{H}_1 : \mu_{\text{odd}} \neq \mu_{\text{even}}
$$

Drawbacks: rough amplitude estimation, strong hypothesis on the distribution, analysis window size