GNSS Receiver Signal Processing Under Spoofing

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ENAC Telecom, Toulouse

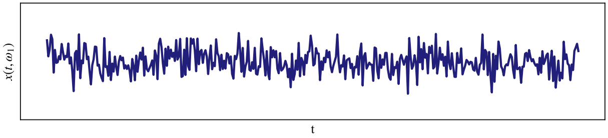
Seminar Presentation

October 17, 2025

A Probabilistic Framework for Non-ergodic Signals

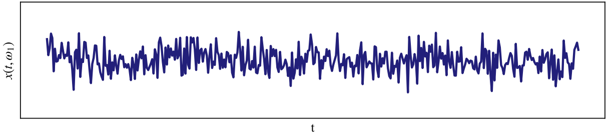
Time-based estimation under non-ergodicity

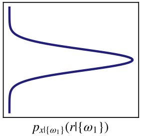
$$y(t, \omega_1) = x(t, \omega_1)$$



Time-based estimation under non-ergodicity

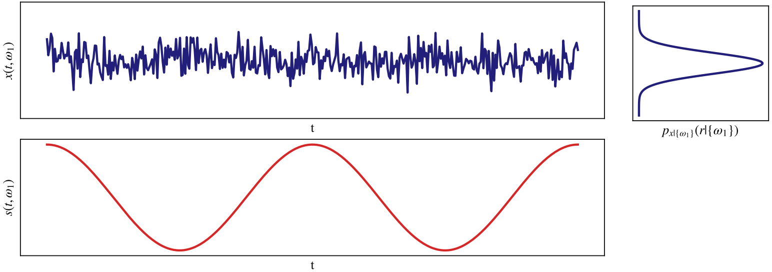
$$y(t,\omega_1) = \underbrace{x(t,\omega_1)}_{\text{Gaussian}}$$





Time-based estimation under non-ergodicity

$$y(t,\omega_1) = \underbrace{x(t,\omega_1)}_{\text{Gaussian}} + \overbrace{s(t,\omega_1)}^{\text{Continuous wave (CW)}}$$



Time-based estimation under non-ergodicity

Continuous wave (CW)

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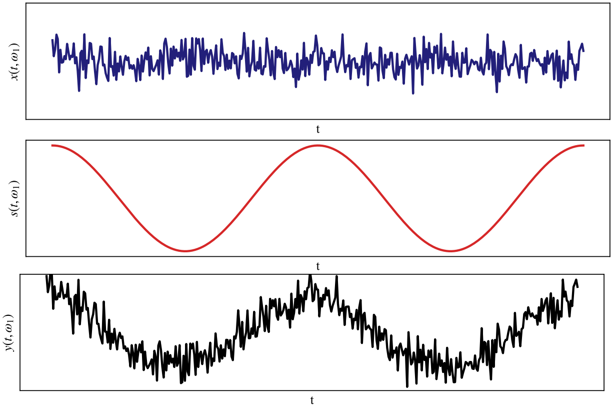
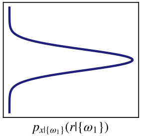
Gaussian





Ergodicity

Ergodicity allows the statistical properties of a signal to be inferred from a single realization, ω_1, observed over a time interval I_t.

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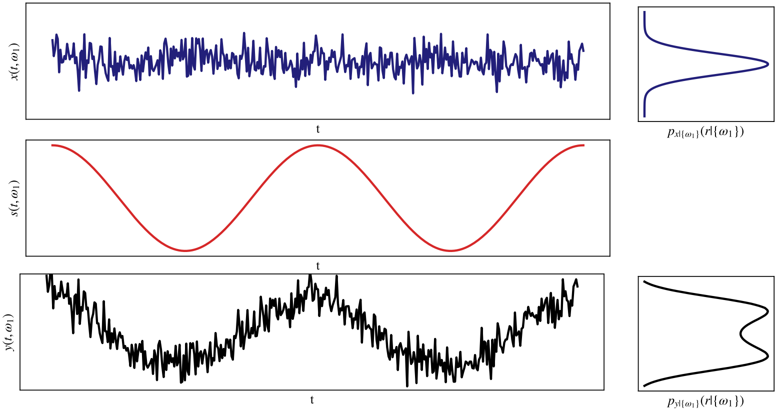

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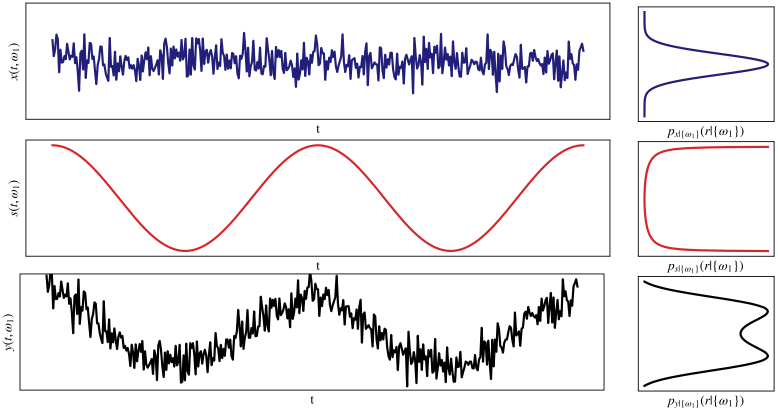

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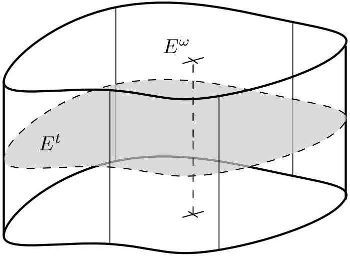
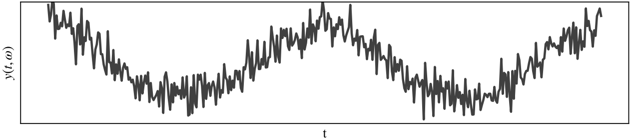
Proposed framework (1/2)

$$\left(I_t \times I_\Omega \,,\, \mathcal{E}_\Omega \otimes \mathcal{E}_t \,,\, \mathbb{P}_{t\times\Omega} \right)$$





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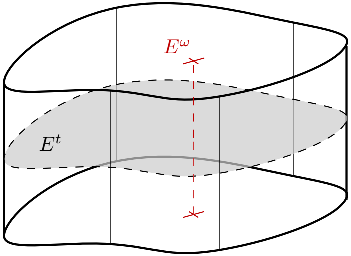

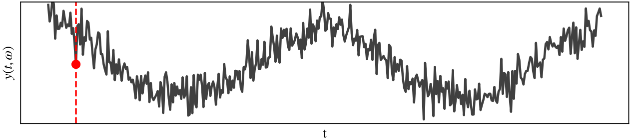

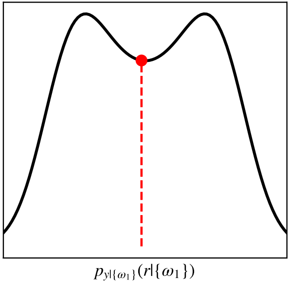

Time probability space:

$$(I_t, \mathcal{E}_t, \mathbb{P}_t)$$

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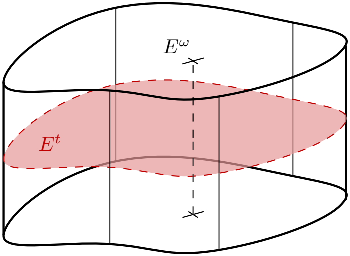
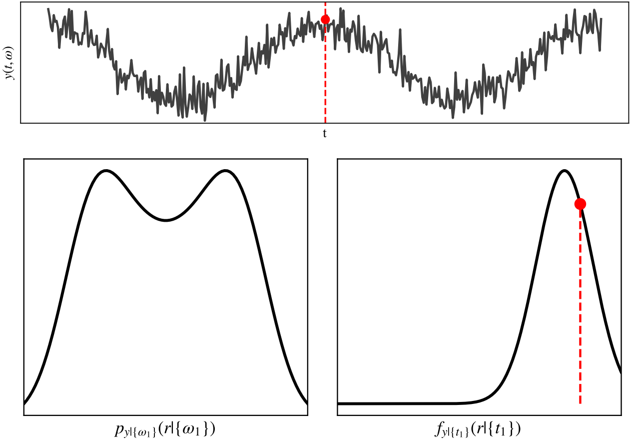


Random probability space:

$$(I_\Omega, \mathcal{E}_\Omega, \mathbb{P}_\Omega)$$



Proposed framework (2/2)

Measure Theory Framework: Consider the time interval as a probability space, with the observed signal represented as a random variable over the total event space:

- **Time Probability Space:** Captures the signal realization over time
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- **Total Probability Space:** Combines the time and random realizations of the signal.

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Measure Theory Framework

- ✓ Probability theory enables formal definitions of power, distribution, and independence within each probability space.
- ✓ Provides a basis for defining properties such as ergodicity and stationarity.
- ✓ Facilitates the analysis of classical signals (e.g., Markov processes, CW, multiple CWs).

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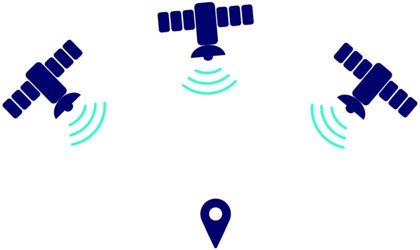

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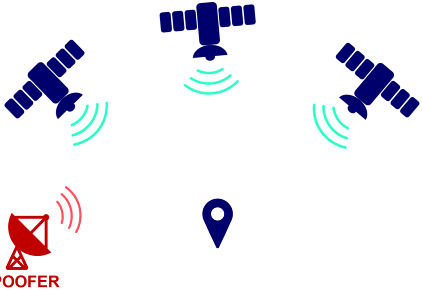

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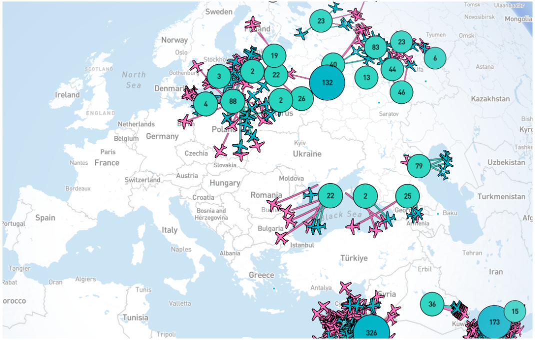

Recorded spoofing events on aircraft receivers (06/11/2025)

https://spoofing.skai-data-services.com/

Which stages of GNSS receivers are affected by spoofing?

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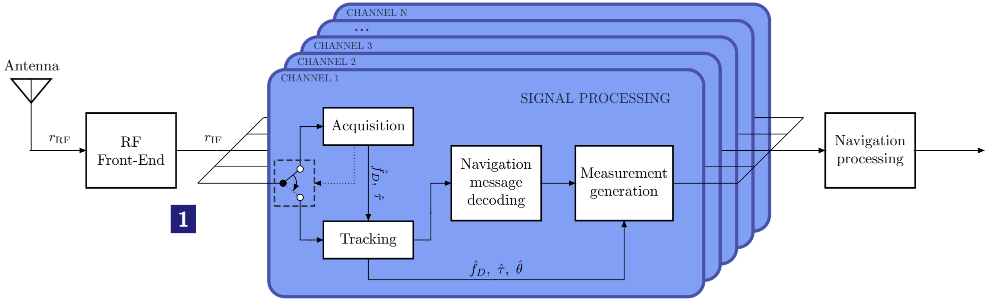

1

- Automatic Gain Control (AGC) [1]
- Signal prior to correlation (power, distribution) [1]

[1] Emile Ghizzo et al. "Assessing jamming and spoofing impacts on GNSS receivers: Automatic gain control (AGC)". In: *Signal Processing* 228 (2025), p. 109762

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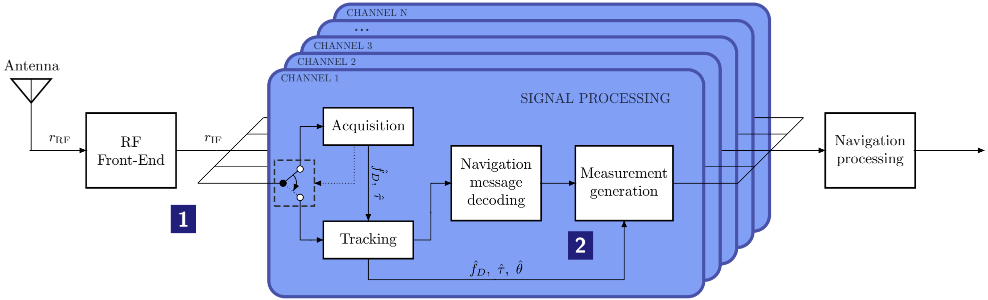

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- Automatic Gain Control (AGC)
- Signal prior to correlation (power, distribution)

2
- Signal post-correlation [2] [4]
- Synchronization estimates [3]
- C/N_0, power [4]

[2] Mathieu Hussong et al. "Impact of Meaconers on Aircraft GNSS Receivers During Approaches". In: *Proc. ION GNSS+ 2023*. Sept. 2023, pp. 856–880
[3] Emile Ghizzo et al. "Assessing Spoofer Impact on GNSS Receivers: Tracking Loops". In: *NAVIGATION: Journal of the Institute of Navigation* 72.4 (2025)
[4] Emile Ghizzo, Julien Lesouple, and Carl Milner. "Assessing jamming and spoofing impacts on GNSS receivers: Carrier-to-Noise density Ratio". In: *Signal Processing* (2025)

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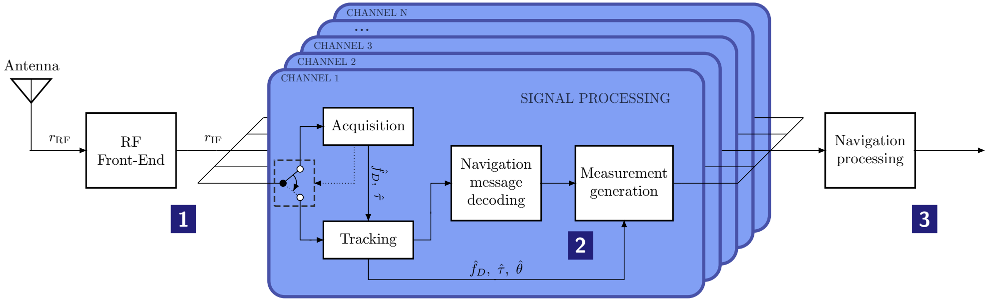

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- Automatic Gain Control (AGC)
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- Synchronization estimates
- C/N_0, power

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- Pseudoranges [5]
- Position [5]
- Residuals

[5] Mathieu Hussong et al. "GNSS performance degradation under meaconing in civil aviation: pseudorange and position models". In: *GPS Solutions* 29.3 (2025), p. 93

Objectives

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Characterize the impact of spoofing on GNSS receiver signal processing, both before and after correlation.

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- Derived theoretical models to **predict spoofing impact both before correlation and after correlation**.

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Characterize the impact of spoofing on GNSS receiver signal processing, both before and after correlation.

Contributions

- Introduced a measure-theoretic framework to **study time-based estimation under non-ergodic conditions.**
- Derived theoretical models to **predict spoofing impact both before correlation and after correlation**.
- Identified specific spoofing situations where GNSS receivers **exhibit harmful distortions**, including tracking loop chaotic behaviors (e.g., high oscillations, bifurcations) and significant C/N_0 degradation.

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Outline

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Received signal model

$$r_{\mathrm{RF}}(t) = \Re\left\{ \left[\sum_{i \in \mathcal{M}_{\mathrm{a}}(t)} \sqrt{C_{\mathrm{a}}^{i}(t)}\; s_i(t - \tau_{\mathrm{a}}^{i}(t))e^{j\,\theta_{\mathrm{a}}^{i}(t)} + \sum_{i \in \mathcal{M}_{\mathrm{s}}(t)} \sqrt{C_{\mathrm{s}}^{i}(t)}\; s_i(t - \tau_{\mathrm{s}}^{i}(t))e^{j\,\theta_{\mathrm{s}}^{i}(t)} \right] \exp(2j\pi f_0 + j\theta_0) + n_{\mathrm{a}}(t) + n_{\mathrm{s}}(t) \right\}$$

$s_i(t)$: GNSS signal of the i^{th} satellite (Navigation message and spreading signal)

Received signal model

Authentic dynamic

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Impact of spoofing on the RFFE

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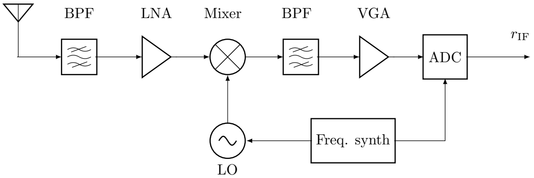
- Characterize the signal at RFFE output
- Characterize the AGC dynamic response

Impact of spoofing on the RFFE



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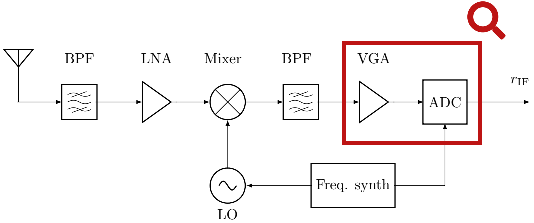

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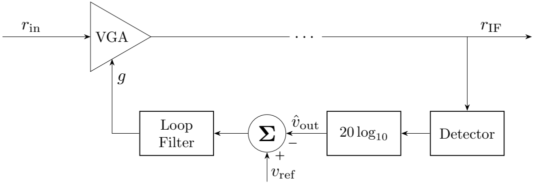

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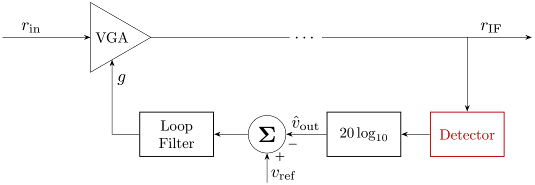

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Two Types of Detectors:[1]

- **Power-based:** Computed from the time-domain power over the estimation time I_t.
- **Distribution-based:** Computed from the time-domain distribution over I_t.

[1] Juan Pablo Alegre Pérez, Santiago Celma Pueyo, and Belén Calvo López. *Automatic Gain Control.* Springer, 2011

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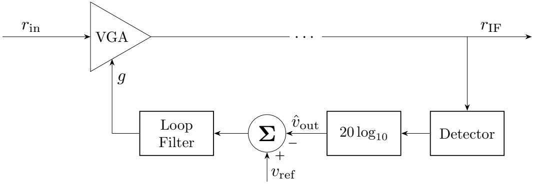

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AGC dynamic model:

$$\overset{\text{VGA gain}}{g(s)} = \frac{F(s)}{1 + F(s)} \left(v_{\text{ref}}(s) - v_{\text{in}}(s) \right)$$

$$v_{\text{out}}(s) = g(s) + v_{\text{in}}(s)$$

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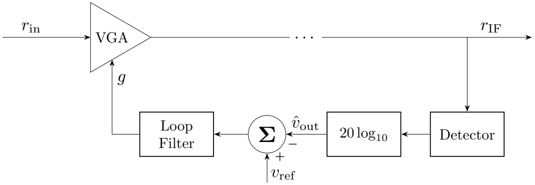

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$$\overbrace{g(s)}^{\text{VGA gain}} = \frac{F(s)}{1 + \underbrace{F(s)}_{\text{Low-pass filter}}} \left(v_{\mathrm{ref}}(s) - v_{\mathrm{in}}(s) \right)$$

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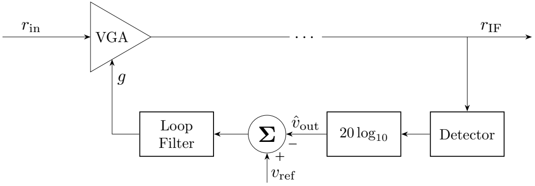
[1] Juan Pablo Alegre Pérez, Santiago Celma Pueyo, and Belén Calvo López. *Automatic Gain Control.* Springer, 2011

Impact of spoofing on the RFFE



Objectives

- Characterize the signal at RFFE output
- Characterize the AGC dynamic response

AGC dynamic model:

$$\overbrace{g(s)}^{\text{VGA gain}} = \frac{F(s)}{1+\underbrace{F(s)}_{\text{Low-pass filter}}}\left(v_{\text{ref}}(s) - v_{\text{in}}(s)\right)$$

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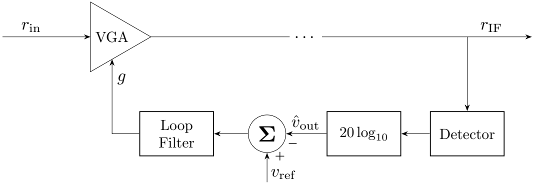

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Spoofing impact on signal level

Signal Level v

Describes the strength of the signal r. It can be defined through the signal power or its distribution within the estimation interval I_t, depending on the type of detector.

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Signal parameters (constant over I_t): $a_{s,m}$, $\tau_{s,m}$, f_m, θ_m

Results

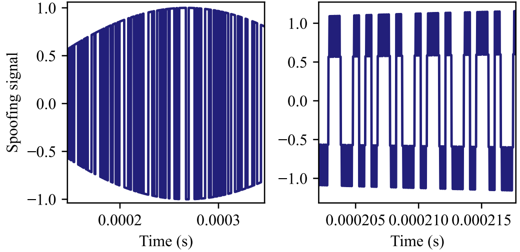
The input signal level v_{in} (and, consequently, the AGC) can be determined by computing its power and distribution.

Individual PRN spoofing signal ($M = 1$)

Results

The individual PRN signal is shown to be equivalent to a weighted **continuous wave (CW)**, modeled through the proposed **measure theory framework**.

Signal time evolution:



(a) BPSK (b) CBOC

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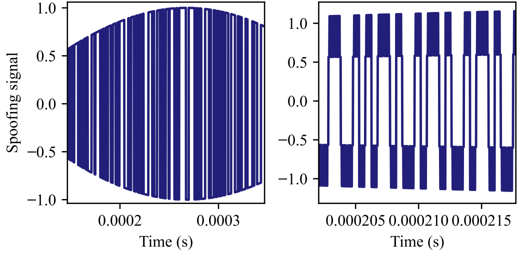

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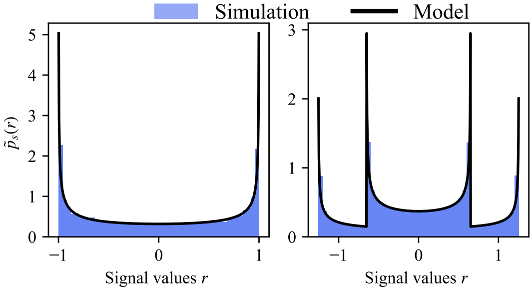

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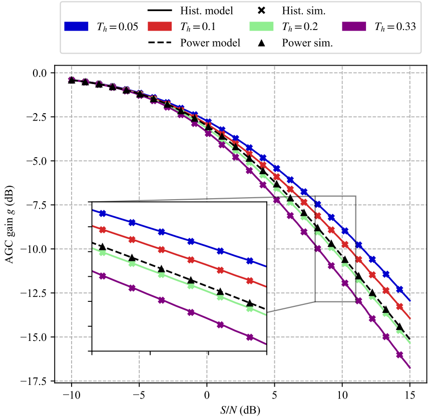
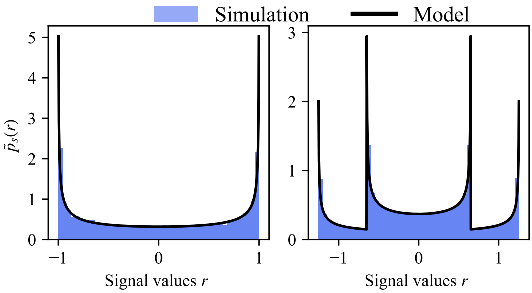




(a) BPSK (b) CBOC

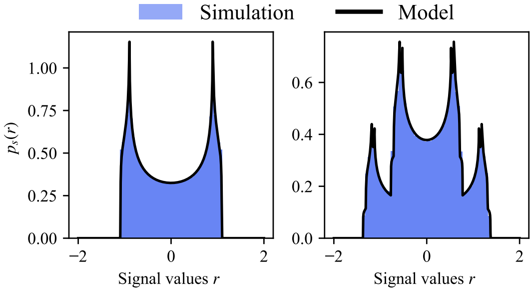
S/N: Spoofing-to-Noise power ratio

Multiple PRN Spoofing Signal ($M > 1$)

Results

- Equivalent to multiple weighted CWs.

Signal distribution (M=2):



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Multiple PRN Spoofing Signal ($M > 1$)

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- Equivalent to multiple weighted CWs.
- Distribution depends on the relative amplitudes between CWs.

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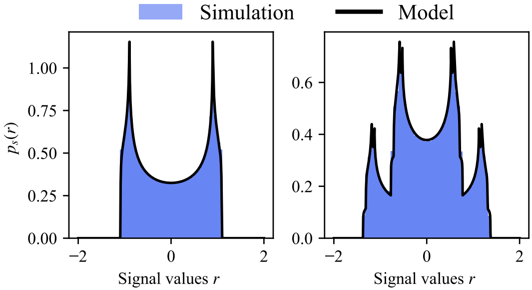

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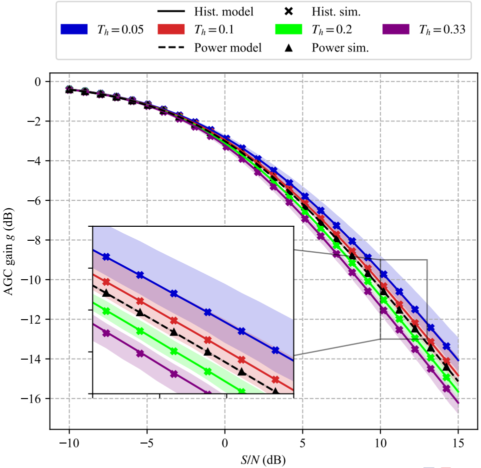
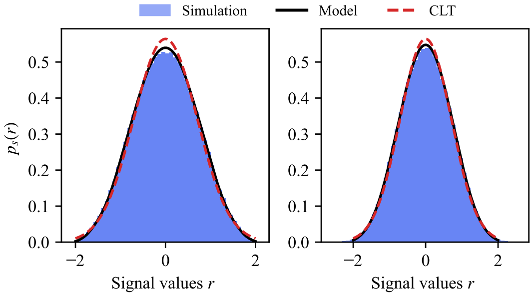


Multiple PRN Spoofing Signal ($M > 1$)

Results

- Equivalent to multiple weighted CWs.
- Distribution depends on the relative amplitudes between CWs.
- The signal converges toward a normal distribution as $M \to +\infty$ (Central Limit Theorem, CLT).

Signal distribution (M=5):



(a) BPSK (b) CBOC

Conclusion on spoofing impact before correlation

Spoofing Impact Before Correlation

The AGC dynamic response can be characterized through the input signal level (and thus signal power and distribution).

The spoofing impact can be modeled as weighted continuous waves (CWs), analyzed similarly to a jammer:

- $M = 1$**:** Equivalent to a single weighted CW.
- $M > 1$**:** Equivalent to multiple weighted CWs.
- $M \to \infty$**:** Converges toward a normal distribution.

Spoofing acts as a multiple weighted CW jammer for the AGC, starting to impact AGC values for $S/N > -5\,\mathrm{dB}$.

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[1] Emile Ghizzo et al. "Assessing jamming and spoofing impacts on GNSS receivers: Automatic gain control (AGC)". In: *Signal Processing* 228 (2025), p. 109762

Outline

1. Context
2. Received signal model under spoofing
3. Impact of spoofing before correlation
4. Impact of spoofing after correlation
   - Correlator output
   - Tracking loops
   - C/N_0
5. Experimentation
6. Conclusion

Outline

Correlator model under spoofing (1/2)

$$\Lambda(\boldsymbol{\eta}) = \frac{1}{T_\mathrm{i}} \int_{k\,T_\mathrm{i}}^{(k+1)\,T_\mathrm{i}} r_\mathrm{IF}(t)\, c^*(t-\tau) \exp\left(-2j\pi t(f_\mathrm{IF} + f) - j(\theta - \pi T_\mathrm{i}\, f)\right) \mathrm{d}t, \quad \forall \boldsymbol{\eta} = [\tau, \theta, f]^\mathsf{T}$$

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$$r_{IF}(t) = \left[\underbrace{r_a(t, \boldsymbol{\eta}_a)}_{\text{Authentic signal}} + \underbrace{r_s(t, \boldsymbol{\eta}_s)}_{\text{Spoofing signal}} + \sum_{i \in \underbrace{\mathcal{X}_a}_{\text{Authentic inter-signal set}}} r_a^i(t) + \sum_{i \in \underbrace{\mathcal{X}_s}_{\text{Spoofing inter-signal set}}} r_s^i(t)\right] \exp\left(2j\pi f_{IF} + j\theta_{IF}\right) + \underbrace{n(t)}_{\text{Total AWGN}}$$

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$\chi(t)$ is shown as a **Gaussian random variable** (CLT)

Correlator model under spoofing (2/2)

$$\Lambda(\boldsymbol{\eta}, \boldsymbol{\eta}_{\mathrm{a}}, \boldsymbol{\eta}_{\mathrm{s}}) = \Lambda_{\mathrm{a}}(\boldsymbol{\eta}_{\mathrm{a}} - \boldsymbol{\eta}) + \Lambda_{\mathrm{s}}(\boldsymbol{\eta}_{\mathrm{s}} - \boldsymbol{\eta}) + \Lambda_{\chi} + \Lambda_{n}$$

Correlator model under spoofing (2/2)

$$\Lambda(\boldsymbol{\eta}, \boldsymbol{\eta}_\mathrm{a}, \boldsymbol{\eta}_\mathrm{s}) = \overbrace{\Lambda_\mathrm{a}(\boldsymbol{\eta}_\mathrm{a} - \boldsymbol{\eta})}^{\text{Authentic peak}} + \Lambda_\mathrm{s}(\boldsymbol{\eta}_\mathrm{s} - \boldsymbol{\eta}) + \Lambda_\chi + \Lambda_n$$

- Authentic peak:
  $\Lambda_\mathrm{a}(\boldsymbol{\eta}_\mathrm{a} - \boldsymbol{\eta}) = \sqrt{C_\mathrm{a}}\, d_k\, \zeta_\tau(\tau_\mathrm{a} - \tau)\, \zeta_f(f_\mathrm{a} - f)\, e^{j(\theta_\mathrm{a} - \theta)}$

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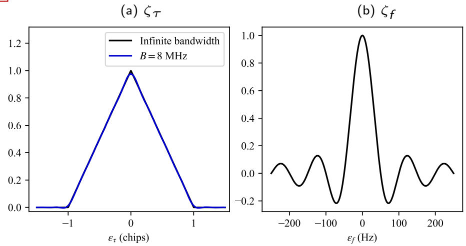

Correlator model under spoofing (2/2)

Authentic peak

$$\Lambda(\eta, \eta_a, \eta_s) = \Lambda_a(\eta_a - \eta) + \Lambda_s(\eta_s - \eta) + \Lambda_\chi + \Lambda_n$$

Spoofing peak

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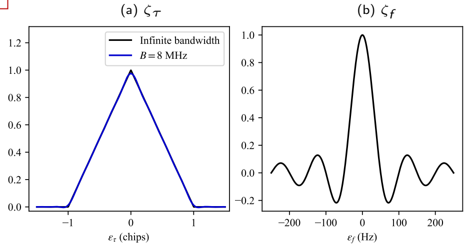

Authentic and spoofing dynamics:

$$\eta_a = \left[\ \tau_a\ ,\ \theta_a\ ,\ f_a\ \right]^\mathsf{T} \text{ and } \eta_s = \left[\ \tau_s\ ,\ \theta_s\ ,\ f_s\ \right]^\mathsf{T}$$

Correlator model under spoofing (2/2)

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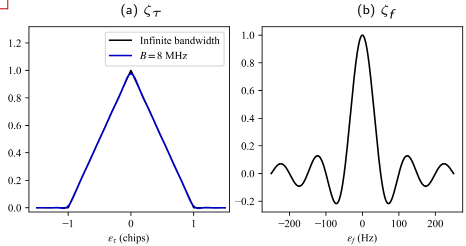

Authentic and spoofing dynamics:

$$\eta_a = [\underbrace{\tau_a}_{\text{Authentic delay}}, \underbrace{\theta_a}_{\text{Authentic phase}}, \underbrace{f_a}_{\text{Authentic frequency}}]^T \text{ and } \eta_s = [\underbrace{\tau_s}_{\text{Spoofing delay}}, \underbrace{\theta_s}_{\text{Spoofing phase}}, \underbrace{f_s}_{\text{Spoofing frequency}}]^T$$

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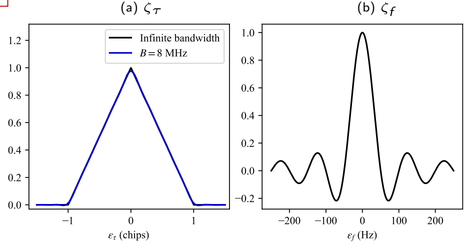
$$\Lambda(\boldsymbol{\eta}, \boldsymbol{\eta}_a, \boldsymbol{\eta}_s) = \underbrace{\Lambda_a(\boldsymbol{\eta}_a - \boldsymbol{\eta})}_{\text{Authentic peak}} + \underbrace{\Lambda_s(\boldsymbol{\eta}_s - \boldsymbol{\eta})}_{\text{Spoofing peak}} + \underbrace{\Lambda_\chi}_{\text{Inter-signal contribution}} + \underbrace{\Lambda_n}_{\text{AWGN contribution}}$$

- Authentic peak:

  $$\Lambda_a(\boldsymbol{\eta}_a - \boldsymbol{\eta}) = \sqrt{C_a}\, d_k\, \zeta_\tau(\tau_a - \tau)\, \zeta_f(f_a - f)\, e^{j(\theta_a - \theta)}$$

- Spoofing peak:

  $$\Lambda_s(\boldsymbol{\eta}_s - \boldsymbol{\eta}) = \sqrt{C_s}\, d_k\, \zeta_\tau(\tau_s - \tau)\, \zeta_f(f_s - f)\, e^{j(\theta_s - \theta)}$$



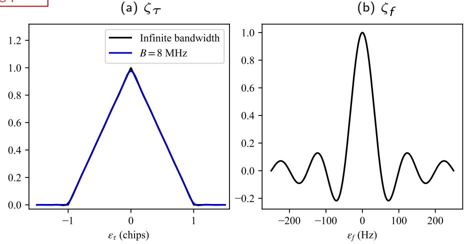
Authentic and spoofing dynamics:

$$\boldsymbol{\eta}_a = [\underbrace{\tau_a}_{\text{Authentic delay}}, \underbrace{\theta_a}_{\text{Authentic phase}}, \underbrace{f_a}_{\text{Authentic frequency}}]^T \text{ and } \boldsymbol{\eta}_s = [\underbrace{\tau_s}_{\text{Spoofing delay}}, \underbrace{\theta_s}_{\text{Spoofing phase}}, \underbrace{f_s}_{\text{Spoofing frequency}}]^T$$

Correlator model under spoofing (2/2)

$$\Lambda(\boldsymbol{\eta},\boldsymbol{\eta}_{\mathrm{a}},\boldsymbol{\eta}_{\mathrm{s}}) = \underbrace{\Lambda_{\mathrm{a}}(\boldsymbol{\eta}_{\mathrm{a}}-\boldsymbol{\eta})}_{\text{Authentic peak}} + \underbrace{\Lambda_{\mathrm{s}}(\boldsymbol{\eta}_{\mathrm{s}}-\boldsymbol{\eta})}_{\text{Spoofing peak}} + \underbrace{\Lambda_{\varsigma}}_{\text{Random contribution}}$$

- Authentic peak:
  $\Lambda_{\mathrm{a}}(\boldsymbol{\eta}_{\mathrm{a}}-\boldsymbol{\eta}) = \sqrt{C_{\mathrm{a}}}\, d_k\, \zeta_\tau(\tau_{\mathrm{a}}-\tau)\,\zeta_f(f_{\mathrm{a}}-f)\, e^{j(\theta_{\mathrm{a}}-\theta)}$
- Spoofing peak:
  $\Lambda_{\mathrm{s}}(\boldsymbol{\eta}_{\mathrm{s}}-\boldsymbol{\eta}) = \sqrt{C_{\mathrm{s}}}\, d_k\, \zeta_\tau(\tau_{\mathrm{s}}-\tau)\,\zeta_f(f_{\mathrm{s}}-f)\, e^{j(\theta_{\mathrm{s}}-\theta)}$
- Random contribution:
  $\Lambda_{\varsigma} = \Lambda_{\chi} + \Lambda_n \;(\Lambda_{\varsigma} \sim \mathcal{CN}(0, P_{\chi}+P_n))$



Authentic and spoofing dynamics:

$$\boldsymbol{\eta}_{\mathrm{a}} = [\underbrace{\tau_{\mathrm{a}}}_{\text{Authentic delay}}, \underbrace{\theta_{\mathrm{a}}}_{\text{Authentic phase}}, \underbrace{f_{\mathrm{a}}}_{\text{Authentic frequency}}]^{\mathsf{T}} \text{ and } \boldsymbol{\eta}_{\mathrm{s}} = [\underbrace{\tau_{\mathrm{s}}}_{\text{Spoofing delay}}, \underbrace{\theta_{\mathrm{s}}}_{\text{Spoofing phase}}, \underbrace{f_{\mathrm{s}}}_{\text{Spoofing frequency}}]^{\mathsf{T}}$$

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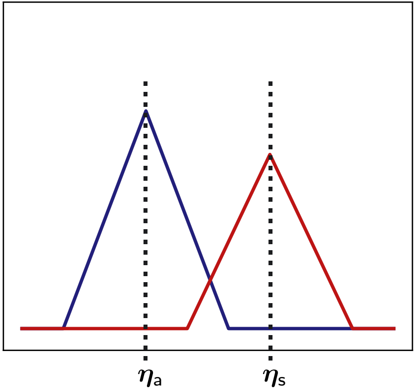
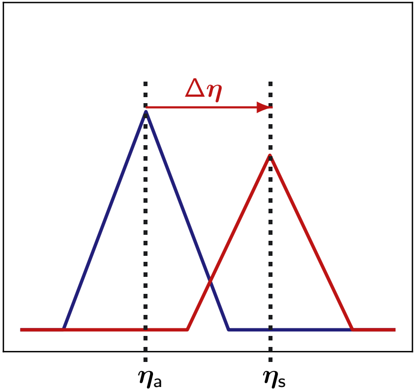
Relative dynamic

$$\Lambda(\eta, \eta_a, \eta_s) = \Lambda_a(\overset{\text{Authentic dynamic}}{\eta_a} - \eta) + \Lambda_s(\underset{\text{Spoofing dynamic}}{\eta_s} - \eta) + \Lambda_\varsigma$$



Relative dynamic

$$\Lambda(\eta, \underbrace{\eta_a}_{}, \Delta\nu) = \Lambda_a(\overset{\text{Authentic dynamic}}{\eta_a} - \eta) + \sqrt{\Delta g}\,\Lambda_a(\eta_a - \eta + \overset{\text{Relative dynamic}}{\Delta\eta}) + \Lambda_\varsigma$$

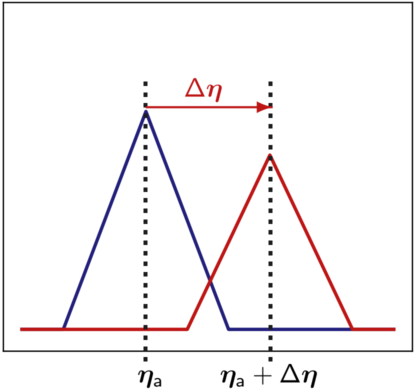


Relative dynamic:

$$\Delta\eta = \eta_s - \eta_a = \left[\overset{\text{Relative delay}}{\Delta\tau}, \overset{\text{Relative phase}}{\Delta\theta}, \underset{\text{Relative frequency}}{\Delta f}\right]^\mathsf{T}$$

Relative dynamic

$$\Lambda(\eta, \overbrace{\eta_a}^{\text{Authentic dynamic}}, \Delta\nu) = \Lambda_a(\overbrace{\eta_a}^{\text{Authentic dynamic}} - \eta) + \sqrt{\Delta g}\,\Lambda_a(\eta_a - \eta + \overbrace{\Delta\eta}^{\text{Relative dynamic}}) + \Lambda_\varsigma$$

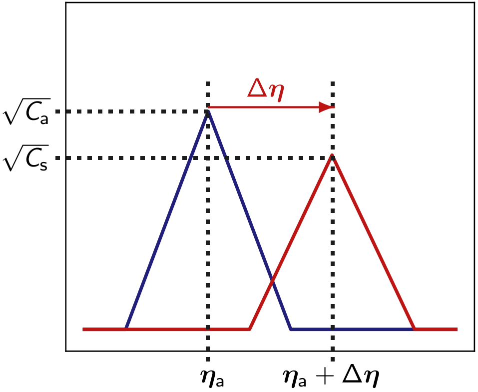


Relative dynamic:

$$\Delta\eta = \eta_s - \eta_a = \left[\overbrace{\Delta\tau}^{\text{Relative delay}}, \overbrace{\Delta\theta}^{\text{Relative phase}}, \underbrace{\Delta f}_{\text{Relative frequency}}\right]^T$$

Relative dynamic

$$\Lambda(\eta, \eta_a, \Delta\nu) = \Lambda_a(\overset{\text{Authentic dynamic}}{\eta_a} - \eta) + \sqrt{\Delta g}\,\Lambda_a(\eta_a - \eta + \overset{\text{Relative dynamic}}{\Delta\eta}) + \Lambda_\varsigma$$

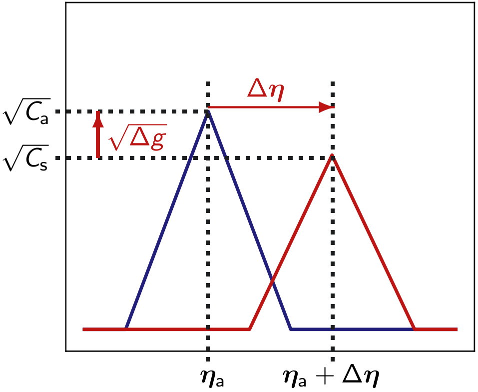


Relative dynamic:

$$\Delta\eta = \eta_s - \eta_a = \left[\overset{\text{Relative delay}}{\Delta\tau}, \overset{\text{Relative phase}}{\Delta\theta}, \underset{\text{Relative frequency}}{\Delta f}\right]^\intercal$$

Relative dynamic

$$\Lambda(\eta, \boxed{\eta_a}, \boxed{\Delta\nu}) = \Lambda_a(\overset{\text{Authentic dynamic}}{\boxed{\eta_a}} - \eta) + \underset{\text{Relative power}}{\boxed{\sqrt{\Delta g}}}\,\Lambda_a(\boxed{\eta_a} - \eta + \overset{\text{Relative dynamic}}{\boxed{\Delta\eta}}) + \Lambda_\varsigma$$



Relative dynamic:

$$\Delta\eta = \eta_s - \eta_a = \left[\overset{\text{Relative delay}}{\boxed{\Delta\tau}}, \overset{\text{Relative phase}}{\boxed{\Delta\theta}}, \underset{\text{Relative frequency}}{\boxed{\Delta f}}\right]^\mathsf{T}$$

Relative parameters:

$$\Delta\nu = \left[\Delta\eta^\mathsf{T}, \overset{\text{Relative power}}{\boxed{\Delta g}}\right]^\mathsf{T} \text{ with } \Delta g = C_s / C_a$$

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Classification

$$\Lambda(\eta, \eta_a, \Delta\nu) = \underbrace{\Lambda_a(\eta_a - \eta)}_{\text{Authentic peak}} + \underbrace{\Lambda_s(\eta_a - \eta, \Delta\nu)}_{\text{Spoofing peak}} + \underbrace{\Lambda_\varsigma}_{\text{Random contribution}}$$

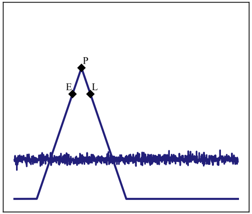
Classification

Authentic peak

Spoofing peak

Random contribution

$$\Lambda(\eta, \eta_a, \Delta\nu) = \Lambda_a(\eta_a - \eta) + \Lambda_s(\eta_a - \eta, \Delta\nu) + \Lambda_\varsigma$$



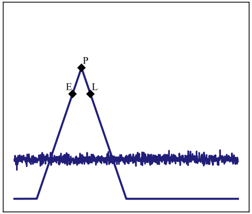
Nominal

$$\Lambda^{(N)} = \Lambda_a + \Lambda_\varsigma$$

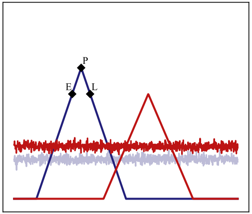
Classification

Authentic peak — Spoofing peak — Random contribution

$$\Lambda(\eta, \eta_a, \Delta\nu) = \Lambda_a(\eta_a - \eta) + \Lambda_s(\eta_a - \eta, \Delta\nu) + \Lambda_\varsigma$$



Nominal

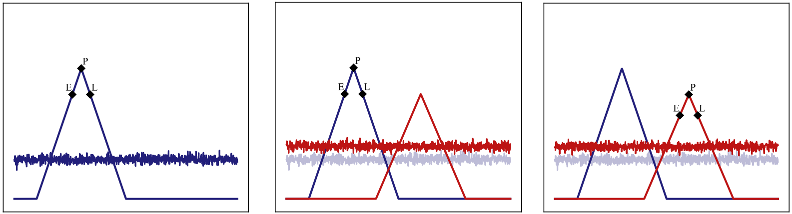


Induced-jamming

$$\Lambda^{(N)} = \Lambda_a + \Lambda_\varsigma \qquad \Lambda^{(J)} = \Lambda_a + \Lambda_\varsigma$$

Classification

$$\Lambda(\eta, \eta_a, \Delta\nu) = \underbrace{\Lambda_a(\eta_a - \eta)}_{\text{Authentic peak}} + \underbrace{\Lambda_s(\eta_a - \eta, \Delta\nu)}_{\text{Spoofing peak}} + \underbrace{\Lambda_\varsigma}_{\text{Random contribution}}$$



Nominal Induced-jamming Induced-spoofing

$$\Lambda^{(N)} = \Lambda_a + \Lambda_\varsigma \qquad \Lambda^{(J)} = \Lambda_a + \Lambda_\varsigma \qquad \Lambda^{(S)} = \Lambda_s + \Lambda_\varsigma$$

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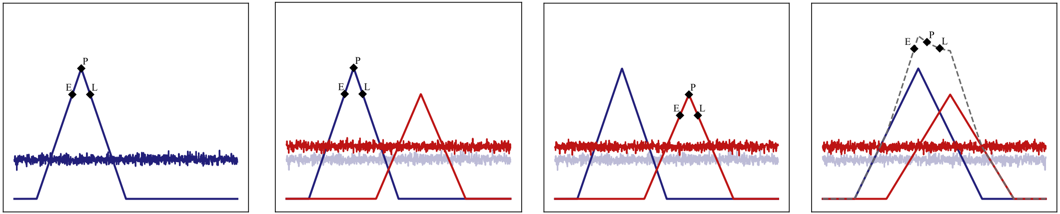
Classification

$$\Lambda(\eta, \eta_a, \Delta\nu) = \overbrace{\Lambda_a(\eta_a - \eta)}^{\text{Authentic peak}} + \overbrace{\Lambda_s(\eta_a - \eta, \Delta\nu)}^{\text{Spoofing peak}} + \overbrace{\Lambda_\varsigma}^{\text{Random contribution}}$$

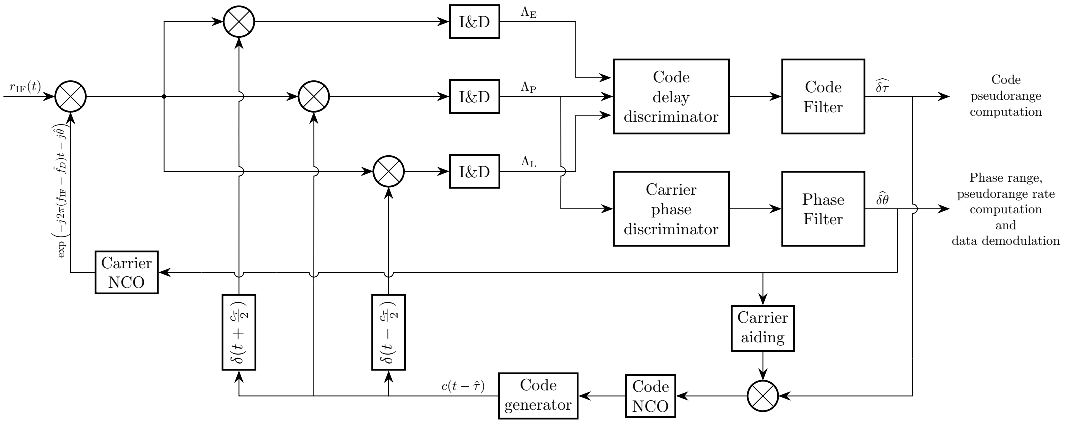


Nominal | Induced-jamming | Induced-spoofing | Induced-multipath

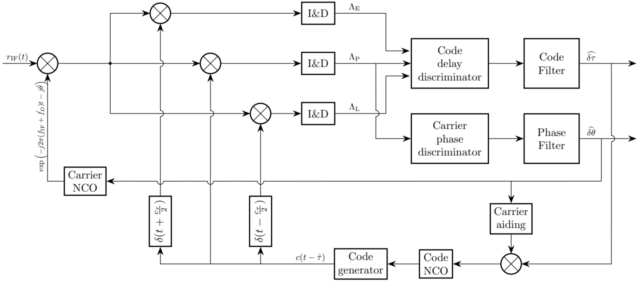
$$\Lambda^{(N)} = \Lambda_a + \Lambda_\varsigma \qquad \Lambda^{(J)} = \Lambda_a + \Lambda_\varsigma \qquad \Lambda^{(S)} = \Lambda_s + \Lambda_\varsigma \qquad \Lambda^{(M)} = \Lambda_a + \Lambda_s + \Lambda_\varsigma$$

Outline

Tracking loops model

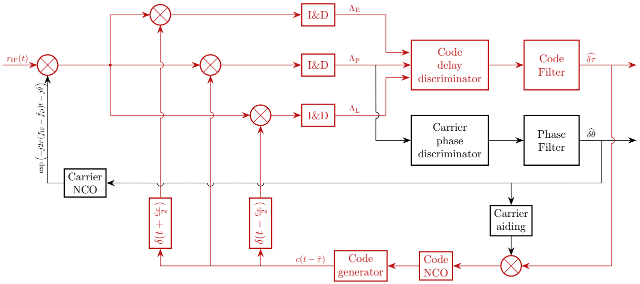


Tracking loops model



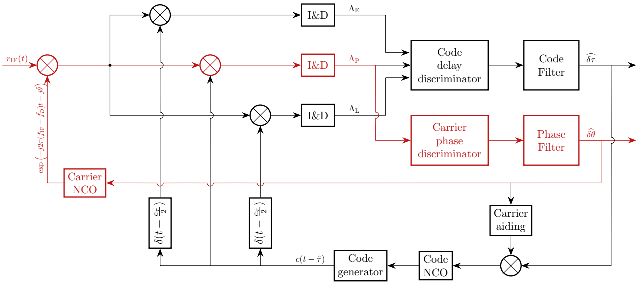
Tracking loops model

$$\begin{cases} \hat{\tau}(z) = \dfrac{z^{-1}}{1 - z^{-1}} T_\mathrm{i}\, F_\tau(z)\, D_\tau\left(\boldsymbol{\eta}_\mathrm{a}(z) - \hat{\boldsymbol{\eta}}(z), \Delta\boldsymbol{\nu}(z)\right) & \text{(DLL)} \end{cases}$$



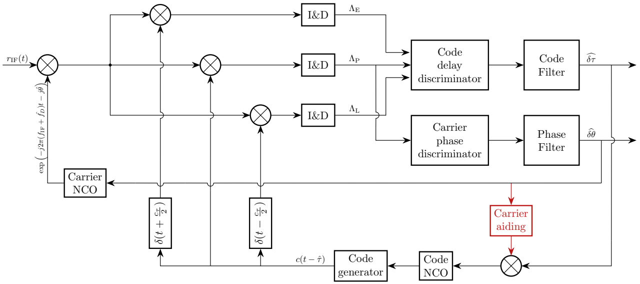
Tracking loops model

$$\begin{cases} \hat{\tau}(z) = \dfrac{z^{-1}}{1-z^{-1}} T_i\, F_\tau(z)\, D_\tau\left(\boldsymbol{\eta}_a(z) - \hat{\boldsymbol{\eta}}(z), \Delta\boldsymbol{\nu}(z)\right) & \text{(DLL)} \\ \hat{\theta}(z) = \dfrac{z^{-1}}{1-z^{-1}} T_i\, F_\theta(z)\, D_\theta\left(\boldsymbol{\eta}_a(z) - \hat{\boldsymbol{\eta}}(z), \Delta\boldsymbol{\nu}(z)\right) & \text{(PLL)} \end{cases}$$



Tracking loops model

$$\begin{cases} \hat{\tau}(z) + \beta\,\hat{\theta}(z) = \dfrac{z^{-1}}{1-z^{-1}}\, T_\mathrm{i}\, F_\tau(z)\, D_\tau\left(\boldsymbol{\eta}_\mathrm{a}(z) - \hat{\boldsymbol{\eta}}(z), \Delta\boldsymbol{\nu}(z)\right) & \text{(DLL)} \\ \hat{\theta}(z) = \dfrac{z^{-1}}{1-z^{-1}}\, T_\mathrm{i}\, F_\theta(z)\, D_\theta\left(\boldsymbol{\eta}_\mathrm{a}(z) - \hat{\boldsymbol{\eta}}(z), \Delta\boldsymbol{\nu}(z)\right) & \text{(PLL)} \end{cases}$$



Tracking loops model

$$\begin{cases} \hat{\tau}(z) + \beta\ \hat{\theta}(z) = \dfrac{z^{-1}}{1-z^{-1}} T_{\mathrm{i}}\ F_{\tau}(z)\ D_{\tau}\left(\boldsymbol{\eta}_{\mathrm{a}}(z) - \hat{\boldsymbol{\eta}}(z)\ ,\ \Delta\boldsymbol{\nu}(z)\right) & \text{(DLL)} \\ \hat{\theta}(z) = \dfrac{z^{-1}}{1-z^{-1}} T_{\mathrm{i}}\ F_{\theta}(z)\ D_{\theta}\left(\boldsymbol{\eta}_{\mathrm{a}}(z) - \hat{\boldsymbol{\eta}}(z)\ ,\ \Delta\boldsymbol{\nu}(z)\right) & \text{(PLL)} \end{cases}$$

Tracking loops model

State vector

$$\begin{cases} \hat{\tau}(z) + \beta\, \hat{\theta}(z) = \dfrac{z^{-1}}{1 - z^{-1}} T_{\mathrm{i}}\, F_{\tau}(z)\;\; D_{\tau}\left(\boldsymbol{\eta}_{\mathrm{a}}(z) - \hat{\boldsymbol{\eta}}(z),\; \Delta\boldsymbol{\nu}(z)\right) & \text{(DLL)} \\ \hat{\theta}(z) = \dfrac{z^{-1}}{1 - z^{-1}} T_{\mathrm{i}}\, F_{\theta}(z)\;\; D_{\theta}\left(\boldsymbol{\eta}_{\mathrm{a}}(z) - \hat{\boldsymbol{\eta}}(z),\; \Delta\boldsymbol{\nu}(z)\right) & \text{(PLL)} \end{cases}$$

Tracking loops model

State vector

$$
\begin{cases}
\hat{\tau}(z) + \beta\,\hat{\theta}(z) = \dfrac{z^{-1}}{1-z^{-1}}\, T_{\mathrm{i}}\, F_{\tau}(z)\;\; D_{\tau}\left(\boldsymbol{\eta}_{\mathrm{a}}(z) - \hat{\boldsymbol{\eta}}(z),\; \Delta\boldsymbol{\nu}(z)\right) & \text{(DLL)} \\
\hat{\theta}(z) = \dfrac{z^{-1}}{1-z^{-1}}\, T_{\mathrm{i}}\, F_{\theta}(z)\;\; D_{\theta}\left(\boldsymbol{\eta}_{\mathrm{a}}(z) - \hat{\boldsymbol{\eta}}(z),\; \Delta\boldsymbol{\nu}(z)\right) & \text{(PLL)}
\end{cases}
$$

Loop filter

Tracking loops model

State vector

$$\begin{cases} \boxed{\hat{\tau}(z)} + \beta\, \boxed{\hat{\theta}(z)} = \dfrac{z^{-1}}{1 - z^{-1}} T_\mathrm{i}\, F_\tau(z)\;\; D_\tau \left(\eta_\mathrm{a}(z) - \boxed{\hat{\eta}(z)},\; \Delta\nu(z)\right) & \text{(DLL)} \\ \boxed{\hat{\theta}(z)} = \dfrac{z^{-1}}{1 - z^{-1}} T_\mathrm{i}\, \underbrace{F_\theta(z)}_{\text{Low-pass filter}}\;\; \underbrace{D_\theta}_{\text{Discriminator}} \left(\eta_\mathrm{a}(z) - \boxed{\hat{\eta}(z)},\; \Delta\nu(z)\right) & \text{(PLL)} \end{cases}$$

Tracking loops model

$$\begin{cases} \underbrace{\hat{\tau}(z)}_{\text{State vector}} + \beta\, \hat{\theta}(z) = \dfrac{z^{-1}}{1 - z^{-1}} T_\mathrm{i}\, F_\tau(z)\ D_\tau \left(\eta_\mathrm{a}(z) - \hat{\eta}(z), \overbrace{\Delta\nu(z)}^{\text{Relative dynamic}}\right) & \text{(DLL)} \\ \hat{\theta}(z) = \dfrac{z^{-1}}{1 - z^{-1}} T_\mathrm{i}\, \underbrace{F_\theta(z)}_{\text{Low-pass filter}}\ \underbrace{D_\theta}_{\text{Discriminator}} \left(\eta_\mathrm{a}(z) - \hat{\eta}(z), \Delta\nu(z)\right) & \text{(PLL)} \end{cases}$$

Tracking loops model

State vector

Relative dynamic

$$\begin{cases} \hat{\tau}(z) + \beta\,\hat{\theta}(z) = \dfrac{z^{-1}}{1-z^{-1}} T_i\, F_\tau(z)\ D_\tau\left(\eta_a(z) - \hat{\eta}(z), \Delta\nu(z)\right) & \text{(DLL)} \\ \hat{\theta}(z) = \dfrac{z^{-1}}{1-z^{-1}} T_i\, F_\theta(z)\ D_\theta\left(\eta_a(z) - \hat{\eta}(z), \Delta\nu(z)\right) & \text{(PLL)} \end{cases}$$

Low-pass filter

Discriminator

Dynamic approach:

- Characterize the dynamic behavior under spoofing through the analysis of the closed-loop model.
- Account for the interconnection between DLL and PLL, incorporating all dynamic aspects (e.g., filter and feedback architecture).
- Propose an analysis of linearity, Stable Equilibria (SE), and stochastic behavior for each spoofing-induced situation.

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Linearity

Definition: State variable $\hat{\phi}$ and its derivatives appear linearly. A linear system satisfies **additivity** and **homogeneity** and can be analyzed though **tranfer function tools**.[1]

[1] Floyd M Gardner. *Phaselock Techniques*. John Wiley & Sons, 2005

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Low-pass filter

Discriminator

Results

- ✓ Maintains its linearity assumption under induced jamming and spoofing situations (near SE).
- × Exhibits non-linearity in both DLL and PLL under induced multipath situation.

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[1] Floyd M Gardner. *Phaselock Techniques*. John Wiley & Sons, 2005

Stable Equilibria (SE) (1/3)

Stable Equilibrium: State where the system has **successfully converged** and **returns to this state after a small perturbation**.[1]

[1] Gennady A. Leonov et al. "Hold-In, Pull-In, and Lock-In Ranges of PLL Circuits: Rigorous Mathematical Definitions and Limitations of Classical Theory". In: *IEEE Trans. Circuits Syst. I, Reg. Papers* 62.10 (2015), pp. 2454–2464

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Objective

- Enable the analysis of **system convergence** (identifying the different **points of convergence** and **potential bifurcations**).
- Facilitate the analysis of the **system's dynamic behavior**, including transient response, tracking error at lock, and stress error.

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Closed-loop model SE:

$$\begin{cases} d_\tau\left(\varepsilon_\eta^l, \Delta\boldsymbol{\nu}\right) = 0, \quad d_\theta\left(\varepsilon_\eta^l, \Delta\boldsymbol{\nu}\right) = 0, \quad \chi_f\left(\varepsilon_\eta^l, \Delta\boldsymbol{\nu}\right) = 0 & \text{(Equilibrium)} \\ \dfrac{\partial\, d_\tau\left(\varepsilon_\eta^l, \Delta\boldsymbol{\nu}\right)}{\partial\varepsilon_\tau} > 0, \quad \dfrac{\partial\, d_\theta\left(\varepsilon_\eta^l, \Delta\boldsymbol{\nu}\right)}{\partial\varepsilon_\theta} > 0, \quad \dfrac{\partial\, \chi_f\left(\varepsilon_\eta^l, \Delta\boldsymbol{\nu}\right)}{\partial\varepsilon_f} > 0 & \text{(Stability)} \end{cases}$$

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- Facilitate the analysis of the **system's dynamic behavior**, including transient response, tracking error at lock, and stress error.

Closed-loop model SE:

$$\begin{cases} \overset{\text{DLL disc.}}{d_\tau}\left(\varepsilon_\eta^I, \Delta\boldsymbol{\nu}\right) = 0, & \overset{\text{PLL disc.}}{d_\theta}\left(\varepsilon_\eta^I, \Delta\boldsymbol{\nu}\right) = 0, & \overset{\text{PLL derivative disc.}}{\chi_f}\left(\varepsilon_\eta^I, \Delta\boldsymbol{\nu}\right) = 0 & \text{(Equilibrium)} \\ \dfrac{\partial\, d_\tau\left(\varepsilon_\eta^I, \Delta\boldsymbol{\nu}\right)}{\partial \varepsilon_\tau} > 0, & \dfrac{\partial\, d_\theta\left(\varepsilon_\eta^I, \Delta\boldsymbol{\nu}\right)}{\partial \varepsilon_\theta} > 0, & \dfrac{\partial\, \chi_f\left(\varepsilon_\eta^I, \Delta\boldsymbol{\nu}\right)}{\partial \varepsilon_f} > 0 & \text{(Stability)} \end{cases}$$

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- Enable the analysis of **system convergence** (identifying the different **points of convergence** and **potential bifurcations**).
- Facilitate the analysis of the **system's dynamic behavior**, including transient response, tracking error at lock, and stress error.

Closed-loop model SE:

SE solutions

$$\begin{cases} d_\tau\left(\varepsilon_\eta^l, \Delta\boldsymbol{\nu}\right) = 0, \quad d_\theta\left(\varepsilon_\eta^l, \Delta\boldsymbol{\nu}\right) = 0, \quad \chi_f\left(\varepsilon_\eta^l, \Delta\boldsymbol{\nu}\right) = 0 & \text{(Equilibrium)} \\ \dfrac{\partial\, d_\tau\left(\varepsilon_\eta^l, \Delta\boldsymbol{\nu}\right)}{\partial\varepsilon_\tau} > 0, \quad \dfrac{\partial\, d_\theta\left(\varepsilon_\eta^l, \Delta\boldsymbol{\nu}\right)}{\partial\varepsilon_\theta} > 0, \quad \dfrac{\partial\, \chi_f\left(\varepsilon_\eta^l, \Delta\boldsymbol{\nu}\right)}{\partial\varepsilon_f} > 0 & \text{(Stability)} \end{cases}$$

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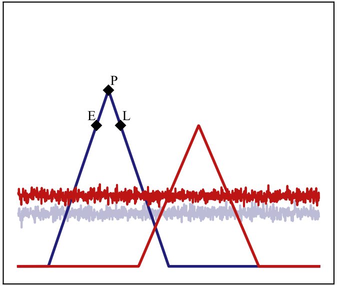
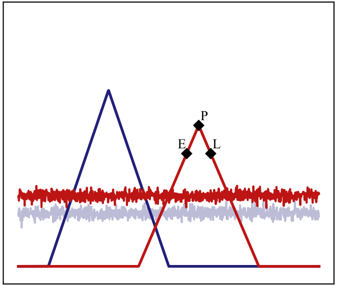
Stable Equilibria (SE) (2/3)

Induced-jamming



Induced-spoofing



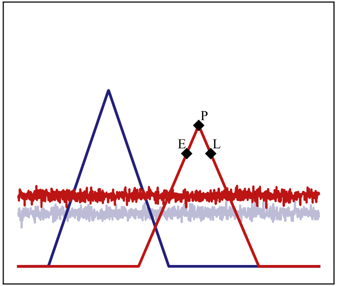
LINEAR ASSUMPTION

Stable Equilibria (SE) (2/3)

Induced-jamming

Tracked dynamic: Authentic dynamic η_{a} or frequency-mismatched version $(1/2T_{\text{i}})$

Induced-spoofing



Stable Equilibria (SE) (2/3)

Induced-jamming

Tracked dynamic: Authentic dynamic η_a or frequency-mismatched version $(1/2T_i)$

Induced-spoofing

Tracked dynamic: Spoofing dynamic η_s or a frequency-mismatched version $(1/2T_i)$

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Stable Equilibria (SE) (2/3)

Induced-jamming

Tracked dynamic: Authentic dynamic η_{a} or frequency-mismatched version $(1/2T_{\mathrm{i}})$

Induced-spoofing

Tracked dynamic: Spoofing dynamic η_{s} or a frequency-mismatched version $(1/2T_{\mathrm{i}})$

Stable Equilibria (SE) (3/3)

System SE under induced-multipath:

$$\begin{cases} d_\tau\left(\boxed{\boldsymbol{\varepsilon}_\eta^l}, \Delta\boldsymbol{\nu}\right) = 0, & d_\theta\left(\boxed{\boldsymbol{\varepsilon}_\eta^l}, \Delta\boldsymbol{\nu}\right) = 0, & \chi_f\left(\boxed{\boldsymbol{\varepsilon}_\eta^l}, \Delta\boldsymbol{\nu}\right) = 0 \\ \dfrac{\partial d_\tau\left(\boxed{\boldsymbol{\varepsilon}_\eta^l}, \Delta\boldsymbol{\nu}\right)}{\partial \varepsilon_\tau} > 0, & \dfrac{\partial d_\theta\left(\boxed{\boldsymbol{\varepsilon}_\eta^l}, \Delta\boldsymbol{\nu}\right)}{\partial \varepsilon_\theta} > 0, & \dfrac{\partial \chi_f\left(\boxed{\boldsymbol{\varepsilon}_\eta^l}, \Delta\boldsymbol{\nu}\right)}{\partial \varepsilon_f} > 0 \end{cases}$$

$$\overbrace{\boxed{\boldsymbol{\varepsilon}_\eta^l}}^{\text{System SE}} = \left[\underbrace{\boxed{{\varepsilon_\tau}^l}}_{\text{Code SE}}, \underbrace{\boxed{{\varepsilon_f}^l}}_{\text{Frequency SE}}, \overbrace{\boxed{{\varepsilon_\theta}^l}}^{\text{Phase SE}}\right]^{\mathsf{T}}$$

⚠ NON-LINEAR AND INTERDEPENDENT ⚠

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Stable Equilibria (SE) (3/3)

System SE under induced-multipath:

Independent of ε_θ

$$\begin{cases} d_\tau\left(\varepsilon_\tau^l, \varepsilon_f^l, \Delta\boldsymbol{\nu}\right) = 0, & d_\theta\left(\boldsymbol{\varepsilon}_\eta^l, \Delta\boldsymbol{\nu}\right) = 0, & \chi_f\left(\varepsilon_\tau^l, \varepsilon_f^l, \Delta\boldsymbol{\nu}\right) = 0 \\ \dfrac{\partial d_\tau\left(\varepsilon_\tau^l, \varepsilon_f^l, \Delta\boldsymbol{\nu}\right)}{\partial \varepsilon_\tau} > 0, & \dfrac{\partial d_\theta\left(\boldsymbol{\varepsilon}_\eta^l, \Delta\boldsymbol{\nu}\right)}{\partial \varepsilon_\theta} > 0, & \dfrac{\partial \chi_f\left(\varepsilon_\tau^l, \varepsilon_f^l, \Delta\boldsymbol{\nu}\right)}{\partial \varepsilon_f} > 0 \end{cases}$$

$$\underbrace{\boldsymbol{\varepsilon}_\eta^l}_{\text{System SE}} = \left[\underbrace{{\varepsilon_\tau}^l}_{\text{Code SE}}, \underbrace{{\varepsilon_f}^l}_{\text{Frequency SE}}, {\varepsilon_\theta}^l\right]^\mathsf{T}$$

⚠ d_τ and χ_f INDEPENDENT OF PHASE ERROR ⚠

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Stable Equilibria (SE) (3/3)

System SE under induced-multipath:

$$\begin{cases} d_\tau\left(\varepsilon_\tau^I, \varepsilon_f^I, \Delta\boldsymbol{\nu}\right) = 0, & d_\theta\left(\boldsymbol{\varepsilon}_\eta^I, \Delta\boldsymbol{\nu}\right) = 0, & \chi_f\left(\varepsilon_\tau^I, \varepsilon_f^I, \Delta\boldsymbol{\nu}\right) = 0 \\ \dfrac{\partial d_\tau\left(\varepsilon_\tau^I, \varepsilon_f^I, \Delta\boldsymbol{\nu}\right)}{\partial \varepsilon_\tau} > 0, & \dfrac{\partial d_\theta\left(\boldsymbol{\varepsilon}_\eta^I, \Delta\boldsymbol{\nu}\right)}{\partial \varepsilon_\theta} > 0, & \dfrac{\partial \chi_f\left(\varepsilon_\tau^I, \varepsilon_f^I, \Delta\boldsymbol{\nu}\right)}{\partial \varepsilon_f} > 0 \end{cases}$$

$$\underbrace{\boldsymbol{\varepsilon}_\eta^I}_{\text{System SE}} = \left[\underbrace{{\varepsilon_\tau}^I}_{\text{Code SE}}, \underbrace{{\varepsilon_f}^I}_{\text{Frequency SE}}, {\varepsilon_\theta}^I\right]^\mathsf{T}$$

⚠ d_τ and χ_f INDEPENDENT OF PHASE ERROR ⚠

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Stable Equilibria (SE) (3/3)

System SE under induced-multipath:

$$\begin{cases} d_\tau\left(\varepsilon^I_\tau, \varepsilon^I_f, \Delta\boldsymbol{\nu}\right) = 0, & d_\theta\left(\boldsymbol{\varepsilon}^I_\eta, \Delta\boldsymbol{\nu}\right) = 0, & \chi_f\left(\varepsilon^I_\tau, \varepsilon^I_f, \Delta\boldsymbol{\nu}\right) = 0 \\ \dfrac{\partial d_\tau\left(\varepsilon^I_\tau, \varepsilon^I_f, \Delta\boldsymbol{\nu}\right)}{\partial \varepsilon_\tau} > 0, & \dfrac{\partial d_\theta\left(\boldsymbol{\varepsilon}^I_\eta, \Delta\boldsymbol{\nu}\right)}{\partial \varepsilon_\theta} > 0, & \dfrac{\partial \chi_f\left(\varepsilon^I_\tau, \varepsilon^I_f, \Delta\boldsymbol{\nu}\right)}{\partial \varepsilon_f} > 0. \end{cases}$$

$$\overset{\text{System SE}}{\boldsymbol{\varepsilon}^I_\eta} = \left[\underset{\text{Code SE}}{{\varepsilon_\tau}^I},\ \underset{\text{Frequency SE}}{{\varepsilon_f}^I},\ {\varepsilon_\theta}^I\right]^{\mathsf{T}}$$

⚠ d_τ and χ_f INDEPENDENT OF PHASE ERROR ⚠

Stable Equilibria (SE) (3/3)

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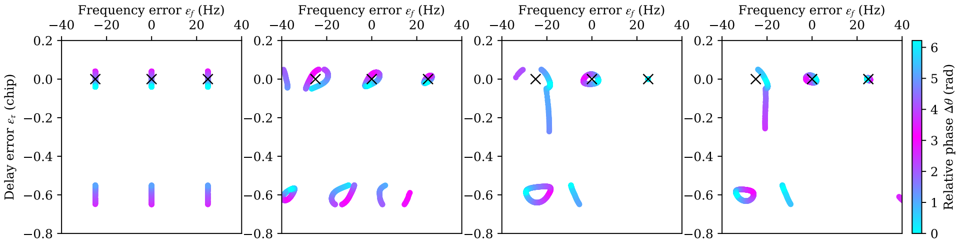

$\Delta f = 0$ Hz $\Delta f = 12$ Hz $\Delta f = 25$ Hz $\Delta f = 31$ Hz

Stable Equilibria (SE) (3/3)

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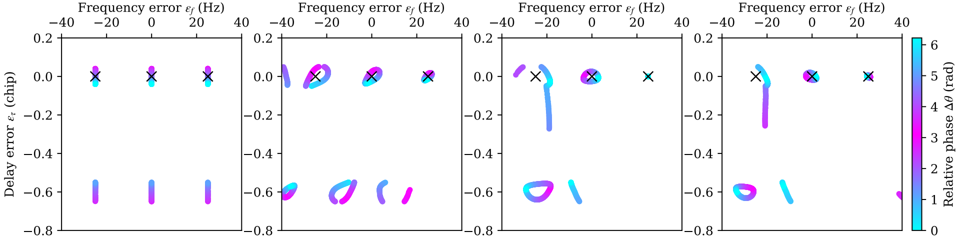

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Induced-Multipath Results (Asynchronous Frequency Spoofer)

Stable Equilibria (SE) (3/3)

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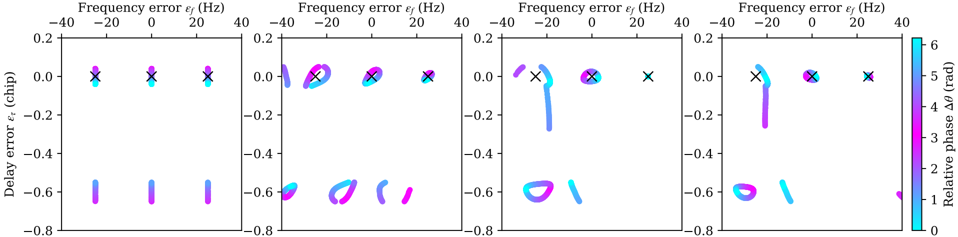

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Induced-Multipath Results (Asynchronous Frequency Spoofer)

Emergence of **multiple possible points of convergence** (chaotic behavior and bifurcations).

Stable Equilibria (SE) (3/3)

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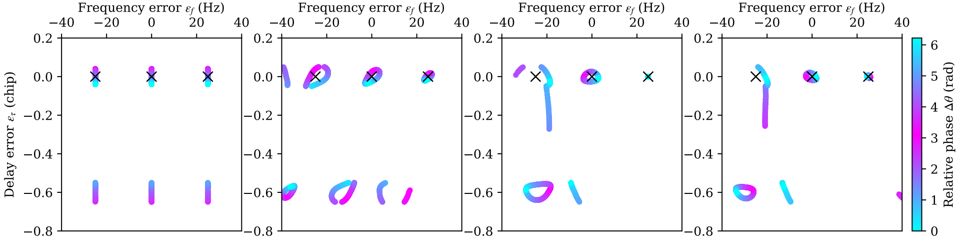

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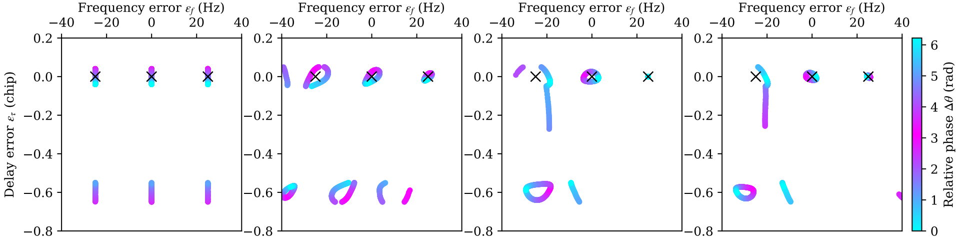
Induced-Multipath Results (Asynchronous Frequency Spoofer)

- Emergence of **multiple possible points of convergence** (chaotic behavior and bifurcations).
- **Interdependence** between the DLL and PLL.

Stable Equilibria (SE) (3/3)

System SE under induced-multipath:



$\Delta f = 0$ Hz $\Delta f = 12$ Hz $\Delta f = 25$ Hz $\Delta f = 31$ Hz

Induced-Multipath Results (Asynchronous Frequency Spoofer)

- Emergence of **multiple possible points of convergence** (chaotic behavior and bifurcations).
- **Interdependence** between the DLL and PLL.
- **Oscillating patterns** over the relative phase $\Delta\theta \in [0, 2\pi]$.

Dynamic and stochastic response (1/5)

Objective

- **Characterize the dynamic response** of the loops, including transient behavior and all dynamic aspects (feedback and filtering).

[1] Someshwar C Gupta. "Phase-Locked loops". In: *Proc. IEEE* 63.2 (1975), pp. 291–306

Dynamic and stochastic response (1/5)

Objective

- **Characterize the dynamic response** of the loops, including transient behavior and all dynamic aspects (feedback and filtering).
- Evaluate the **impact of noise** on the dynamic response.[1]

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Dynamic and stochastic response (1/5)

Objective

- **Characterize the dynamic response** of the loops, including transient behavior and all dynamic aspects (feedback and filtering).
- Evaluate the **impact of noise** on the dynamic response.[1]

Closed-loop model:

$$\begin{cases} \hat{\tau}(z) + \beta\, \hat{\theta}(z) = \dfrac{z^{-1}}{1 - z^{-1}} T_{\mathrm{i}}\, F_{\tau}(z)\, D_{\tau}\left(\boldsymbol{\eta}_{\mathrm{a}}(z) - \hat{\boldsymbol{\eta}}(z), \Delta\boldsymbol{\nu}(z)\right) & \text{(DLL)} \\ \hat{\theta}(z) = \dfrac{z^{-1}}{1 - z^{-1}} T_{\mathrm{i}}\, F_{\theta}(z)\, D_{\theta}\left(\boldsymbol{\eta}_{\mathrm{a}}(z) - \hat{\boldsymbol{\eta}}(z), \Delta\boldsymbol{\nu}(z)\right) & \text{(PLL)} \end{cases}$$

[1] Someshwar C Gupta. "Phase-Locked loops". In: *Proc. IEEE* 63.2 (1975), pp. 291–306

Dynamic and stochastic response (2/5)

Induced-Jamming and Induced-Spoofing Results

Linear Assumption:

LINEAR ASSUMPTION

[1] John W Betz and Kevin R Kolodziejski. "Generalized theory of code tracking with an early-late discriminator part II: Noncoherent processing and numerical results". In: *IEEE Transactions on Aerospace and Electronic Systems* 45.4 (2009), pp. 1557–1564.

Dynamic and stochastic response (2/5)

Induced-Jamming and Induced-Spoofing Results

Linear Assumption:

✓ The dynamic response can be analyzed through the **transfer function** of the equivalent linear closed-loop model $H_\phi(z)$:

$$\hat{\phi}^{(\mathrm{J})}(z) = H_\phi(z)\,\phi_{\mathrm{a}}, \qquad \hat{\phi}^{(\mathrm{S})}(z) = H_\phi(z)\,\phi_{\mathrm{s}}.$$

LINEAR ASSUMPTION

[1] John W Betz and Kevin R Kolodziejski. "Generalized theory of code tracking with an early-late discriminator part II: Noncoherent processing and numerical results". In: *IEEE Transactions on Aerospace and Electronic Systems* 45.4 (2009), pp. 1557–1564.

Dynamic and stochastic response (2/5)

Induced-Jamming and Induced-Spoofing Results

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$$\hat{\phi}^{(\mathrm{J})}(z) = H_\phi(z)\,\phi_\mathrm{a}, \qquad \hat{\phi}^{(\mathrm{S})}(z) = H_\phi(z)\,\phi_\mathrm{s}.$$

- ✓ The noise is considered **additive and homogeneous** and can be modeled as presented for the nominal situation in the literature[1].

LINEAR ASSUMPTION

[1] John W Betz and Kevin R Kolodziejski. "Generalized theory of code tracking with an early-late discriminator part II: Noncoherent processing and numerical results". In: *IEEE Transactions on Aerospace and Electronic Systems* 45.4 (2009), pp. 1557–1564.

Dynamic and stochastic response (3/5)

Resolution Under Induced-Multipath Situation

Non-linear:

- × The system **cannot be modeled using a transfer function**.

$$\begin{cases} \hat{\tau}(z) + \beta\, \hat{\theta}(z) = \dfrac{z^{-1}}{1 - z^{-1}} T_\mathrm{i}\, F_\tau(z)\, D_\tau\Big(\boldsymbol{\eta}_\mathrm{a}(z) - \hat{\boldsymbol{\eta}}(z)\, , \boxed{\Delta\boldsymbol{\nu}(z)}\Big) & \text{(DLL)} \\ \hat{\theta}(z) = \dfrac{z^{-1}}{1 - z^{-1}} T_\mathrm{i}\, F_\theta(z)\, D_\theta\Big(\boldsymbol{\eta}_\mathrm{a}(z) - \hat{\boldsymbol{\eta}}(z)\, , \boxed{\Delta\boldsymbol{\nu}(z)}\Big) & \text{(PLL)} \end{cases}$$

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Dynamic and stochastic response (3/5)

Resolution Under Induced-Multipath Situation

Non-linear:

- × The system **cannot be modeled using a transfer function**.

Random state vector $\hat{\phi}$ (p_ϕ)

$$\begin{cases} \hat{\tau}(z) + \beta\,\hat{\theta}(z) = \dfrac{z^{-1}}{1-z^{-1}} T_\mathrm{i}\, F_\tau(z)\, D_\tau\Big(\boldsymbol{\eta}_\mathrm{a}(z) - \hat{\boldsymbol{\eta}}(z), \Delta\boldsymbol{\nu}(z)\Big) & \text{(DLL)} \\ \hat{\theta}(z) = \dfrac{z^{-1}}{1-z^{-1}} T_\mathrm{i}\, F_\theta(z)\, D_\theta\Big(\boldsymbol{\eta}_\mathrm{a}(z) - \hat{\boldsymbol{\eta}}(z), \Delta\boldsymbol{\nu}(z)\Big) & \text{(PLL)} \end{cases}$$

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Dynamic and stochastic response (3/5)

Resolution Under Induced-Multipath Situation

Non-linear:

- × The system **cannot be modeled using a transfer function**.
- × **Non-additivity:** Input dynamics and noise response cannot be separated.

Random state vector $\hat{\phi}$ (p_{ϕ})

$$
\begin{cases}
\hat{\tau}(z) + \beta\, \hat{\theta}(z) = \dfrac{z^{-1}}{1 - z^{-1}}\, T_{\mathrm{i}}\, F_{\tau}(z)\, D_{\tau}\left(\boldsymbol{\eta}_{\mathrm{a}}(z) - \hat{\boldsymbol{\eta}}(z), \Delta\boldsymbol{\nu}(z)\right) & \text{(DLL)} \\
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\end{cases}
$$

⚠ NON-LINEAR AND INTERDEPENDENT ⚠

Dynamic and stochastic response (3/5)

Resolution Under Induced-Multipath Situation

Non-linear:

- × The system **cannot be modeled using a transfer function**.
- × **Non-additivity:** Input dynamics and noise response cannot be separated.
- × **Heterogeneity:** Noise variance depends on the system's actual state.

Random state vector $\hat{\phi}$ (p_{ϕ})

$$\begin{cases} \hat{\tau}(z) + \beta\, \hat{\theta}(z) = \dfrac{z^{-1}}{1 - z^{-1}} T_{\mathrm{i}}\, F_{\tau}(z)\, D_{\tau}\Big(\boldsymbol{\eta}_{\mathrm{a}}(z) - \hat{\boldsymbol{\eta}}(z), \Delta\boldsymbol{\nu}(z)\Big) & \text{(DLL)} \\ \hat{\theta}(z) = \dfrac{z^{-1}}{1 - z^{-1}} T_{\mathrm{i}}\, F_{\theta}(z)\, D_{\theta}\Big(\boldsymbol{\eta}_{\mathrm{a}}(z) - \hat{\boldsymbol{\eta}}(z), \Delta\boldsymbol{\nu}(z)\Big) & \text{(PLL)} \end{cases}$$

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Dynamic and stochastic response (3/5)

Resolution Strategy

Quasi-Harmonic Behavior:

Random state vector $\hat{\phi}$ (p_ϕ)

$$\begin{cases} \hat{\tau}(z) + \beta\, \hat{\theta}(z) = \dfrac{z^{-1}}{1-z^{-1}}\, T_{\mathrm{i}}\, F_\tau(z)\, D_\tau\Big(\boldsymbol{\eta}_{\mathrm{a}}(z) - \hat{\boldsymbol{\eta}}(z),\, \Delta\boldsymbol{\nu}(z)\Big) & \text{(DLL)} \\ \hat{\theta}(z) = \dfrac{z^{-1}}{1-z^{-1}}\, T_{\mathrm{i}}\, F_\theta(z)\, D_\theta\Big(\boldsymbol{\eta}_{\mathrm{a}}(z) - \hat{\boldsymbol{\eta}}(z),\, \Delta\boldsymbol{\nu}(z)\Big) & \text{(PLL)} \end{cases}$$

⚠ NON-LINEAR AND INTERDEPENDENT ⚠

Dynamic and stochastic response (3/5)

Resolution Strategy

Quasi-Harmonic Behavior:

- The system's SE exhibit oscillations along the relative phase $\Delta\theta$.

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⚠ NON-LINEAR AND INTERDEPENDENT ⚠

Dynamic and stochastic response (3/5)

Resolution Strategy

Quasi-Harmonic Behavior:

- The system's SE exhibit oscillations along the relative phase $\Delta\theta$.
- Other relative parameters, $\Delta\boldsymbol{\nu}$, can be assumed constant over $\Delta\theta \in [0, 2\pi]$.

Random state vector $\hat{\phi}$ (p_{ϕ})

$$\begin{cases} \hat{\tau}(z) + \beta\, \hat{\theta}(z) = \dfrac{z^{-1}}{1 - z^{-1}}\, T_{\mathrm{i}}\, F_{\tau}(z)\, D_{\tau}\Big(\boldsymbol{\eta}_{\mathrm{a}}(z) - \hat{\boldsymbol{\eta}}(z), \Delta\boldsymbol{\nu}(z)\Big) & \text{(DLL)} \\ \hat{\theta}(z) = \dfrac{z^{-1}}{1 - z^{-1}}\, T_{\mathrm{i}}\, F_{\theta}(z)\, D_{\theta}\Big(\boldsymbol{\eta}_{\mathrm{a}}(z) - \hat{\boldsymbol{\eta}}(z), \Delta\boldsymbol{\nu}(z)\Big) & \text{(PLL)} \end{cases}$$

⚠ NON-LINEAR AND INTERDEPENDENT ⚠

Dynamic and stochastic response (3/5)

Resolution Strategy

Quasi-Harmonic Behavior:

- The system's SE exhibit oscillations along the relative phase $\Delta\theta$.
- Other relative parameters, $\Delta\boldsymbol{\nu}$, can be assumed constant over $\Delta\theta \in [0, 2\pi]$.
- The system behavior can be characterized by its quasi-harmonic response for any $\Delta\boldsymbol{\nu}$.

Random state vector $\hat{\phi}$ (p_ϕ)

Constant except $\Delta\theta \in [0, 2\pi]$

$$\begin{cases} \hat{\tau}(z) + \beta\,\hat{\theta}(z) = \dfrac{z^{-1}}{1 - z^{-1}}\, T_{\mathrm{i}}\, F_\tau(z)\, D_\tau\Big(\boldsymbol{\eta}_{\mathrm{a}}(z) - \hat{\boldsymbol{\eta}}(z), \Delta\boldsymbol{\nu}\Big) & \text{(DLL)} \\ \hat{\theta}(z) = \dfrac{z^{-1}}{1 - z^{-1}}\, T_{\mathrm{i}}\, F_\theta(z)\, D_\theta\Big(\boldsymbol{\eta}_{\mathrm{a}}(z) - \hat{\boldsymbol{\eta}}(z), \Delta\boldsymbol{\nu}\Big) & \text{(PLL)} \end{cases}$$

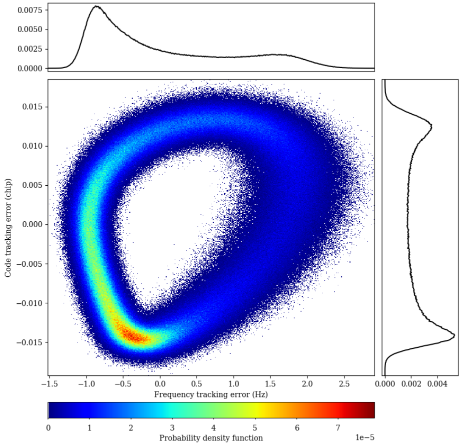
⚠ NON-LINEAR AND INTERDEPENDENT ⚠

Dynamic and stochastic response (4/5)

Induced-Multipath Results

Quasi-harmonic behavior:

Dynamic and stochastic response (4/5)



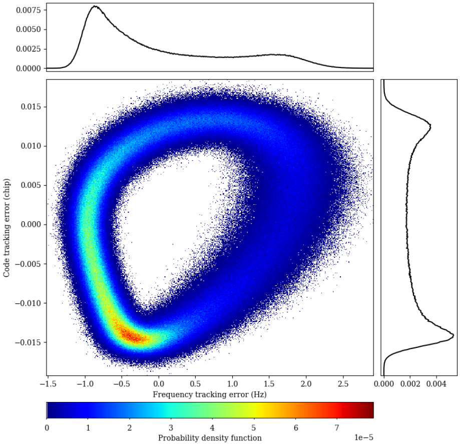
Tracking error PDF p_ϕ ($\Delta\tau = 0.5$ chip, $\Delta f = 2$ Hz, $\Delta g = 0.64$ and $C/N_0 = 50$ dB.Hz).

Induced-Multipath Results

Quasi-harmonic behavior:

- ✓ Enable the characterization of the tracking error under induced multipath as a function of the relative dynamics $\Delta\nu$ and receiver parameters.

Dynamic and stochastic response (4/5)



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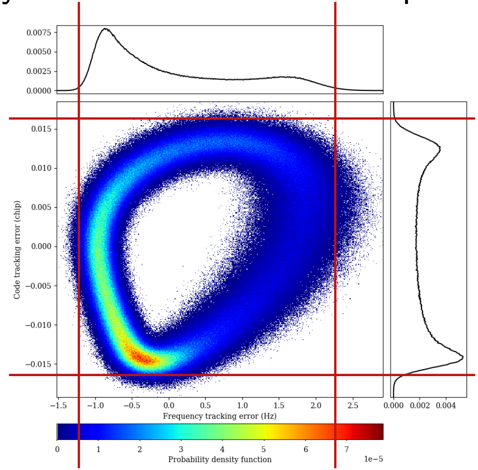
Induced-Multipath Results

Quasi-harmonic behavior:

- ✓ Enable the characterization of the tracking error under induced multipath as a function of the relative dynamics $\Delta\nu$ and receiver parameters.
- ✓ Accounts for the **system's non-linearity, filtering, and stochastic aspects**.

Dynamic and stochastic response (4/5)



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Induced-Multipath Results

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- ✓ Enable the characterization of the tracking error under induced multipath as a function of the relative dynamics $\Delta\nu$ and receiver parameters.
- ✓ Accounts for the **system's non-linearity, filtering, and stochastic aspects**.
- ✓ Enables the derivation of a more accurate **Spoofing Error Envelope (SEE)**.

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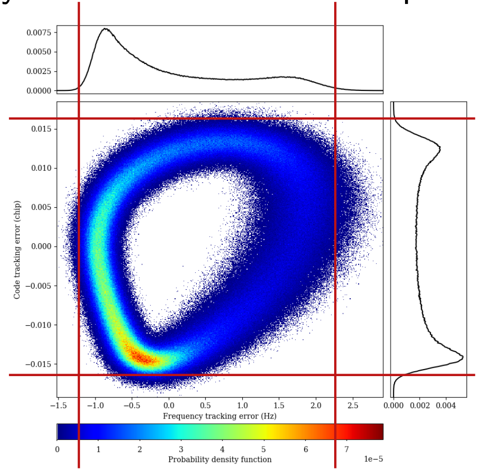
Dynamic and stochastic response (4/5)



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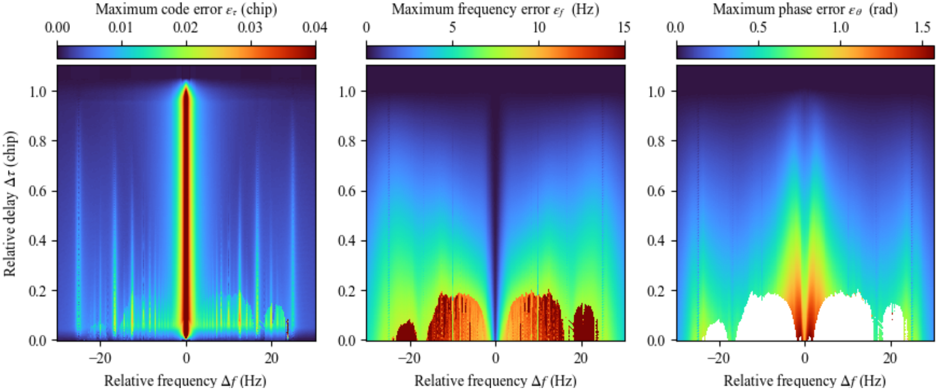
Induced-Multipath Results

Quasi-harmonic behavior:

- ✓ Enable the characterization of the tracking error under induced multipath as a function of the relative dynamics $\Delta\boldsymbol{\nu}$ and receiver parameters.
- ✓ Accounts for the **system's non-linearity, filtering, and stochastic aspects**.
- ✓ Enables the derivation of a more accurate **Spoofing Error Envelope (SEE)**.
- × Does not model **bifurcation or transient response**.

Dynamic and stochastic response (5/5)



Code Frequency Phase

Absolute Spoofing Error Envelope ($\Delta g = 0.64$ and authentic C/N_0 of 55 dB.Hz).

Outline

C/N_0 definition

Objective

Characterize the true and estimated C/N_0 under each induced-spoofing situation.

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Characterize the true and estimated C/N_0 under each induced-spoofing situation.

True C/N_0 (over $\Omega \times I_t$):

$$\left(\frac{C}{N_0}\right)_{\text{true}} \triangleq \frac{1}{T_\mathrm{i}} \frac{P_\mathrm{d}}{P_\varsigma}$$

C/N_0 definition

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C/N_0 definition

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Characterize the true and estimated C/N_0 under each induced-spoofing situation.

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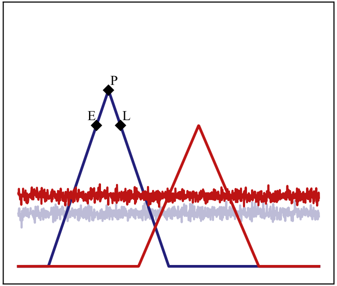
Estimated C/N_0 (measured on the prompt correlator outputs over I_t):
Three algorithms:

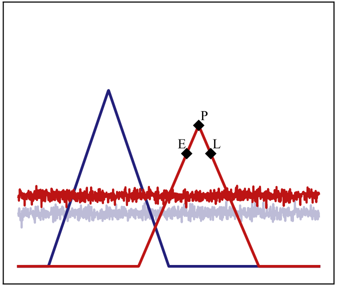
- Moment Method (MM) (equivalent to Variance Summing Method (VSM),
- Narrow Wideband Power Ratio (NWPR),
- Beaulieu method.

C/N_0 under spoofing (1/4)

Induced-jamming



Induced-spoofing



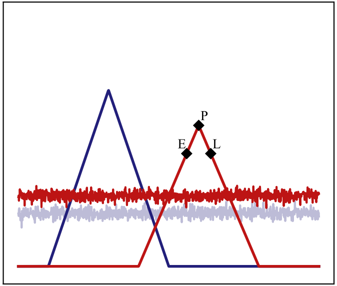
C/N_0 under spoofing (1/4)

Induced-jamming

True C/N_0:

$$\left(\frac{C}{N_0}\right)^{(\mathrm{J})}_{\mathrm{true}} = \frac{1}{T_\mathrm{i}}\frac{P_\mathrm{d}}{P_\varsigma} = \frac{\zeta(\varepsilon_\eta)^2}{T_\mathrm{i}}\frac{C_\mathrm{a}}{P_n + P_\chi}$$

Induced-spoofing



C/N_0 under spoofing (1/4)

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$$\left(\frac{C}{N_0}\right)^{(\mathrm{J})}_{\mathrm{true}} = \frac{1}{T_\mathrm{i}}\frac{P_\mathrm{d}}{P_\varsigma} = \frac{\zeta(\varepsilon_\eta)^2}{T_\mathrm{i}}\frac{C_\mathrm{a}}{P_n + P_\chi}$$

Induced-spoofing

True C/N_0:

$$\left(\frac{C}{N_0}\right)^{(\mathrm{S})}_{\mathrm{true}} = \frac{1}{T_\mathrm{i}}\frac{P_\mathrm{d}}{P_\varsigma} = \frac{\zeta(\varepsilon_\eta + \Delta\boldsymbol{\eta})^2}{T_\mathrm{i}}\frac{\Delta g\, C_\mathrm{a}}{P_n + P_\chi}$$

C/N_0 under spoofing (1/4)

Induced-jamming

True C/N_0:

$$\left(\frac{C}{N_0}\right)^{(\mathrm{J})}_{\mathrm{true}} = \frac{1}{T_\mathrm{i}} \frac{\overbrace{P_\mathrm{d}}^{\text{Authentic power}}}{\underbrace{P_\varsigma}_{\text{Random contribution power}}} = \frac{\zeta(\varepsilon_\eta)^2}{T_\mathrm{i}} \frac{\overbrace{C_\mathrm{a}}^{\text{Authentic power}}}{\underbrace{P_n + P_\chi}_{\text{Random contribution power}}}$$

Induced-spoofing

True C/N_0:

$$\left(\frac{C}{N_0}\right)^{(\mathrm{S})}_{\mathrm{true}} = \frac{1}{T_\mathrm{i}} \frac{\overbrace{P_\mathrm{d}}^{\text{Spoofing power}}}{\underbrace{P_\varsigma}_{\text{Random contribution power}}} = \frac{\zeta(\varepsilon_\eta + \Delta\eta)^2}{T_\mathrm{i}} \frac{\overbrace{\Delta g\, C_\mathrm{a}}^{\text{Spoofing power}}}{\underbrace{P_n + P_\chi}_{\text{Random contribution power}}}$$

C/N_0 under spoofing (1/4)

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Induced-Jamming and Induced-Spoofing Results

The estimators are unbiased and converge to the true C/N_0 value.

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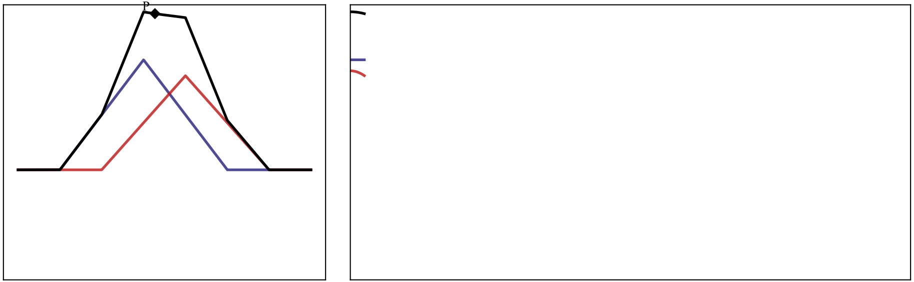
C/N_0 under spoofing (2/4)

Induced-multipath:

$$\Lambda(\eta, \eta_a, \Delta\eta) = \underbrace{\Lambda_a(\eta_a - \eta)}_{\text{Authentic peak}} + \underbrace{\Lambda_s(\eta_a - \eta + \Delta\eta)}_{\text{Spoofing peak}} + \underbrace{\Lambda_\varsigma}_{\text{Random contribution}}$$

C/N_0 under spoofing (2/4)

Prompt correlator evolution:



C/N_0 under spoofing (2/4)

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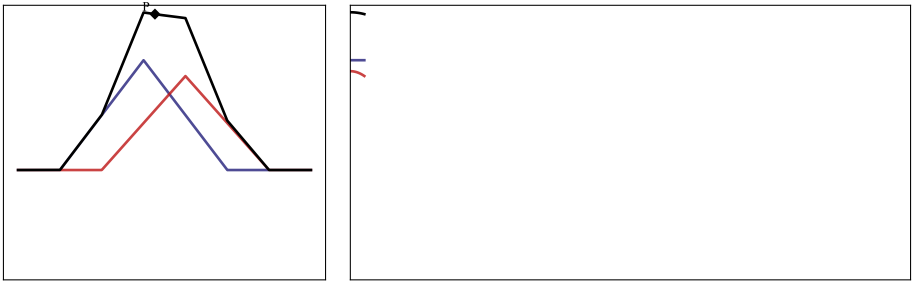

True C/N_0:

Individual power Interaction power

$$\left(\frac{C}{N_0}\right)^{(\mathrm{M})}_{\mathrm{true}} = \frac{1}{T_\mathrm{i}}\frac{P_\mathrm{d}}{P_\varsigma} = \frac{1}{T_\mathrm{i}}\frac{\left(\mu_d + \omega_d\right) C_\mathrm{a}}{P_n + P_\chi}.$$

Random contribution power

C/N_0 under spoofing (3/4)

Induced-Multipath Results

C/N_0 under spoofing (3/4)

Induced-Multipath Results

- Derivation of **analytic expressions** as a function of $\Delta\eta$.

MM:

$$\mathbb{E}_\Omega\left[\frac{\widehat{C}}{N_0}\right]^{(\mathrm{M})}_{\mathrm{MM}} \approx \frac{1}{T_\mathrm{i}}\frac{\sqrt{{P_\mathrm{d}}^2 - {C_\mathrm{a}}^2 \overbrace{\sigma_\mathrm{d}^2}^{\text{Interaction random power variance}}}}{P_\varsigma + P_\mathrm{d} - \sqrt{{P_\mathrm{d}}^2 - {C_\mathrm{a}}^2 \sigma_\mathrm{d}^2}}$$

Beaulieu:

$$\mathbb{E}_\Omega\left[\frac{\widehat{C}}{N_0}\right]^{(\mathrm{M})}_{\mathrm{B}} = \left[\frac{P_\varsigma + \overbrace{\delta_B}^{\text{Correlator difference power}}}{P_\mathrm{d}} - \underbrace{\varepsilon_B}_{\text{Beaulieu Non-ergodic error}}\right]^{-1}$$

NWPR:

$$\mathbb{E}_\Omega\left[\frac{\widehat{C}}{N_0}\right]^{(\mathrm{M})}_{\mathrm{NWPR}} \approx \frac{1}{T_\mathrm{i}}\frac{P_\mathrm{d} + \sigma_{\mathrm{WB}} - \varepsilon_{\mathrm{PR}}}{P_\varsigma - \underbrace{\sigma_{\mathrm{WB}}}_{\text{NW band error}} + \underbrace{\varepsilon_{\mathrm{PR}}}_{\text{PR non-ergodic error}} / L}$$

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C/N_0 under spoofing (3/4)

Induced-Multipath Results

- Derivation of **analytic expressions** as a function of $\Delta\eta$.
- The MM, Beaulieu, and NWPR estimators are **biased** and do not reflect the true C/N_0.

MM:

$$\mathbb{E}_\Omega\left[\frac{\widehat{C}}{N_0}\right]_{\mathrm{MM}}^{(\mathrm{M})} \approx \frac{1}{T_\mathrm{i}} \frac{\sqrt{{P_\mathrm{d}}^2 - {C_\mathrm{a}}^2 \overbrace{\sigma_\mathrm{d}^2}^{\text{Interaction random power variance}}}}{P_\varsigma + P_\mathrm{d} - \sqrt{{P_\mathrm{d}}^2 - {C_\mathrm{a}}^2 \sigma_\mathrm{d}^2}}$$

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NWPR:

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C/N_0 under spoofing (3/4)

Induced-Multipath Results

- Derivation of **analytic expressions** as a function of $\Delta\eta$.
- The MM, Beaulieu, and NWPR estimators are **biased** and do not reflect the true C/N_0.
- Spoofing distortion **varies depending on the type of estimator**.

MM:

$$\mathbb{E}_\Omega\left[\frac{\widehat{C}}{N_0}\right]^{(\mathrm{M})}_{\mathrm{MM}} \approx \frac{1}{T_\mathrm{i}} \frac{\sqrt{{P_\mathrm{d}}^2 - {C_\mathrm{a}}^2 \overbrace{\sigma_\mathrm{d}^2}^{\text{Interaction random power variance}}}}{P_\varsigma + P_\mathrm{d} - \sqrt{{P_\mathrm{d}}^2 - {C_\mathrm{a}}^2 \sigma_\mathrm{d}^2}}$$

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NWPR:

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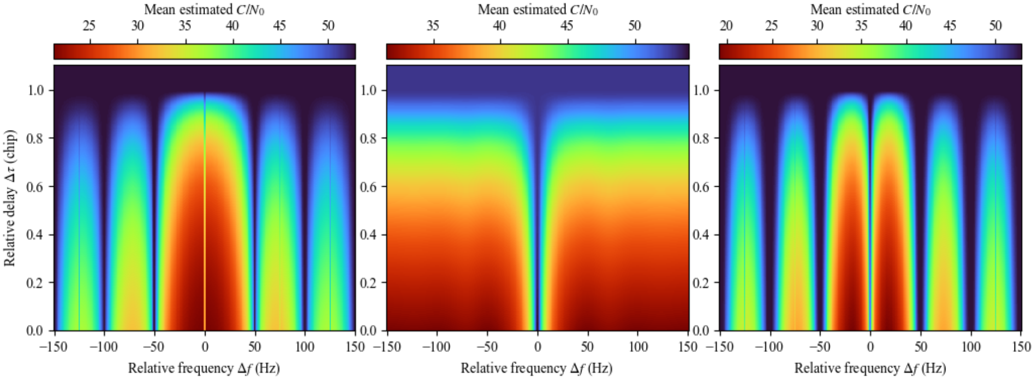
C/N_0 under spoofing (4/4)



MM NWPR Beaulieu

Mean estimated C/N_0 in dB.Hz ($\Delta g = 0.64$ and authentic C/N_0 of 55 dB.Hz).

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Conclusion

Spoofing Impact After Correlation

The impact of spoofing on post-correlation stages can be classified into four situations:

- **Nominal:** The reference scenario with no spoofing impact.
- **Induced-Jamming:** The receiver tracks the authentic dynamics, and the spoofing impact is **reduced to re-radiated noise**.
- **Induced-Spoofing:** The receiver tracks the spoofing dynamics and **adapts to the characteristics of the spoofing signal**.
- **Induced-Multipath:** The receiver tracks a composite of both signals:
  - ⚠ The tracking loops exhibit **chaotic behavior**, including multiple SE and non-linear effects, inducing varying tracking errors and possible bifurcations.
  - ⚠ The C/N_0 estimators **suffer significant degradation**, potentially leading to **loss-of-lock**.

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Outline

Experiment presentation

Objectives

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- Observe the distortions predicted by the theoretical model in a **realistic spoofing environment** using **real GNSS receivers**.

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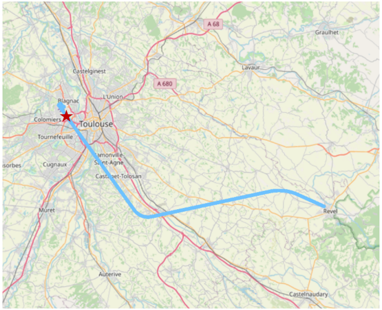

Experiment presentation

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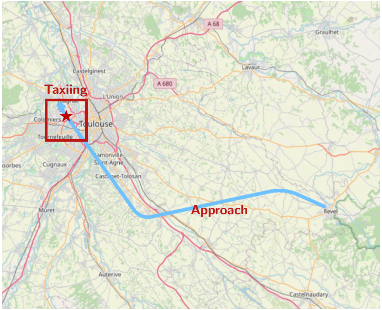

Experiment presentation

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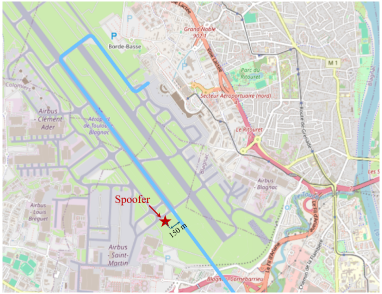
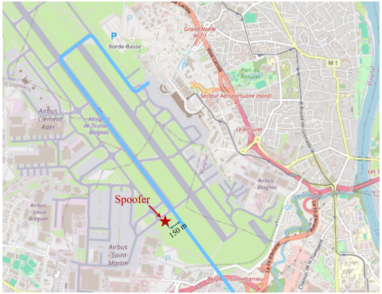


Experiment presentation

Objectives

- Observe the distortions predicted by the theoretical model in a **realistic spoofing environment** using **real GNSS receivers**.
- **Evaluate** the theoretical predictions against experimental results and **compare** different receiver architectures.



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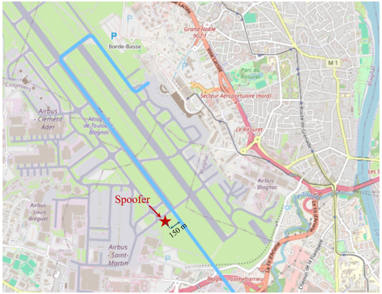

Examined Observables:

- **Theoretical Models**
- **Simulator:** Implements correlator, C/N_0 estimator, and tracking loops.
- **Real Receivers:**
  - GNSS SDR
  - Ifen SX3

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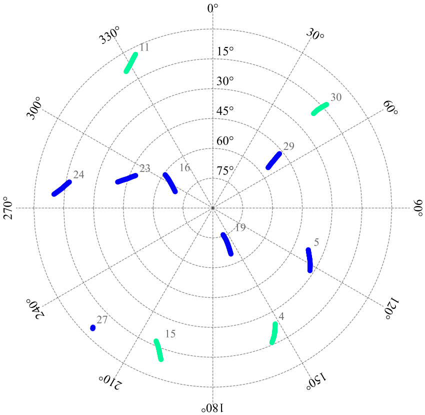
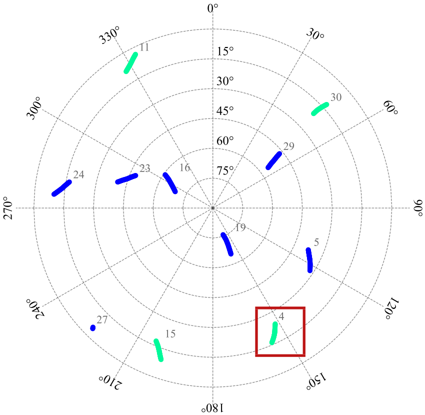
Experiment scenario (1/2)



Experiment scenario (1/2)

PRN 4 analyzed (PRN 11, 15 and 30 in the manuscript)

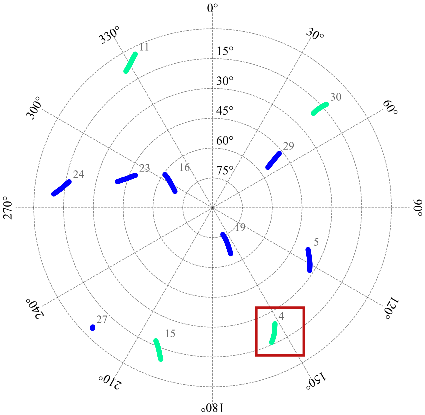


Experiment scenario (1/2)

PRN 4 analyzed (PRN 11, 15 and 30 in the manuscript)

Experiment Scenarios:

- **Scenario 1:** Nominal scenario (no spoofing).
- **Scenario 2:** Low-power sophisticated repeater without re-radiated noise.
- **Scenario 3:** High-power sophisticated repeater without re-radiated noise.



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PRN 4 analyzed (PRN 11, 15 and 30 in the manuscript)

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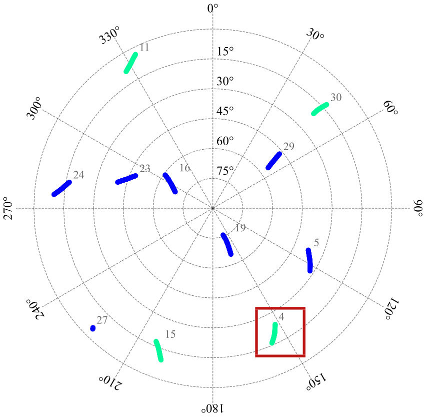
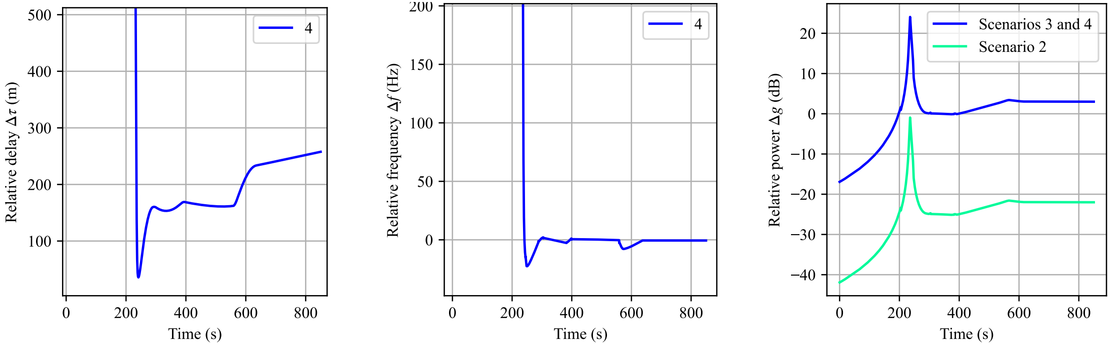
Same authentic dynamics: $\tau_a(t)$, $\theta_a(t)$, and **same spoofing dynamics:** $\tau_s(t)$, $\theta_s(t)$ across each scenario.

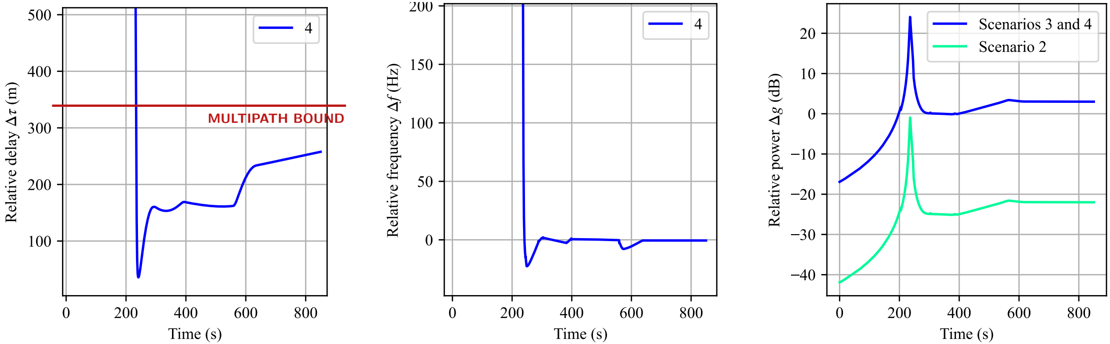


Experiment scenario (2/2)



Relative parameters (code, frequency, power) for PRN 4.

Experiment scenario (2/2)



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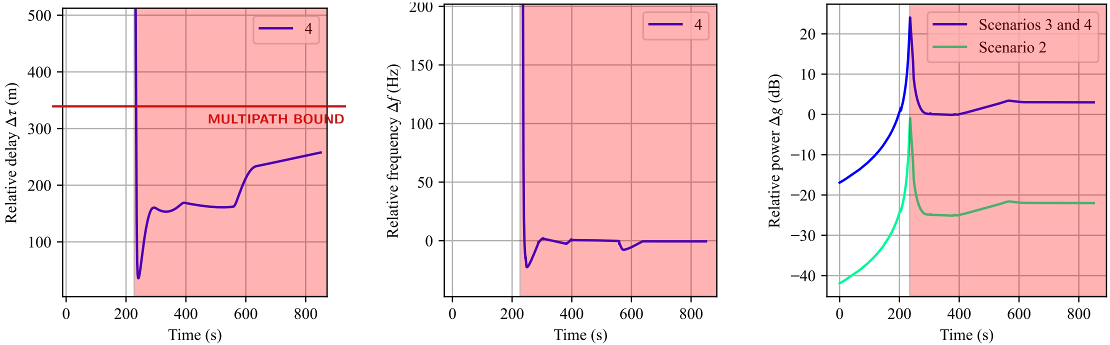

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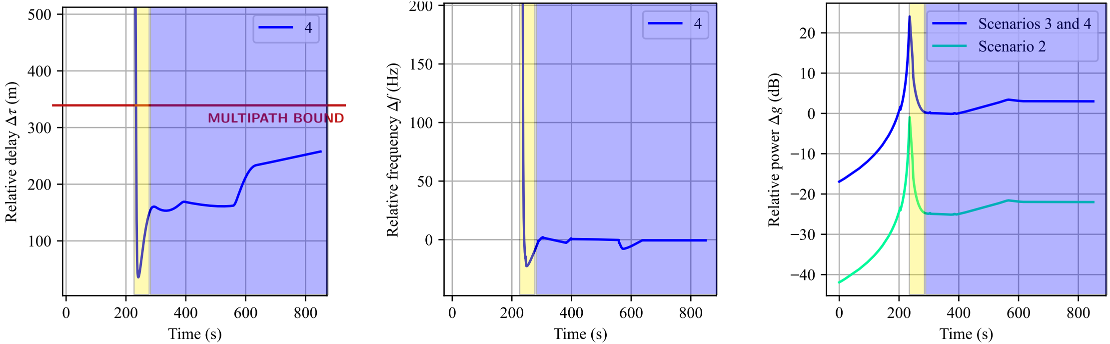
Experiment scenario (2/2)



Relative parameters (code, frequency, power) for PRN 4.

: Flying over the spoofer / Landing (high dynamics)

: Taxiing (low dynamics)

Experiment setup

<table>
<tr><th colspan="2"></th><th>SX3</th><th>GNSS-SDR</th><th>Simulator</th></tr>
<tr><td colspan="2" rowspan="2">Acquisition</td><td colspan="2">Doppler $= \pm 5000$ Hz</td><td>Doppler $= \pm 3500$ Hz</td></tr>
<tr><td colspan="3">Coherent time $= 1$ ms, Incoherent number $= 10$, $P_{\mathrm{Fa}} = 10^{-3}$</td></tr>
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<tr><td rowspan="4">DLL</td><td rowspan="2">Discriminator</td><td>EMLP</td><td>EMLE</td><td>EMLP</td></tr>
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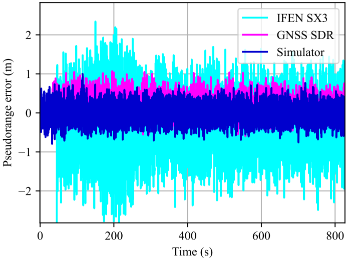
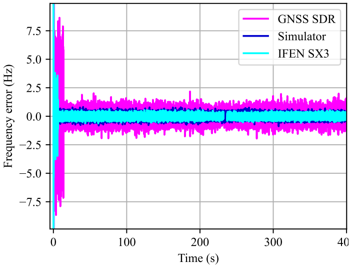
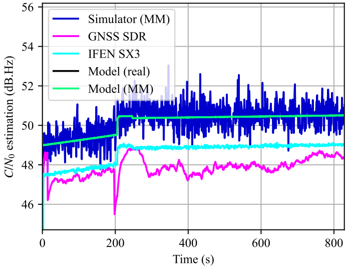


Code error

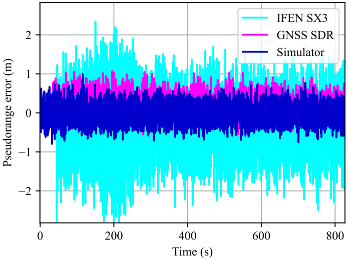


Frequency error



C/N_0

Nominal scenario



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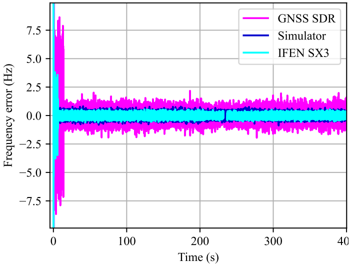

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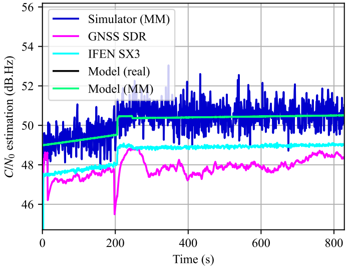
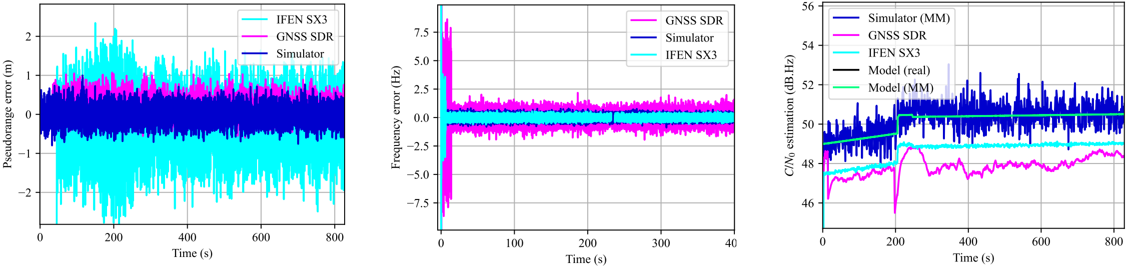


C/N_0

Experimental Errors:

Nominal scenario

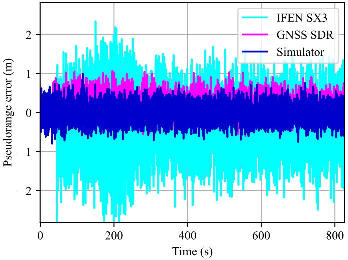


Code error Frequency error C/N_0

Experimental Errors:

- **Additional noise in the SX3 code tracking error** (possibly due to clock correction of the pseudorange).

Nominal scenario



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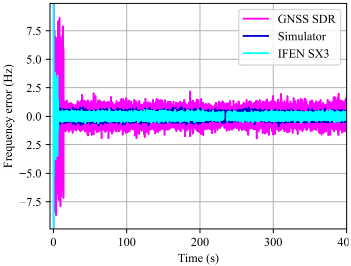
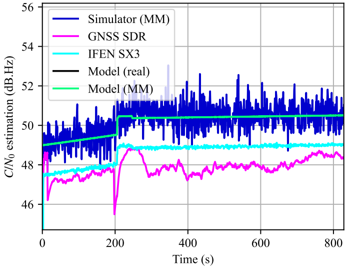


Frequency error

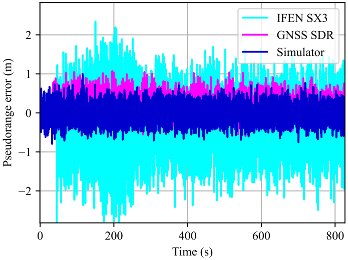


C/N_0

Experimental Errors:

- **Additional noise in the SX3 code tracking error** (possibly due to clock correction of the pseudorange).
- The SX3 and GNSS-SDR C/N_0 estimators **are biased** (constant bias but show the same trend as the expected C/N_0).

Nominal scenario



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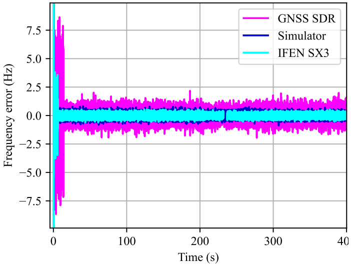
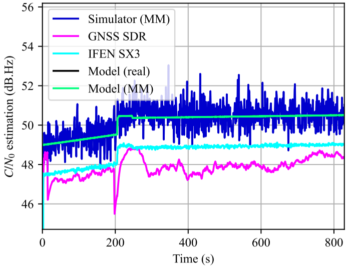


Frequency error

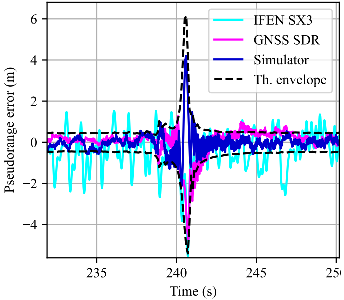


C/N_0

Experimental Errors:

- **Additional noise in the SX3 code tracking error** (possibly due to clock correction of the pseudorange).
- The SX3 and GNSS-SDR C/N_0 estimators **are biased** (constant bias but show the same trend as the expected C/N_0).
- The GNSS-SDR C/N_0 estimators **appear to be low-pass filtered**.

Low-power repeater scenario - Landing phase



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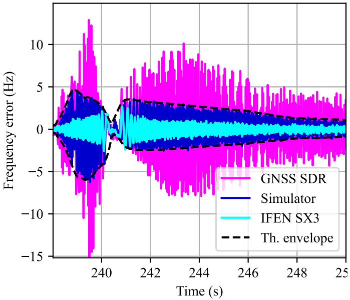

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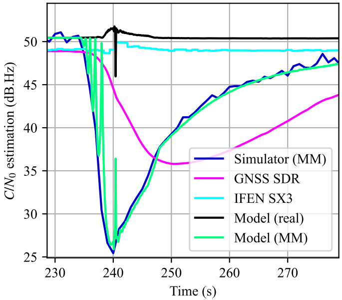

C/N_0

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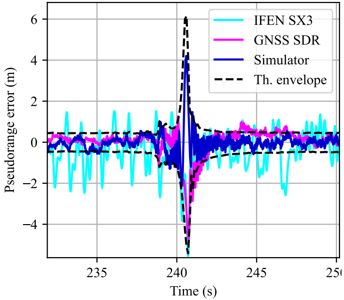

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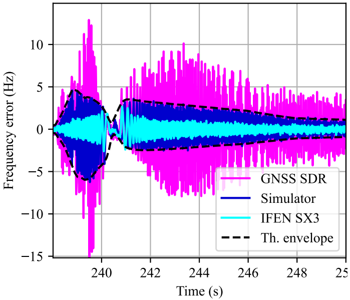

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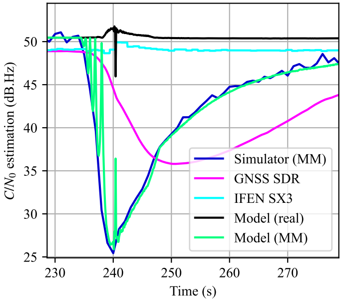
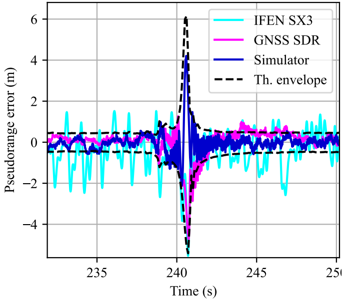


C/N_0

Observations:

Low-power repeater scenario - Landing phase



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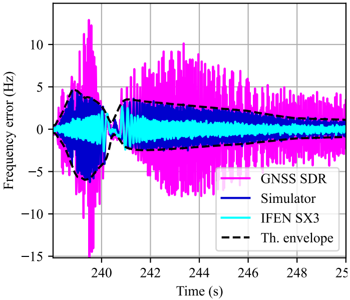

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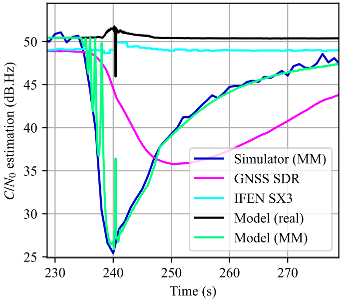

C/N_0

Observations:

- Tracking Errors: **oscillations** observed (dependence on the **receiver architecture**).

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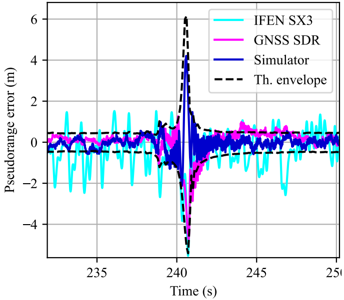

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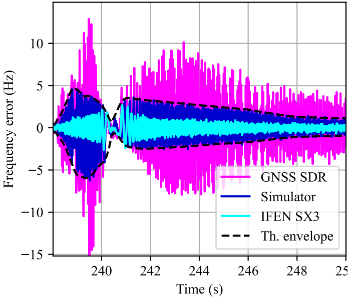

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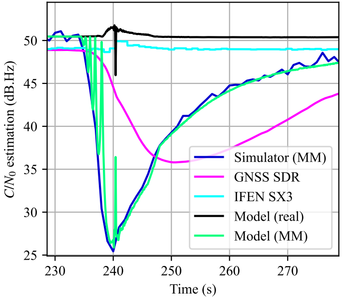

C/N_0

Observations:

- Tracking Errors: **oscillations** observed (dependence on the **receiver architecture**).
- C/N_0 Degradation: **Significant degradation** observed (strong **dependence** on the estimator method).

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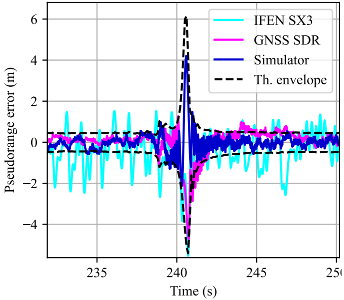

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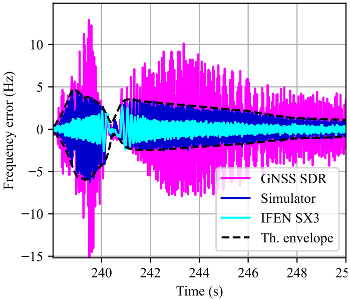

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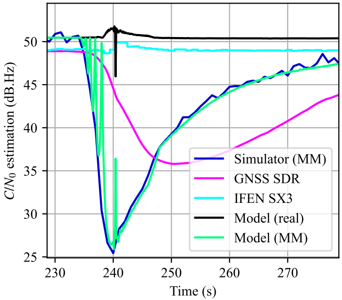

C/N_0

Observations:

- Tracking Errors: **oscillations** observed (dependence on the **receiver architecture**).
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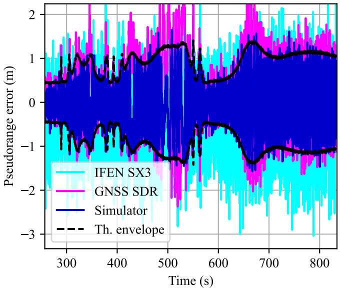
- The theoretical models **accurately predict receiver behavior under spoofing conditions**.

Low-power repeater scenario - Taxiing phase



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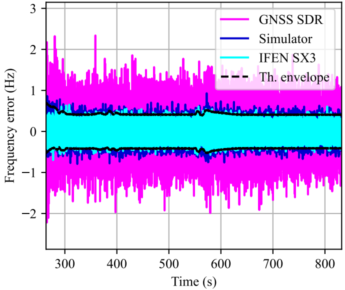

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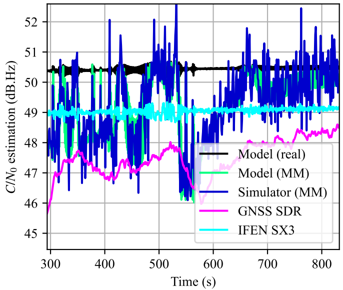

C/N_0

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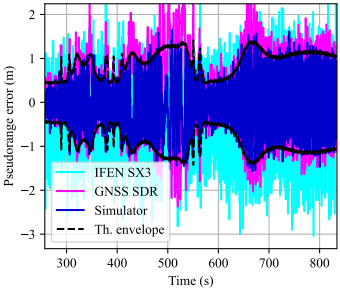

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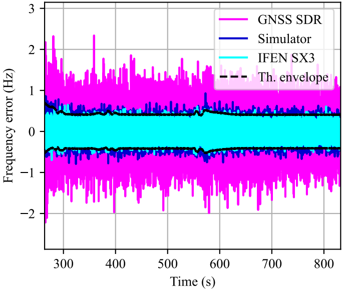

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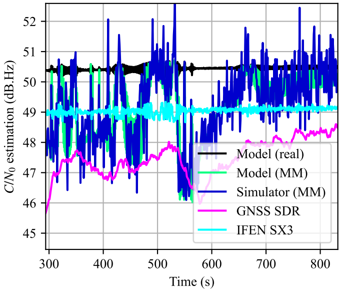

C/N_0

Observations:

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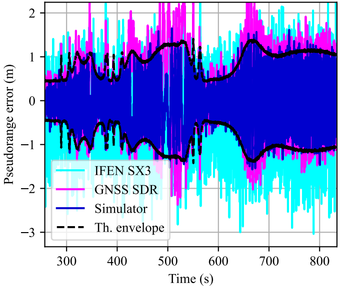

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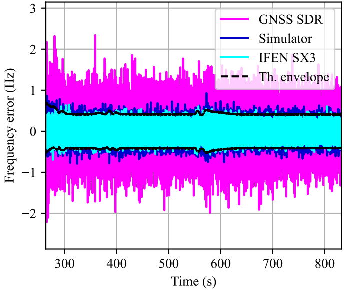

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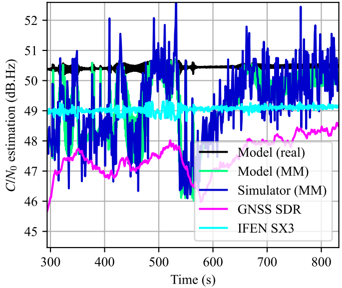
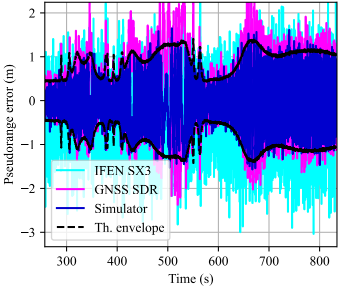


C/N_0

Observations:

- Low oscillations in tracking errors and C/N_0 degradation ranging from 0.5 to 2 dB (dependence on the **receiver architecture**).

Low-power repeater scenario - Taxiing phase



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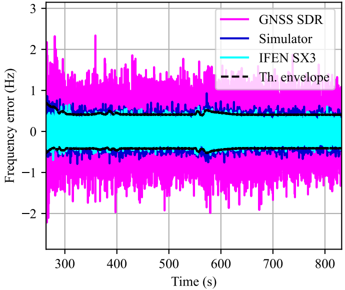

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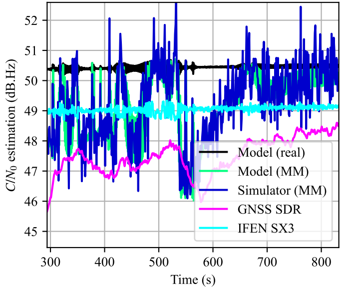
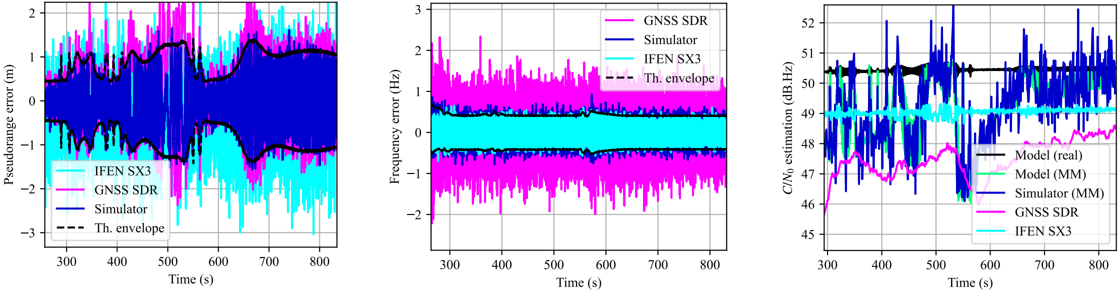


C/N_0

Observations:

- Low oscillations in tracking errors and C/N_0 degradation ranging from 0.5 to 2 dB (dependence on the **receiver architecture**).
- Observed distortion even at **very low spoofing received power**, $\Delta g < -20$ dB.

Low-power repeater scenario - Taxiing phase



Code error | Frequency error | C/N_0

Observations:

- Low oscillations in tracking errors and C/N_0 degradation ranging from 0.5 to 2 dB (dependence on the **receiver architecture**).
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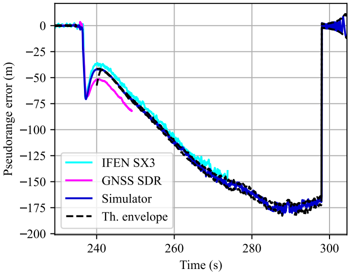
- The theoretical models **accurately predict receiver behavior under spoofing conditions**.

High-power repeater scenario - Landing phase



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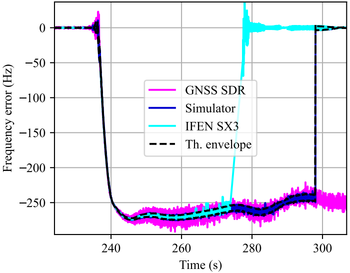

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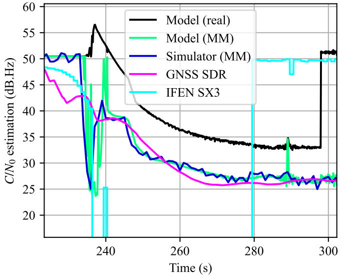

C/N_0

Observations:

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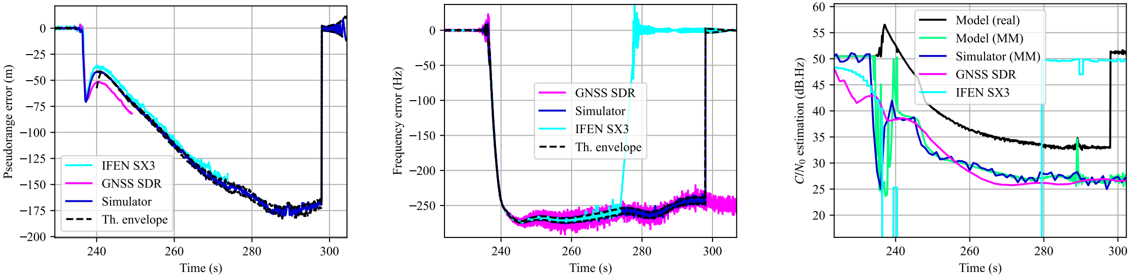
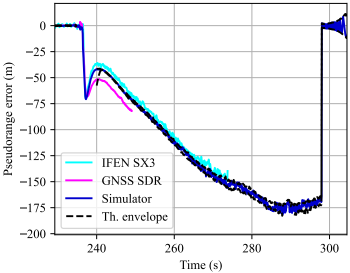


Code error | Frequency error | C/N_0

Observations:

- **Bifurcation** toward the 250 Hz-mismatched spoofing SE.

High-power repeater scenario - Landing phase



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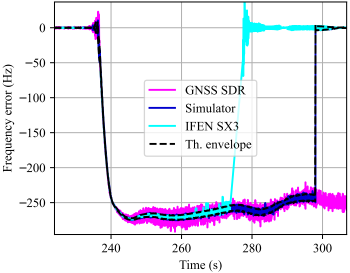
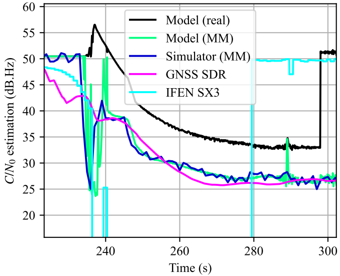


Frequency error



C/N_0

Observations:

- **Bifurcation** toward the 250 Hz-mismatched spoofing SE.
- **Loss-of-lock** as the received spoofing power decreases, followed by **re-acquisition** on the authentic SE.

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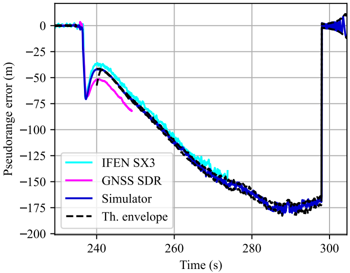
High-power repeater scenario - Landing phase



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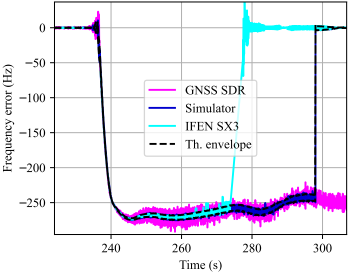

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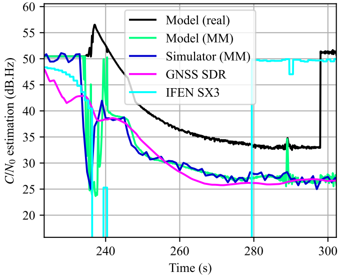

C/N_0

Observations:

- **Bifurcation** toward the 250 Hz-mismatched spoofing SE.
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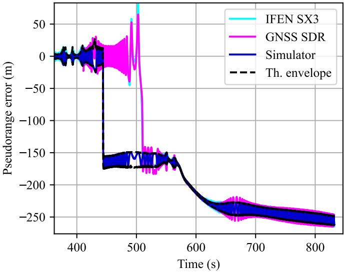
- The model predicts the possible SE to which the system converges, as well as the C/N_0 behavior.

High-power repeater scenario - Taxiing phase



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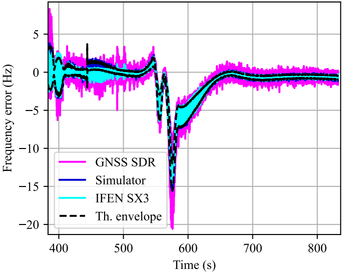

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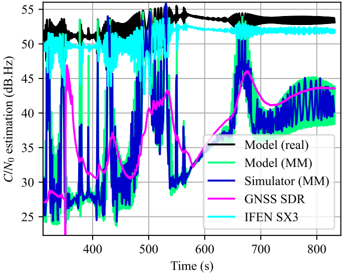

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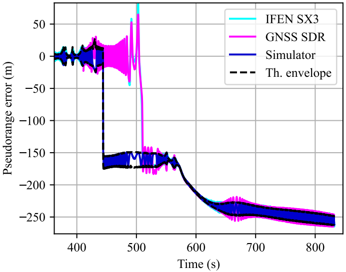

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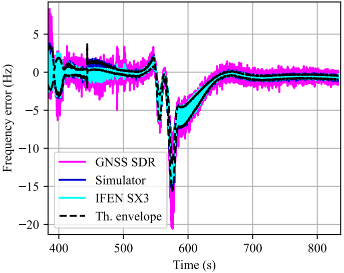

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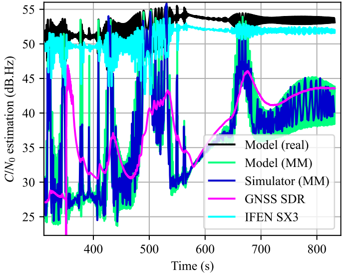

C/N_0

Observations:

- Tracking errors: **Strong oscillations** and **bifurcation** toward the spoofing SE (**chaotic behavior** with high dependence on the receiver architecture and environment).

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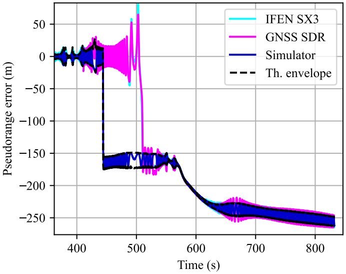

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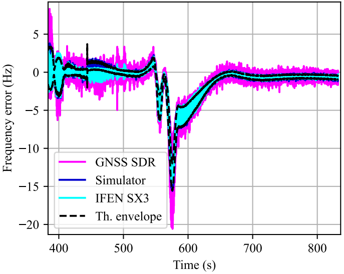

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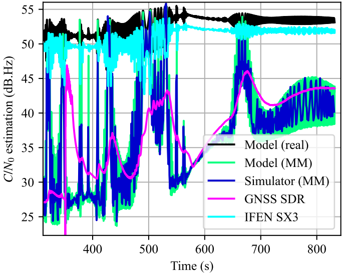

C/N_0

Observations:

- Tracking errors: **Strong oscillations** and **bifurcation** toward the spoofing SE (**chaotic behavior** with high dependence on the receiver architecture and environment).
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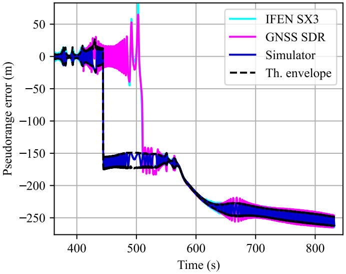

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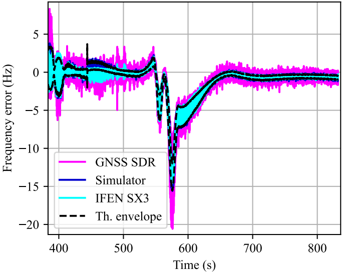

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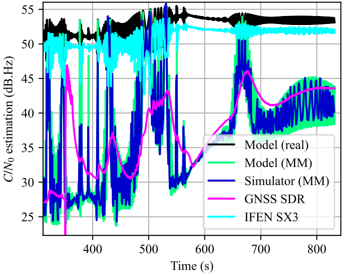

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Observations:

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Conclusion

Experiment observations

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⚠ The harmful distortion observed under induced-multipath situation, as exhibited in the theoretical models, **is also observed in realistic spoofing environments**.

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- ⚠ The experiments highlight the **strong chaotic behavior** of the GNSS receiver under such distortion, including tracking loop bifurcations, loss-of-lock, and a strong dependence on both the receiver architecture and the spoofing environment.

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- ⚠ The harmful distortion observed under induced-multipath situation, as exhibited in the theoretical models, **is also observed in realistic spoofing environments**.
- ⚠ The experiments highlight the **strong chaotic behavior** of the GNSS receiver under such distortion, including tracking loop bifurcations, loss-of-lock, and a strong dependence on both the receiver architecture and the spoofing environment.
- ✓ However, the models proposed in this thesis **provide accurate predictions** of the possible loop SE and C/N_0 degradation, **offering a means to understand and quantify the impact of spoofing**.

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Outline

General Conclusions

Measure-Theory Framework

Introduction of a measure-theoretic framework for estimation under non-ergodic conditions:

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Derivation of the theoretical model for the impact of spoofing before correlation (AGC, IF signal) and after correlation (Correlator outputs, tracking loops, and C/N_0):

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- **Pre-Correlation:** Spoofing acts as a multiple **weighted CW jammer** for the AGC, starting to impact AGC values when $S/N > -5\,\text{dB}$.

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Receiver Behavior Under Spoofing

Derivation of the theoretical model for the impact of spoofing before correlation (AGC, IF signal) and after correlation (Correlator outputs, tracking loops, and C/N_0):

- **Pre-Correlation:** Spoofing acts as a multiple **weighted CW jammer** for the AGC, starting to impact AGC values when $S/N > -5\,\text{dB}$.
- **Post-Correlation:** The impact of spoofing is categorized into four situations, including a harmful case where the receiver exhibits chaotic behavior, inducing **high tracking errors** and **significant C/N_0 degradation**, as well as, possible **tracking bifurcations** and **loss-of-locks**.

Thank You!

emile.ghizzo@alumni.enac.fr