Optimisation of internet throughput in constellations of satellites

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Summary

- A constellation of satellites
- Unsplittable flows
    - Problem presentation
    - Randomized rounding
    - Sequential randomized rounding
    - Results and proofs
- Dynamic unsplittable flows
    - Problem presentation
    - One-timestep methods
    - Multi-timestep methods

A constellation of satellites

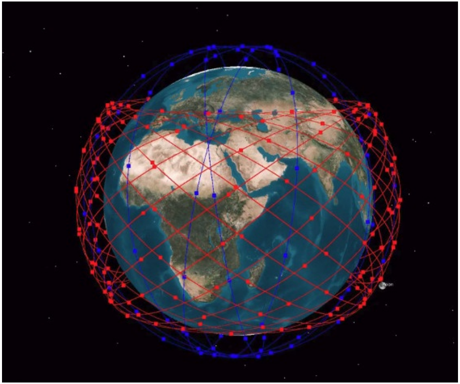


Figure – Telesat constellation

Telecommunication constellation :

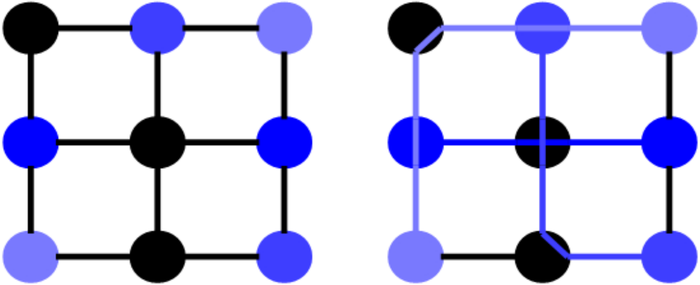
- Connect users to terrestrial networks
- Provide internet access in sparsely populated areas

Managing its telecommunications is a challenging problem !

Unsplittable flows

- Satellites → Nodes
- Inter-satellites-links → Arcs
- Users demands → Commodities

Objective : Minimize the used capacity of the most used arc while routing each commodity on a **single** path.



Linear relaxation : Multi-commodity flows : each commodity route its flow on **several** paths

Discussion on the problem

- NP-Hard problem :
  - Knapsack problem as a sub-problem
  - Edge-disjoint paths problem as a sub-problem
- Size of the instances in the applications : 400 nodes, 2000 arcs, 10 000 commodities
- Exact algorithms : 30 nodes, 80 arcs, 100 commodities
- Multi-commodity flows :
  - Polynomial problem
  - Solvable with linear programming or approximation algorithms

Randomized rounding [1]

- Solve the linear relaxation :
  - Each commodity uses several paths
  - Flow distribution on the paths : $(x_{pk})_{p \in P_k, k \in K}$
- Independent randomized rounding of each commodity :
  - Commodity k chooses path p with probability x_{pk}
  - Fix $x_{pk} = 1$ in the unsplittable solution

2. Prabhakar Raghavan and Clark D Tompson. Randomized rounding : a technique for provably good algorithms and algorithmic proofs. Combinatorica, 7(4) :365–374, 1987.

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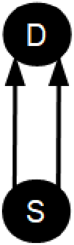
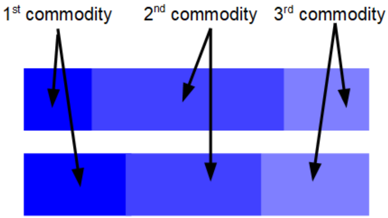


(a) Toy example : 2 nodes, 2 arcs



(b) A linear solution

Randomized rounding

- Independent randomized rounding of each commodity :
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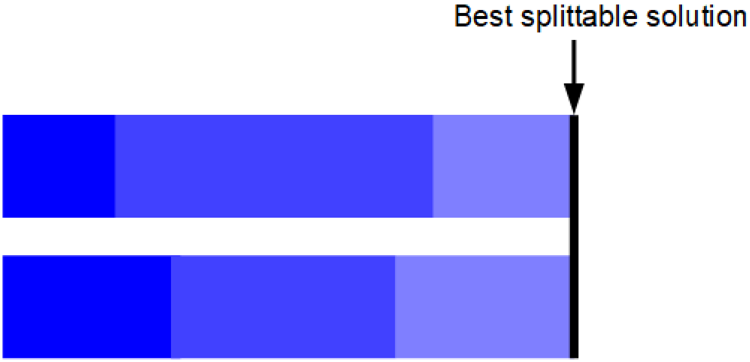

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Figure – Example of rounding

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Randomized rounding [2]

- Solve the linear relaxation :
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  - Flow distribution on the paths : $(x_{pk})_{p \in P_k, k \in K}$

- Independent randomized rounding of each commodity :
  - Commodity k chooses path p with probability x_{pk}
  - Fix $x_{pk} = 1$ in the unsplittable solution

- Provable approximation factor : $O(\frac{log\ m}{log\ log\ m})$ $m =$ number of arcs
- This factor is optimal

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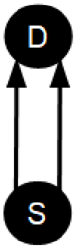
- Solve the linear relaxation
- Then alternate between :
  - Fixing the path of a commodity through randomized rounding
  - Actualizing the linear relaxation
- Round the biggest commodities first



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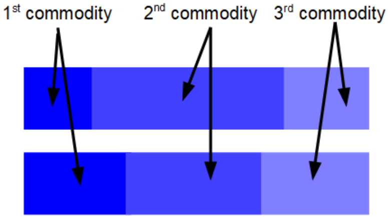

(b) A linear solution

Sequential randomized rounding (SRR)

- Alternate between :
  - Fixing the path of a commodity through randomized rounding
  - Actualizing the linear relaxation

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Sequential randomized rounding (SRR)

- Solve the linear relaxation
- Then alternate between :
  - Fixing the path of a commodity through randomized rounding
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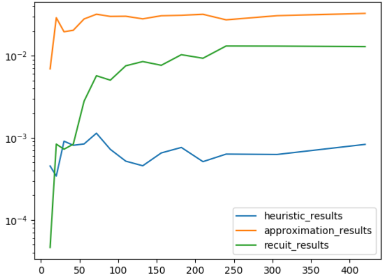
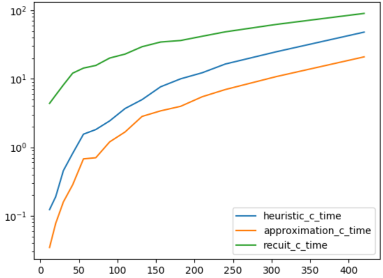
- The order of rounding now has an impact : round the biggest commodities first

- Approximation factor : $O(\frac{log\ m}{log\ log\ m})$ $\quad m =$ number of arcs

Experimental results



(a) Solution quality



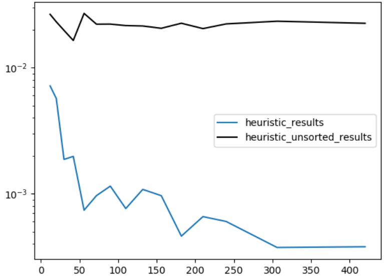
(b) Computing time

Graphs of increasing size

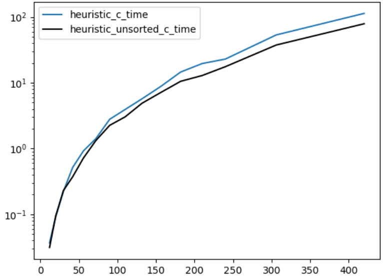
- SRR yield the solutions of best quality
- SRR has longer computing time than pure randomized rounding

Experimental results

Impact of the rounding order

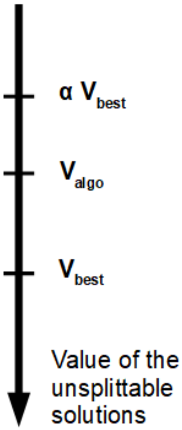


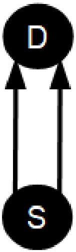
(a) Solution quality



(b) Computing time

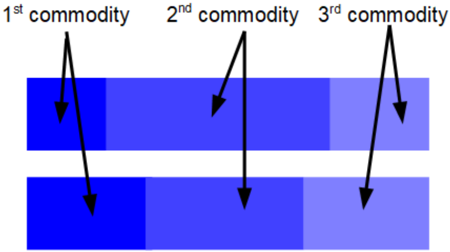
Approximation proof





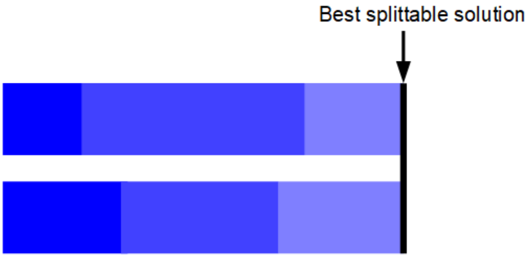
A simple example with 2 arcs and 3 commodities.

Approximation proof



Each commodity distributes its flow between the two arcs.

Approximation proof



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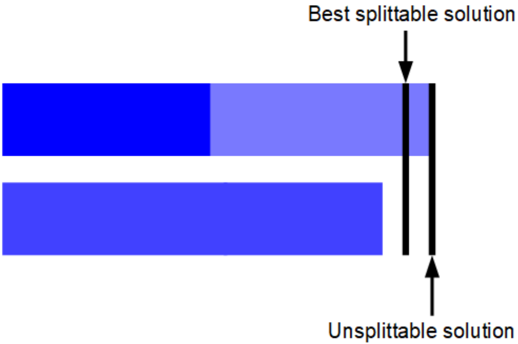
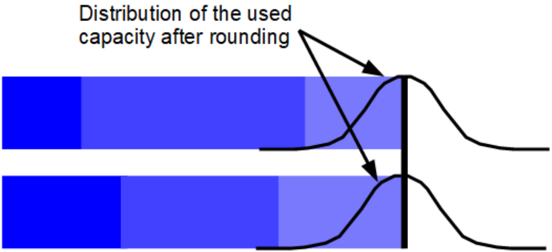


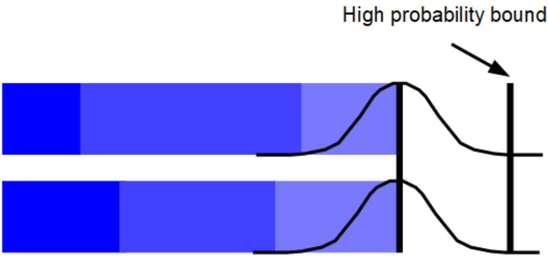
Every unsplittable solution is worse than the best splittable solution.

Approximation proof



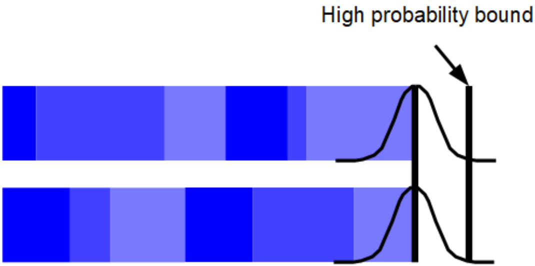
Rounding selected at random $\rightarrow$ distribution of the total flow.

Approximation proof



We can bound how far the roundings can deviate from the mean.
(Chernoff bound)

Approximation proof

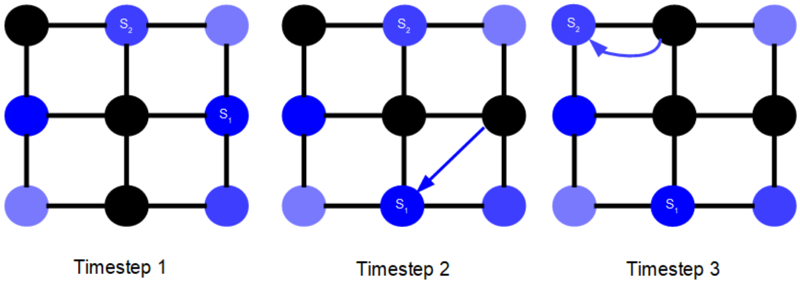


The smaller the commodities, the more concentrated the distribution and the lower the bound

Dynamic unsplittable flows

- The constellation moves around the earth
- Source/destination of the commodities are moving
- Time discretization : several timesteps

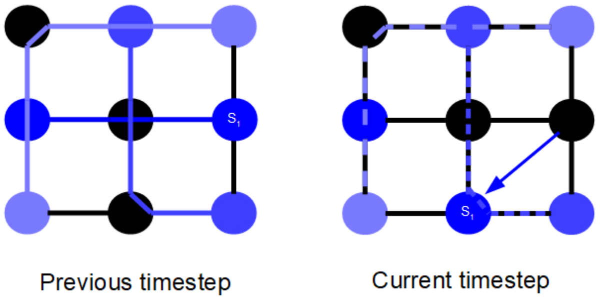
Bi-objective :
Route the commodities on a **single** path inside the capacities.
A **penalty** is paid when a commodity change its path.



One-timestep methods

- Consider one timestep at a time : rolling horizon
- Problem = Static unsplittable flows + preferred path

- Solvers :
  - Sequential Randomized Rounding
  - Commercial MILP solver (very efficient in this case)



Multi-timestep methods

- One-timestep : choose a path per commodity
- Multi-timestep : choose a sequence of paths per commodity

- Solvers :
  - Sequential Randomized Rounding
  - Require the use of column generation to solve the linear relaxation

Experimental results

- One-timestep methods :
  - Faster → can use heavier methods to solve each timestep
  - Does not require column generation → Commercial solvers are very effective
- Multi-timestep methods :
  - Slower
  - Can find significant improvement when close to optimality

Conclusion

- Routing in a constellation of satellites → Dynamic unsplittable flow problem
- Study of a new algorithm : Sequential Randomized Rounding
  - Very good results on the static problem
  - Extension of approximation factor
- Unmentioned :
  - How to solve the linear relaxation in multi-timestep methods
  - How to get a tighter linear relaxation (polyhedral analysis)