Tight Integration of GNSS and a 3D City Model for Robust Positioning in Urban Canyons

A. Bourdeau[1], M. Sahmoudi[1], J.-Y. Tourneret[2]
[1]*Institut Supérieur de l'Aéronautique et de l'Espace (ISAE/SUPAERO), TéSA, Toulouse, France*
[2]*Université de Toulouse (ENSEEIHT/TéSA), Toulouse, France*

BIOGRAPHIES

Aude Bourdeau received an engineer degree in mathematics and numerical modeling from INSA Toulouse in 2010. Since October 2010, she is preparing PhD at the French Institute of Aeronautics and Space (ISAE). Her research interest includes signal processing, GNSS navigation in challenging environment and GNSS signal tracking.

Mohamed Sahmoudi received a PhD in signal processing and communications from Paris Sud University and Telecom Paris in 2004, and an M. S. degree in statistics from Pierre and Marie Curie University in 2000. During his PhD, he was an assistant lecturer at Ecole Polytechnique, then a lecturer at Paris Dauphine University. From 2005 to 2007, he was a post-doc researcher on GPS signal processing at Villanova University, PA, USA. In august 2007, he joined the ETS School of Engineering at Montreal, Canada, to work on GNSS RTK for precise positioning. In december 2009, he became an associate professor at the French Institute of Aeronautics and Space (ISAE), Toulouse, France. His research interest includes weak multi-GNSS signals processing, multipath mitigation and multi-sensor fusion.

Jean-Yves Tourneret (SM'08) received the Ingénieur degree in electrical engineering from the ENSEEIHT (Ecole Nationale Supérieure d'Electronique, d'Electrotechnique, d'Informatique, d'Hydraulique, et des Télécommunications de Toulouse), France, in 1989 and the Ph.D. degree from the National Polytechnic Institute, Toulouse, France, in 1992. He is currently a Professor in the University of Toulouse (ENSEEIHT), France, and a member of the IRIT laboratory (UMR5505 of the CNRS). His research activities are centered around statistical signal processing, with a particular interest to Bayesian and Markov chain Monte Carlo methods. Dr. Tourneret has been involved in the organization of several conferences, including the European Conference on Signal Processing (EUSIPCO) in 2002 (as the program chair), the International Conference on Acoustics, Speech and Signal Processing (ICASSP) in 2006 (in charge of plenaries) and the Statistical Signal Processing Workshop (SSP) in 2012 (for international liaisons). He has been a member of different technical committees, including the Signal Processing Theory and Methods (SPTM) Committee of the IEEE Signal Processing Society from 2001 to 2007 and from 2010 to present. He served as an Associate Editor for the IEEE TRANSACTIONS ON SIGNAL PROCESSING from 2008 to 2011.

ABSTRACT

Positioning and navigation by GNSS in urban context are always challenging tasks, because of signal propagation problems such as shadowing effects and multipath. When not enough GNSS signals are received in line-of-sight (LOS), classical approaches mitigating multipath effects become insufficient because there is not enough reliable information available. Consequently, positioning errors can be about tens of meters, especially in urban canyons.

In this paper, we introduce a GNSS positioning approach that uses constructively non-line-of-sight (NLOS) signals in order to have enough information to compute the user's position. In this work, we use the SE-NAV software to predict the geometric paths of NLOS signals using a high realistic 3D model of the environment. More precisely, we propose a new version of the extended Kalman filter augmented by the information provided by SE-NAV, referred to as 3D AEKF, for GNSS navigation in NLOS context. In the proposed approach, the measurement model traditionally based on the trilateration equations is constructed from the received paths estimated by SE-NAV. The Jacobian of the measurement model is calculated through knowledge of the objects on which the reflections have occured. To use even less reliable measurements, we propose a robust version of the 3D AEKF. Simulations conducted in realistic scenarios allow the performance of the proposed method to be evaluated.

INTRODUCTION

The number of global navigation satellite system (GNSS) applications has steadily increased over the last decades, in particular for personal mobility (e.g., GNSS-enabled mobilephones, smartphones and services). Intelligent systems of transportation are also an important segment of the GNSS market including in-car navigation and road user charging. However, the urban environment presents significant challenges for satellite positioning. On the one hand, the user is expecting for a positioning accuracy greater than that obtained in open sky areas, because of the proximity of the various points of interest and intersections. On the other hand, the urban environment creates difficulties in the GNSS signal reception, particularly because of satellite masking and multipath phenomena. As a consequence, the receiver delivers a position that can be biased by an error of several tens of meters [1], [2], when it is not totally impossible to calculate a position. This is particularly true in the context of urban canyons, i.e., when the streets are very narrow and/or the buildings are very high.

The main undesirable phenomena encountered in urban areas are attenuations, multipath and shadowing effects. Multipath propagation occurs when GNSS signals bounce off buildings and reach the receiver's antenna via different paths with a travelling time longer than that of the LOS path. Multipath signals can be very strong and have small relative delays which makes them difficult to be distinguished from the desired path signal. Examples of efficient in-receiver multipath mitigation methods include the narrow correlator, the strobe correlator, the multipath estimating delay lock loop, the multipath elimination technology, the vision correlator and the fast iterative maximum-Likelihood algorithm (see [1], [3]–[5] for more details). However, in urban canyon environment, the number of LOS satellites is very low and the position dilution of precision (PDOP) of these satellites is usually unsatisfactory. We suggest in this work to investigate the constructive use of multipath signals instead of simply mitigating those reflections as in most current GNSS receivers. If the user is in an NLOS context, the receiver will consider the received GNSS signal as a LOS and will estimate a pseudorange from the satellite biased by up to several tens of meters. It has been recognized that a stand-alone GNSS receiver is not enough to provide reliable location service in severely obstructed signal conditions. To deal with these difficulties, the GNSS receiver can be assisted by several sources of information such as inertial navigation sensors, wireless network or vision devices, requiring additional infrastructure and complex hybridization technologies. Another possibility is to exploit all the available information for improving the positioning performance in these harsh environments. One solution consists of comparing visible satellites with an a priori knowledge of the shadowed satellites [6]. Another solution is to use NLOS constructively rather than just deleting them. However, the difficulty in using NLOS signals is the capability of modelling the length of the indirect paths. Without this knowledge, it is difficult to correct the distance error carried by the signal that has undergone multipath. In [7] a geometric path model is used, whose parameters are estimated by a nonlinear filter. In [8] and [9], paths are calculated by laser scanning of the environment. In [10], we have proposed a new navigation strategy based on the augmentation of GNSS measurements by a 3D model of the environment. This approach tightly integrates the 3D model information in an extended Kalman filter (EKF) for positioning computation.

In this paper, we adapt the ideas presented in [10] in a high realistic simulation using a 3D model of Toulouse downtown. The SE-NAV software [11] is used to predict the signal reception of systems such as GPS and GALILEO into 3D virtual scenes of known urban areas. This software is based on a geometrical ray-tracing algorithm that computes the shadowing effects and the multipath generated by the objects of a given environment. To use even less reliable measurements, a robust version of the proposed filter is also introduced. The robust approach proposed is adapted to the special context of urban canyons and to the tight integration of GNSS and the 3D city model.

PROBLEM FORMULATION

GNSS positioning is based on the geometrical principle of trilateration using radio waveforms received from satellites. After computing the satellite positions using the navigation message and estimating the ranges between the receiver and the satellites, we can compute the user's position with a simple least squares algorithm or a Kalman filter [1].

In radio positioning, one of the dominant limitation factors is the NLOS that happens when the direct path between the transmitter and receiver is blocked, such as in dense urban environments. NLOS signals travel a longer distance and thus are characterized by a longer propagation time with power reduction and angle bias. The identification of NLOS is challenging and permits to discard NLOS measurements when there are enough measurements identified as LOS signals. Our interest is focused on harsh situations when there are less than four LOS signals available at the receiver. Our objective is to exploit these NLOS signals. In recent work NLOS signals have been processed jointly with LOS measurements (with larger weights for LOS Signals). We propose to estimate directly the NLOS measurements by using 3D modeling [10].

If we use an EKF to compute the position, a problem is the Jacobian matrix necessary to compute the update of the position. To calculate this matrix, we have to know the derivative of the function which gives the measurement as function of the receiver position. Receiver stand-alone is not able to determine the bias of a NLOS signal and moreover its evolution in space. The multipath trajectory estimation needs the knowledge of the receiver geometric environment and in particular the plans on which the signal could have been reflected. We have chosen to obtain this knowledge from a 3D model of the environment, developed by a specialized compagny.

In [10], we introduced a positioning approach exploiting NLOS GPS signals, based on the integration of a 3D model in the navigation algorithm. We showed that this method gives better results than a robust EKF alone in the context of a simple simulated 3D model. The main contribution of this paper is to consider a more realistic 3D model of the reception environment in order to test the performance of the algorithm introduced in [10] for real data. All simulations will be conducted using the SE-NAV software [11].

Simulating GNSS Signals with a 3D Model

The SE-NAV software has been developed by the company OKTAL-SE to simulate GNSS signal reception in stringent environment. SE-NAV simulates the propagation of a GNSS signal in a 3D virtual scene, using a ray-tracing algorithm to compute the shadowing effects and the multipath generated by objects of the environment. It uses geometric optics to calculate reflected, diffracted and transmitted rays. In this paper, only the reflected signals are considered for reason of simplicity. Fig. 1 presents a SE-NAV simulation of reflected signals in Toulouse downtown. Signals in white are LOS signals and signals in blue or red are multipath signals.

3D virtual scenes can be loaded in SE-NAV from the most classical 3D formats if the environment is represented as plans. For this paper, we have worked with a scene provided by the SE-NAV software, which corresponds to Capitole Square in Toulouse. After the configuration of the 3D virtual scene, SE-NAV can take as inputs the satellite and receiver positions and speeds. The software outputs are the geometrical configurations of received signals and Dopplers for all signals. If the received signal is a multipath, SE-NAV provides also the coordinates of the reflection points and the equations of the reflection plans.

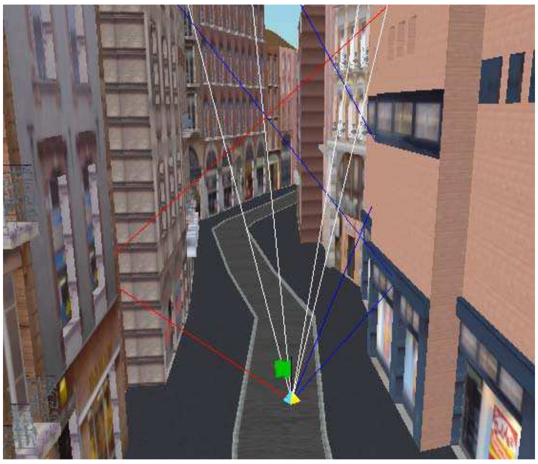

Figure 1 – SE-NAV simulation in Toulouse downtown by OKTAL-SE.

In summary, the SE-NAV simulator provides deterministic geometric information for the received GNSS signals at a certain position and for a given time instant. Note that the physical aspect of signal propagation is not considered in this paper.

CONSTRUCTIVE USE OF NLOS SIGNALS

Geometric Modeling of NLOS

To be able to compute the Jacobian matrix of the measurements, we have to express the NLOS paths as functions of the receiver position. For this purpose, we adopt temporarily the notation of Fig. 2, where S and R are the satellite and receiver positions. I_1 and I_2 are the reflection points on walls 1 and 2 respectively. The walls are defined by their normal vectors N_1 and N_2 and their coefficients d_1 and d_2, through the equations

$$\begin{cases} N_x^1 X + N_y^1 Y + N_z^1 Z + d_1 = 0 \\ N_x^2 X + N_y^2 Y + N_z^2 Z + d_2 = 0 \end{cases} \quad (1)$$

N_1, N_2, d_1 and d_2 can be determined thanks to SE-NAV information.

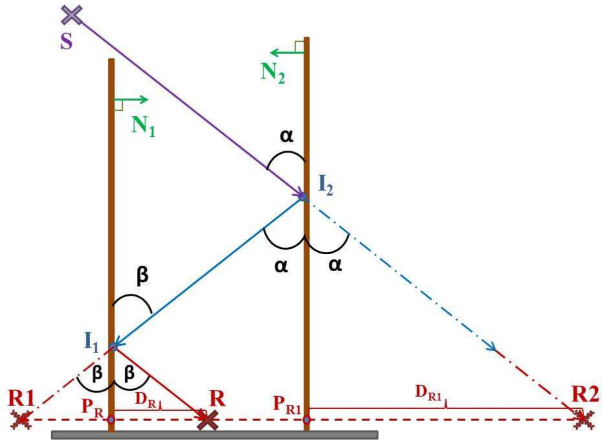

Figure 2 – Geometrical path of a signal reflected on two walls (S is the satellite and R is the receiver)

The pseudorange ρ associated with a multipath signal can be written

$$\rho = \|S - I_2\| + \|I_2 - I_1\| + \|I_1 - R\|. \quad (2)$$

It depends on the receiver position R and on known elements such as the satellite position S and the equations of the plans (1). Thanks to geometrical optics laws, we can use the equations of planar symmetry in order to avoid the use of I_1 and I_2. If we denote P_R the projection of R on the wall 1, the distance D_R between R and P_R can be calculated as

$$\begin{cases} R - D_R N_1 = P_R \\ N_1^T P_R + d_1 = 0 \end{cases} \rightarrow D_R = \frac{d_1 + N_1^T R}{N_1^T N_1} \quad (3)$$

The symmetric point of R relative to the wall 1 is defined by

$$\begin{aligned} R1 &= R - 2 D_R N_1 \\ &= \left(I - 2\frac{N_1 N_1^T}{\|N_1\|^2}\right) R - 2\frac{d_1 N_1}{\|N_1\|^2}. \end{aligned} \quad (4)$$

Planar symmetry has the property to keep unchanged the distances, hence

$$\|I_1 - R\| = \|I_1 - R1\|. \quad (5)$$

As the three angles β are equal, the angle $\widehat{I_2 I_1 R1}$ is equal to π. As a consequence, using (5) we obtain

$$\|I_2 - I_1\| + \|I_1 - R\| = \|I_2 - R1\|. \quad (6)$$

Using the same approach for R2 (the symmetric of R1 with respect to the wall 2), we can express (2) as

$$\begin{aligned} \rho &= \|S - R2\| \\ \rho &= \Bigg\| S - \Bigg(\left(I - 2\frac{N_2 N_2^T}{\|N_2\|^2} \right) \left(I - 2\frac{N_1 N_1^T}{\|N_1\|^2} \right) R \\ &\qquad - \left(I - 2\frac{N_2 N_2^T}{\|N_2\|^2} \right) \frac{2 d_1 N_1}{\|N_1\|^2} - \frac{2 d_2 N_2}{\|N_2\|^2} \Bigg) \Bigg\|. \end{aligned} \quad (7)$$

The differentiation of (7) with respect to R yields

$$\frac{\partial \rho}{\partial R} = -\left(I - 2\frac{N_2 N_2^T}{\|N_2\|^2} \right)\left(I - 2\frac{N_1 N_1^T}{\|N_1\|^2} \right)\frac{S - R2}{\rho}. \quad (8)$$

Integration of a 3D Model in the Kalman Filter

Fig. 3 presents the principle of an EKF dedicated to satellite navigation. In this figure, $\hat{X}_{k|k}$ is the estimated state vector at time instant k using the measurements up to to time k and $P_{k|k}$ is its covariance matrix. The measurement vector Y_k consists of the pseudoranges ρ_k^i, for $i = 1, \cdots, n$ (where n is the number of in view satellites) resulting from the visible satellites at times instant k. The matrices Q and R are the covariance matrices of the state and measurement noises, both assumed to be white Gaussian.

As we have simulated a trajectory in city downtown, we consider a random-walk as evolution model. As a consequence, the state vector X_k is defined as

$$X_k = [x_k \, y_k \, z_k \;\, b]^T$$

where $(x_k \, y_k \, z_k)$ are the three receiver coordinates and b is the receiver clock bias. The corresponding state transition matrix Φ is equal to the identity matrix.

Finally, h_k and Jh_k denote the nonlinear function of the measurement model and its Jacobian, respectively.

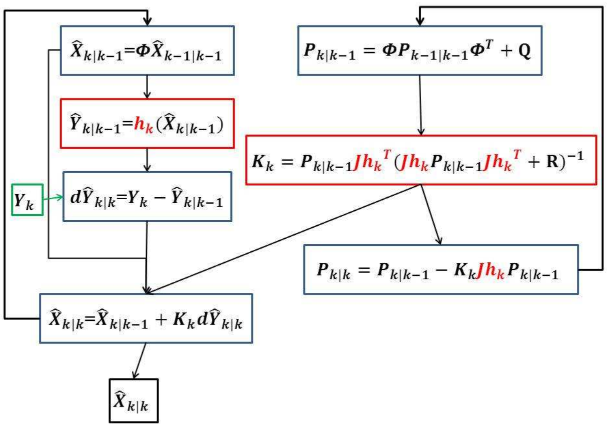

Figure 3 – Extended Kalman Filter.

In what we call thereafter the trilateration version of the navigation filter, the trilateration equations and their derivatives are used for h_k and Jh_k, i.e.

$$h_k^i(\hat{X}_{k|k-1}) = \hat{\rho}_{k|k-1}^i = \left\| X_{sat}^{i,k} - \hat{X}_{rec}^{k|k-1} \right\| + b \tag{9}$$

$$Jh_k^i = \left(-\frac{X_{sat}^{i,k} - \hat{X}_{rec}^{k|k-1}}{\left\| X_{sat}^{i,k} - \hat{X}_{rec}^{k|k-1} \right\|} \quad 1 \right) \tag{10}$$

where $X_{sat}^{i,k}$ and $\hat{X}_{rec}^{k|k-1}$ are the vectors containing the coordinates of the i^{th} observed satellite at time instant k and the receiver predicted coordinates.

Eq. (9) provides a good measurement model when the signal is received in LOS conditions. It represents the geometric distance between the satellite and the receiver, with an additive receiver clock bias. However, if the signal has been received after one ore several reflections, this model no longer corresponds to the geometrical reality. In this case, we propose to use the 3D city model to determine the true geometric path travelled by the signal. Thanks to the estimation of the walls on which the reflections occur, we can replace (9) and (10) by equations based on (7) and (8). In the case of two reflections, we obtain for h_k

$$h_k^i(\hat{X}_{k|k-1}) = \hat{\rho}_{k|k-1}^i \tag{11}$$

$$= \left\| X_{sat}^{i,k} - \left(\left(I - 2\frac{N_2 N_2^T}{\|N_2\|^2} \right) \left(I - 2\frac{N_1 N_1^T}{\|N_1\|^2} \right) \hat{X}_{rec}^{k|k-1} - \left(I - 2\frac{N_2 N_2^T}{\|N_2\|^2} \right) \frac{2d_1 N_1}{\|N_1\|^2} - \frac{2d_2 N_2}{\|N_2\|^2} \right) \right\| + b.$$

Note that the navigation filters using the 3D city model will be referred to as 3D augmented navigation filter (3D ANF) and 3D augmented extended Kalman filter (3D AEKF).

Robust Kalman Filter for Improved Performance

Modeling multipath presents a major constraint: the NLOS trajectory is not a continuous function of the receiver position. On the contrary, it is a highly variable and discontinuous function, as we can see in Fig. 4. As a consequence, the SE-NAV prediction is correct only if the predicted receiver position is close enough to the true position. As it is not always the case, we cannot have absolute confidence in SE-NAV prediction of signal paths. To improve the performance of the 3D ANF, two solutions are proposed to search the better signal modeling at each step, and one solution is proposed to detect and mitigate outliers.

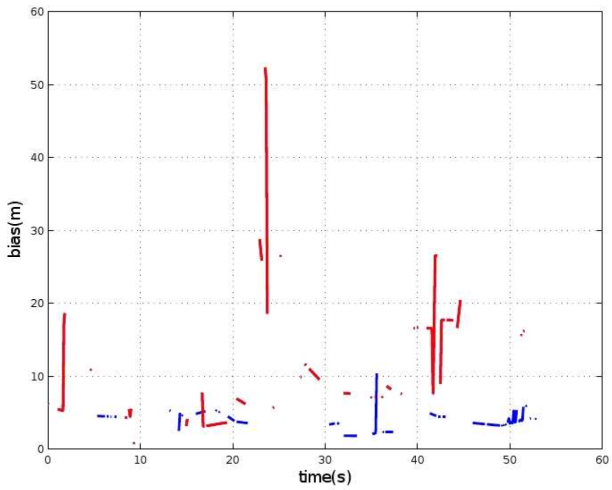

Figure 4 – Bias of signals received by the receiver during a trajectory in city downtown (In blue: satellite 4. In red: satellite 17).

In a first step, several positions around the predicted receiver position are submitted to SE-NAV in order to know the predicted signal paths for each of these positions. Then we retain for each satellite the predicted signal paths which have the pseudorange the closest to the measurement. To keep the integrity of the EKF, we recompute at the predicted receiver position the pseudorange from the signal path configuration retained, thanks to (7). We use this solution because of the highly variability of the multipath. Simulations from SE-NAV show that even a position error less than one meter can change the path predictions. So we test some positions around the predicted position in order to improve our chance to find the right path prediction.

In a second step, paths predicted as multipath are replaced by the LOS modeling (9) if the pseudorange predicted by (9) is closest to the measurement than the one predicted by SE-NAV. If SE-NAV does not provide any prediction for a received signal, we also use (9) to model its pseudorange, as we have no other information.

The third step consists of making the EKF robust to outliers. We add to the Kalman filter an adaptive stochastic method using a robust M-estimation approach [12]. This method uses a weighting function to adapt and correct the contribution of the updated parameters in the Kalman filter. Instead of minimizing the sum of residual squares $\left(d\hat{Y}_{k|k}^{i}\right)^2$, the M-estimation method minimizes a so-called influence function defined as [13]

$$\psi\left(d\hat{Y}\right) = \begin{cases} d\hat{Y} & if\ \left|d\hat{Y}\right| < a \\ a & if\ a \leq \left|d\hat{Y}\right| < c. \\ \left(\frac{a}{c}\right)\left|d\hat{Y}\right| e^{1-\frac{d\hat{Y}^2}{c^2}} & if\ \left|d\hat{Y}\right| \geq c \end{cases} \quad (12)$$

The associated diagonal weight matrix is defined from (12) by

$$D\left(d\hat{Y}\right) = \frac{\psi\left(d\hat{Y}\right)}{\left|d\hat{Y}\right|}. \quad (13)$$

The robust EKF equations are identical to those of the EKF except for the innovation step, where the residuals are weighted by the matrix D

$$V_k = D\left(Y_k - \hat{Y}_{k|k-1}\right). \quad (14)$$

Since the residuals with low weights have reduced reliability, the robust processing associates a higher value to their estimated noise. Consequently, the Kalman gain matrix for the robust version can be calculated as

$$K_k = P_{k|k-1} Jh_k^T \big(Jh_k P_{k|k-1} Jh_k^T + D^{-1} R \left(D^{-1}\right)^T\big)^{-1}. \quad (15)$$

The parameters a and c are chosen to keep the good measurements and at the same time to eliminate efficiently the outliers. We use the well known Mahalanobis distance to determine a and c

$$a = \left(Jh_k P_{k|k-1} Jh_k^T + R\right)^{\frac{1}{2}} \quad (16)$$
$$c = 2a.$$

NAVIGATION USING 3D MODEL INTEGRATION

Simulation Scenario

The proposed simulation is conducted in a high realistic 3D virtual scene of Toulouse downtown (France), around the Capitole Square. In Fig. 5, the simulated trajectory has been represented in yellow with its beginning at the top of the scene. Real ephemeris of eight satellites recorded during a measurement campaign have been used to simulate the satellite positions.

A first run of the trajectory is performed to simulate real measurements. We consider that only one signal can be received for each satellite (if a path exists between the satellite and the receiver), i.e. the receiver measures only the pseudorange of the most powerful signal received for each satellite. Additional multipath signals that can be observed in practical situations are included in the noise. Receiver clock bias is added to the measurements thanks to a simulator provided in the Akos book [14].

Figure 5 – Trajectory (in yellow) simulated inside the 3D virtual scene of the city downtown.

Simulation Results

We compare positioning results obtained with the trilateration EKF and the proposed 3D AEKF in our realistic environment. A white Gaussian measurement noise with standard deviation $\sigma = 3m$ affects the measured pseudoranges.. Fig. 6 shows that the positioning accuracy obtained with the 3D AEKF is better in the lower part of the trajectory, which corresponds to an area with strong multipath. This result is confirmed in Fig. 7 where we can see that the 3D AEKF error is lower than the trilateration EKF error at the end of the simulation. Note that the errors very similar in the beginning of the simulation. Fig. 8 shows the same results for the estimation of the receiver clock bias. Another important property of the proposed strategy can be observed in Fig.'s 7 and 8, where we can see that the uncertainty of the 3D AEKF is always smaller than that of the trilateration EKF. All these results are confirmed in Tab. 1 showing some error statistics for both filters. The mean and standard deviation of the 3D AEKF errors are clearly lower than those obtained with the trilateration EKF.

CONCLUSIONS

This paper has developed a new approach for GNSS navigation in critical NLOS environments. A tight integration of a 3D high realistic model of the environment with an extended Kalman filter allows NLOS signals to be used constructively. A robust version of the resulting navigation algorithm has also been investigated to use all the available information, even the less reliable measurements. High realistic simulation results showed that the proposed robust 3D AEKF outperforms a robust trilateration EKF. These results are currently under validation using real data. In particular, simulation results shown in this paper have been obtained with good signal to noise ratios (SNRs). With real data, SNRs are generally lower for reflected signals than for the line of sight signals. This will need a specific study.

ACKNOWLEDGMENTS

This work was supported by DGA (the French Defense Agency) and Thales Alenia Space, Toulouse, France. Authors would like to thank Benoit Priot and Damien Vivet for their technical assistance.

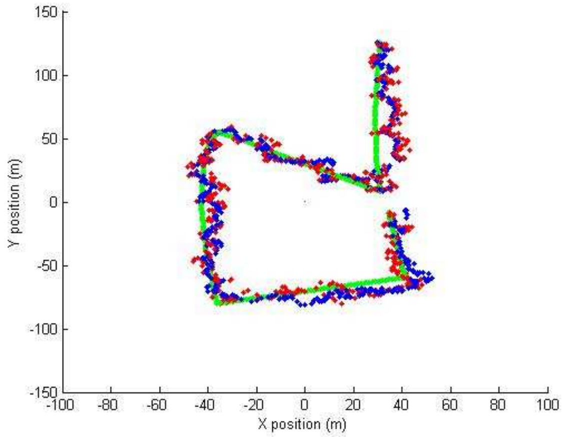

Figure 6 – Positioning results for an additive Gaussian noise with standard deviation σ=3m (In green: real position. In blue: the trilateration EKF solution. In red: the 3D AEKF solution).

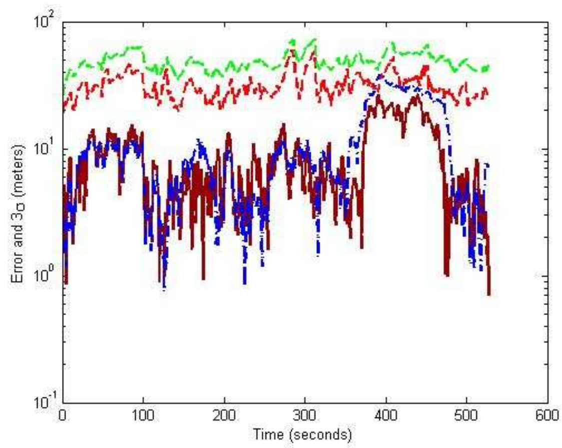

Figure 7 – Error on the position estimation, for an additive Gaussian noise with standard deviation σ=3m (In green dashdot: 3σ value at each time for the trilateration EKF. In red dashdot: 3σ value at each time for the 3D AEKF. In blue: trilateration EKF position error. In dark red: 3D AEKF position error).

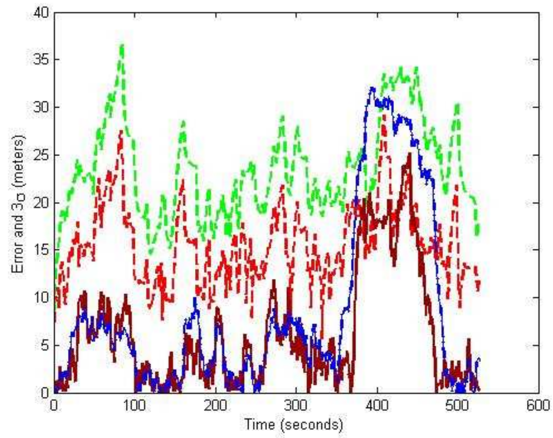

Figure 8 – Error on the receiver clock bias estimation, for an additive Gaussian noise with standard deviation σ=3m (In green dashdot: 3σ value at each time for the trilateration EKF. In red dashdot: 3σ value at each time for the 3D AEKF. In blue: trilateration EKF bias error. In dark red: 3D AEKF bias error).

Table 1 – Statistics of each filter errors

	3D AEKF	Trilateration EKF
Position error: standard deviation (m)	6.04	9.40
Position error: mean (m)	8.46	10.85
Position error: maximum (m)	26.88	38.44
Clock bias error: standard deviation (m)	6.05	9.46
Clock bias error: mean (m)	6.02	8.58
Clock bias error: maximum (m)	25.18	32.16

REFERENCES

[1] Kaplan, E., *Understanding GPS: Principles and Application*, 2nd Edition, Artech House, Norwood, MA, 2006

[2] Ercek, E., De Doncker, P. and Grenez, F., "NLOS-Multipath Effects on Pseudo-range Estimation in Urban Canyons for GNSS Applications", *Proceedings of EuCAP-2006*, Nice, France, 2006

[3] Van Dierendonck, A., "Theory and Performance of Narrow Correlator Spacing in a GPS Receiver", *Navigation, Journal of the Institute of Navigation*, vol.39, no.3, pp. 265-283, Fall 1992

[4] Braasch, M., "Performance Comparison of Multipath Mitigating Receiver Architectures", *Proceedings of IEEE Aerospace Conference*, vol.3 Big Sky, MT, 2001

[5] Sahmoudi, M. and Amin, A., "Fast Iterative Maximum-Likelihood Algorithm (FIMLA) for

Multipath Mitigation in Next Generation of GNSS Receivers", *IEEE Transactions in Wireless Communications*, vol.11, no.7, pp. 4352-4374, Nov. 2008

[6] Groves, P.D., "Shadow Matching: A New GNSS Positioning Technique for Urban Canyons", *Journal of the Royal Institute of Navigation*, vol.64, pp. 417-430, 2011

[7] Gustafson, D., Elwell, J. and Soltz, J., "Innovative Indoor Geolocation Using RF Multipath Diversity", *Proceedings of IEEE/ION Position Location and Navigation Symposium*, San Diego, CA, April 2006

[8] Soloviev, A. and Van Graas, F., "Utilizing Multipath Reflections in Deeply Integrated GPS/INS Architecture for Navigation in Urban Environments", *Proceedings of IEEE/ION Position Location and Navigation Symposium*, Monterey, CA, May 2008

[9] Soloviev, A. and Van Graas, F., "Use of Deeply Integrated GPS/INS Architecture and Laser Scanners for the Identification of Multipath Reflections in Urban Environments", *IEEE Journal of Selected Topics in Signal Processing*, vol.3, no.5, pp. 786-797, Oct. 2009

[10] Bourdeau, A., Sahmoudi, M. and Tourneret, J.-Y., "Constructive Use of GNSS NLOS-Multipath: Augmenting the Navigation Kalman Filter with a 3D Model of the Environment", *Proceedings of International Conference on Information Fusion (FUSION 2012)*, Singapore, July 2012

[11] http://www.oktal-se.fr/

[12] Rao, K.D., Swamy, M.N.S. and Plotkin, I., "GPS Navigation with Increased I mmunity to Modeling Errors", *IEEE Transaction of Aerospace and Electronique Systems*, vol.40, no.1, pp. 2-11, Jan. 2004

[13] Gao, Y., "A New Algorithm of Receiver Autonomous Integrity Monitoring (RAIM) for GPS Navigation", *Proceedings of ION GPS 1991*, Albuquerques, NM, 1991

[14] Borre, K., Akos, D., Bertelsen, N., Rinder, P. and Jenson, S.H., *A Software-Defined GPS and Galileo Receiver: A Single-Frequency Approach*, Birkhaüser, Boston, 2007