Robust Multi Sensor Fusion for State Estimation

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Thanks to our team and may more!

Multi Sensor Systems Group

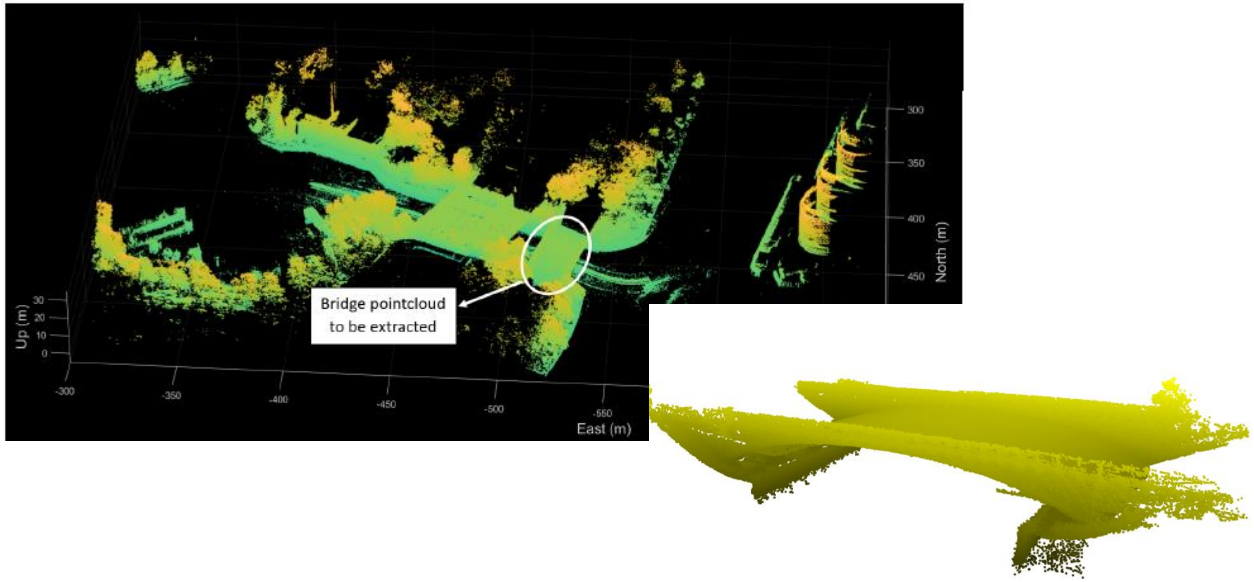
- **Daniel Medina** – Group leader, GNSS & *robust estimation*
- **Christoph Lass** – deputy leader, *Precise GNSS*
- **Andrea Bellés** – *Precise GNSS & Machine Learning*
- **Hakan Uyanik** – *GNSS Jamming Detection*
- **Iulian Filip** – *LiDAR and Visual SLAM*
- **Alonso Llorente** – *SLAM and DigitalTwin simulations*
- **Filippo Rizzi** – *Multi-Sensor Architectures for PNT*

Further collaborators

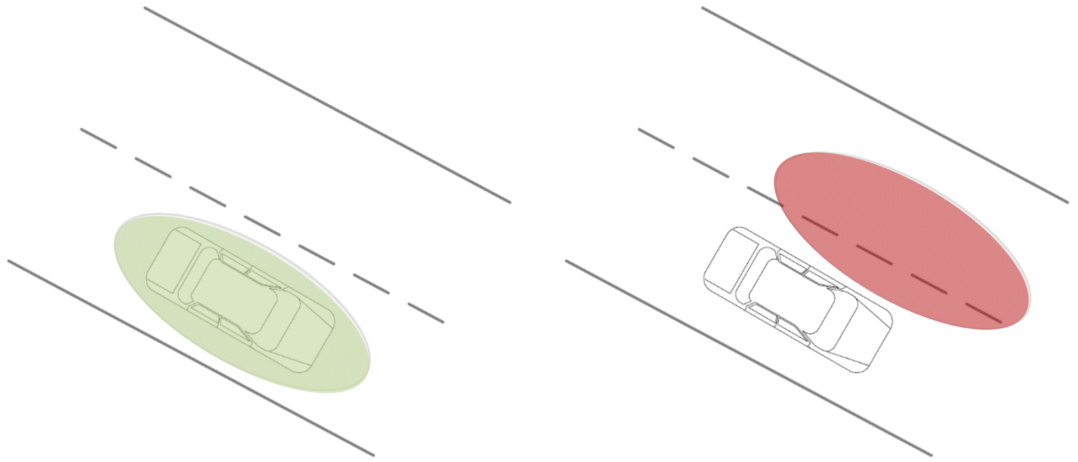
- **My PhD advisor: Jordi Vilà-Valls**
- **My dear French collaborators**: Éric Chaumette, Lorenzo Ortega, Paul Chauchat, Samy Labsir, François Vincent
- **Northeastern University:** Pau Closas, Haoqing Li, Helena Calatrava
- **Spanish colleagues:** Juan Manuel Gandarias, Jorge García
- **Former colleagues:** Anja, Xiangdong, JuanMar, Lukas, Iván, etc.

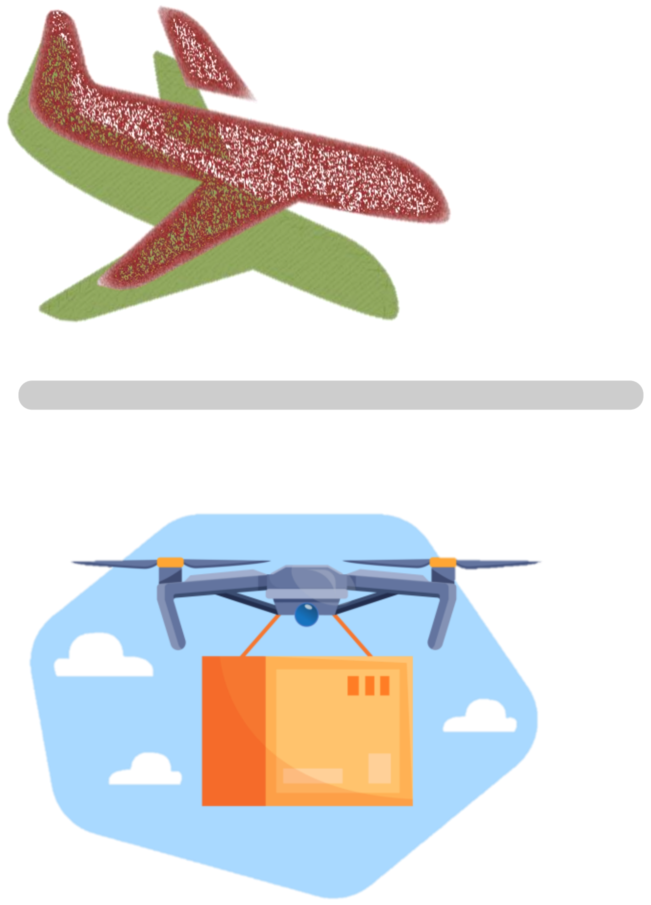
Thank you very much TéSA, Lorenzo and Julien for bringing me here!

State Estimation



Need for Robustness





- Our models & data are far from perfect!
- Imperfect sensor calibration, contaminated measurements, intentional interferences, …
- “conventional” estimators are designed for nominal conditions

In today's seminar…

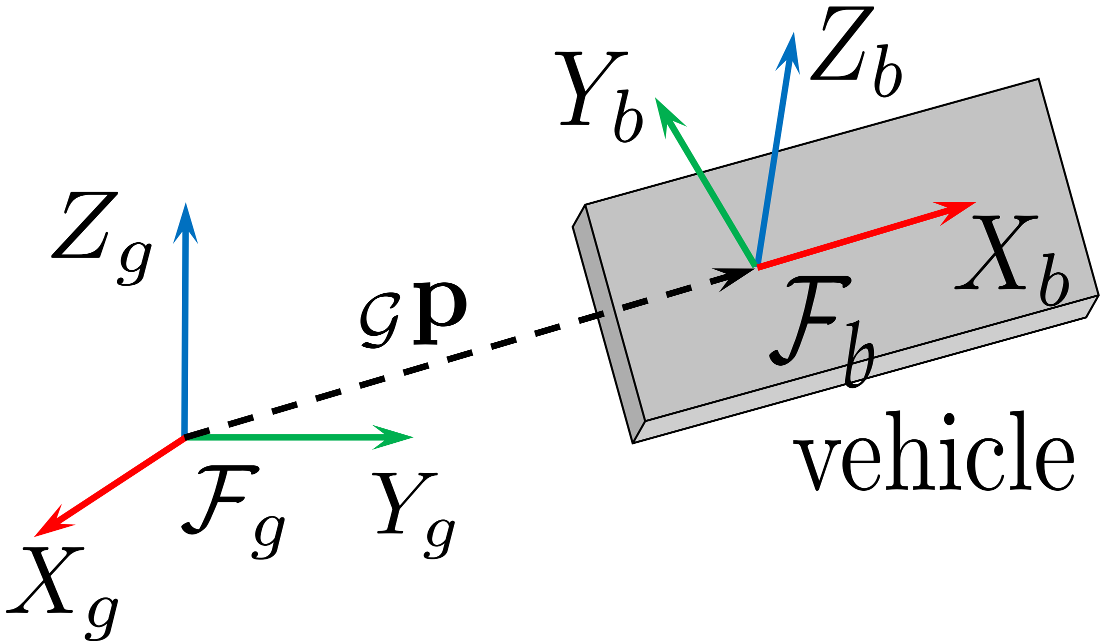
- **Basics of state estimation**: factor form, filtering, smoothing, batch estimation
- **The problem of model mismatch**: outliers, wrong noise distribution, incorrect model parameters, etc.
- **Robust Estimation and Filtering**: going beyond the Gaussian assumption?
- **Robust Estimation for GNSS:** better estimates for a better world
- **Research & industry perspectives**

Outline

1	Basics of State Estimation
2	Estimation under model mismatch
3	Robust Estimation and Filtering
4	Research & industry perspectives

State Estimation

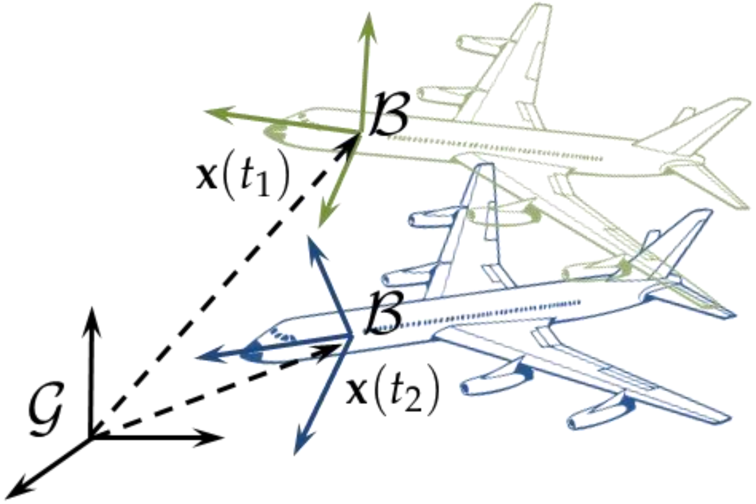
Describing a vehicle's dynamics



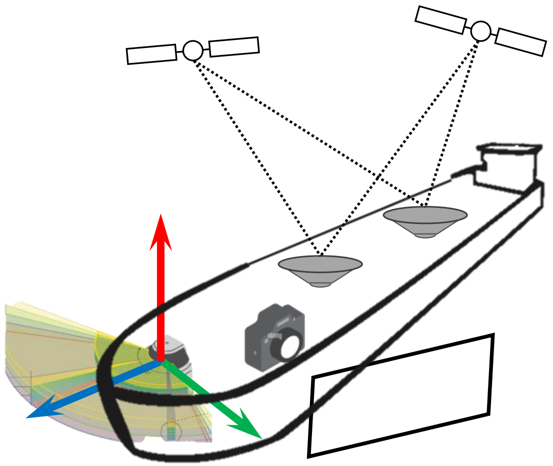
The principles

- The ***state*** of a robot / vehicle / target is a set of quantities (position, speed, orientation) that, if known, fully describe that robot's motion over time

$$\mathbf{x} = \begin{bmatrix} \mathbf{p} \\ \mathbf{v} \\ \mathbf{q} \end{bmatrix} \in \mathbb{R}^3 \times \mathbb{R}^3 \times SO(3)$$



- **Interoceptive perception** – sensors that *feel* your own movement: accelerometers, gyroscopes, wheel encoders
- **Exteroceptive perception** – sensors activated from an external stimuli: GNSS, camera, LiDAR, RADAR, etc.



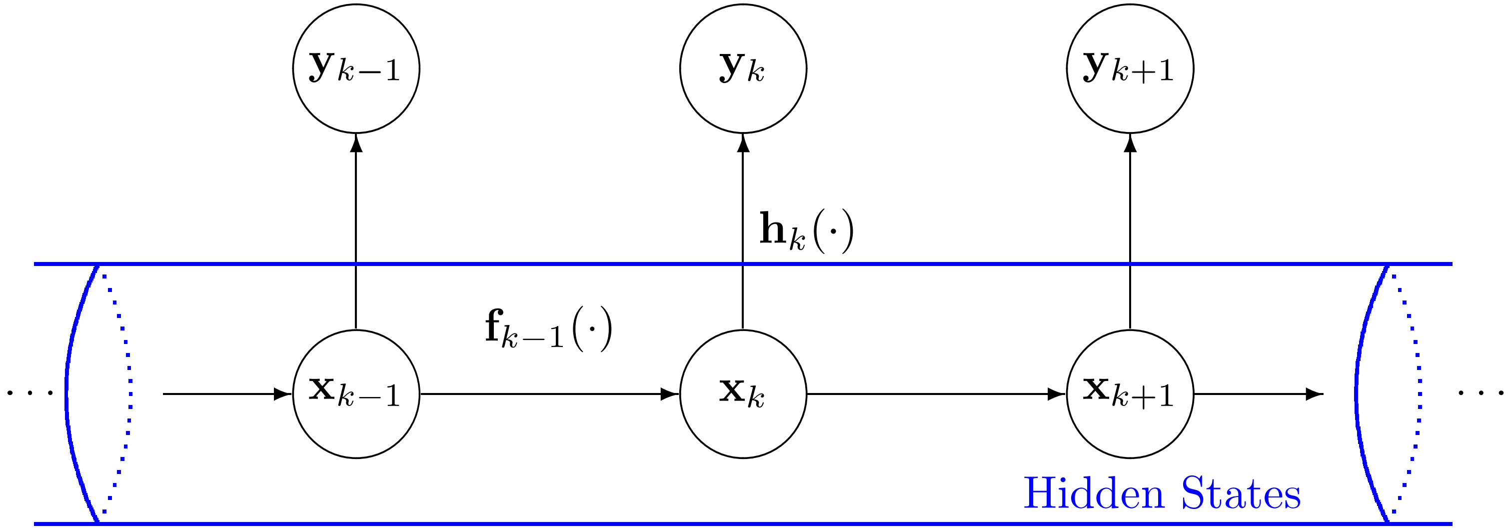
State Space Modeling

- The **discrete state-space** model can be expressed as

$$\begin{cases} \mathbf{x}_k = \mathbf{f}_{k-1}(\mathbf{x}_{k-1}) + \mathbf{v}_k \Rightarrow \text{dynamic model} \\ \mathbf{y}_k = \mathbf{h}_k(\mathbf{x}_k) + \mathbf{n}_k \Rightarrow \text{measurement model} \end{cases}$$

- equivalently, can be seen as a Hidden Markov Model (HMM) where states follow a first-order Markov chain

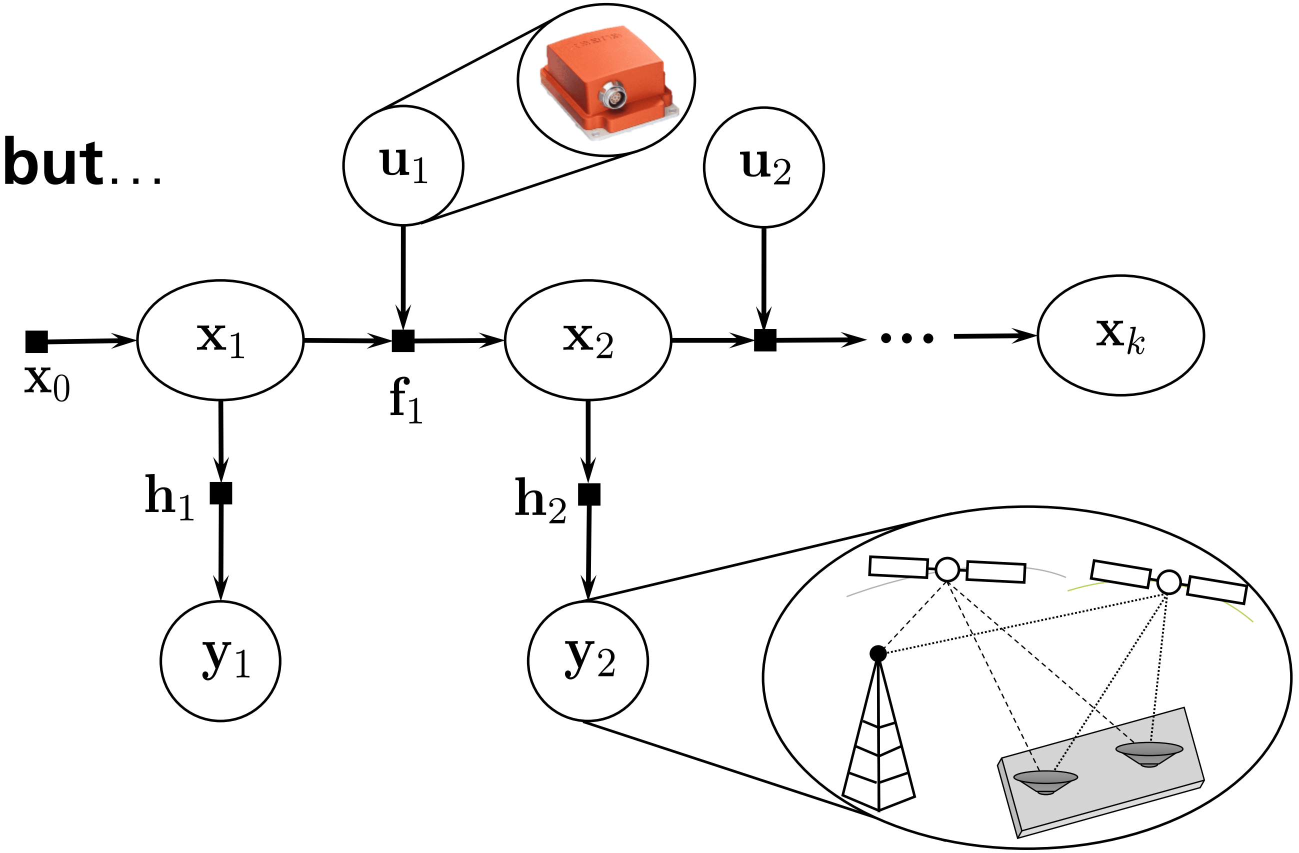
- **Filtering**
- **Smoothing**
- **Batch estimation**



- Often, dynamical models have inputs (i.e., **I**nertial **M**easurement **U**nit)

$$\begin{cases} \mathbf{x}_k = \mathbf{f}_{k-1}(\mathbf{x}_{k-1}, \mathbf{u}_k, \mathbf{v}_k) \\ \mathbf{y}_k = \mathbf{h}_k(\mathbf{x}_k) + \mathbf{n}_k \end{cases}$$

- IMU-aided solutions are well-known **but**…
  - Careful with noise modeling!
  - Influence of gravity (bad orientation)



Kalman Filtering
Inertial Aiding & Lie Groups

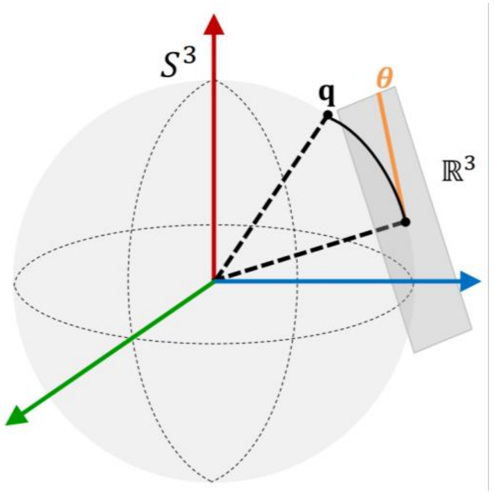
- The KF update step optimization reads:

$$\hat{\mathbf{x}}_k = \arg\min_{\mathbf{x}_k \in \mathcal{M}} \left(\left\| \mathbf{x}_k \ominus \hat{\mathbf{x}}_{k|k-1} \right\|^2_{\mathbf{P}_{k|k-1}} + \left\| \mathbf{y}_k - \mathbf{h}\left(\mathbf{x}_k\right) \right\|^2_{\boldsymbol{\Sigma}_k} \right)$$

$\mathcal{M}$ a generic manifold, e.g., $\mathcal{S}^3 \times \mathbb{R}^p$

- Thanks to **Lie Theory** → the **composition operator combine Manifold & Euclidean spaces**

Composition operation $\mathbf{x} = \widehat{\mathbf{x}} \oplus \delta\mathbf{x}$



Euclidean – Algebra – Manifold relationship

$$\boldsymbol{\theta} \in \mathbb{R}^3 \underset{(\cdot)^\wedge}{\mapsto} \mathbf{u}\theta \in \mathfrak{so}(3) \underset{\exp(\cdot)}{\mapsto} \mathbf{q} \in \mathcal{S}^3$$

$$(\boldsymbol{\theta})^\wedge \triangleq \begin{cases} \mathbf{u} = \boldsymbol{\theta}/\|\boldsymbol{\theta}\|^2 \\ \theta = \|\boldsymbol{\theta}\| \end{cases}, \quad \exp(\mathbf{e}\theta) \triangleq \begin{bmatrix} \cos(\theta/2) \\ \mathbf{e}\sin(\theta/2) \end{bmatrix}$$

Isomorphism + exponential mapping

Formalization on model mismatch

- **Observation/Prediction model mismatch**

$$\begin{cases} \mathbf{x}_k = \mathbf{f}_{k-1}(\mathbf{x}_{k-1}, \boldsymbol{\kappa}) + \mathbf{v}_k, & \mathbf{v}_k \sim \mathcal{N}(0, \mathbf{Q}_k) \\ \mathbf{y}_k = \mathbf{h}_k(\mathbf{x}_k, \boldsymbol{\theta}) + \mathbf{n}_k, & \mathbf{n}_k \sim \mathcal{N}(0, \boldsymbol{\Sigma}_k) \end{cases}$$

$$\begin{cases} \mathbf{x}_k = \mathbf{f}_{k-1}(\mathbf{x}_{k-1}, \tilde{\boldsymbol{\kappa}}) + \mathbf{v}_k, & \mathbf{v}_k \sim \mathcal{N}(0, \mathbf{Q}_k) \\ \mathbf{y}_k = \mathbf{h}_k(\mathbf{x}_k, \tilde{\boldsymbol{\theta}}) + \mathbf{n}_k, & \mathbf{n}_k \sim \mathcal{N}(0, \boldsymbol{\Sigma}_k) \end{cases}$$

- **Mismatch on stochastic modeling**

$$\begin{cases} \mathbf{x}_k = \mathbf{f}_{k-1}(\mathbf{x}_{k-1}) + \mathbf{v}_k, & \mathbf{v}_k \sim \mathcal{N}(0, \mathbf{Q}_k) \\ \mathbf{y}_k = \mathbf{h}_k(\mathbf{x}_k) + \mathbf{n}_k, & \mathbf{n}_k \sim \mathcal{N}(0, \boldsymbol{\Sigma}_k) \end{cases}$$

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Next…

- Protection against mis-specified system parameters
  - ➢ Linearly Constrained (Kalman) Filtering

- Protection against outliers, out-of-distribution and unknown noises
  - ➢ Robust Statistical-based Filtering

Outline

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Model Mismatch

- Consider a discrete state-space model, with dynamical and observation functions:

True:

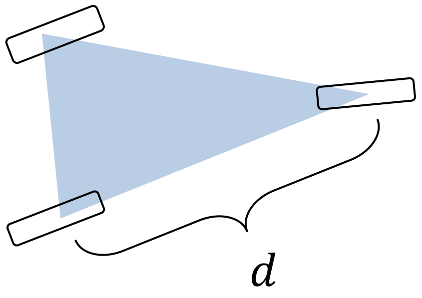
$$\mathbf{x}_k = \mathbf{f}\left(\mathbf{x}_{k-1}, \boldsymbol{\omega}_{k-1}\right) + \mathbf{v}_k\,,$$
$$\mathbf{y}_k = \mathbf{h}\left(\mathbf{x}_k, \boldsymbol{\theta}_k\right) + \mathbf{w}_k\,,$$

Mismatched:

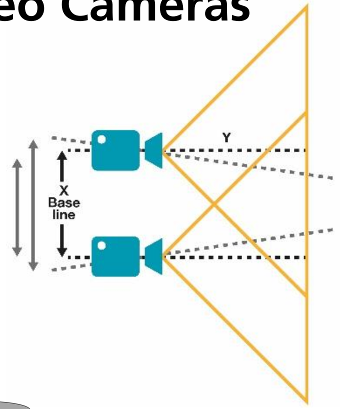
$$\hat{\mathbf{x}}_k = \mathbf{f}\left(\mathbf{x}_{k-1}, \hat{\boldsymbol{\omega}}_{k-1}\right) + \mathbf{v}_k\,,$$
$$\mathbf{y}_k = \mathbf{h}\left(\hat{\mathbf{x}}_k, \hat{\boldsymbol{\theta}}_k\right) + \mathbf{w}_k\,,$$

With $\boldsymbol{\omega}_{k-1}, \boldsymbol{\theta}_k$ certain parameters which we assumed as perfectly known. For example:

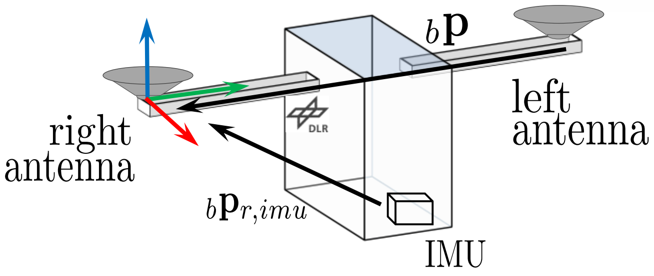
3-Wheeled Model



Stereo Cameras



✗ Poor estimation when these parameters are not perfectly known (calibration, body deformations, etc.)



Inter-antenna & IMU lever arms

Dealing with mis-specified System Parameters

One may think that there is a mismatch on a model parameter, such that

$$d\boldsymbol{\theta}_k = \boldsymbol{\theta}_k - \widehat{\boldsymbol{\theta}}_k$$

This is **what we believe** the parameter to be

This is the **true** parameter (**which we do not perfectly know**)

One could include $\boldsymbol{d\theta}_k$ as part of the state estimate ($\boldsymbol{x}_k$), although…

- Maintain a low state dimensionality
- Unavailability of prior distribution for $\widehat{\boldsymbol{\theta}_k}$
- No knowledge of the time evolution of the parameters

Linearly constrained Filtering is an attractive paradigm!

Vilà-Valls, Jordi, et al. "Recursive linearly constrained Wiener filter for robust multi-channel signal processing." *Signal Processing* 167 (2020): 107291.

Linearly Constrained Filtering

The impact of the mismatch over the error can be characterized (if $d\boldsymbol{\theta}_k$ is small) by:

$$\mathbf{y}_k \simeq \underbrace{\mathbf{h}(\mathbf{x}_k, \widehat{\boldsymbol{\theta}}_k)}_{\widehat{\mathbf{y}}_k} + \left.\frac{\partial \mathbf{h}}{\partial \boldsymbol{\theta}}\right|_{\mathbf{x}_k, \widehat{\boldsymbol{\theta}}_k} d\boldsymbol{\theta}_k$$

Contribution of a mis-specification on the observation model

Idea: deriving a new "Kalman" gain for a mismatch update step:

$$\mathbf{L}_k = \arg\min_{\mathbf{L}} \left\{ \mathbb{E}\left[\mathbf{e}_{k|k}\mathbf{e}_{k|k}^\top\right]\right\}, \qquad \text{s.t.} \quad \mathbf{L}\boldsymbol{\Delta}_k = \mathbf{0}, \; \boldsymbol{\Delta}_k = \left.\frac{\partial \mathbf{h}}{\partial \boldsymbol{\theta}}\right|_{\widehat{\mathbf{x}}_{k|k-1}, \widehat{\boldsymbol{\theta}}_k}$$

The correction step for the linearly constrained (LC) Kalman Filter:

$$\widehat{\mathbf{x}}_{k|k} = \widehat{\mathbf{x}}_{k|k-1} + \mathbf{L}_k(\mathbf{y}_k - \mathbf{h}(\widehat{\mathbf{x}}_{k|k-1}, \widehat{\theta}))$$

- ✓ Low computation overhead
- ✓ Easy to implement

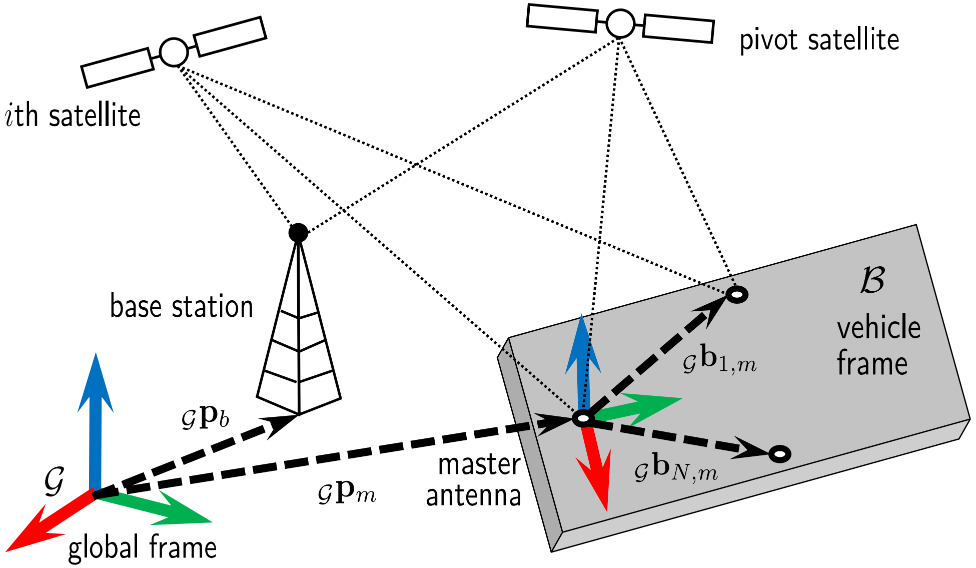
Linearly Constrained Filtering on Manifolds
Application to GNSS Joint Position and Attitude (JPA)

- Consider the **Joint Position and Attitude (JPA)** problem:

$$\mathbf{x}_k^\top = \left[\mathbf{p}_k^\top, \mathbf{v}_k^\top, \mathbf{q}_k^\top, \mathbf{a}_k^\top, \mathbf{b}_k^\top\right],$$
$$\text{with } (\mathbf{p}_k, \mathbf{v}_k, \mathbf{q}_k, \mathbf{a}_k, \mathbf{b}_k) \in \mathbb{R}^3 \times \mathbb{R}^3 \times \mathcal{S}^3 \times \mathbb{Z}^M \times \mathbb{R}^L$$

- The measurement model depends on the baseline vectors in the body frame

$$y_{i,j} = -\mathbf{u}_i^\top \mathbf{R}(\mathbf{q})_{\mathcal{B}}\mathbf{b}_j + \lambda \cdot a_{i,j} + \epsilon_{i,j}$$



The manifold-compatible correction form reads:

$$\widehat{\mathbf{x}}_{k|k} = \widehat{\mathbf{x}}_{k|k-1} \oplus \mathbf{K}_k\left(\mathbf{y}_k - \mathbf{h}\left(\widehat{\mathbf{x}}_{k|k-1}, \boldsymbol{\theta}\right)\right)$$

Linearly Constrained Filtering on Manifolds
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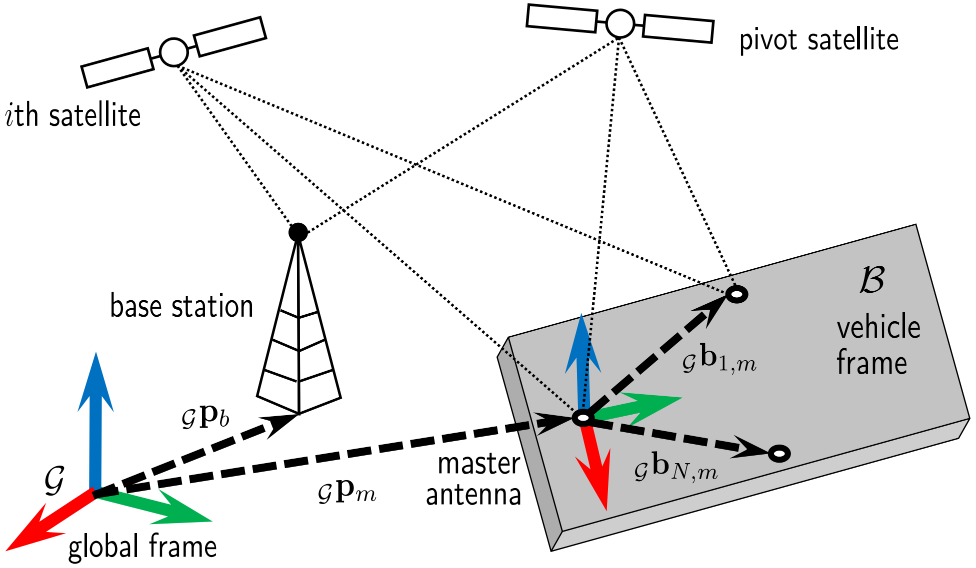
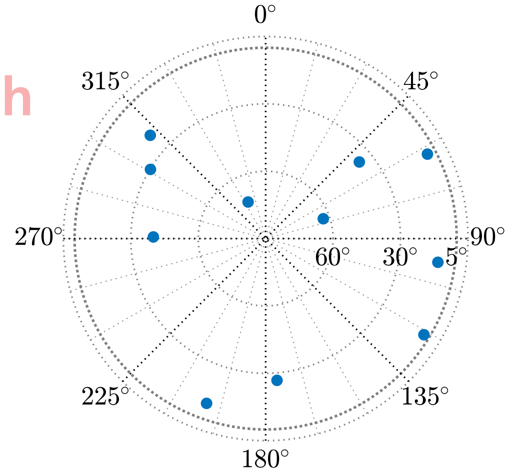


The Linearly-constrained Kalman Filter on Manifolds is now

$$\boxed{\widehat{\mathbf{x}}_{k|k} = \widehat{\mathbf{x}}_{k|k-1} \oplus \mathbf{L}_k \left(\mathbf{y}_k - \mathbf{h}\left(\widehat{\mathbf{x}}_{k|k-1}, \hat{\theta}\right)\right)}$$

Chauchat, P., Medina, D., Vilà-Valls, J., & Chaumette, E. (2021, November). Robust Linearly Constrained Filtering for GNSS Position and Attitude Estimation under Antenna Baseline Mismatch. In *2021 IEEE 24th International Conference on Information Fusion (FUSION)* (pp. 1-7). IEEE.

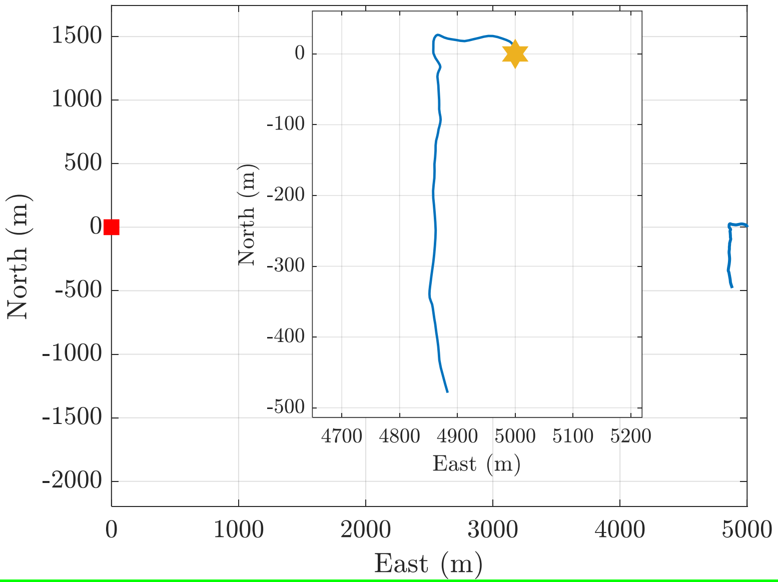
Linearly Constrained Filtering for JPA
Monte Carlo Characterization

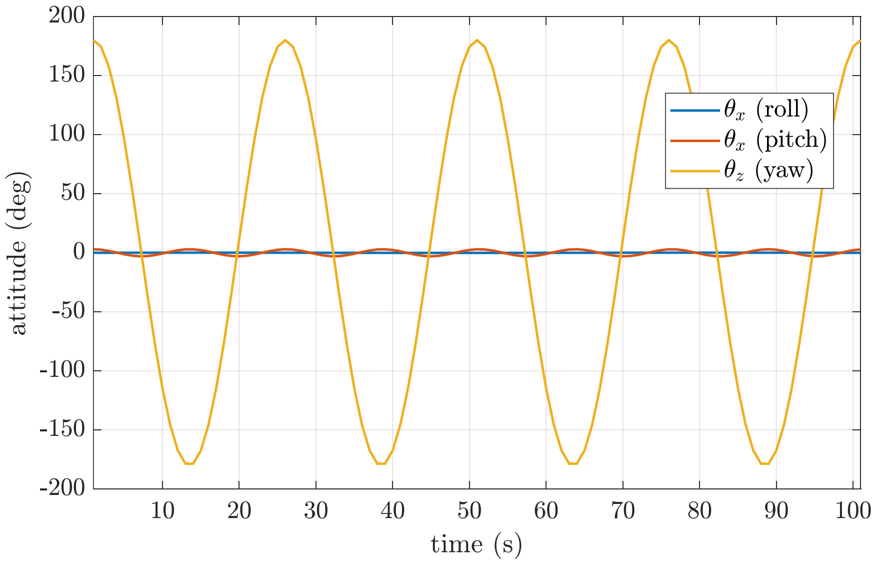


➢ Assessing the **sensitivity** of JPA estimation **under antenna baseline mismatch**

Experimental setup:

- A medium-sized vehicle (e.g., a vessel/airplane) with 4 antennas and inertial unit
- Antenna separation of 5 meters & **mismatch of 5 and 20%**





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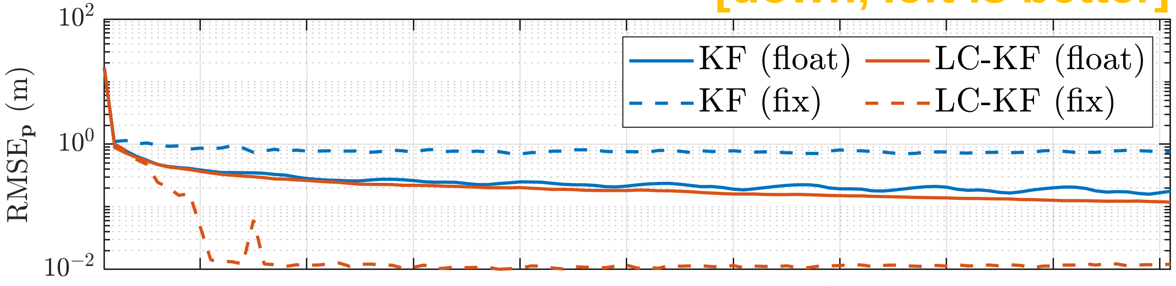
Linearly Constrained Filtering for Precise Navigation
Monte Carlo Characterization

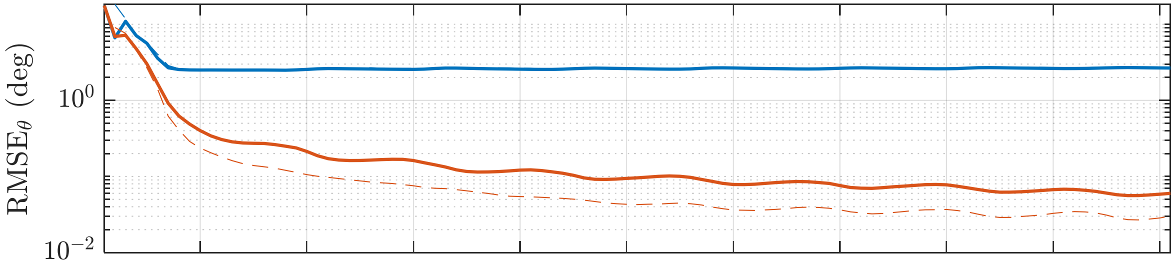
5% Mismatch – Positioning performance

[down, left is better]



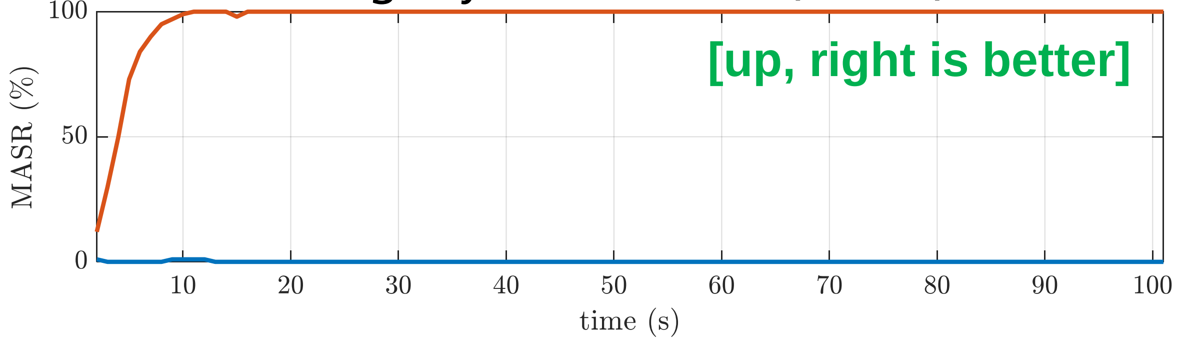
Attitude performance

[down, left is better]



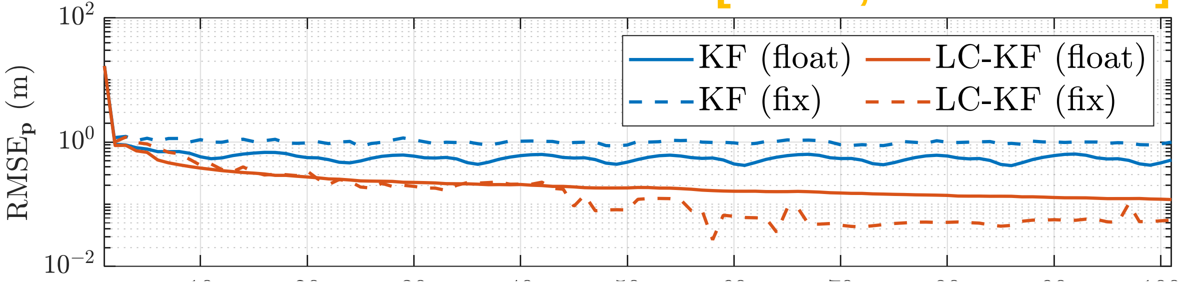
Mean Ambiguity Success Rate (MASR)

[up, right is better]



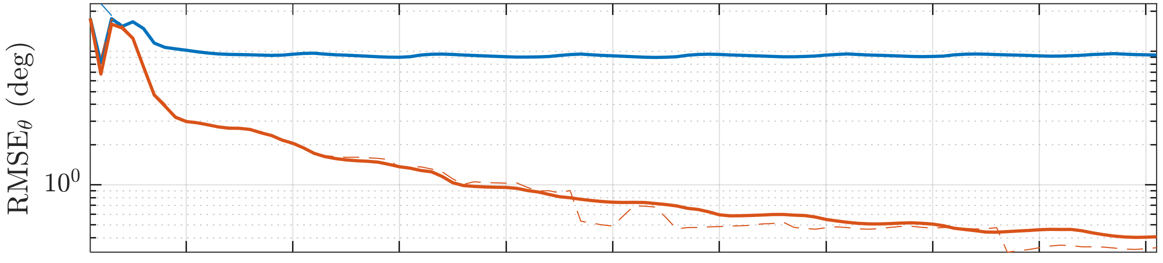
20% Mismatch – Positioning performance

[down, left is better]



Attitude performance

[down, left is better]



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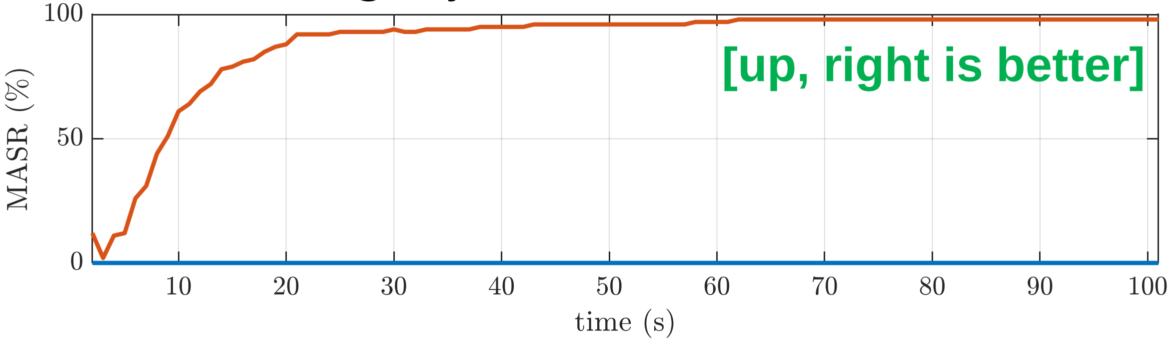

Linearly Constrained Filtering for Precise Navigation
Monte Carlo Characterization

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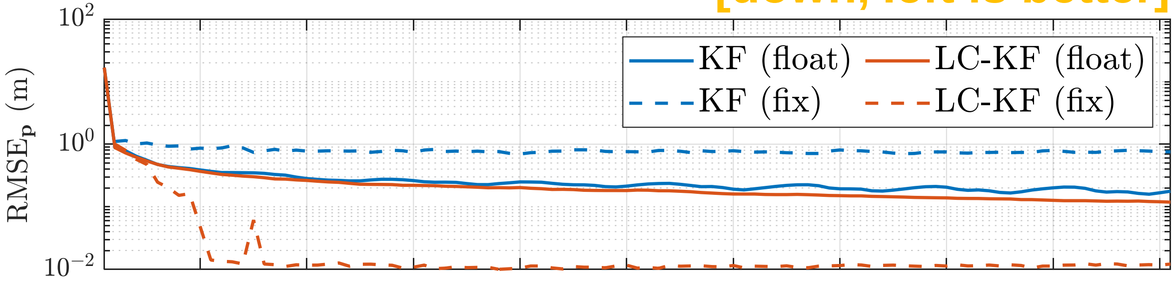

Attitude performance

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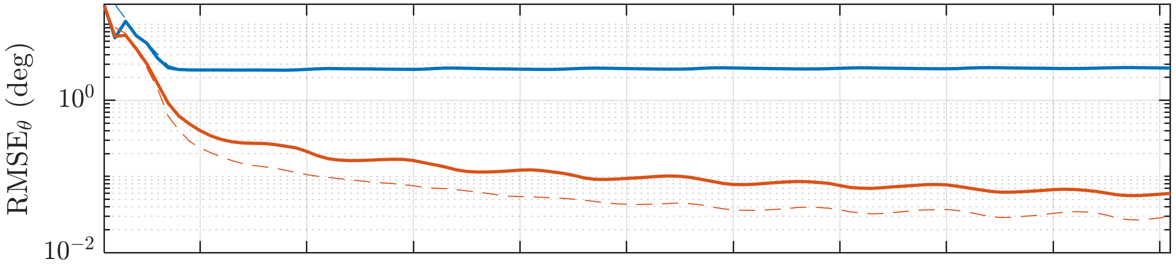

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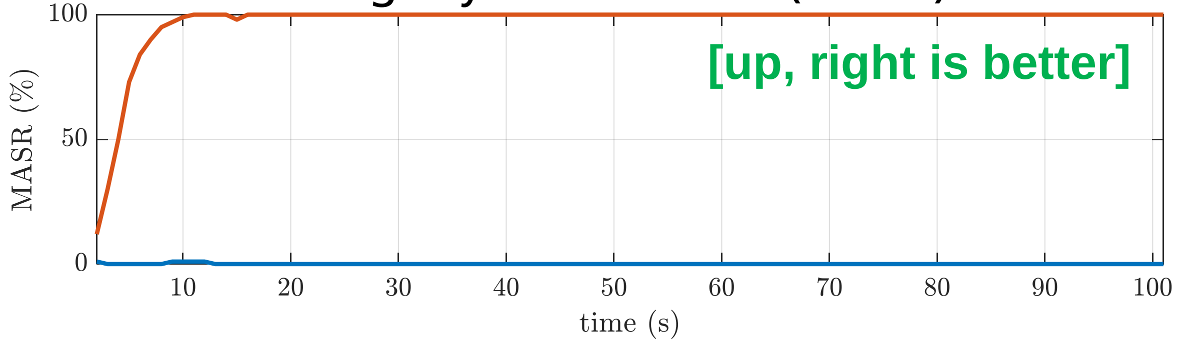

Wrapping it up…

- **Linearly-constrained filtering can be successfully applied to manifold spaces**
- The performance is conditional on the dimension of the mis-specification
- LC-KF JPA can **correctly fix** ambiguities with **20% baseline mismatch**

Chauchat, P., Medina, D., Vilà-Valls, J., & Chaumette, E. (2021, November). Robust Linearly Constrained Filtering for GNSS Position and Attitude Estimation under Antenna Baseline Mismatch. In *2021 IEEE 24th International Conference on Information Fusion (FUSION)* (pp. 1-7). IEEE.

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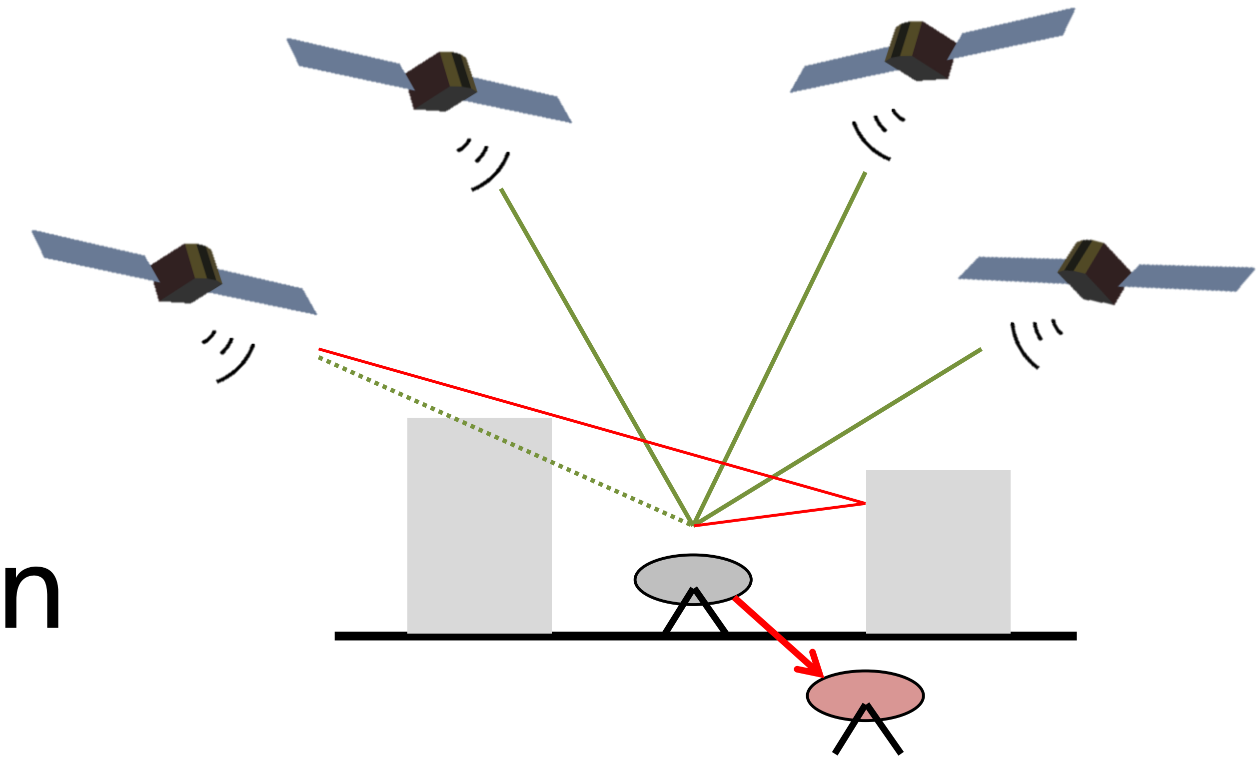
Outline

Robust Estimation

Beyond the Gaussian assumption



Motivation for Robustness

- **Classical Estimation Methods are designed under the Gaussian assumption**
  - Relatively easy to derive
  - Optimal when the assumption is hold *exactly*

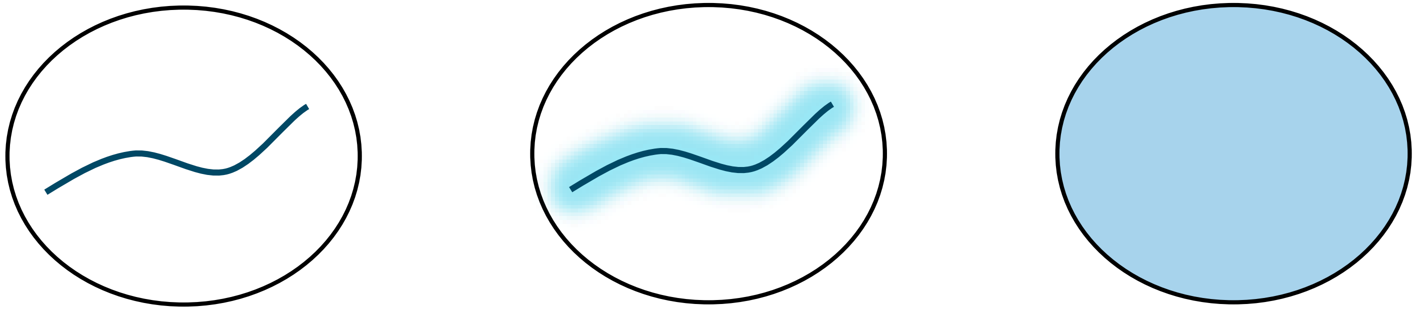
"Everyone believes it: the experimenters imagine that it is a mathematical theorem, and the mathematicians that it is an experimental fact."
Henry Poincaré (1854 – 1912)

But… what happens with the Gaussian assumption fails?

- Heavy-tailed noise have been shown in data collection across multiple fields
- The effect of a single *outlier* is unbounded → estimation can be completely spoiled!

Family of Robust Estimation

	Parametric	Robust	Nonparametric
Description	Model specified by a set of parameters	Parametric model allowing for deviations	Model specified in terms of general properties
Ideal performance	Excellent (optimal*)	Good	Mediocre
Range of validity	Small	Medium	Large



Muma, Michael E. "Robust estimation and model order selection for signal processing." (2014).

What is Robust Statistics?

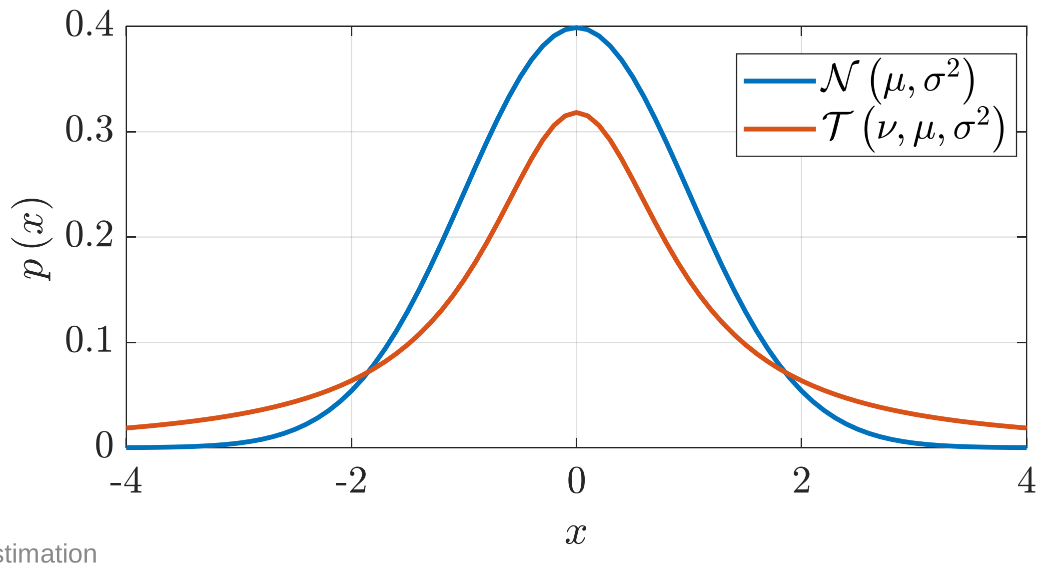
- **Robust Statistics** aim at deriving estimators which are:
  - *nearly* optimal when the Gaussian assumption holds
  - *nearly* optimal under heavy-tailed/contaminated distributions

- There are different ways to express noise distributions:

Ideal / exact distribution

$$\varepsilon \sim \mathcal{N}\left(\mu, \sigma^2\right)$$

Heavy-Tailed distribution



Normal Mixture distribution

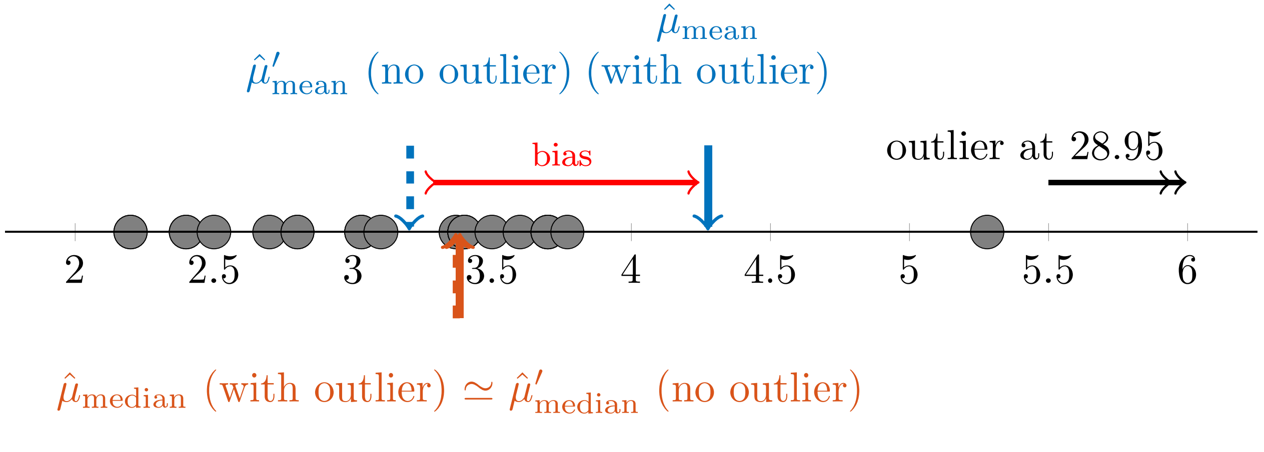
$$\varepsilon \sim (1-\epsilon)\, G + \epsilon\, H$$

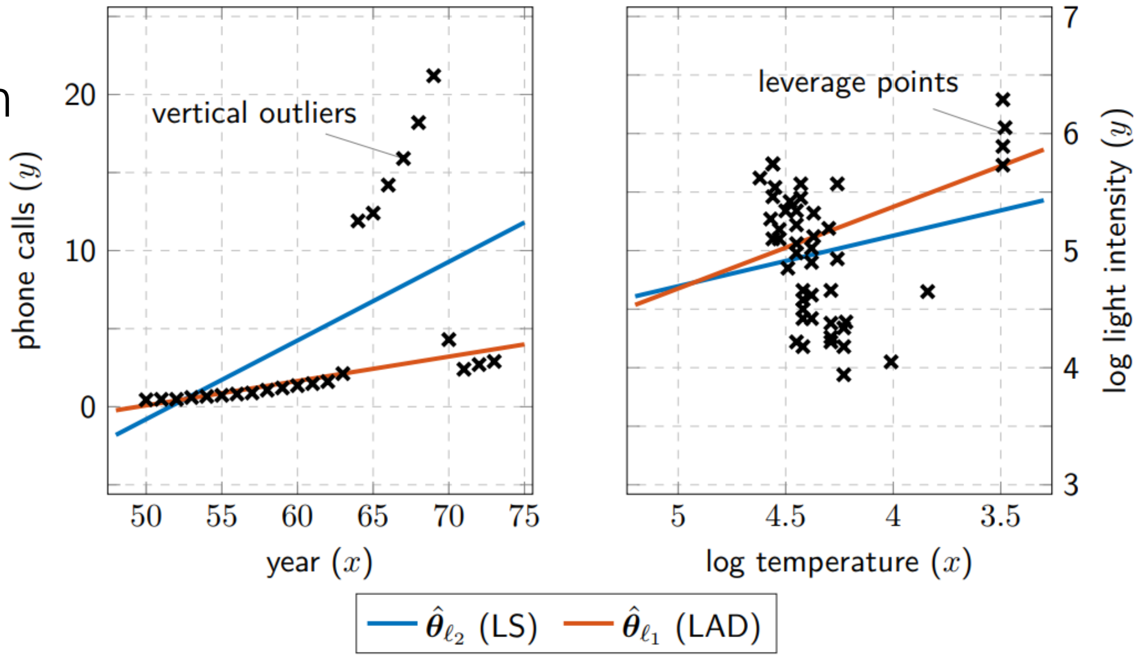
$$G = \mathcal{N}\left(\mu, \sigma^2\right)$$

$$H = ???$$

A brief history on Robust Estimation…

- Initial thoughts on the 60s by **Huber and Hampel**, looking for better estimates for linear regression
- Within the Signal Processing community: Zoubir & Muma
- Robust estimation is not necessarily the only solution:
  - Variational Inference
  - Estimators for heavy tailed distribution
  - Statistical tests for fault detection and exclusion
  - …





Robust Statistics Dictionary

- *Robustness* → Capability of an estimator that:
  i. Does not suffer a large impact under the presence of an erroneous observation, even if it takes an arbitrary value
  ii. Remains without catastrophic effects, even when larger deviations from the model occur

- *Breakdown point* → the smallest percentage of contamination that can cause an estimator to take on arbitrarily large aberrant values.

- *Inlier / Outlier* → healthy observations / observations that are well separated from the majority of the data

- *Relative Efficiency* → performance similarity of a method wrt. an optimal method (e.g., the LS) under nominal normal-distributed noise

Robust Estimators Working Principle

- Let us consider a linear regression problem:

$$y_i = x_i^\top \boldsymbol{\beta} + \varepsilon_i, \qquad \mathbf{r} = \mathbf{y} - \mathbf{x}^\top \boldsymbol{\beta}$$

Classical Least Squares

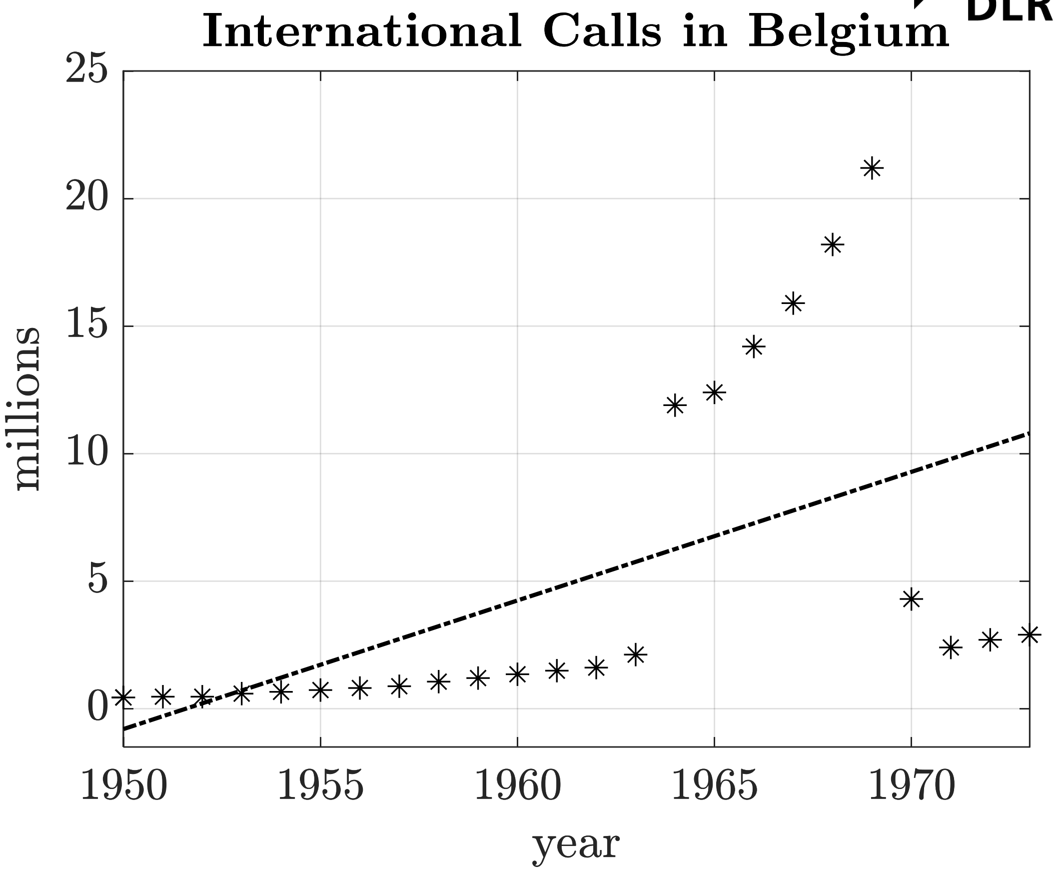
$$\hat{\boldsymbol{\beta}}_{LS} = \arg\min_{\boldsymbol{\beta}} \|\mathbf{y} - \mathbf{x}^\top \boldsymbol{\beta}\|^2 \Rightarrow \arg\min_{\boldsymbol{\beta}} \sum_{i=1}^{n} \left(r_i(\boldsymbol{\beta})\right)^2$$

Let's call this *loss function* $\rho_{LS}(\bullet) = r^2$

Least Absolute Deviation

$$\hat{\boldsymbol{\beta}}_{\ell_1} = \arg\min_{\boldsymbol{\beta}} \sum_{i=1}^{n} \left| \frac{r_i(\boldsymbol{\beta})}{\sigma_i} \right|$$

$$\rho_{\ell_1}(\bullet) = |r|$$



[Rousseeuw84]

Robust Estimators Working Principle

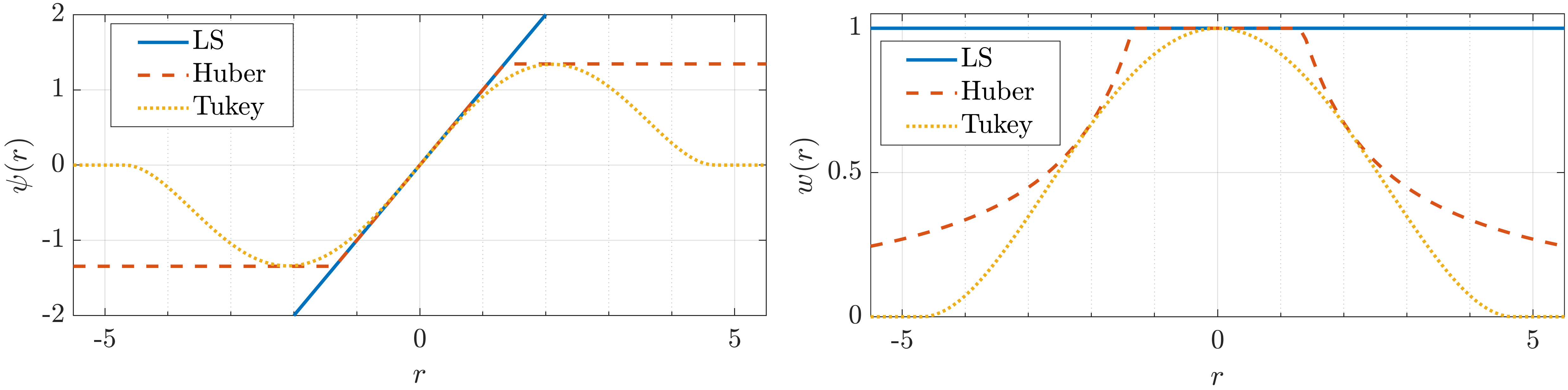
M estimation

- ***Huber*** *and* ***Hampel*** proposed replacing the original loss functions for other that bound the influence of contaminated observations

$$\hat{\boldsymbol{\beta}}_M = \arg\min_{\boldsymbol{\beta}} \sum_{i=1}^{N} \rho\left(r_i(\boldsymbol{\beta})\right)$$

$$\textit{influence function} \rightarrow \psi\left(x\right) = \frac{\partial \rho(x)}{\partial x}$$

$$\textit{weighting function} \rightarrow w\left(x\right) = \psi(x)/x$$



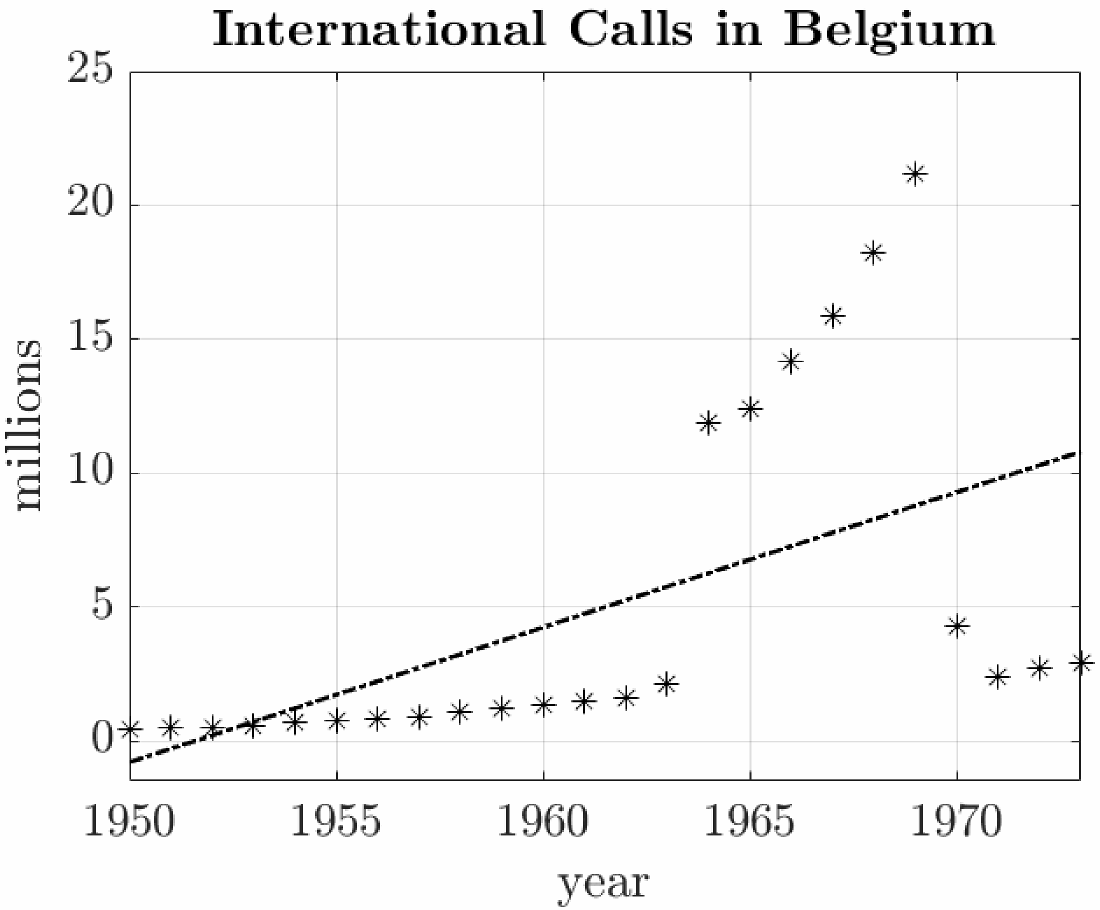
Robust Estimators Working Principle

M estimation

$$\hat{\boldsymbol{\beta}}_M = \arg\min_{\boldsymbol{\beta}} \sum_{i=1}^{N} \rho\left(r_i(\boldsymbol{\beta})\right)$$

$$\sum_{i=1}^{n} \psi\left(r_i\left(\boldsymbol{\beta}\right)\right) \frac{\partial r_i\left(\boldsymbol{\beta}\right)}{\partial \boldsymbol{\beta}} = 0$$

$$\sum_{i=1}^{n} w\left(r_i\left(\boldsymbol{\beta}\right)\right) r_i\left(\boldsymbol{\beta}\right) \frac{\partial r_i\left(\boldsymbol{\beta}\right)}{\partial \boldsymbol{\beta}} = 0$$



This is exactly a weighted least squares!!
The robust estimation turns the problem →
Iteratively **R**eweighted **L**east **S**quares (**IRLS**)

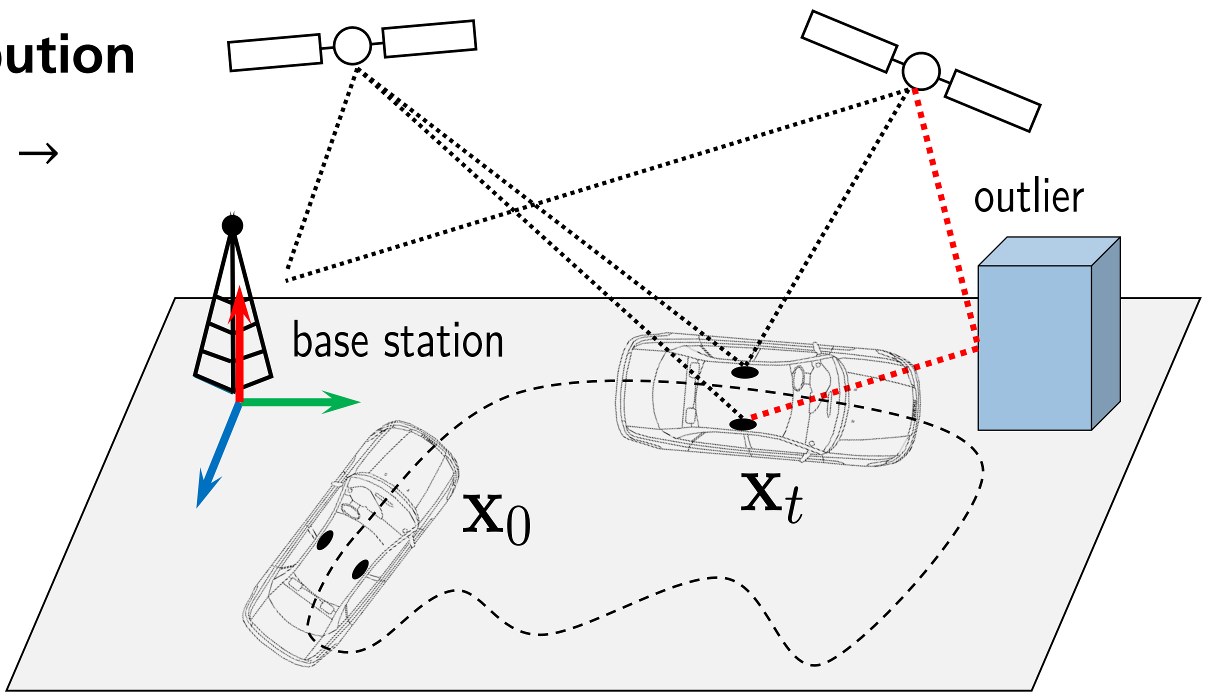
Robust Filtering

From a Kalman Filter (KF) point of view…

$$\hat{\mathbf{x}}_t = \arg\min_{\mathbf{x}_t} \left(\left\| \mathbf{x}_t - \mathbf{x}_{t|t-1} \right\|^2_{\mathbf{P}_{t|t-1}} + \left\| \mathbf{y}_t - \mathbf{H}_t \mathbf{x}_t \right\|^2_{\boldsymbol{\Sigma}_t} \right)$$

KF optimal for Gaussian noises of known distribution

- ✗ Mismatch between assumed and true distribution → **unbounded errors!**



How to… Robust Filtering
Robust Statistics-based Solutions

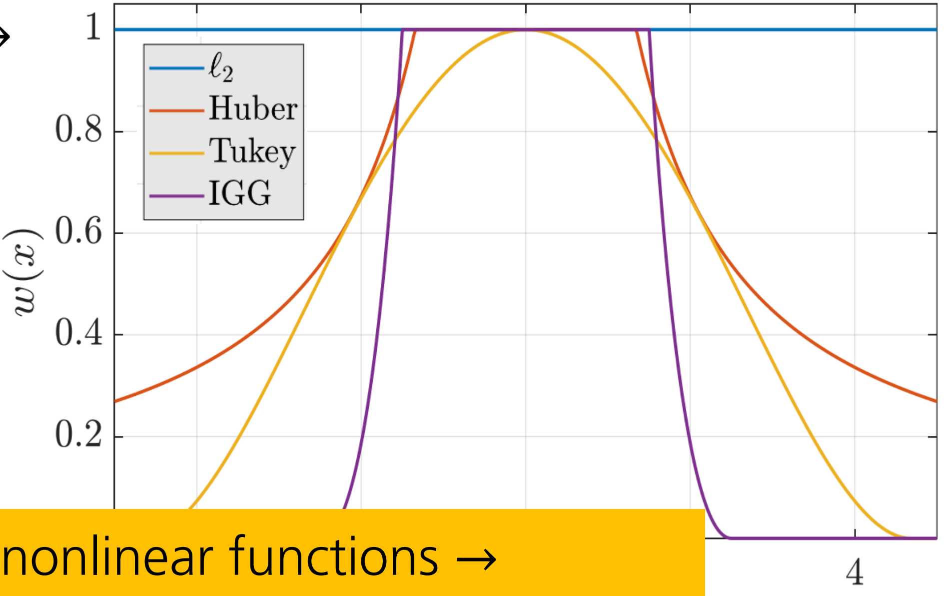
Unlike KF, the **assumption** on our observations' distribution is **flexible**

$$\hat{\mathbf{x}}_t = \arg\min_{\mathbf{x}_t} \left(\left\| \mathbf{x}_t - \mathbf{x}_{t|t-1} \right\|^2_{\mathbf{P}_{t|t-1}} + \left\| \mathbf{y}_t - \mathbf{h}\left(\mathbf{x}_t\right) \right\|^2_{\bar{\boldsymbol{\Sigma}}_t} \right)$$

Iteratively, the observations become "**re-weighted**" → how good they fit to the underlying model

$$\bar{\boldsymbol{\Sigma}}_t = \boldsymbol{\Sigma}_t^{1/2} \mathbf{W}_{\mathbf{y}}^{-1} \boldsymbol{\Sigma}_t^{\top/2},$$

$$\mathbf{W}_{\mathbf{y}} = \mathrm{diag}\left[\mathbf{w}\left(\boldsymbol{\Sigma}_t^{-1/2} \left(\mathbf{y}_t - \mathbf{h}(\mathbf{x}_t) \right) \right) \right]$$



The devil is in the details

The implementation of robust filtering + nonlinear functions → cascaded iterative procedures and KF re-formulations

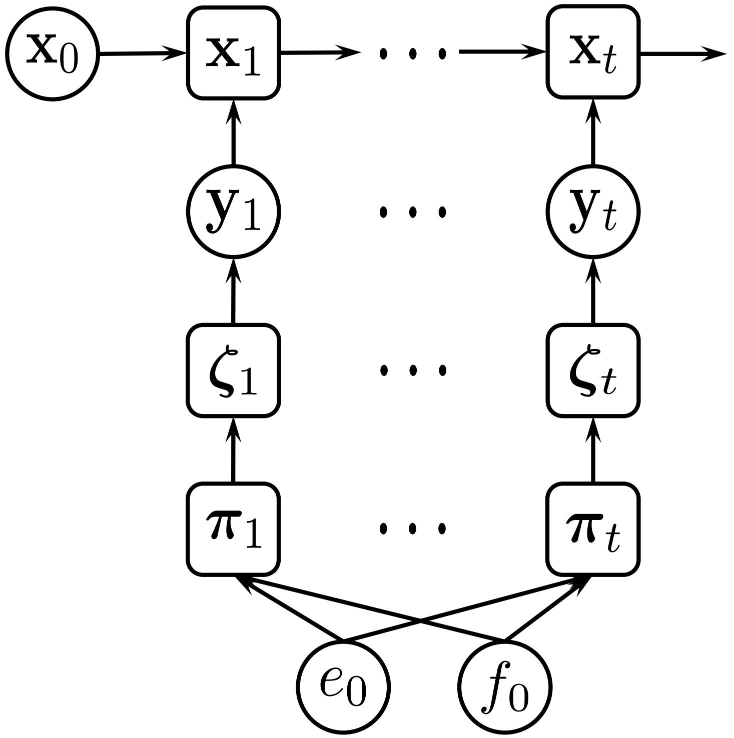
How to… Robust Filtering
Robust Variational-based Filtering

Using **Variational Inference** → **detect** presence of **outliers**

$$\mathbf{x}_t = \mathbf{f}\left(\mathbf{x}_{t-1}\right) + \boldsymbol{\epsilon}_t$$

$$\mathbf{y}_t = \begin{cases} \mathbf{h}\left(\mathbf{x}_t\right) + \boldsymbol{\eta}_t, & \text{under } \mathcal{M}_0 \\ \mathbf{h}\left(\mathbf{x}_t\right) + \boldsymbol{\eta}_t + \boldsymbol{o}_t, & \text{under } \mathcal{M} \end{cases}$$

Graphical model for Variational Filtering



Scalar Variational KF (S-VKF)

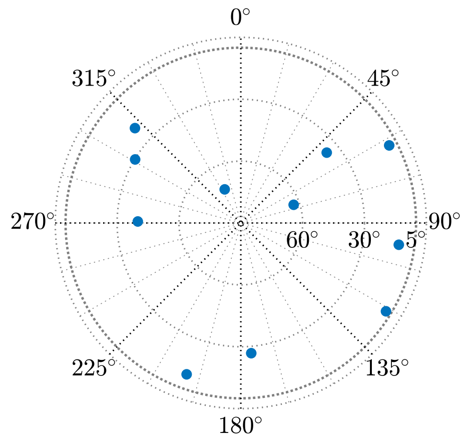
$$p(\mathbf{y}_t|\mathbf{x}_t, \zeta_t) = \mathcal{N}(\mathbf{y}_t; \mathbf{h}(\mathbf{x}_t), \boldsymbol{\Sigma}_t)^{\zeta_t}$$

Independent Variational KF (I-VKF)

$$p(\mathbf{y}_t|\mathbf{x}_t, \boldsymbol{\zeta}_t) = \prod_{i=1}^{n_y} \mathcal{N}(y_t^{(i)}; \mathbf{h}^{(i)}(\mathbf{x}_t), [\boldsymbol{\Sigma}_t]_{ii})^{\zeta_t^{(i)}}$$

Li, H., Medina, D., Vilà-Valls, J., & Closas, P. (2020). Robust variational-based Kalman filter for outlier rejection with correlated measurements. *IEEE Transactions on Signal Processing*, *69*, 357-369.

A Showcase: Robust Filtering for RTK

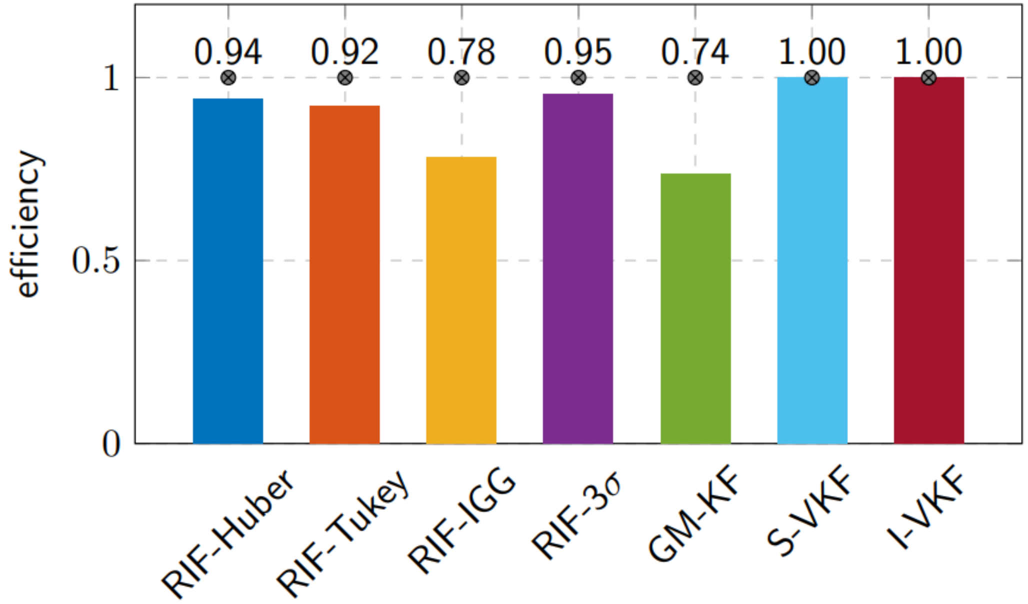


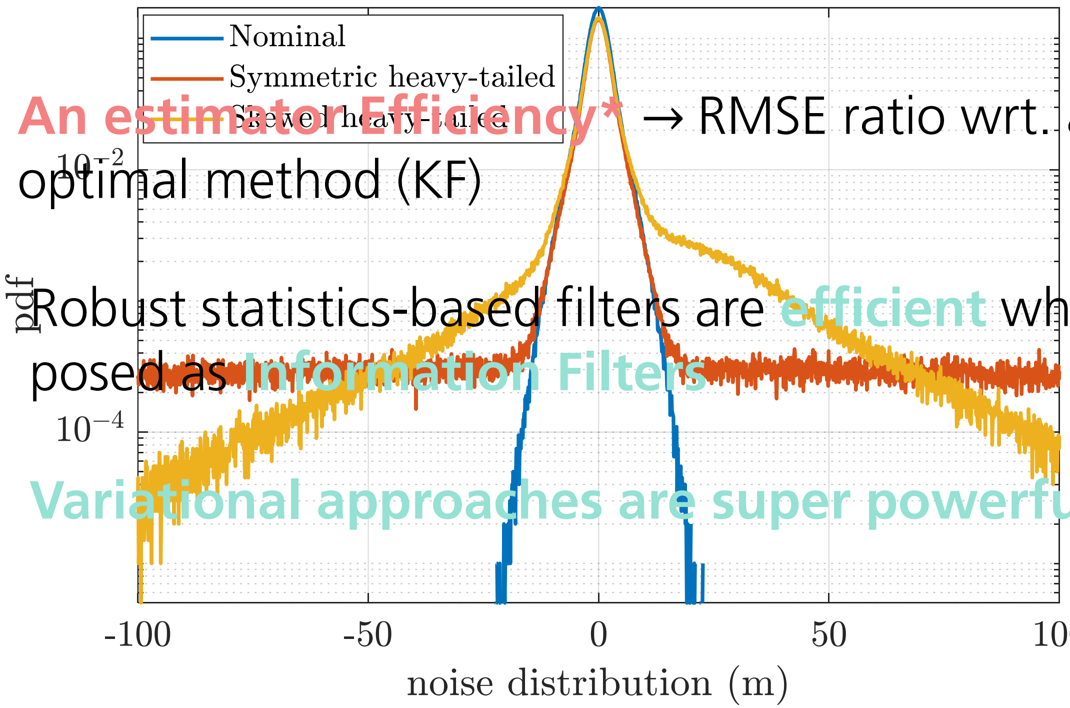
Addressing the performance of estimators under:

I. Nominal (Gaussian) noise → test efficiency

II. Symmetric heavy-tailed

III. Skewed heavy-tailed



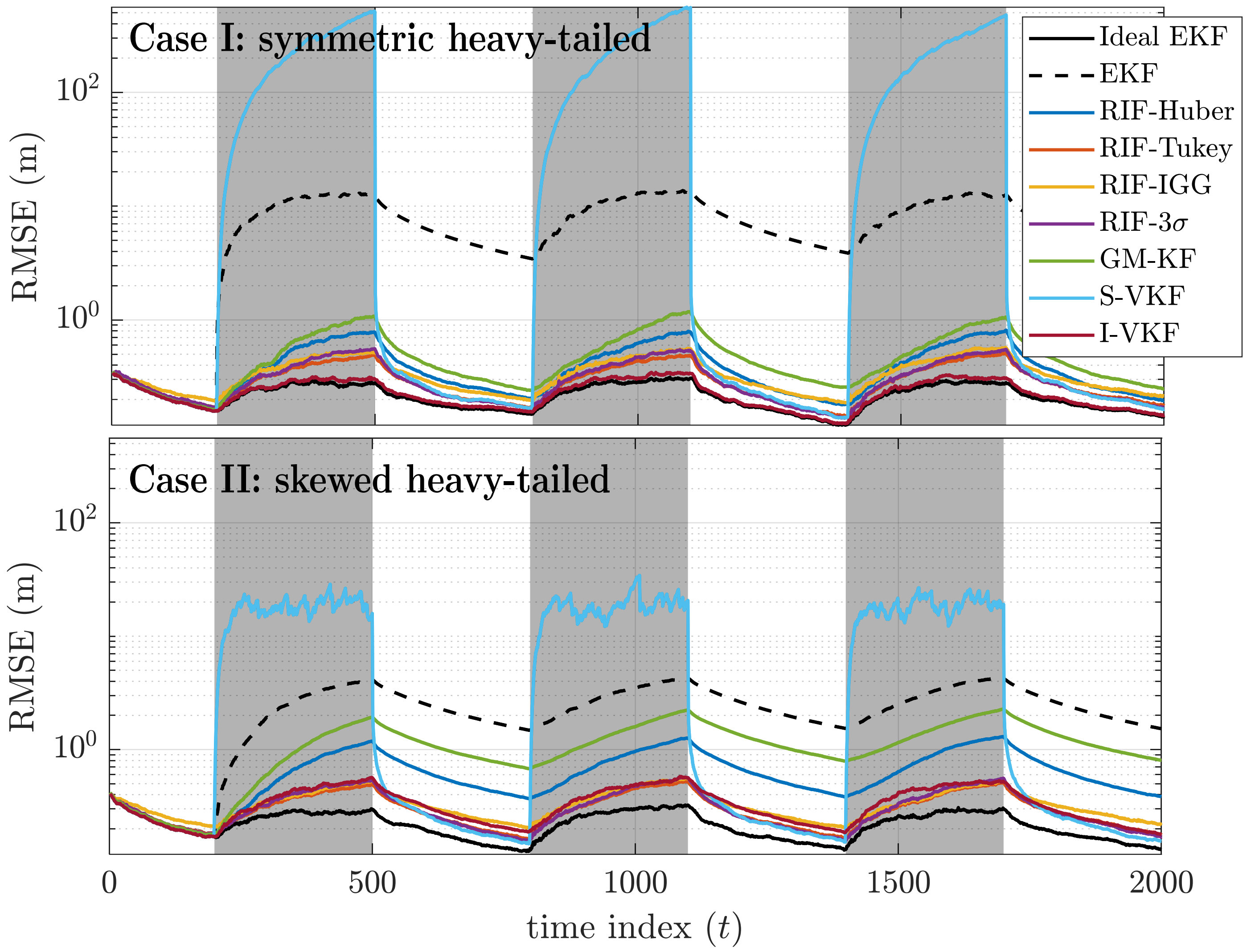


- An estimator Efficiency* → RMSE ratio wrt. an optimal method (KF)
- Robust statistics-based filters are efficient when posed as Information Filters
- Variational approaches are super powerful!

Li, H., Medina, D., Vilà-Valls, J., & Closas, P. (2020). Robust variational-based Kalman filter for outlier rejection with correlated measurements. *IEEE Transactions on Signal Processing*, *69*, 357-369.

Medina, D., Li, H., Vilà-Valls, J., & Closas, P. (2021). Robust filtering techniques for RTK positioning in harsh propagation environments. *Sensors*, *21*(4), 1250.

Robust Filtering for RTK



- The performance of robust statistics-based filters → solution form
- GM-KF "underperforms" but resilient against dynamical mismatches
- Variational filters promising but computationally demanding

Li, H., Medina, D., Vilà-Valls, J., & Closas, P. (2020). Robust variational-based Kalman filter for outlier rejection with correlated measurements. *IEEE Transactions on Signal Processing*, *69*, 357-369.

Medina, D., Li, H., Vilà-Valls, J., & Closas, P. (2021). Robust filtering techniques for RTK positioning in harsh propagation environments. *Sensors*, *21*(4), 1250.

Robust Estimation for Integrity Monitoring ?
Basics on Integrity Monitoring

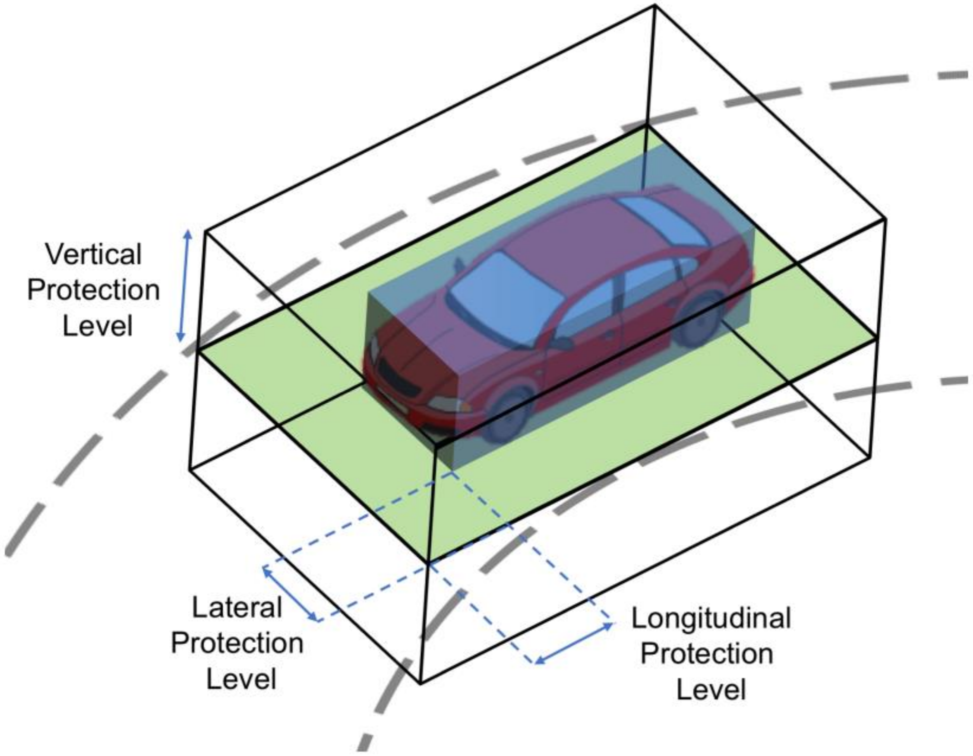
Integrity monitoring measures the trust on the navigation estimates & provides timely warnings when an unacceptable fault occurs / system is unreliable

Navigational requirements

- Accuracy
- Continuity
- Availability

Integrity components

- Alert Limit
- Integrity Risk
- Time to Alert
- Protection Level



Reid, Tyler GR, et al. "Localization requirements for autonomous vehicles." *arXiv preprint arXiv:1906.01061* (2019).

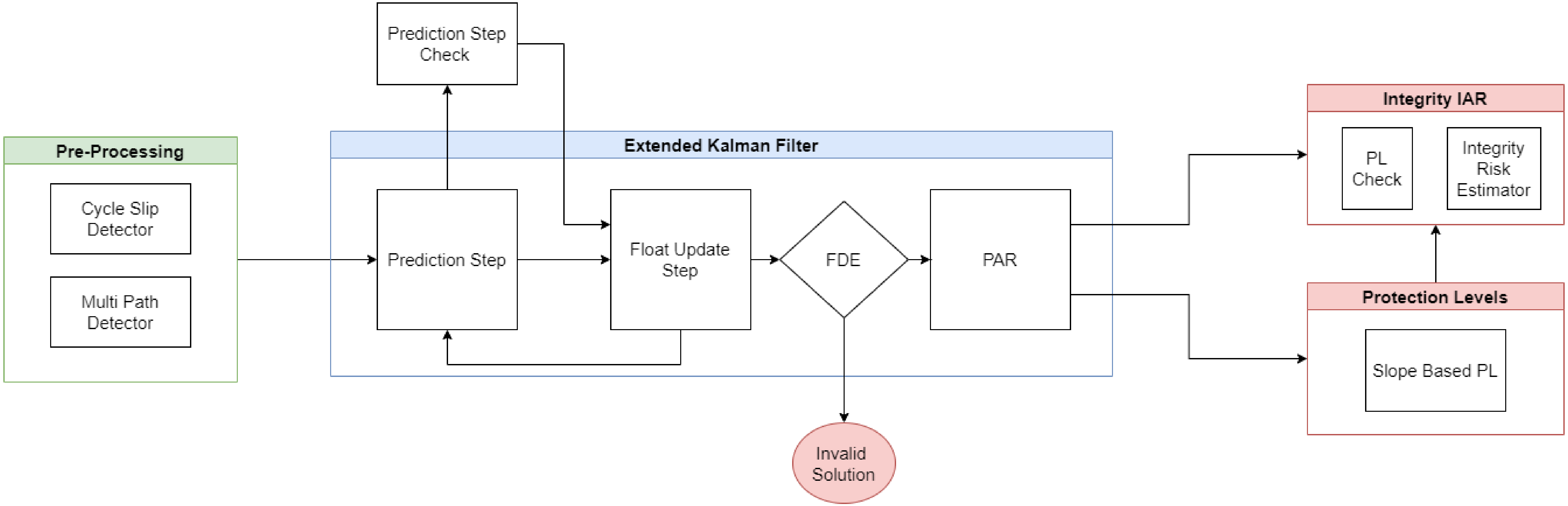
State of the Art on Integrity Monitoring: the limitations

- Standard solutions are derived specifically for aviation purposes:
  - open sky assumption
  - **very low number of faults** (only due to satellite faults)
  - not applicable to landing / take-off maneuvers
- Typically, **only code observations** are used (or code-carrier smoothing)
- **Only snapshot solutions** are considered (no recursive estimation)
- Multi sensor integration and related challenges are not contemplated
- Availability of Integrity Support Message (ISM), meaning “perfect” stochastic modeling

There is a need for new methods on Integrity Monitoring!

So… what is the least we could do?

1. **pre-processing**
2. **estimator + Fault Detection and Exclusion (FDE)** mechanism (+ Test statistic)
3. **error bounding** (protection level / integrity risk)



Multi Hypothesis-based Filtering for Integrity Monitoring

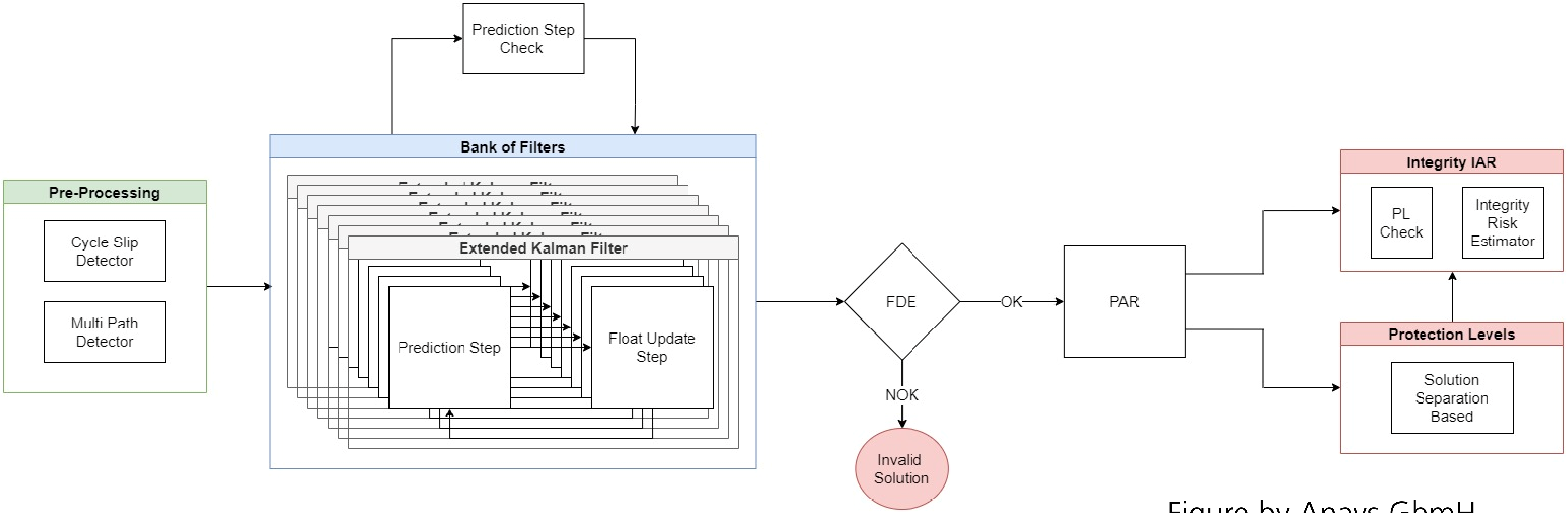
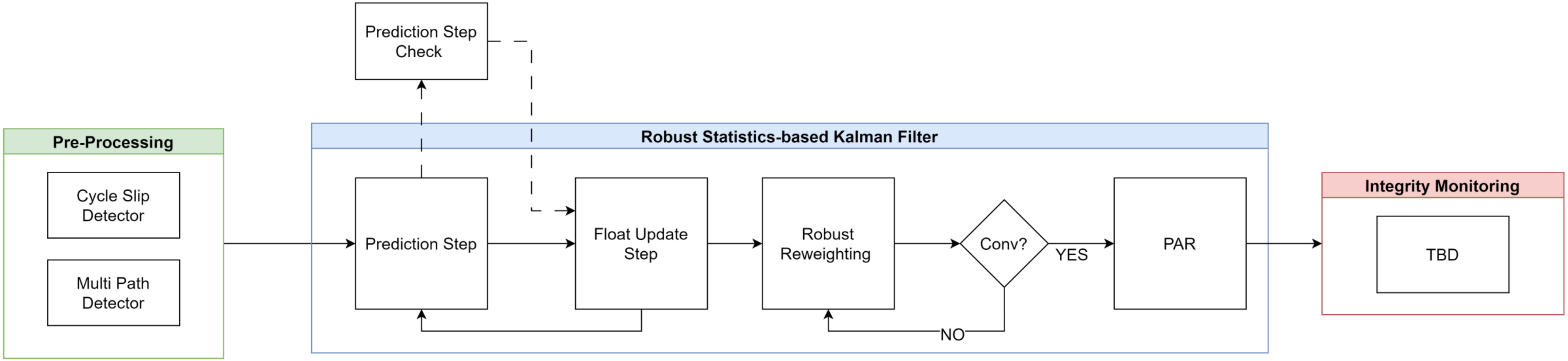


Figure by Anavs GbmH

Pros
- Lower PLs expected
- Fault Detection and Exclusion inherently covered

Cons
- Computational Cost → Operational feasibility?

Robust Filtering for Integrity Monitoring



Pros

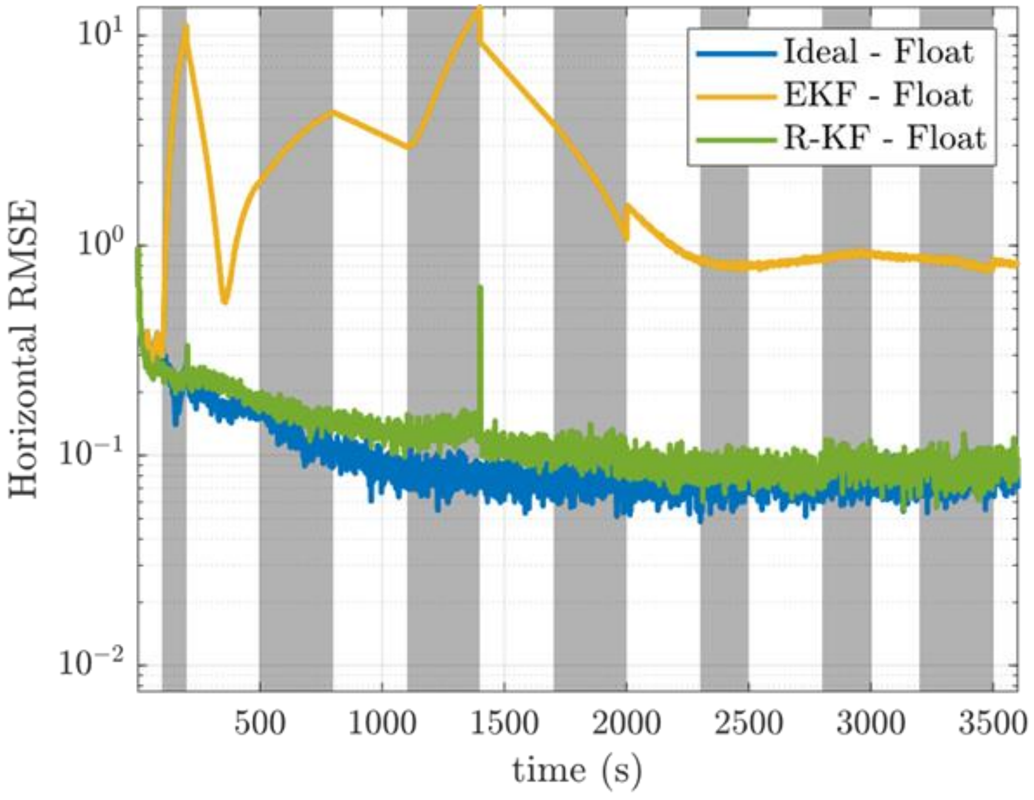
- Performance scalable with the number of measurements
- Inherently support observations' covariance estimation
- Certain flexibility against "poor dynamics"

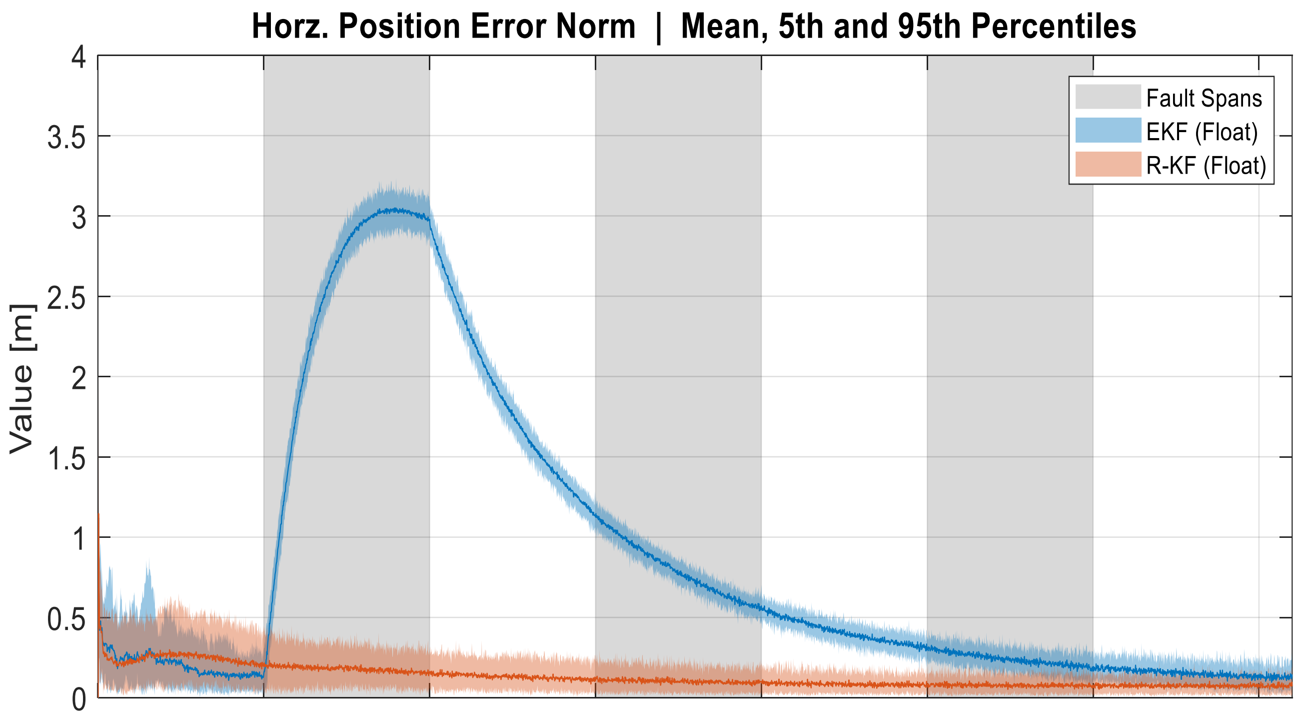
Cons

- It may resort to non-convex optimization
- Conventional Integrity Monitoring *may* not apply (observations' covariance matrix is estimated)

Results on Robust Integrity Monitoring

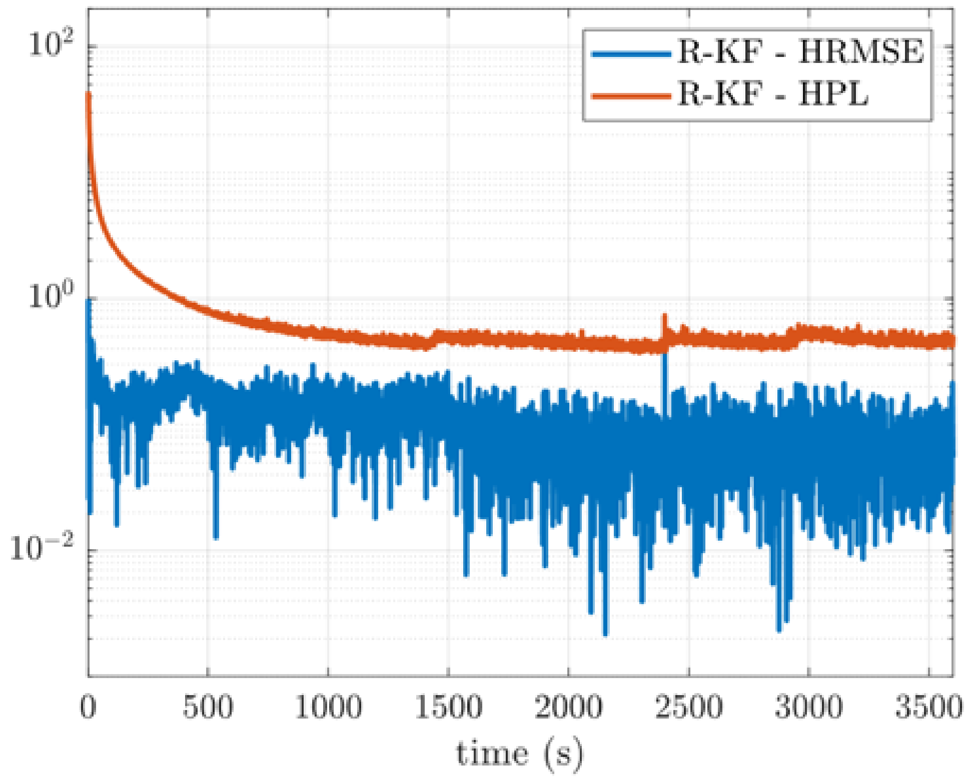
- We inject ramp errors (gray regions)
- Comparison baseline (EKF will info on outliers), conventional EKF (no FDE) and Robust

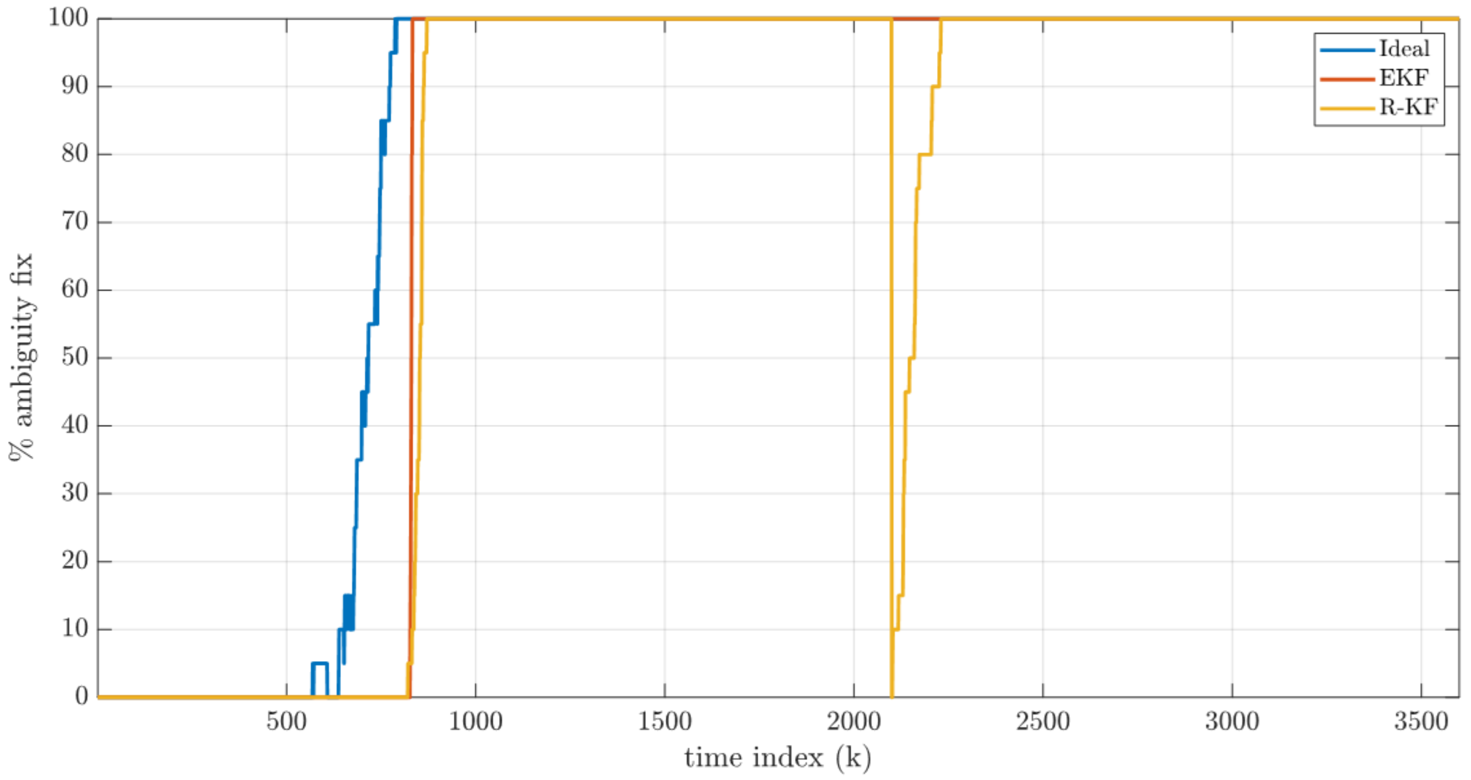




Results on Robust Integrity Monitoring

- Integer Estimation for the EKF passes the validity test (despite being biased)
- Regardless of whether we fix or not the ambiguities, the estimated covariance matrices for filters using carrier phase are unrealistic





Robust Filtering for Lie Groups

Error State Kalman Filter (**ESKF**)

$$\mathbf{x}_k = \widehat{\mathbf{x}}_k \oplus \delta\mathbf{x}_k = \begin{cases} \widehat{\mathbf{q}}_k \circ \delta\mathbf{q}_k \\ \widehat{\mathbf{b}}_{\omega,k} + \delta\mathbf{b}_{\omega,k} \end{cases}, \quad \delta\mathbf{x}_k = \mathbf{x}_k \ominus \widehat{\mathbf{x}}_k$$

Iterated ESKF (**I-ESKF**) → **but NOT robust**

$$\widehat{\mathbf{x}}_{k|k}^{(i+1)} = \widehat{\mathbf{x}}_{k|k-1} \oplus \mathbf{K}_k^{(i)} \left(\mathbf{y}_k - \mathbf{h}_k(\widehat{\mathbf{x}}_{k|k}^{(i)}) + \mathbf{H}_k^{(i)}(\widehat{\mathbf{x}}_{k|k}^{(i)} \ominus \widehat{\mathbf{x}}_{k|k-1})\right)$$

Iterative Gauss-Newton process to find the best linearization point → avoids the EKF loss of optimality due to linearization errors!

Robust Filtering for Lie Groups

Robust I-ESKF (**RI-ESKF**): Merges the iterated linearization, the quaternion state estimation and a robust weighting function

- 1^{st} loop → linearization + 2^{nd} loop → robust weights

Alg. 1: Iterated ESKF (**I-ESKF**)

$$\widehat{\mathbf{x}}_{k|k}^{(i+1)} = \widehat{\mathbf{x}}_{k|k-1}$$
$$\oplus\, \mathbf{K}_k^{(i)} \left(\mathbf{y}_k - \mathbf{h}_k(\widehat{\mathbf{x}}_{k|k}^{(i)}) + \mathbf{H}_k^{(i)}(\widehat{\mathbf{x}}_{k|k}^{(i)} \ominus \widehat{\mathbf{x}}_{k|k-1})\right)$$

Algorithm 2: RI-ESKF Correction Step

Input : $\widehat{\mathbf{x}}_{k|k-1}$, $\mathbf{P}_{k|k-1}$, $\mathbf{y}_k$, $\mathbf{\Sigma}_k$

Initialize $\widehat{\mathbf{x}}^{(0)} = \widehat{\mathbf{x}}_{k|k-1}$, $\mathbf{W}^{(0)} = \mathbf{I}$

for $j = 1, 2, \ldots$ *until convergence* **do**

- Update $\bar{\mathbf{\Sigma}}_k = \mathbf{\Sigma}_k^{1/2} \left(\mathbf{W}^{(j-1)}\right)^{-1} \mathbf{\Sigma}_k^{\top/2}$
- Obtain $\widehat{\mathbf{x}}^{(j)}$, $\mathbf{P}^{(j)}$ from Alg. 1 with inputs $\widehat{\mathbf{x}}^{(j-1)}$, $\mathbf{P}_{k|k-1}$, $\mathbf{y}_k$, $\bar{\mathbf{\Sigma}}_k$
- Compute $\mathbf{W}^{(j)}$ from (9) with $\widehat{\mathbf{x}}^{(j)}$

Return: $\widehat{\mathbf{x}}_{k|k} = \widehat{\mathbf{x}}^{(j)}$, $\mathbf{P}_{k|k} = \mathbf{P}^{(j)}$

Bellés, A., Medina, D., Chauchat, P., Labsir, S., & Vilà-Valls, J. (2023, September). Robust M-Type Error-State Kalman Filters for Attitude Estimation. In *2023 31st European Signal Processing Conference (EUSIPCO)* (pp. 840-844). IEEE.
Bellés, A., Medina, D., Chauchat, P., Labsir, S., & Vilà-Valls, J. (2023). Robust Error-State Kalman-Type Filters for Attitude Estimation.

Results on Robust Estimation for Lie Groups

Parameters:

- Number of 3D baselines: N = 4

Initial std. dev.	Attitude: 1 [deg] Gyroscope bias: 0.1 [deg/h]
Process noise std. dev.	Gyroscope: 0.02 [deg / $\sqrt{s^3}$] Bias random walk: $2 \cdot 10^{-5}$ [deg / $\sqrt{s^3}$]
Obs. noise std. dev.	Code zenith-referenced (σ_ρ): 0.1 [m]
Outliers	25%, $\alpha = 100$, $\sigma = \alpha\sigma_\rho$

Generated as

- In the results, grey areas: **outliers**

Performance metric:

- **I**ntrinsic **m**ean **s**quared **e**rror (**IMSE**)

$$\frac{1}{M}\sum_{i=1}^{M} \mathrm{Log}^{\vee}_{SO(3)}\left(\mathbf{R}_k^{-1}\left(\hat{\mathbf{R}}_k\right)_i\right)^{\top} \mathrm{Log}^{\vee}_{SO(3)}\left(\mathbf{R}_k^{-1}\left(\hat{\mathbf{R}}_k\right)_i\right)$$

Where $\mathbf{R}_k$: true rotation matrix at epoch k

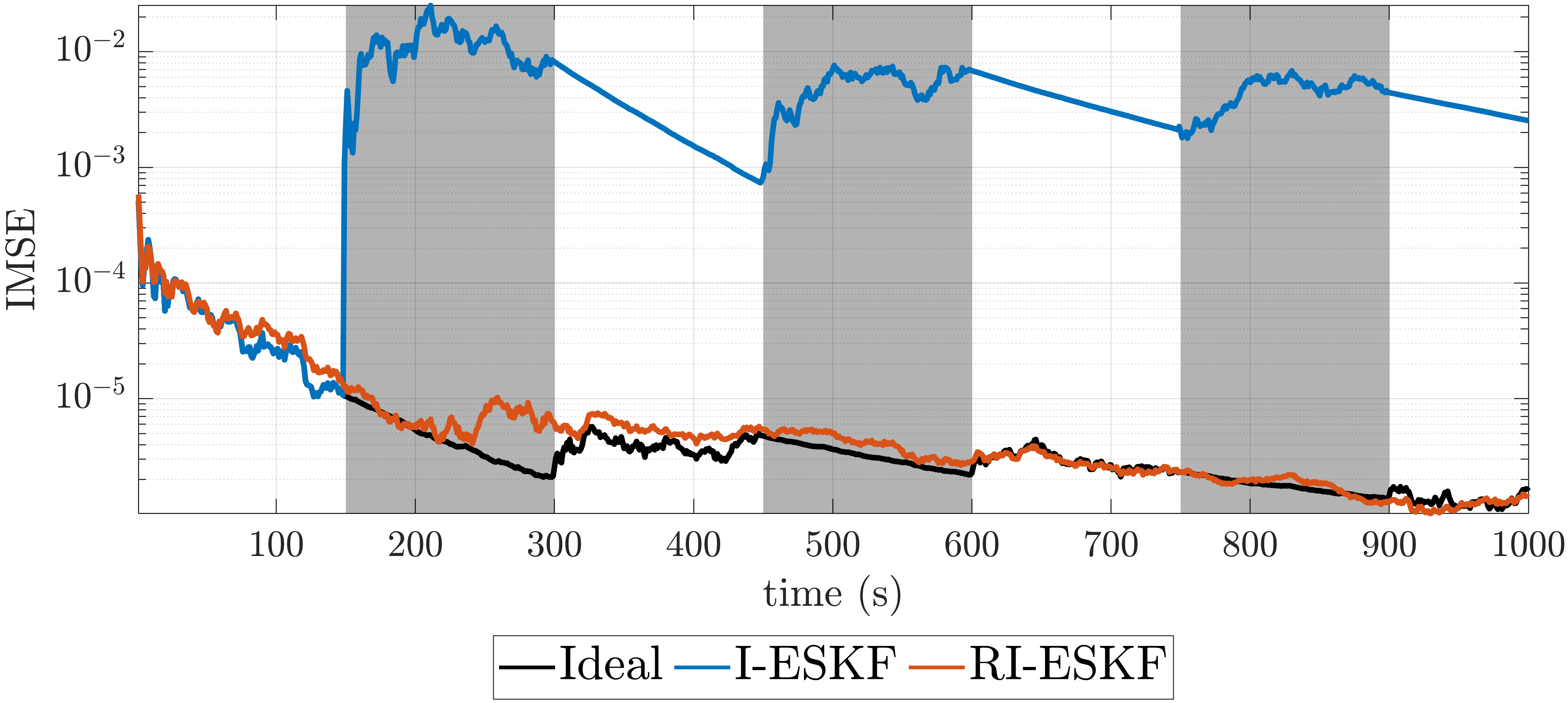
$\hat{\mathbf{R}}_k$: estimated rotation matrix at ep. k

$\mathrm{Log}^{\vee}_{SO(3)}$: shortcut to map directly from the manifold to the vector space $\mathbb{R}^3$

Units of ISME: very close to radians

Results on Robust Estimation for Lie Groups

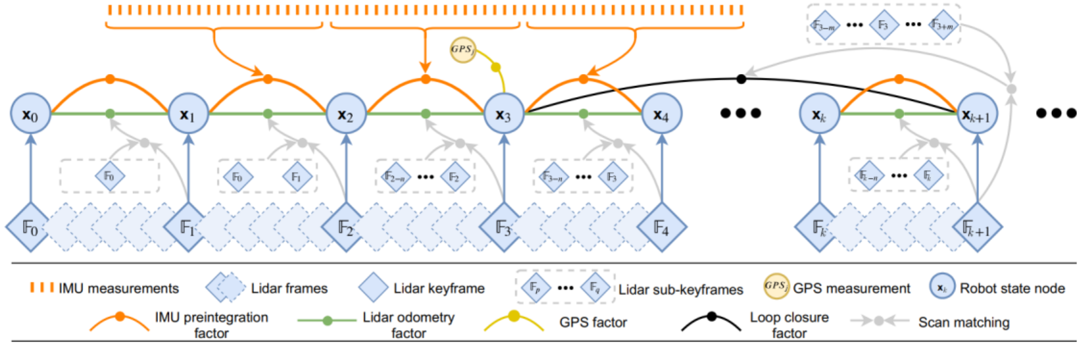
IMSE of the quaternion estimate:



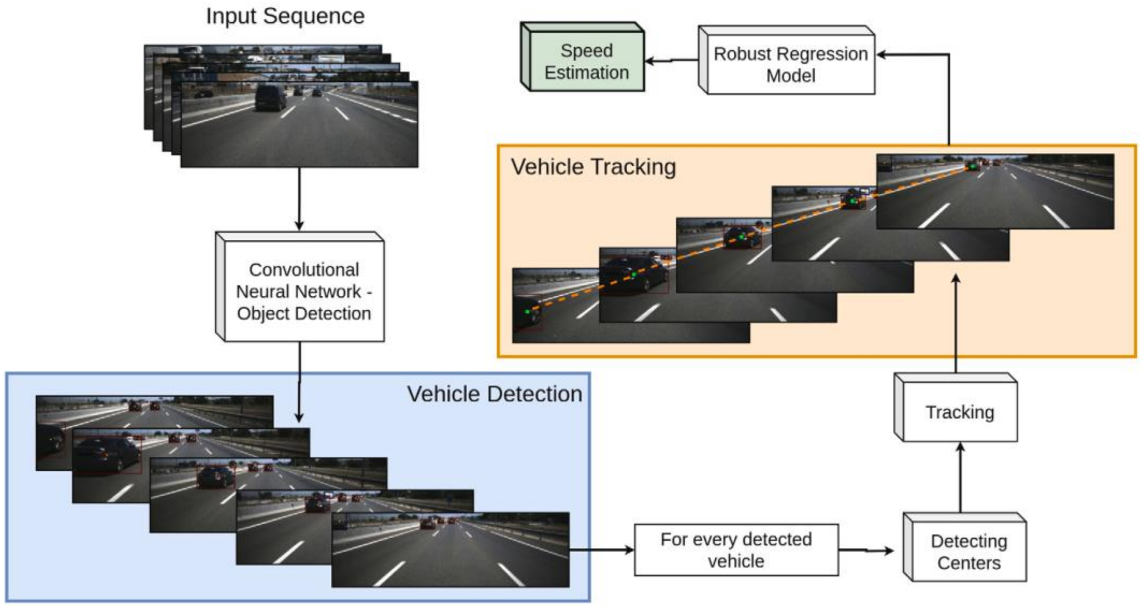
Bellés, A., Medina, D., Chauchat, P., Labsir, S., & Vilà-Valls, J. (2023, September). Robust M-Type Error-State Kalman Filters for Attitude Estimation. In *2023 31st European Signal Processing Conference (EUSIPCO)* (pp. 840-844). IEEE.

Further Applications for Robust Estimation

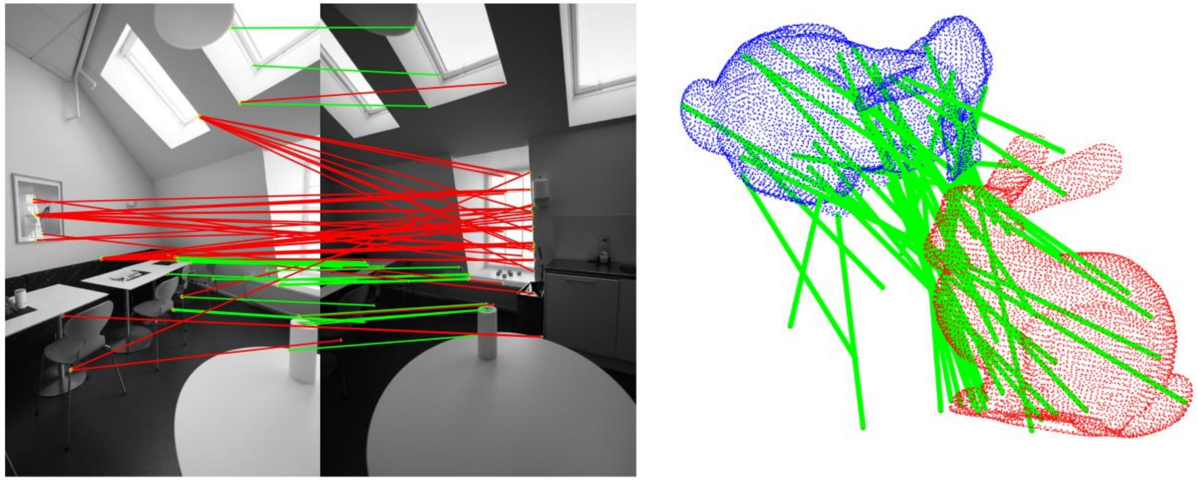
- **Robust SLAM**
- **Robust Point Cloud Registration**
- **Perception subtasks**



Shan, Tixiao, et al. "Lio-sam: Tightly-coupled lidar inertial odometry via smoothing and mapping." *2020 IEEE/RSJ international conference on intelligent robots and systems (IROS)*. IEEE, 2020.



García-Aguilar, Iván, et al. "Detection of dangerously approaching vehicles over onboard cameras by speed estimation from apparent size." *Neurocomputing* 567 (2024): 127057.



Yang, Heng, and Luca Carlone. "A quaternion-based certifiably optimal solution to the Wahba problem with outliers." *Proceedings of the IEEE/CVF International Conference on Computer Vision*. 2019.

Outline

1	Basics of State Estimation
2	Estimation under model mismatch
3	Robust Estimation and Filtering
4	Research & industry perspectives

Industry and Research Perspectives

From industry:

- Towards the GNSS people: the time for Least Squares is over. Robustness is here!
- Certifiable GNSS (integrity monitoring) will be highly dependent on today's discussion
- Robust theory showcased so often in Computer Vision (Apple, Microsoft, Flir, etc., etc.)

From research:

- **Statistical properties for Robust Estimation?** Breakdown point? Convergence guarantees? Sensitivity?
- **Robust Statistical Testing?** When to trust Robustness?
- **Adaptive Robust Functions** (follow the work from dear Paul Chauchat)

- The Gaussian model assumption is outdated…
- There is no field insensitive to model mismatch!

Say hi at:
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Thanks for your attention!

Impressum

Title: **High Precision Satellite-based Navigation**
Theory and Applications

Date: 2024-07-05

Author: Daniel Medina (daniel.ariasmedina@dlr.de)

Institut: Communications and Navigation