

Delay Optimization of Conventional Non-Coherent Differential CPM Detection

Anouar Jerbi, Karine Amis, Frédéric Guilloud, and Tarik Benaddi

Abstract—The conventional non-coherent differential detection of continuous phase modulations (CPM) is quite robust to channel impairments such as phase and Doppler shifts. Its implementation is on top of that simple. It consists in multiplying the received baseband signal by its conjugate version delayed by one symbol period. However it suffers from a signal-to-noise ratio gap compared to the optimum coherent detection. In this paper, we improve the error rate performance of the conventional differential detection by using a delay higher than one symbol period. We derive the trellis description as well as the branch and cumulative metrics that take into account a delay of K symbol periods. We then determine an optimized delay K_{opt} based on the minimum Euclidean distance between two differential signals for some popular CPM formats. The optimized values are confirmed by error rate simulations.

Index Terms—CPM, differential detection, Doppler shift, phase shift

I. INTRODUCTION

CONTINUOUS PHASE MODULATIONS (CPM) are a class of non-linear constant-envelope modulations with a limited spectral occupancy. The constant envelope is interesting when the channel includes a strong non-linearity like e.g. in satellite communications. Moreover, non-coherent CPM detection enables to face the possible phase distortion introduced by the channel, without carrying out any phase synchronisation¹. Combined with the energy efficiency of CPM, these properties make this kind of waveform a good candidate for Internet of Things (IoT) [1], [2], especially for Satellite IoT communications which arouse our interest in this paper. Non-coherent CPM detectors can be grouped into two families depending on the criterion they are based on. The first one is derived from the generalized maximum-likelihood criterion [3]–[5] and only requires the knowledge of the phase distribution. Algorithms proposed either in [6] or in [7] with an uniformly-distributed phase assumption belong to it. The second one preprocesses the received signal to neutralize the phase contribution making possible the application of the

maximum-likelihood criterion for coherent detection on the resulting signal.

Numerous papers based on differentially-preprocessed signals can be found in the state-of-the-art. The common feature is the use of the product of the received baseband signal and a conjugate time-delayed version of it, yielding a signal that we will refer to as differential signal in the remaining of the paper. Different algorithms are proposed and apply either on time-discrete differential signals (see e.g. [8]) or on time-continuous differential signals (see e.g. [9]–[12]). In [9] applied to tamed frequency modulation (TFM) and in [12] extended to CPM signals, a detection metric is defined from a set of multiple sampled differential signal versions of the original one (differing from the delay value). The sampling rate is set to the symbol rate, which yields insufficient statistics and possible severely degraded performance. Simulations are used to compare different set definitions (with a maximum delay equal to three symbol periods) for the TFM and the Gaussian minimum shift keying (GMSK). However, the most used representative of the second class is the conventional differential detection as defined in [10], [11] and applied with one symbol period as the delay value. Differential detection is also robust against Doppler shifts, which is particularly interesting in the context of Satellite IoT. The main drawback of standard differential detection is the signal-to-noise ratio (SNR) gap as compared to the optimum coherent detection. To reduce this SNR gap, we propose in this paper to modify the differential detection as defined in [10]. Our contributions are threefold, (i) the theoretical extension of the usual CPM differential detection algorithm to consider a delay higher than one symbol period (including the description of the phase trellis and the derivation of the equations of the branch and cumulative metrics), (ii) the systematic determination of an optimized delay value based on the application of the minimum Euclidean distance criterion between two CPM differential signals and, (iii) the optimized delay values for different CPM formats (modulation index, frequency pulse length, frequency pulse type).

This work has been financially supported by Thales Alenia Space and Brest Métropole.

Anouar Jerbi is with the Mathematical and Electrical Engineering (MEE) Department, IMT Atlantique, Lab-STICC, UMR CNRS 6285, 29238 Brest, France. He is also with TêSA, 31000 Toulouse, France. (e-mail: anouar.jerbi@imt-atlantique.fr).

Karine Amis and Frédéric Guilloud are with the Mathematical and Electrical Engineering (MEE) Department, IMT Atlantique, Lab-STICC, UMR CNRS 6285, 29238 Brest, France (e-mail: firstname.lastname@imt-atlantique.fr). Tarik Benaddi is with Thales Alenia Space, 31000 Toulouse, France (e-mail : tarik.benaddi@thalesaleniaspace.com)

¹Let us mention that differential encoding can be used at the transmitter to deal with the phase ambiguity at the receiver. Such techniques are out of scope of this paper.

The remainder of this paper is organized as follows: in Section II, the system model is presented and the notations are introduced. In Section III, the differential detection using a delay of K symbol periods is exposed, followed by the optimization of K in Section IV. The simulations and the resulting tables for different CPM formats are presented in Section V. A conclusion is drawn in Section VI which ends the paper.

II. SYSTEM MODEL AND NOTATIONS

We consider a sequence of N independent and identically distributed (i.i.d.) information symbols $\mathbf{a} = \{a_i\}_{0 \leq i \leq N-1}$ to be transmitted. Given M an even positive integer, a_i takes on values in the M -ary alphabet $\mathcal{M} = \{\pm 1, \pm 3, \dots, \pm (M-1)\}$ with equal probabilities. Note that all the signals considered in this paper are assumed to be causal, hence $t \geq 0$, unless otherwise specified. The complex envelope of the CPM-modulated signal is given by:

$$s(t, \mathbf{a}) = \sqrt{\frac{2E_s}{T_s}} e^{j\theta(t, \mathbf{a})}, \quad (1)$$

where E_s is the average symbol energy, T_s is the symbol period and $\theta(t, \mathbf{a})$ is the signal phase which depends on the information symbols. It is defined by:

$$\theta(t, \mathbf{a}) = 2\pi h \sum_{i=0}^{N-1} a_i q(t - iT_s) \quad (2)$$

where h is the modulation index and $q(t)$ is the phase smoothing-response whose expression is $q(t) = \int_{-\infty}^t g(u) du$ with $g(t)$ the frequency pulse. In practice, $g(t)$ has a finite duration LT_s . Without loss of generality, we consider that $g(t) = 0, \forall t \notin [0, LT_s)$ and it satisfies the following conditions:

$$\begin{cases} g(t) = g(LT_s - t), & 0 \leq t < LT_s, \\ \int_0^t g(\tau) d\tau = q(LT_s) = \frac{1}{2}, & \forall t \geq LT_s. \end{cases} \quad (3)$$

Our interest is satellite communications whose transmission channel can be considered as non-frequency selective and Gaussian with Doppler effect as the main propagation issue. In the following, we neglect the Doppler effect and we will investigate the receiver robustness against Doppler shift in Section V. We thus assume that the modulated signal is transmitted over a Gaussian channel. The equivalent baseband received signal, denoted by $r(t)$, is given by:

$$r(t) = s(t, \mathbf{a}) e^{j\psi} + n(t), \quad (4)$$

where ψ is an arbitrary phase introduced by the channel and supposed to be uniformly distributed in $[0, 2\pi)$. $n(t)$ is the realization of a zero-mean wide-sense stationary complex circularly symmetric Gaussian noise, independent of the signal, and with double-sided power spectral density $2N_0$.

III. DIFFERENTIAL DETECTION OF CPM

A. K -delay based differential receiver

Let us consider a delay equal to K symbol periods. At the receiver side, a differential signal denoted by $R_K(t)$ is generated using the received signal $r(t)$ and its delayed version $r(t - KT_s)$. It can be decomposed as the sum of two signals:

$$R_K(t) = \frac{1}{2} r(t) r^*(t - KT_s) = S_K(t, \mathbf{a}) + N_K(t), \quad (5)$$

where the first term does not include any noise contribution:

$$S_K(t, \mathbf{a}) = \frac{1}{2} s(t, \mathbf{a}) s^*(t - KT_s, \mathbf{a}) = \frac{E_s}{T_s} e^{j\Theta_K(t, \mathbf{a})} \quad (6)$$

with $\Theta_K(t, \mathbf{a}) = \theta(t, \mathbf{a}) - \theta(t - KT_s, \mathbf{a})$.

The second term, denoted by $N_K(t)$, consists of all noise-dependent components. It is decomposed as

$$N_K(t) = U_K(t) + W_K(t)$$

with

$$\begin{aligned} U_K(t) &= \frac{1}{2} (s(t, \mathbf{a}) e^{j\psi} n^*(t - KT_s) + \dots \\ &\quad n(t) s^*(t - KT_s, \mathbf{a}) e^{-j\psi}), \\ W_K(t) &= \frac{1}{2} (n(t) n^*(t - KT_s)). \end{aligned} \quad (7)$$

The computation of its autocorrelation leads to the following expression:

$$E[N_K(t) N_K^*(t - \tau)] = (N_0^2 + A^2 N_0) \delta(\tau) \quad (8)$$

with $A = |s(t, \mathbf{a})| = \sqrt{\frac{2E_s}{T_s}}$ and $\delta(t)$ the delta function. The random process $N_K(t)$ is wide-sense stationary with zero mean and constant power spectral density (PSD) equal to $(N_0^2 + A^2 N_0)$. From now on, it will be assumed to follow a Gaussian distribution as in [12].

B. Phase trellis description

Let $t = \tau + nT_s$, with $0 \leq \tau < T_s$. Taking into account the properties of the frequency pulse given in (3), the phase introduced in (6) can be decomposed as the sum of a time-independent term and a time-dependent term:

$$\Theta_K(\tau + nT_s, \mathbf{a}) = \phi_n + 2\pi h a_n q(\tau) + \varphi_n(\tau), \quad (9)$$

with $\phi_n = \pi h \sum_{i=0}^{K-1} a_{n-L-i}$ and

$$\begin{aligned} \varphi_n(\tau) &= 2\pi h \sum_{i=1}^{L-1} (a_{n-i} - a_{n-K-i}) q(\tau + iT_s) \dots \\ &\quad - a_{n-K} q(\tau). \end{aligned} \quad (10)$$

$\varphi_n(\tau)$ represents a time-dependent contribution which corresponds to the last L memory symbols of both the signal and its delayed version. The term ϕ_n represents the time-independent part. $\varphi_n(\tau)$ and ϕ_n are completely determined by the set of symbols $(a_{n-i})_{1 \leq i \leq L+K-1}$. As a consequence, ϕ_n doesn't need to be stored, contrary to the original CPM trellis description which comprises the cumulative phase as a defining state parameter. We can thus define the state $\Sigma_n = [a_{n-L-K+1}, \dots, a_{n-1}]$ for the n -th section of the trellis representation of $\Theta_K(t, \mathbf{a})$. Note that there are M^{K+L-1} different possible states.

C. Maximum likelihood (ML)-based detection

The ML criterion is applied to detect the information symbols from $R_K(t)$. Given the constant amplitude property of CPM, it consists in maximizing the correlation between $R_K(t)$ and all possible realizations of $S_K(t, \mathbf{a})$. The inner product between $R_K(t)$ and a specific realization $S_K(t, \tilde{\mathbf{a}})$, denoted by $\Gamma_N(\tilde{\mathbf{a}})$, is defined as

$$\Gamma_N(\tilde{\mathbf{a}}) = \text{Re} \left[\int_0^{NT_s} R_K(t) S_K^*(t, \tilde{\mathbf{a}}) dt \right], \quad (11)$$

which can be recursively computed:

$$\Gamma_n(\tilde{\mathbf{a}}) = \Gamma_{n-1}(\tilde{\mathbf{a}}) + \Lambda_n(\tilde{\mathbf{a}}) \quad (12)$$

with

$$\Lambda_n(\tilde{\mathbf{a}}) = \text{Re} \left[\int_{(n-1)T_s}^{nT_s} R_K(t) S_K^*(t, \tilde{\mathbf{a}}) dt \right]. \quad (13)$$

The Viterbi algorithm is applied on the trellis. At the n -th section, it computes for each state the maximum cumulative metric (12) among all the paths arriving at this state.

The complexity of the differential detector can be estimated in terms of the number of trellis states $S = M^{K+L-1}$ and the number of multiplications per trellis section involved in the metric calculations $Q = \rho SM$, where ρ samples per symbol are used to calculate the metrics. Note that the complexity per symbol is not affected by the frame length.

IV. DELAY OPTIMIZATION

In this section, we aim at tuning K to improve the detection error probability. Let us consider the following error event when $s(t, \mathbf{a})$ is transmitted, $s(t, \tilde{\mathbf{a}})$ is detected and $\mathbf{a} \neq \tilde{\mathbf{a}}$. Given the ML-based detection criterion and the independence between R_K and N_K , it means that:

$$\int_0^{NT_s} |R_K(t) - S_K(t, \tilde{\mathbf{a}})|^2 dt \leq \int_0^{NT_s} |R_K(t) - S_K(t, \mathbf{a})|^2 dt \quad (14)$$

which can be reformulated as:

$$Z_K \geq \frac{1}{2} \Delta_K^2(\mathbf{a}, \tilde{\mathbf{a}}), \quad (15)$$

where $Z_K = \int_0^{NT_s} \text{Re}[(S_K(t, \mathbf{a}) - S_K(t, \tilde{\mathbf{a}})) N_K^*(t)] dt$. $\Delta_K(\mathbf{a}, \tilde{\mathbf{a}}) = \sqrt{\int_0^{NT_s} |S_K(t, \mathbf{a}) - S_K(t, \tilde{\mathbf{a}})|^2 dt}$ is the Euclidean distance between the two differential signals $S_K(t, \mathbf{a})$ and $S_K(t, \tilde{\mathbf{a}})$ corresponding to the symbol sequences \mathbf{a} and $\tilde{\mathbf{a}}$. Z_K has zero mean. Assuming that Z_K is Gaussian, the probability of an error event is given by

$$P_e(\mathbf{a}; \tilde{\mathbf{a}}) = Q \left(\sqrt{\frac{\varepsilon_b}{2(N_0^2 + A^2 N_0)} d_K^2(\mathbf{a}, \tilde{\mathbf{a}})} \right) \quad (16)$$

where Q is the Q -function and where $d_K(\mathbf{a}, \tilde{\mathbf{a}}) = \frac{\Delta_K(\mathbf{a}, \tilde{\mathbf{a}})}{\sqrt{2\varepsilon_b}}$ is the normalized Euclidean distance, ε_b denoting the average energy per information bit in the differential symbol sequence. Proceeding as in [13, Chapter 2, Paragraph 2.1.2], a union bound on the probability of error is obtained at reasonably high SNR. The error probability is thus approximated by

$$P_e \propto Q \left(\sqrt{\frac{\varepsilon_b}{2(N_0^2 + A^2 N_0)} d_{\min}^2(K)} \right) \quad (17)$$

where:

$$d_{\min}^2(K) = \min_{\substack{\mathbf{a}, \tilde{\mathbf{a}} \\ a_0 \neq \tilde{a}_0}} (d_K^2(\mathbf{a}, \tilde{\mathbf{a}})). \quad (18)$$

By applying the same reasoning as in [13], we obtain:

$$d_K^2(\mathbf{a}, \tilde{\mathbf{a}}) = \frac{\log_2(M)}{T_s} \int_0^{NT_s} [1 - \cos(\Theta_K(t, \mathbf{e}))] dt \quad (19)$$

where $\mathbf{e} = \mathbf{a} - \tilde{\mathbf{a}}$ is the so-called difference symbol sequence.

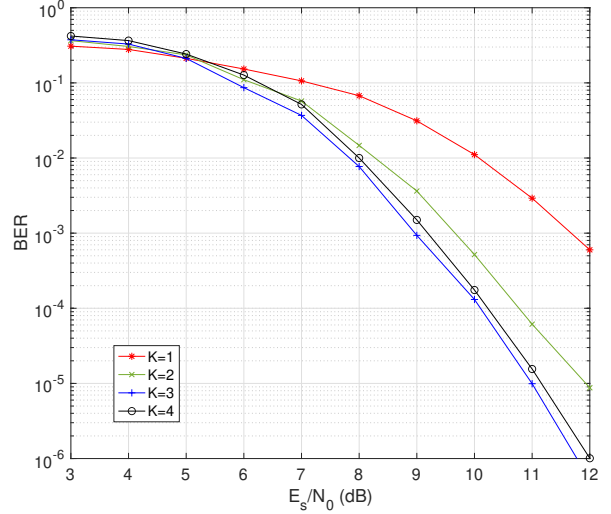


Fig. 1. BER of differential detection for the CPM scheme 3REC with modulation index $h = 0.75$ for different values of delay K .

Finding the minimum Euclidean distance is done by searching over all possible pairs of sequences \mathbf{a} and $\tilde{\mathbf{a}}$. In practice, these pairs are those whose respective paths on a phase tree diverge at time 0 and merge again as soon as possible. Proceeding as in [13], the phase difference tree is a good method to determine the difference symbol sequences to be considered and the corresponding pairs of symbol sequences.

For each value of the delay, a corresponding value of the minimum Euclidean distance d_{\min} is obtained. Since we are looking for minimizing the error probability, the best choice of the delay is the value that yields the highest d_{\min} .

V. NUMERICAL RESULTS

In this section, we study different CPM formats. We focus on Satellite IoT which involves short frame communications over a non-frequency selective channel mainly disturbed by Doppler effects. In the simulation setup, we thus consider $N = 120$ (note that this choice does not affect the conclusions) and an AWGN channel without Doppler shift in Sections V-A, V-B, V-C and with Doppler shift in Section V-D.

A. Influence of K on the detection performance

We first illustrate the influence of K on the detection performance. We consider a CPM format with rectangular frequency pulse, $L = 3$ and $h = 0.75$. The delay K takes on values in $\{1, 2, 3, 4\}$. The bit error rate (BER) is plotted as a function of E_s/N_0 in Fig. 1. We observe that $K = 3$ is the delay that yields the best BER. A gain of 3 dB is obtained compared to the receiver with $K = 1$ and almost 1 dB compared to the receiver with $K = 2$ while the receiver with $K = 4$ exhibits a slight degradation of performance.

B. Optimization of K from the minimum Euclidean distance criterion

The optimization of K based on the Euclidean distance computation in (19) is run by Monte-Carlo simulations. For

TABLE I
OPTIMIZED VALUES OF K FOR RC CPM

Freq. pulse length L	Modulation index		
	$h = 1/3$	$h = 1/2$	$h = 3/4$
1	$K = 2$	$K = 2$	$K = 3$
3	$K = 3$	$K = 3$	$K = 3$
5	$K = 4$	$K = 4$	$K = 4$

TABLE II
OPTIMIZED VALUES OF K FOR REC CPM

Freq. pulse length L	Modulation index		
	$h = 1/3$	$h = 1/2$	$h = 3/4$
1	$K = 2$	$K = 2$	$K = 4$
3	$K = 4$	$K = 4$	$K = 3$
5	$K = 5$	$K = 5$	$K = 5$

each CPM format (g, L, h) , several possible pairs of sequences are considered which yield different realizations of \mathbf{e} . The optimized value of K is provided in Tables I, II, III for raised cosine (RC), rectangular (REC) and Gaussian (GFSK) frequency pulses respectively. We consider several modulation indices h and several frequency pulse lengths L . The consistency of the optimized delay has also been checked by BER simulations for all CPM formats. Note that when several values of K provide the same best error rate, then the displayed value is simply the lowest one to reduce the complexity of the decoder.

C. Comparison with some state-of-the-art receivers

In Fig. 2, we show a comparison between the optimized differential receiver (K_{opt}), the conventional differential receiver ($K = 1$), and also the coherent receiver. This comparison is performed for 2 different CPM families: GFSK with $h = 0.5$ and $BT = 0.3$ (GMSK), and 5RC with $h = 0.5$. For GMSK, there is almost 4 dB between the coherent BER and the conventional differential detection ($K = 1$). Using the optimized $K = 3$ reduces this gap by almost 2 dB. For the 5RC CPM, using the optimized $K = 4$ delay reduces the gap to coherent BER from around 6 dB down to 2 dB. Note that the curves for the coherent and the optimized differential receivers are quasi-parallel which means that the diversity gain is almost the same and the difference between the two is mainly in the noise variance which is higher for the differential receiver.

D. Comparison in presence of Doppler shift

The differential detector is especially interesting in applications where the Doppler shift affects the communication. When dealing with a constant Doppler shift f_D , the received signal is expressed as:

$$r(t) = s(t, \mathbf{a})e^{j(\psi + 2\pi f_D t)} + n(t).$$

The impact of f_D is relative to the symbol duration T_S and so the product $f_D T_S$ is considered as a variable parameter in the following to illustrate the influence of a constant Doppler shift.

TABLE III
OPTIMIZED VALUES OF K FOR GFSK ($BT = 0.3$)

Freq. pulse length L	Modulation index		
	$h = 1/3$	$h = 1/2$	$h = 3/4$
3	$K = 3$	$K = 3$	$K = 4$
5	$K = 3$	$K = 3$	$K = 4$

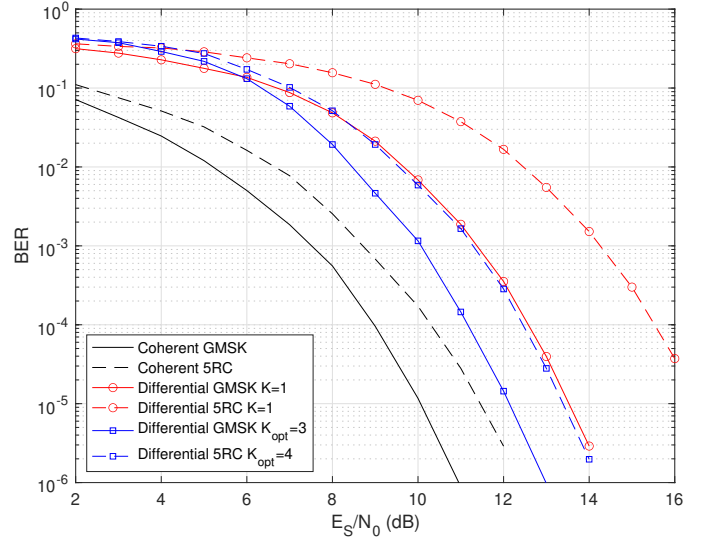


Fig. 2. BER comparison between coherent and differential detection for two CPM schemes: GMSK with $BT = 0.3$, and 5RC with $h = 0.5$.

In Fig. 3, a performance comparison between the differential detector and the coherent one in terms of BER is illustrated for the rectangular pulse with $h = 0.5$ and $L = 5$ in presence of a small Doppler shift. We observe a huge performance degradation for the coherent detector whereas the differential detector is not affected for the considered Doppler shift values.

VI. CONCLUSION

In this paper, the increase of the delay used in the conventional non-coherent differential detection of CPM has been shown to have an impact on the error rate. We have therefore proposed to optimize this delay based on the minimum Euclidean distance between two differential signals. We have obtained an optimized delay ranging from 2 to 5 symbol periods depending on the considered CPM format. Simulations have confirmed the choice of the optimized delay value which offers a gain from 2 to 4 dB on the error rate performance compared to a single symbol duration delay. In a future work, CPM with optimized differential detection will be investigated as an alternative candidate waveform in the context of limited-power Satellite Internet of Things (Satellite IoT) where Doppler shift is an issue.

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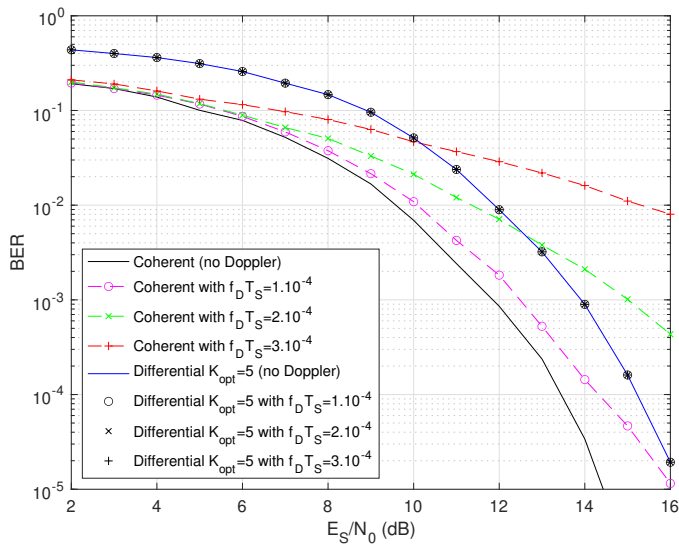


Fig. 3. BER comparison between coherent and differential detection for 5REC with $h = 0.5$ in the presence of a Doppler shift.

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