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LOCALIZATION IN A SWARM OF SATELLITES

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21/05/2026

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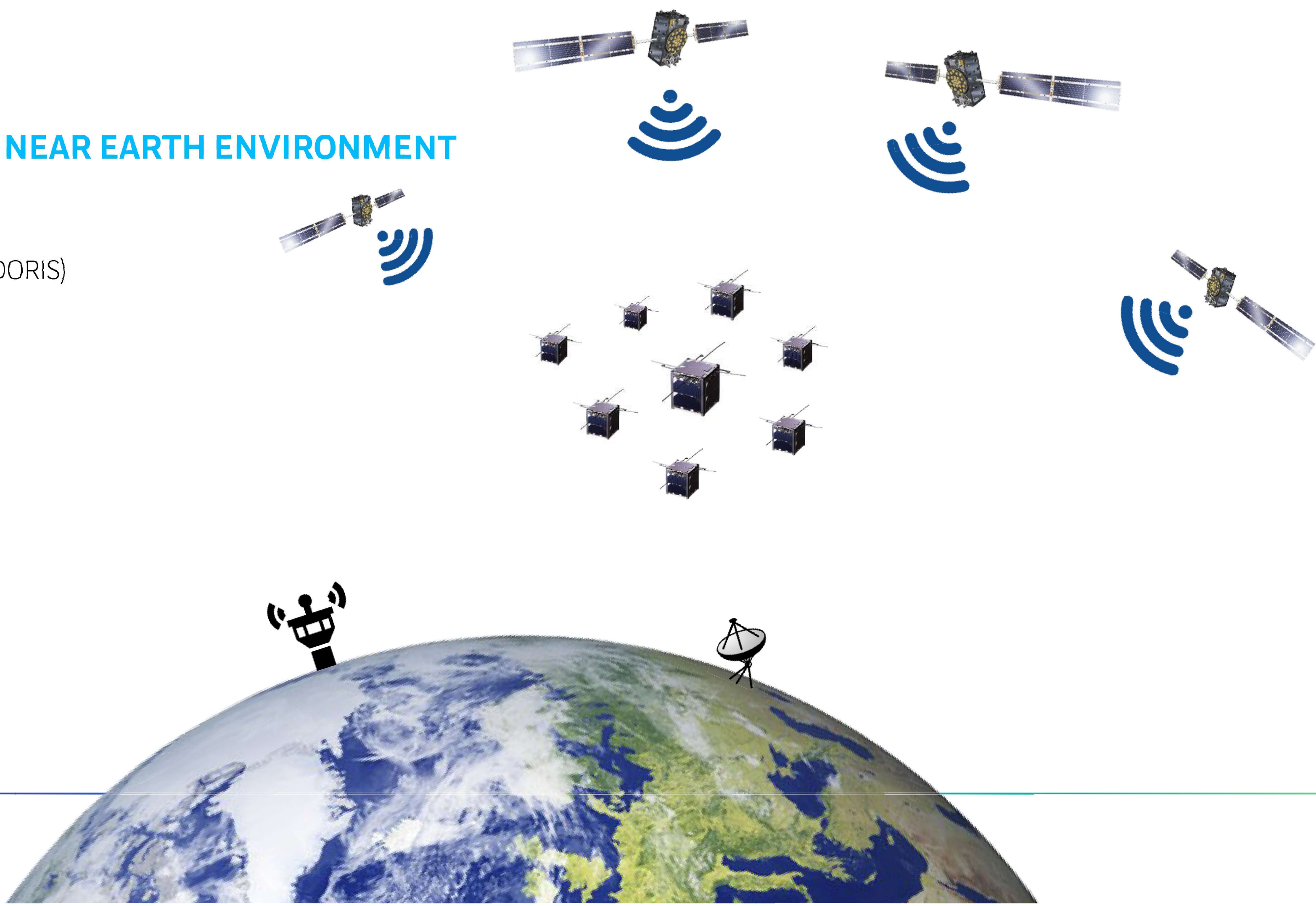
SUMMARY

INTRODUCTION

LOCALIZATION IN NEAR EARTH ENVIRONMENT

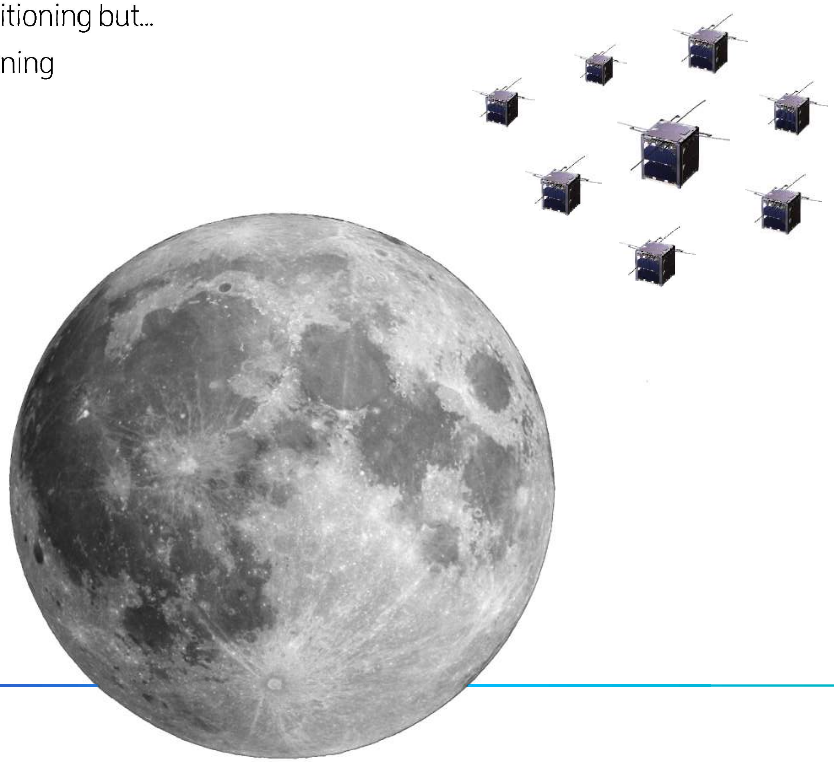
- GNSS
- Ground stations
- Reference beacons (DORIS)



INTRODUCTION

LOCALIZATION IN DEEP SPACE

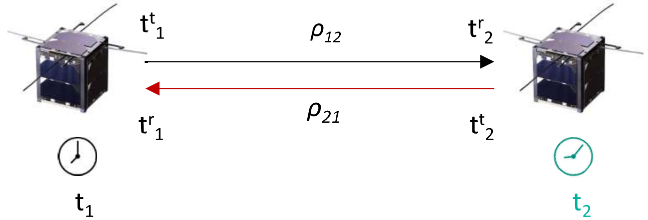
- Access to Earth infrastructure still possible, but more complex
- No need for accurate absolute positioning but...
- ... need for accurate relative positioning



INTRODUCTION

LOCALIZATION IN DEEP SPACE

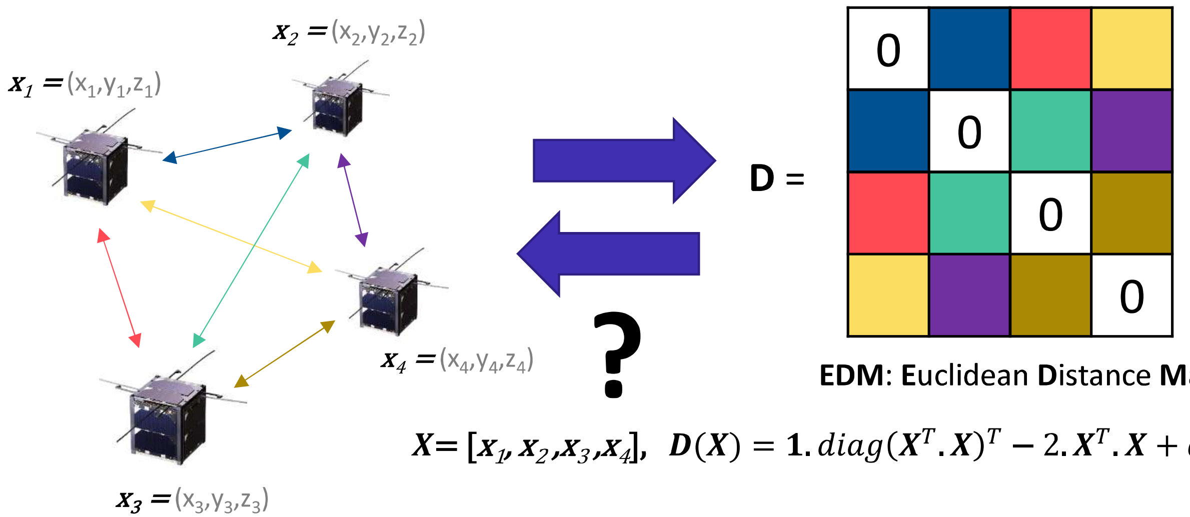
- Using inter-satellite measurements to improve relative positioning
  - Inter-distances
  - Relative velocities



$d_{12} = \frac{\rho_{12}+\rho_{21}}{2}$ → distance estimation

$\Delta t_{12} = \frac{\rho_{12}-\rho_{21}}{2.c}$ → time offset estimation

- Initial idea: use of EDM based methods



EDM: Euclidean Distance Matrix

$X = [x_1, x_2, x_3, x_4]$, $D(X) = \mathbf{1}.diag(X^T.X)^T - 2.X^T.X + diag(X^T.X).\mathbf{1}^T$

INTRODUCTION

SUMMARY OF ACTIVITIES REALIZED SO FAR

Performance characterization under ideal conditions (2021)

- Assumptions: all distances available, same noise
- Results: Cramer-Rao bound calculation, establishment of an efficient estimator

Performance characterization under degraded conditions (2022)

- Assumptions: some distances are missing, noise model as function of link budget
- Results: efficient EDM reconstruction method

Time offset estimation (2023)

- Assumptions: use of pseudo-distance if distance is not available
- Results: efficient estimation of time offset

Simple motion model (2024-2026)

- Assumptions: relative velocitiy measurements are available, simple orbital model
- Results: efficient estimation of instantaneous position and velocity

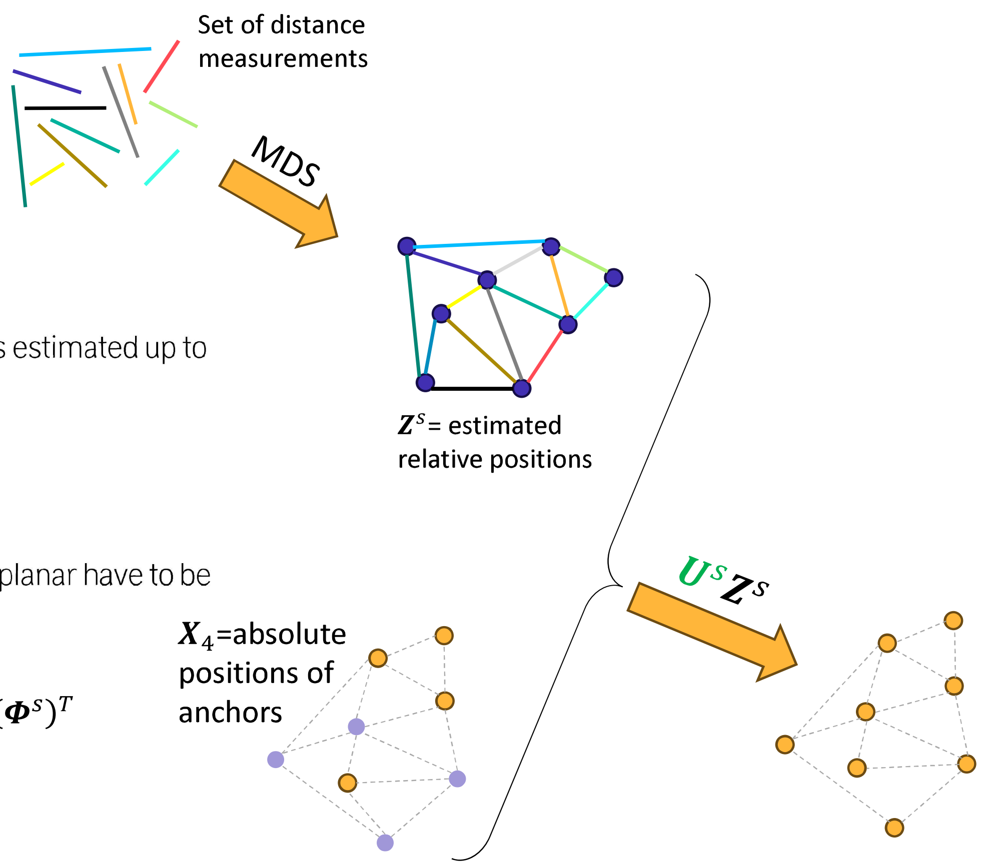
LOCALIZATION METHOD USING EUCLIDEAN DISTANCE MATRIX (EDM)

MDS PRINCIPLE

- Change of origin: $\boldsymbol{Y}^s = \boldsymbol{X}\left(\boldsymbol{I}_N - \frac{1}{N}\boldsymbol{1}_N\boldsymbol{1}_N^T\right) = \boldsymbol{U}^s\boldsymbol{\Sigma}^s\boldsymbol{V}^s$
- Centered EDM: $\boldsymbol{D}^s = (\boldsymbol{Y}^s)^T\boldsymbol{Y}^s = -\frac{1}{2}\left(\boldsymbol{I}_N - \frac{1}{N}\boldsymbol{1}_N\boldsymbol{1}_N^T\right)\boldsymbol{D}(\boldsymbol{I}_N - \frac{1}{N}\boldsymbol{1}_N\boldsymbol{1}_N^T)$
- Eigen value decomposition: $\boldsymbol{D}^s = \boldsymbol{V}^s((\boldsymbol{\Sigma}^s)^T\boldsymbol{\Sigma}^s)(\boldsymbol{V}^s)^T$
- Position estimation : $\boldsymbol{Z}^s = \boldsymbol{\Sigma}_3^s(\boldsymbol{V}_3^s)^T \Leftrightarrow \boldsymbol{Y}^s = \boldsymbol{U}^s\boldsymbol{Z}^s \Leftrightarrow \boldsymbol{X} = \boldsymbol{U}^s\boldsymbol{Z}^s + \frac{1}{N}\boldsymbol{X}\boldsymbol{1}_N \rightarrow \boldsymbol{X}$ is estimated up to a translation and unitary transform

MAP PRINCIPLE

- To estimate absolute position of each node, the absolute position of at least 4 non-coplanar have to be known (or measured)
- We search $\boldsymbol{U}^s$ such that: $\boldsymbol{X}_4 = \boldsymbol{U}^s\boldsymbol{Z}_4^s$
- Singular Value Decomposition: $\boldsymbol{Z}_4^s\left(\boldsymbol{I}_4 - \frac{1}{4}\boldsymbol{1}_4\boldsymbol{1}_4^T\right)\boldsymbol{X}_4 = \boldsymbol{\Phi}^s\boldsymbol{\Upsilon}^s(\boldsymbol{\Psi}^s)^T \Rightarrow \boldsymbol{U}^s = \boldsymbol{\Psi}^s(\boldsymbol{\Phi}^s)^T$



LOCALIZATION METHOD USING EUCLIDEAN DISTANCE MATRIX (EDM)

MAXIMUM LIKELIHOOD ESTIMATOR

- In case of noisy measurements of distance and anchor positions, the MDS+MAP method still works. But can it be improved ?

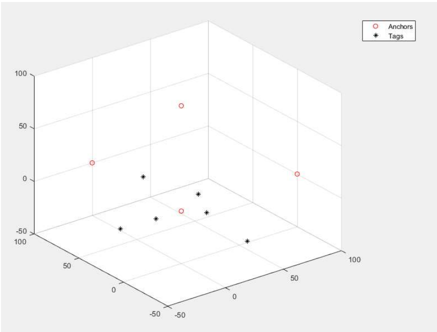
- Maximum likelihood estimator [1]:
  - $\widehat{\boldsymbol{\theta}}_{MV} = \arg\max_{\theta} \left\{ p(\widehat{\sqrt{\boldsymbol{D}}}, \widehat{\boldsymbol{X}_4}; \boldsymbol{\theta}) \right\}$, no analytical expression
  - $\widehat{\boldsymbol{\theta}}_{MV} \in \mathbb{R}^{3N} \rightarrow$ grid search impossible
  - Use of MDS+MAP as a starting point of a Gauss-Newton iterative algorithm (Fisher Scoring Method)

- Carmér-Rao bound has been computed to evaluate estimator efficiency

[1] E. Chaumette, J. Vilà-Valls and Y. Gregoire, "Centralized Cooperative Positioning From Euclidean Distance Matrices: Multidimensional Scaling and Maximum Likelihood," *2025 33rd European Signal Processing Conference (EUSIPCO)*, Palermo, Italy, 2025, pp. 840-84

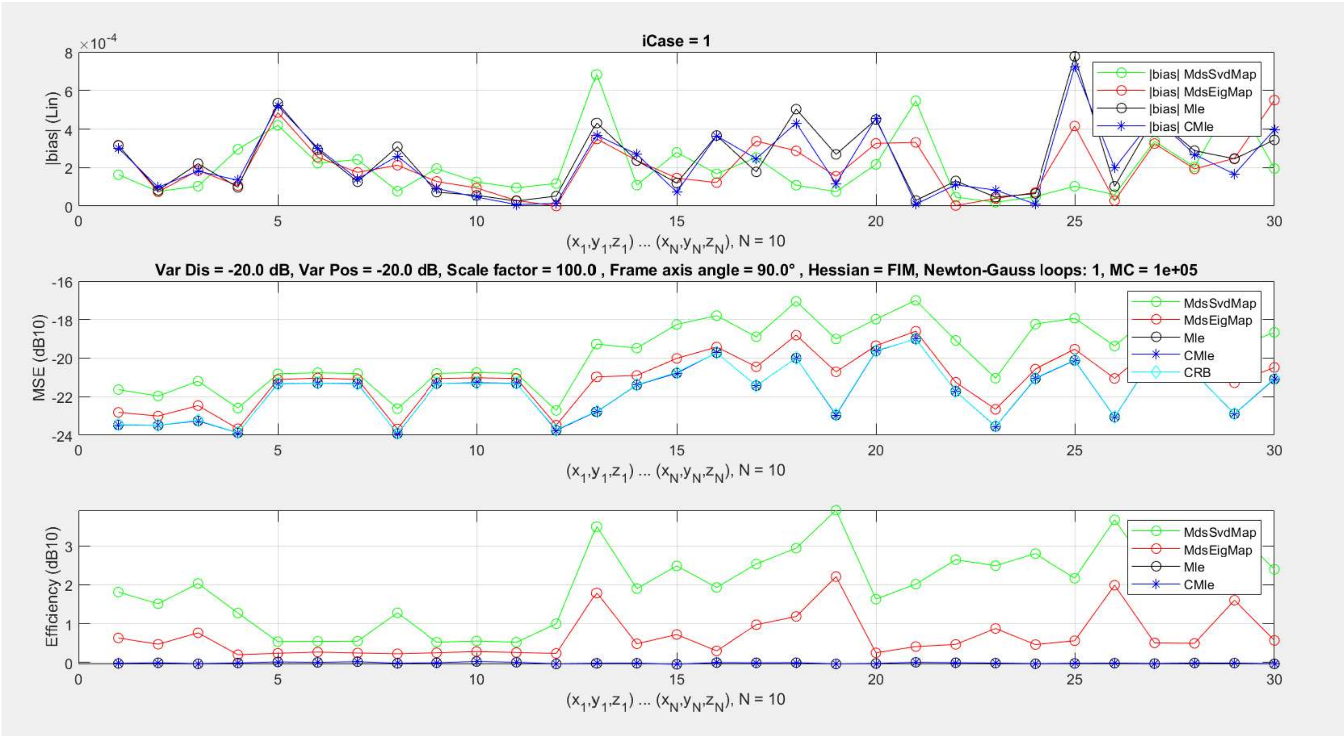
LOCALIZATION METHOD USING EUCLIDEAN DISTANCE MATRIX (EDM)

RESULT EXAMPLE



Example with 10 nodes:

- 4 anchors
- 6 tags



Unbiased estimation

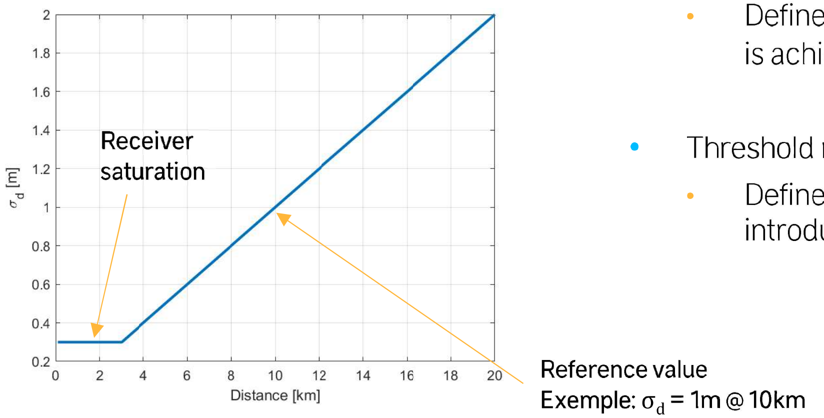
MLE is efficient and outperforms MDS+MAP method

LOCALIZATION METHOD USING EUCLIDEAN DISTANCE MATRIX (EDM)

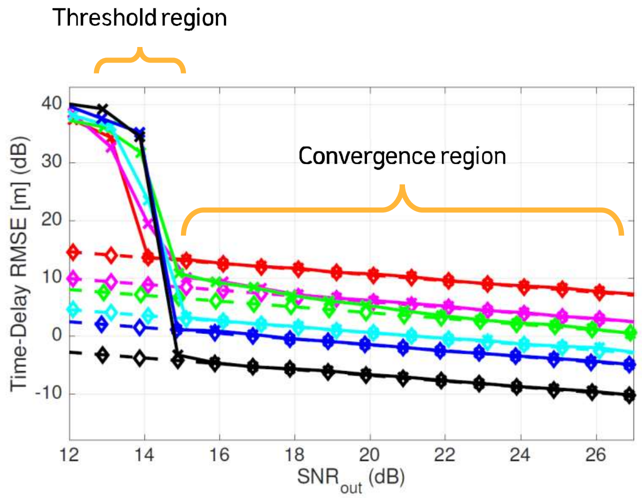
REFINEMENTS: LINK BUDGET

- Link budget considerations
  - Longer distance = low SNR = more noise on distance measurement, abnormal distance measurement (outlier) or distance measurement is missing
  - Short distance = high SNR = less noise up to a certain SNR (receiver saturation model effect)

- Model summary:
  - Covariance vs distance:

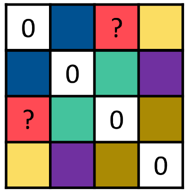


  - Probability of detection:
    - Define distance d_{P_0} at which P_0 detection probability is achieved
  - Threshold region:
    - Define how abnormal distance measurements are introduced



LOCALIZATION METHOD USING EUCLIDEAN DISTANCE MATRIX (EDM)

REFINEMENTS: MISSING DISTANCES



- **Missing distances**: how can we use EDM with unknown distances ?
  - Main idea:
    - EDM completion for MDS+MAP initialization
    - MLE applied to completed EDM
  - 2 strategies envisaged for EDM completion:
    - Optimistic case : each tag is connected to 4+ anchors ⟶ guarantee of EDM completion
    - Pessimistic case: at least one tag is connected to 4+ anchors and all tags are 4-connected ⟶ iterative process with no guarantee of EDM completion
  - MLE expression can easily manage missing distance
    - Cost function to minimize:

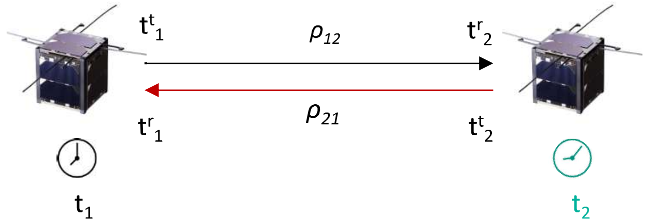
$$\mathcal{C}^d\left(\widehat{\sqrt{\boldsymbol{D}}};\boldsymbol{\theta}\right) = \frac{1}{2}\sum_{n=1}^{N}\sum_{l=1,l\neq n}^{N}\frac{\left(\hat{d}_{n,l} - \|x_n - x_l\|\right)^2}{\Sigma_{n,l}}$$

Covariance term is set at $+\infty$ if distance is missing

LOCALIZATION METHOD USING EUCLIDEAN DISTANCE MATRIX (EDM)

OTHER REFINEMENTS

- Estimation of time offsets:
  - Useful metrics for swarm synchronization



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- Addition of pseudo-velocities measurements:
  - Accurate relative velocities may help to better estimate swarm geometry
  - $\frac{\underline{v}_{n,l}+\underline{v}_{l,n}}{2} = (\boldsymbol{v}_n - \boldsymbol{v}_l)^T \left(\frac{\boldsymbol{x}_n-\boldsymbol{x}_l}{\|\boldsymbol{x}_n-\boldsymbol{x}_l\|}\right)$, with $\underline{v}_{n,l}$ = one way pseudo-velocity measurement
  - Pseudo-velocities are added to MLE

- Final version of MLE implementation allows estimating:
  - Absolute positions
  - Absolute velocities

  } Absolute position/velocity from at least 4 anchors

  - Absolute time offsets
  - Absolute frequency offsets

  } Absolute time/frequency offset from at least 1 anchor

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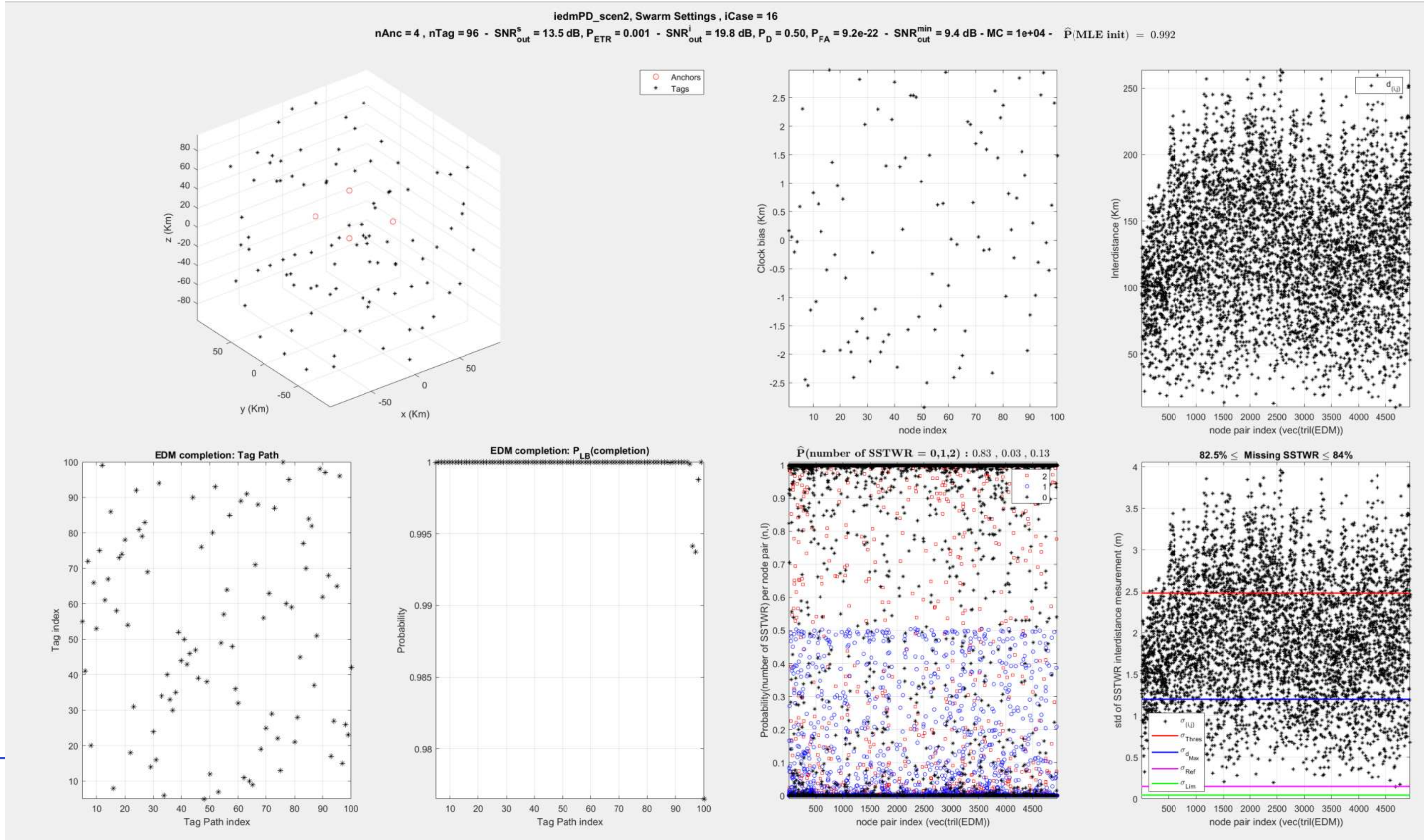
LOCALIZATION METHOD USING EUCLIDEAN DISTANCE MATRIX (EDM)

RESULTS EXAMPLE

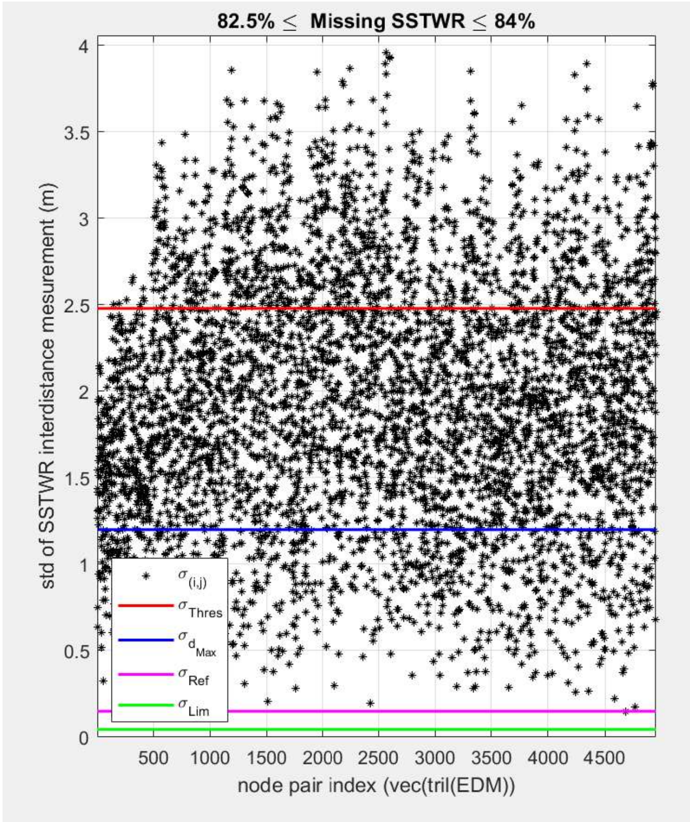
- 100 nodes



LOCALIZATION METHOD USING EUCLIDEAN DISTANCE MATRIX (EDM)

RESULTS EXAMPLE

- 100 nodes
- Success probability of EDM completion = 99.2 %
- Missing distances > 82 %



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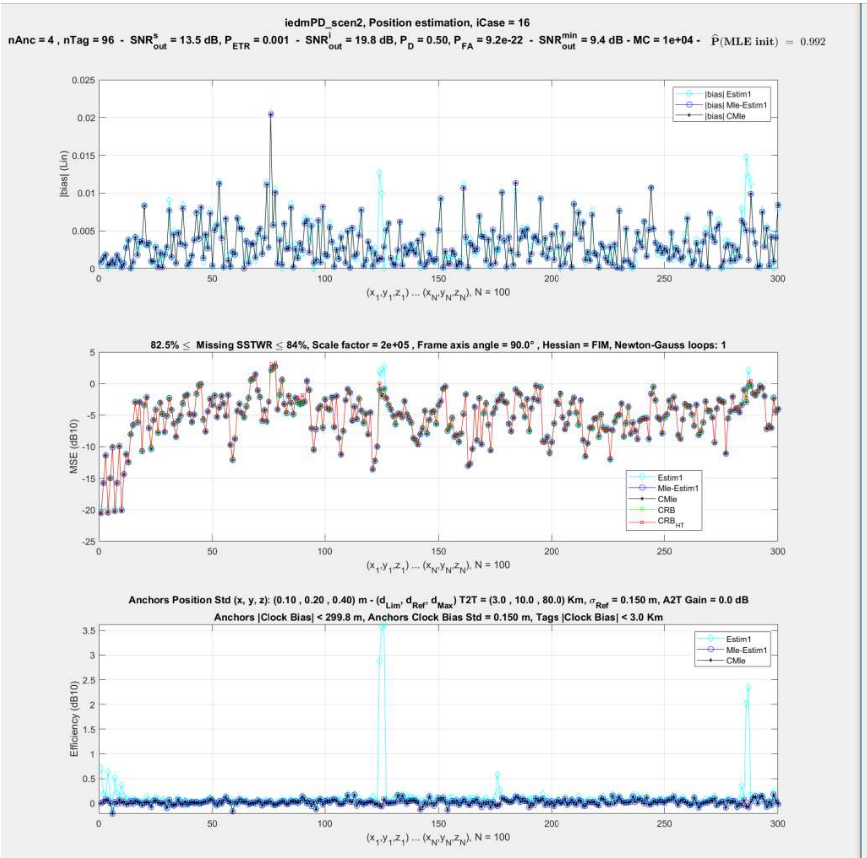

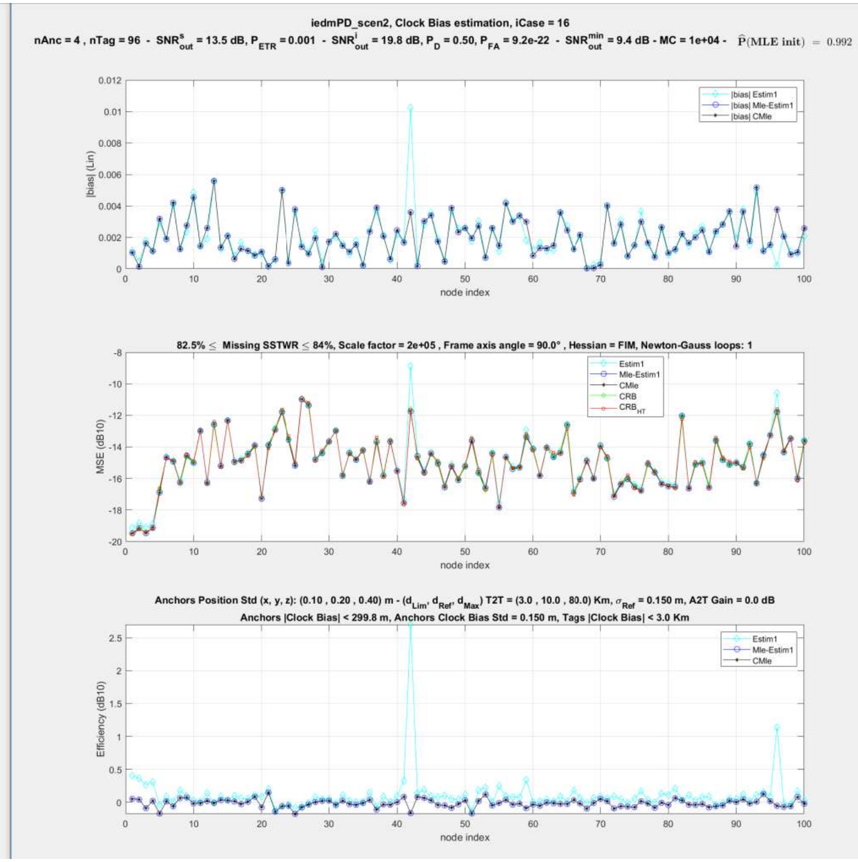

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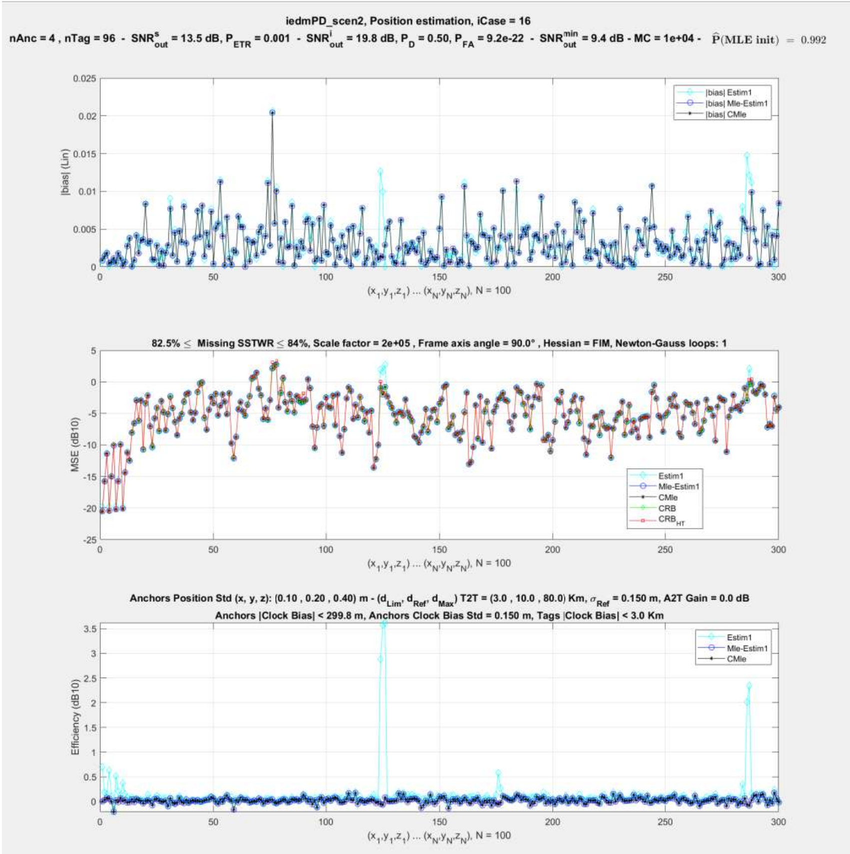

Unbiased estimation of position

MLE is efficient for position estimation

LOCALIZATION METHOD USING EUCLIDEAN DISTANCE MATRIX (EDM)

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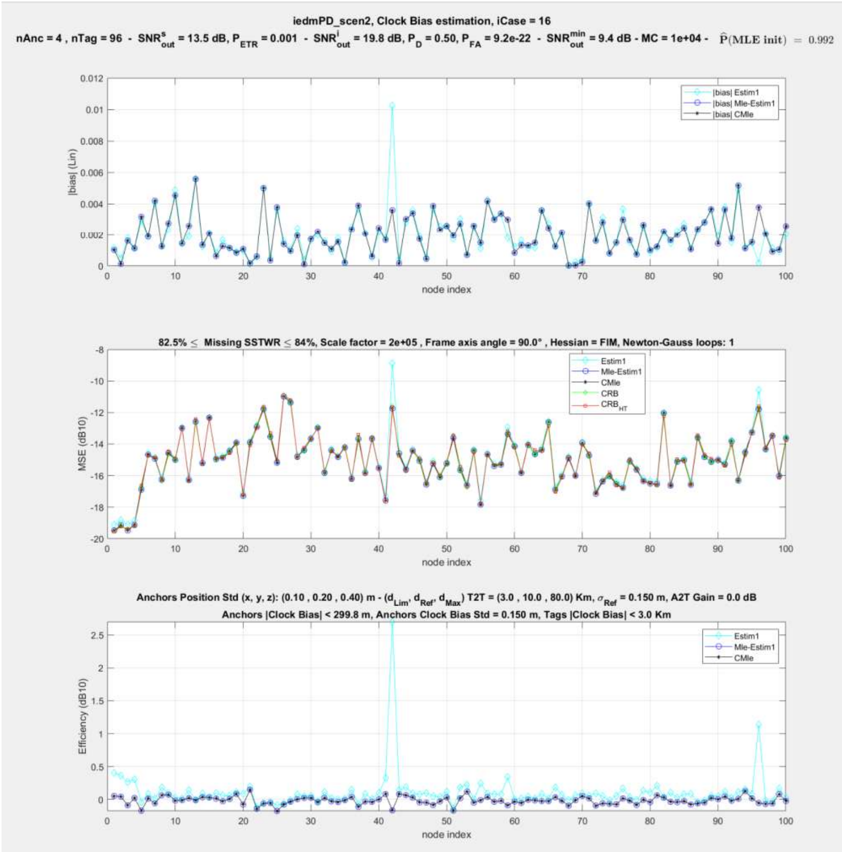


Unbiased estimation of time offsets

MLE is efficient for time offsets estimation

LOCALIZATION METHOD USING EUCLIDEAN DISTANCE MATRIX (EDM)

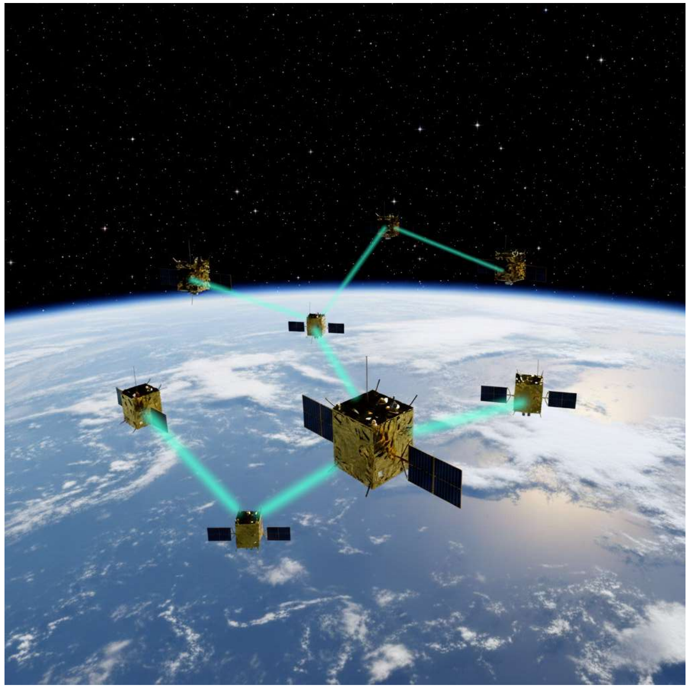
CONCLUSION

- An efficient method (in the MSE sense) has been developed for estimating the instantaneous absolute positions of a swarm of satellites from :
  - The partial knowledge of inter-distances measurement
  - The measurement of absolute position of at least 4 anchors

Klaus Barbie

- The method has been extended to:
  - Integrate an advanced link budget model (noise level vs distance, probability of detection, outliers generation model)
  - Integrate pseudo-velocities measurements
  - Estimate: velocities, time offsets and frequency offsets

- Next step: integration of a simple orbital model for satellite motion



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Localization in a Swarm of satellites

Consideration of simple orbital dynamics

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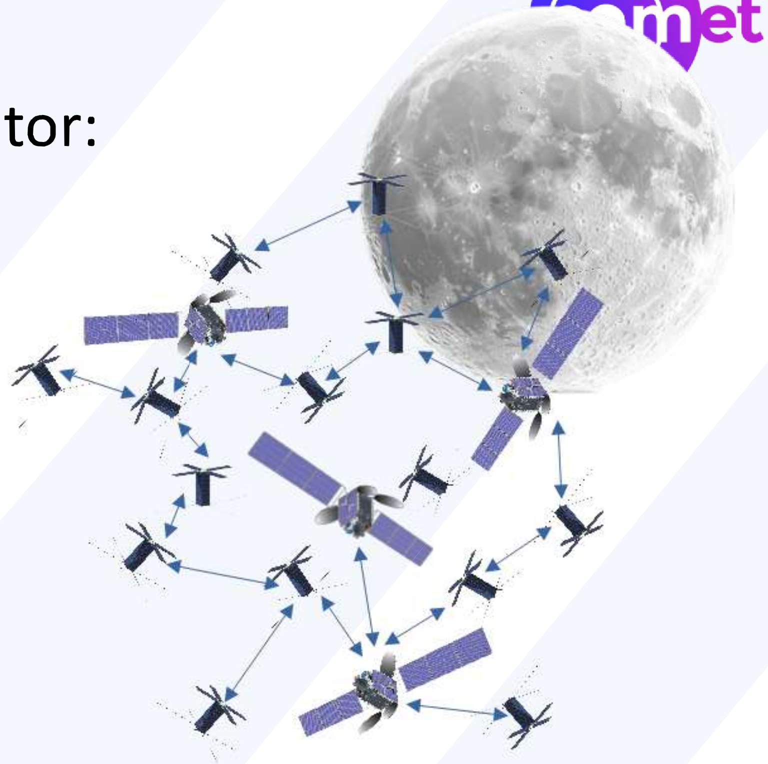
1. Interface with MLE

- Specificities of the swarm position-velocity estimator:
  - Swarm structure:
    - A few anchors (min. 4) forming a trihedron;
    - Several tens of tags.
  - Measurements considered:
    - Anchors positions, velocities, clock bias and clock drift,
    - ISL pseudo-range and pseudo-velocity measurements
  - Estimator operation:
    - Snapshot estimation of tags positions and velocities from a set of measurements performed at a single epoch,
    - Estimation is performed at a regular time step.

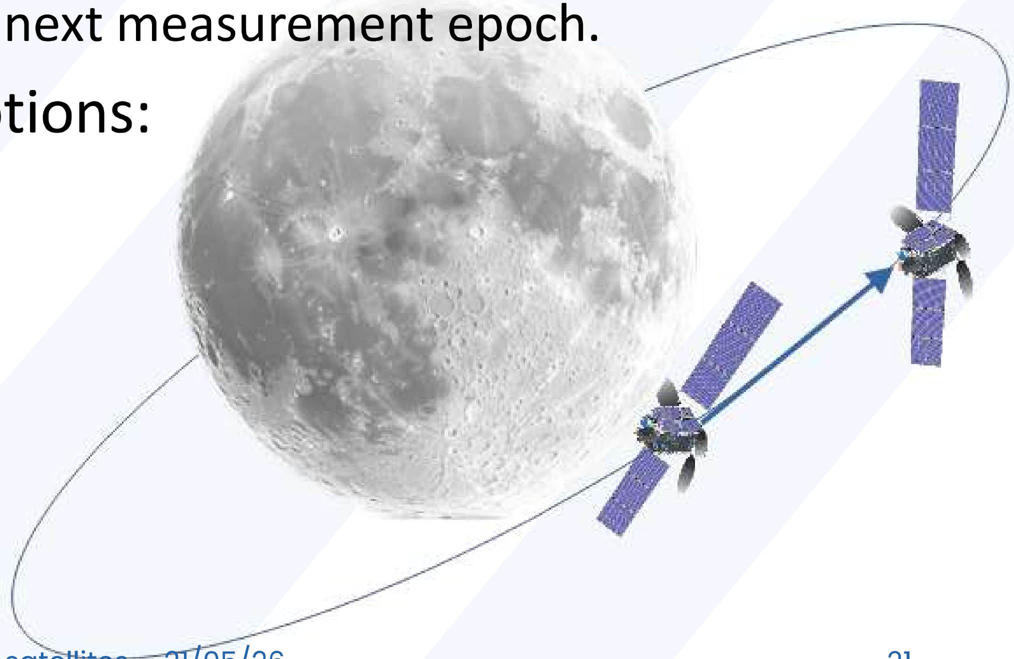


The evolution over time of the states of the swarm satellites is not considered, while it is strongly constrained by orbital mechanics.

2. Swarm trajectory

- **Assumption:** satellite trajectories are essentially defined by the gravitational field to which they are submitted.
  - The gravitational field provides a differential equation which accounts for the satellites' dynamics
  - Solving this equation, initialized from the MLE position and velocity, provides a prediction of position and velocity at the next measurement epoch.
- A simple example based on two assumptions:
  - Gravitation is the only force applied to the satellite
  - The gravitational field is consistent with the two-body model (i.e. it is due to a single massive body with spherical symmetry).

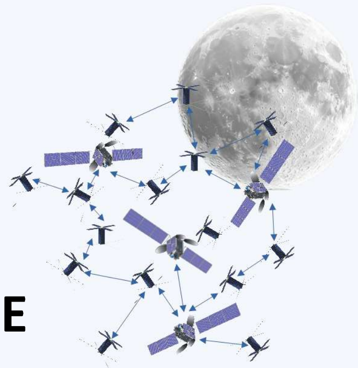
$$\vec{a} = -\frac{\mu}{\|\vec{p}\|^3}\vec{p}$$



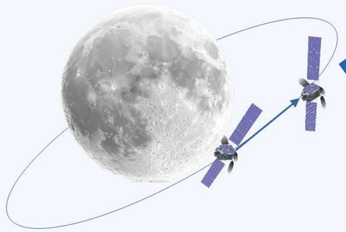
3. Aim of the proposed filter

- **Aim:** fuse the orbital movement model with the measurements provided by the snapshot MLE.

- **Positions, velocities, clocks measured by the MLE**



- **Orbital model**



Kalman filter

- **Improved positions, velocities, clocks**

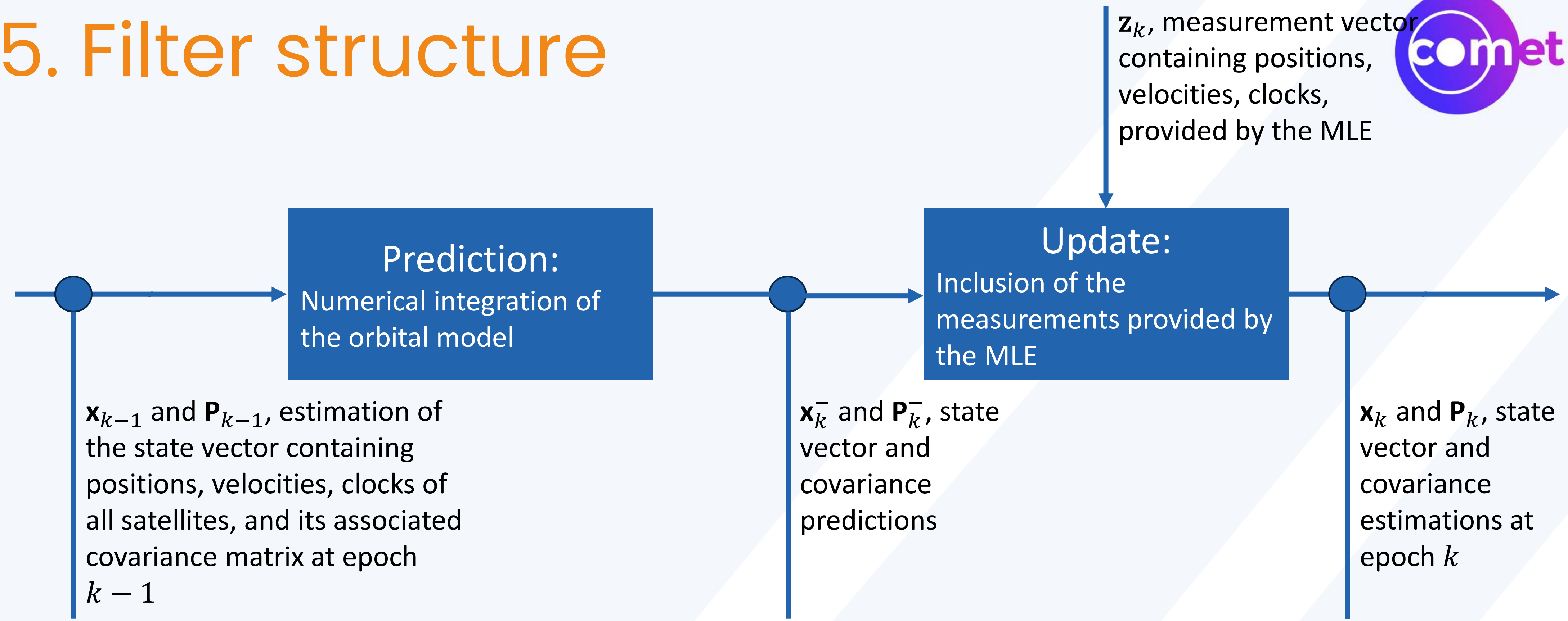
TéSA

4. Algorithmic expression of the system

$$\mathbf{x}_k = \mathbf{f}(\mathbf{x}_{k-1}, \mathbf{u}_{k-1}) + \mathbf{w}_{k-1}$$
$$\mathbf{z}_k = \mathbf{h}(\mathbf{x}_k, \mathbf{u}_k) + \mathbf{v}_k$$

- k: time index;
- $\mathbf{x}_k$: state vector, i.e. position, velocity, clock of each satellite in the swarm;
- $\mathbf{z}_k$: measurement vector, i.e. position, velocity and clock measurements provided by the MLE;
- $\mathbf{u}_k$: control vector ; $\mathbf{w}_{k-1}$: Gaussian zero-mean model noise, covariance $\mathbf{Q}_{k-1}$; $\mathbf{v}_k$: Gaussian zero-mean measurement noise, covariance $\mathbf{R}_k$
- **State model f :** prediction of position and velocity through numerical integration of the gravitational model. Numerical integrator used: Runge-Kutta.
- **Measurement model h :** identity, since the MLE directly measures the state vector parameters.

5. Filter structure



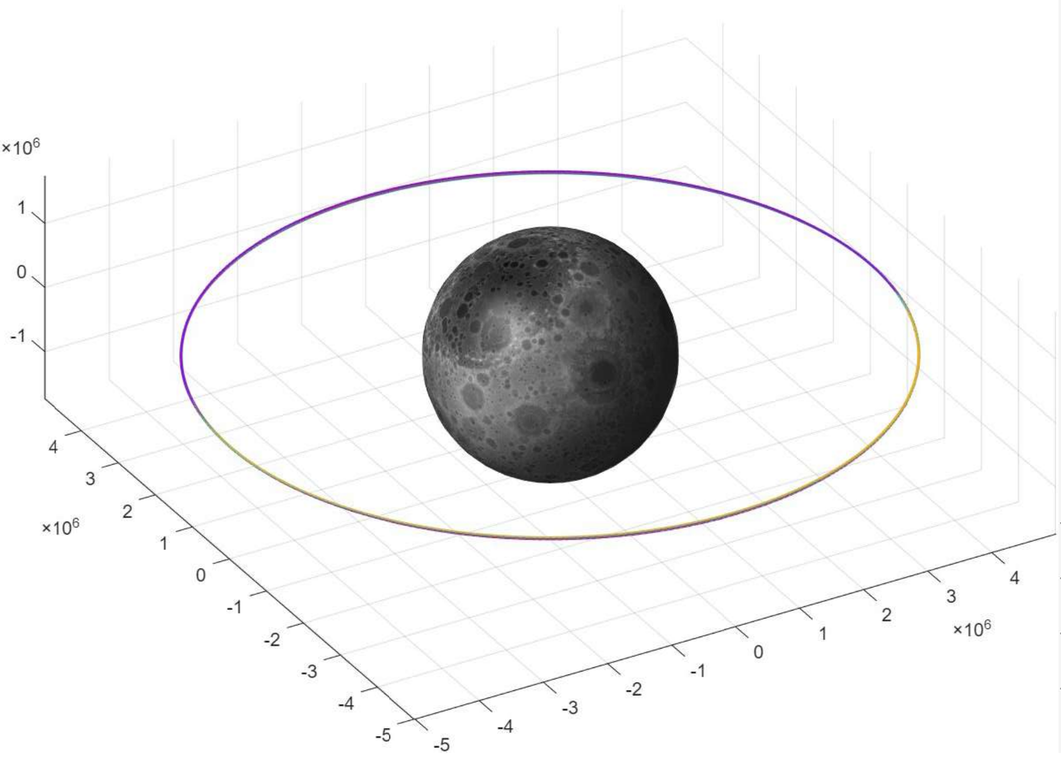
$\mathbf{x}_k$ is estimated by a *Cubature Kalman Filter*: the CKF estimates the state of a non-linear system by propagating a set of deterministic points through state and measurement models, to approximate means and covariances without requiring explicit linearization.

6. Swarm modelling

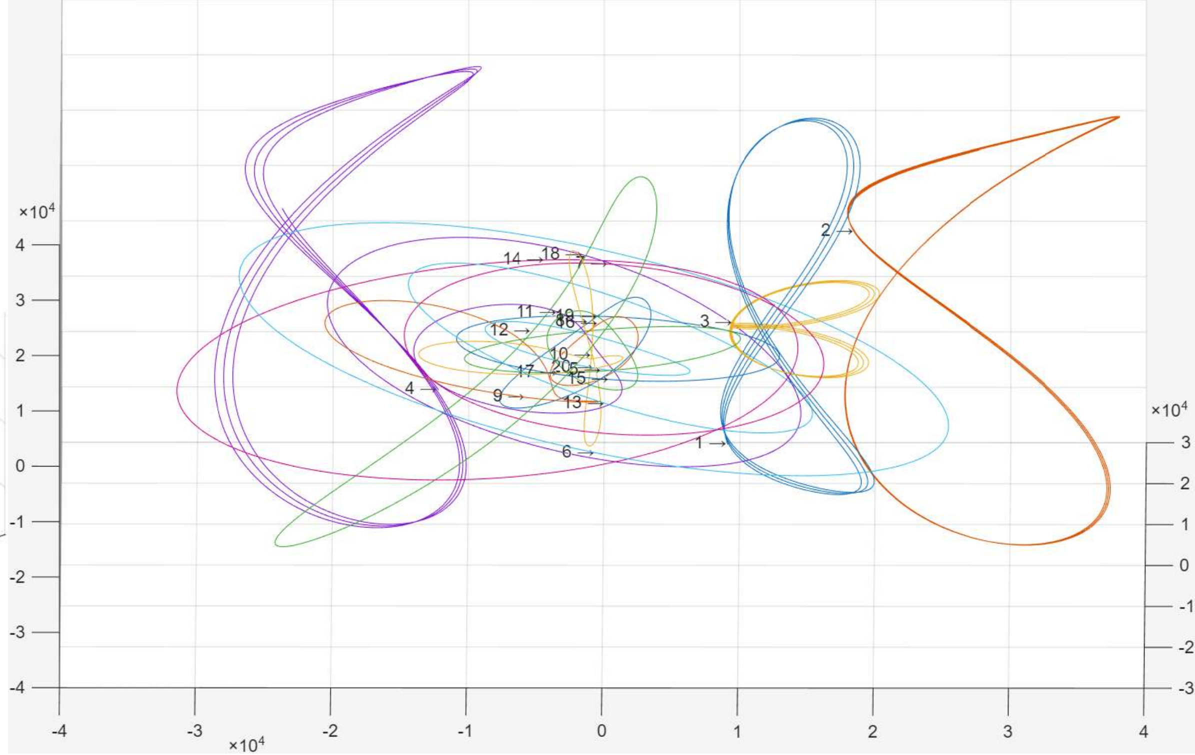
Aim: generate satellite trajectories consistent with the considered gravitational model

Example of the two-body model:

Moon-centric trajectories



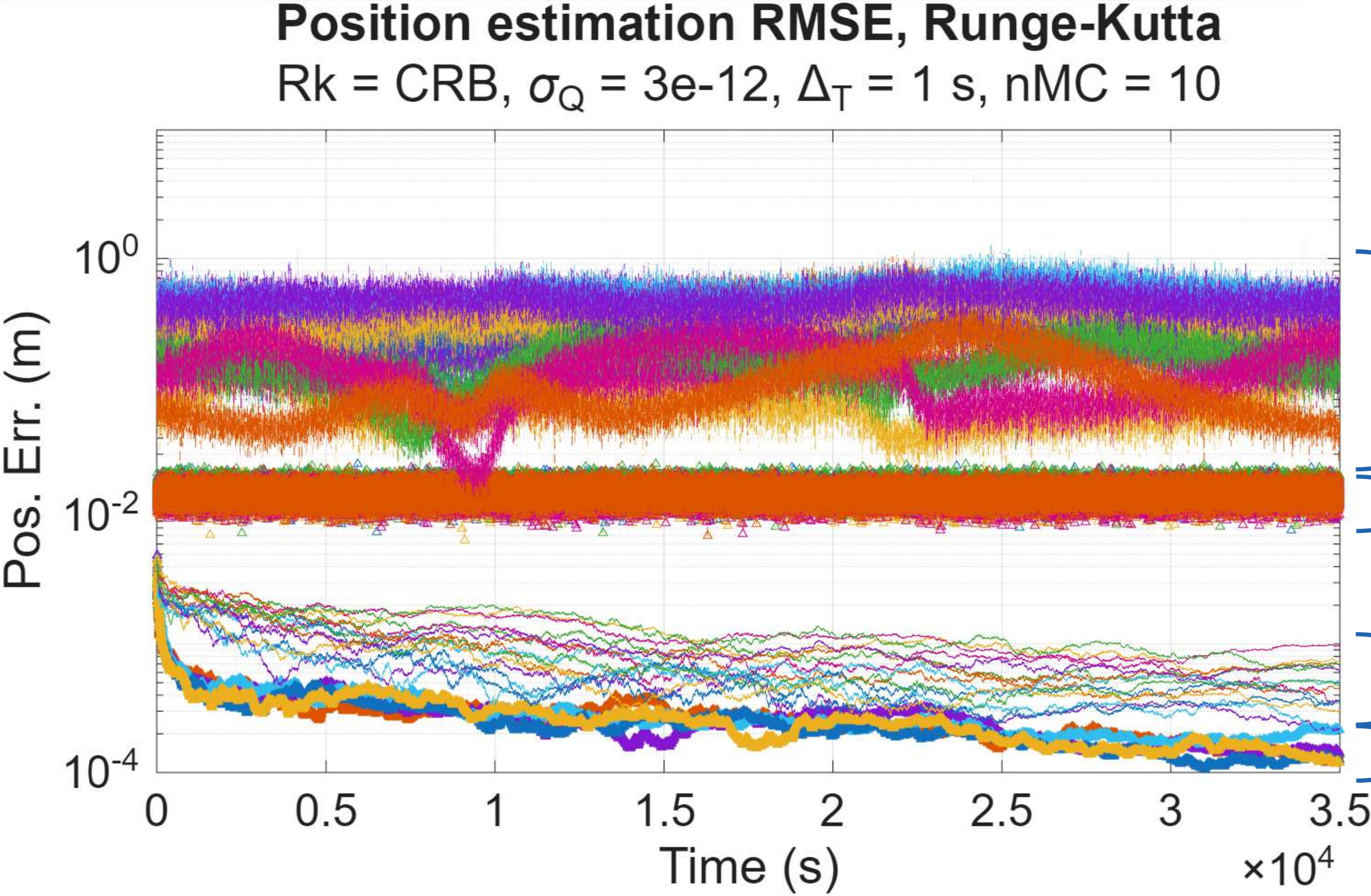
Barycentric trajectories



7. Simulation conditions

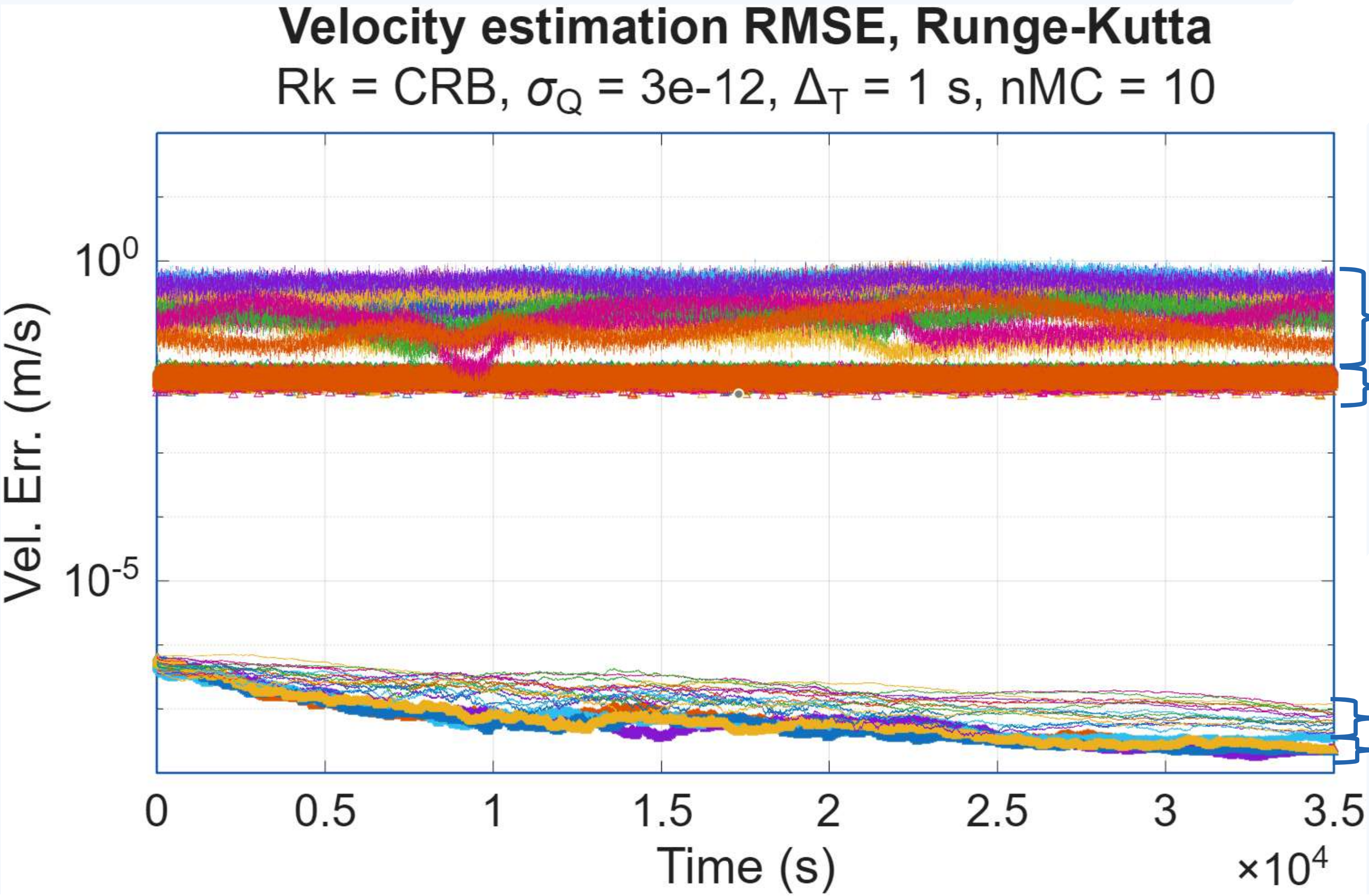
- Physical model: trajectories are generated under a two-body model assumption, which is also integrated to the filter's state model.
- Simulation scenario:
  - Swarm: 20 satellites (5 anchors, 15 tags)
  - Measurements: no outlying measurement, the MLE converges correctly at each measurement epoch.
  - Trajectories: total duration is 35000s, with a 1 second measurement period. Each trajectory is repeated 10 times.
  - Model noise: $\mathbf{Q}_k$, estimated a priori
  - Measurement noise: $\mathbf{R}_k$ computed from the Cramér-Rao Bound provided by the MLE
- Performance criterion:
  - For each satellite, the position and velocity root mean square errors are estimated and compared to the MLE output.

8. Simulation Results



- Tag position RMSE at MLE output
- Anchor position RMSE at MLE output
- Tag position RMSE at Kalman filter output
- Anchor position RMSE at Kalman filter output

8. Simulation Results



- Tag velocity RMSE at MLE output
- Anchor velocity RMSE at MLE output
- Tag velocity RMSE at Kalman filter output
- Anchor velocity RMSE at Kalman filter output

8. Conclusion

- Using a Cubature Kalman Filter to hybridize orbital model with MLE measurements allows reducing estimation error:
  - By a factor 100 for position,
  - By a factor 10^5 for velocity.

- However, these results are obtained under favourable conditions:
  - The MLE's performance is optimal in the result example, but in case of outlying measurements, the CKF's performance is degraded: an outlier detection test will need to be integrated in the filter update.
  - Trajectory generation and prediction both use the same simple orbital model: it will be necessary to update this model to account for more realistic trajectories.