Introduction à la cryptographie post-quantique

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November 30, 2023

Outline

Disclaimer !!

(dans l'éventualité où nous n'aurions pas le temps)

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What are the alternatives to classical cryptography in presence of an adversary equipped with a large scale quantum computer?

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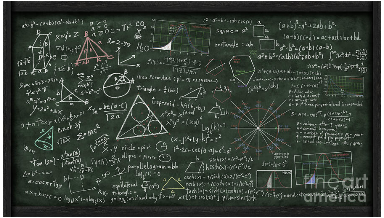
- Post-Quantum Cryptography



Recalls on hash functions

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Demo with SHA-1 (OpenSSL).

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- Resistance to the second pre-image :
  Given x (this time x is known), it is computationally infeasible to find a $y \neq x$ such that $\mathcal{H}(y) = \mathcal{H}(x)$.

Security of hash functions

Security of a hash function (collision-resistance), birthdays' paradox

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How many people do you need to gather at least to have more than a one in two chance of having two born on the same day?

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How many people do you need to gather at least to have more than a one in two chance of having two born on the same day?

Answer

23 people for more than one chance in two. The odds increase to $\sim 90\%$ with only 41 people.

(the magic of the exponential!)

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23 people: explanation

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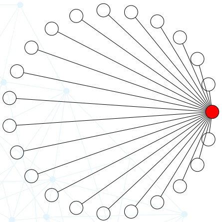

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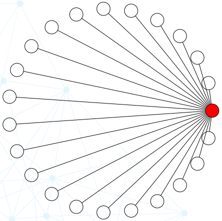

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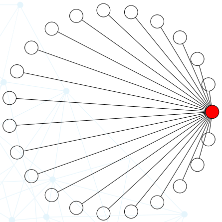

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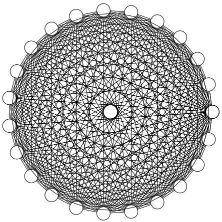

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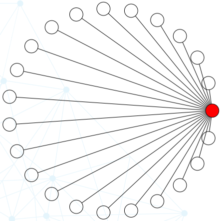

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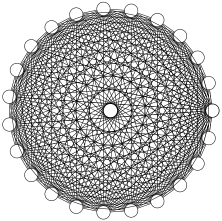

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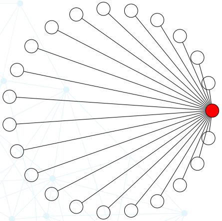

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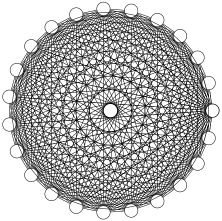

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Demo (real if sufficiently many, simulation otherwise)

Security of hash functions

Application

$\mathcal{H} : \{0,1\}^* \longrightarrow \{0,1\}^\ell$. How many elements of $\{0,1\}^*$ will we have to sample before finding two elements with the same hash (*i.e.* a collision) with a good probability?

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Only $2^{n/2}$!

... but what does it mean? How hard is 2^{100} for instance?

Computing power / security level

Computing power in 2020

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For a standard machine: 64-bit architecture

2^6

Computing power / security level

Computing power in 2020

For a standard machine: 16 cores

$2^6 \times 2^4$

Computing power / security level

Computing power in 2020

For a standard machine: 4 GHz

$2^6 \times 2^4 \times 2^2 \times 10^9$

Computing power / security level

Computing power in 2020

Let it run for a year

$$2^6 \times 2^4 \times 2^2 \times 10^9 \times 60 \times 60 \times 24 \times 365$$

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Imagine that a government agency can easily acquire 10,000 machines

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A relatively concrete example

Globally, and with a substantial investment, there was 2^{89} hash SHA-256 made on the blockchain BitCoin in 2019...

Cryptographic protocol recall/presentation

Corollary

In 2020, for short-term security, it is recommended to use parameters providing a security level equivalent to 128 bits.

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Paranoid, you'll become...

In 2020, prefer algorithms offering a security level equivalent to 256 bits.

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Paranoid, you'll become...

In 2020, prefer algorithms offering a security level equivalent to 256 bits.

Deprecated!

This increase in computing power makes some "old" algorithms obsolete, such as:

- DES (64 secret key bits, 56 only for security)
- MD≤ 5 (128 bits output for MD5 (paradox + weaknesses))
- SHA-≤ 1 (160 bits output)...

Outline

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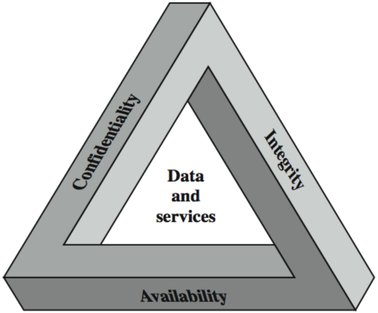
Sécurité informatique (security)

National Institute of Standards and Technology
Technology Administration, U.S. Department of Commerce

Définition du NIST:
The protection afforded to an automated information system in order to attain the applicable objectives of preserving the **integrity**, **availability** and **confidentiality** of information system resources (includes hardware, software, firmware, information/data, and telecommunications)



http://csrc.nist.gov/publications/fips/fips199/FIPS-PUB-199-final.pdf

Sécurité informatique (security)

3 piliers fondamentaux de la security

- **Confidentialité**
  → Restrictions concernant l'**accès** aux informations et leur **divulgation** aux **seules personnes autorisées**, y compris les moyens de protéger la **vie privée** et les informations exclusives.
- **Intégrité**
  → Protection contre la **modification** ou la **destruction** des informations, y compris la garantie de la **non-répudiation** et de l'**authenticité** des informations.
- **Disponibilité**
  → Garantir un **accès rapide** et **fiable** aux informations et leur utilisation.

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Cryptography embodies the theoretical aspects of *security*.

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wishes to send message to 

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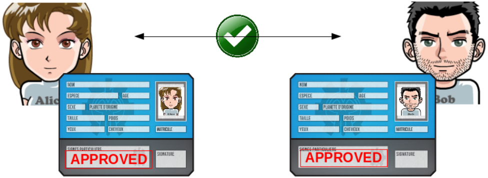

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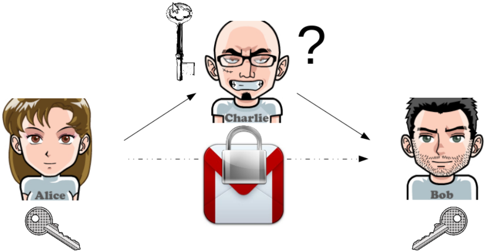

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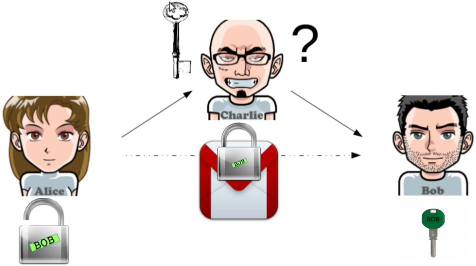

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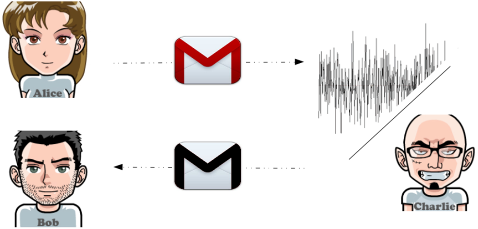

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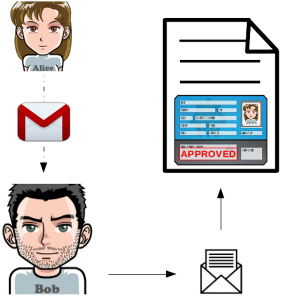

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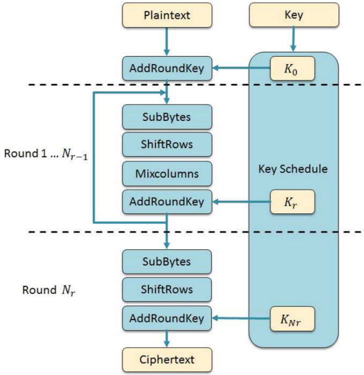
Crypto symétrique : AES

Advanced **E**ncryption **S**tandard



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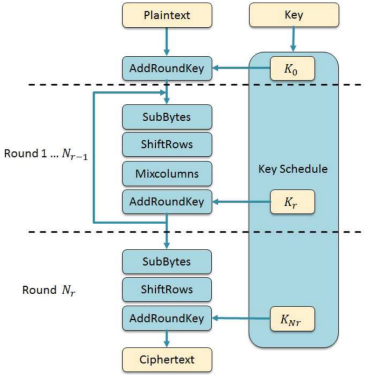
Crypto symétrique : AES

Advanced Encryption Standard



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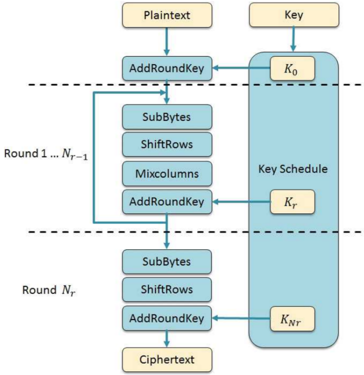

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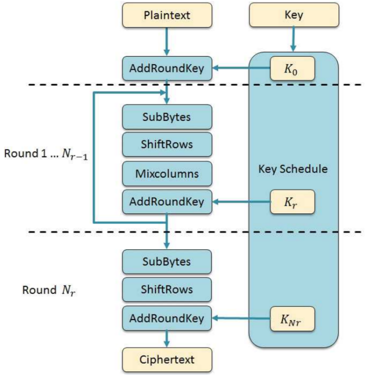

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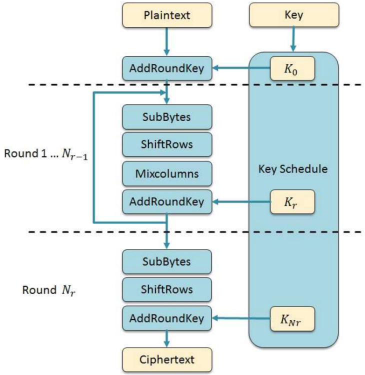

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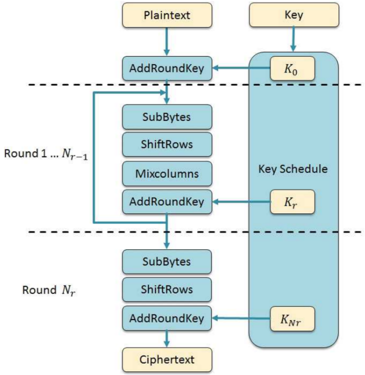

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- Best known attacks only marginally affect security

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Crypto asymétrique : New directions in cryptography

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$d = e^{-1} \mod \varphi(N)$

$(m^e)^d \mod N = m$

message $m \in \mathbb{Z}/N\mathbb{Z}$

N, e →

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N, e $\longrightarrow$

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Fermat's (little) theorem

If p is prime, then $\forall a \in \mathbb{Z}/p\mathbb{Z}$, it holds that : $a^{p-1} = 1 \mod p$.

Cryptanalyse de ces standards sur une architecture classique (1/2)

Problèmes mathématiques sous-jacents :

- problème RSA : *étant donnés* N, c, *et* e, *trouver* m *tel que* $m^e = c \mod N$
- problème DH : *étant donnés* p, g, $g^a \mod p$ *et* $g^b \mod p$, *trouver* $g^{ab} \mod p$

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Problèmes connexes :

- FACT : *étant donné* $N = pq$ *avec* p *et* q *premiers distincts, trouver* p *et* q
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Cryptanalyse de ces standards sur une architecture classique (1/2)

Problèmes mathématiques sous-jacents :

- problème RSA : *étant donnés* N, c, *et* e, *trouver* m *tel que* $m^e = c \mod N$
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Lien entre ces problèmes :

- Si FACT est facile, Alors RSA est facile *(comprendre RSA cassé)*
- Si DL est facile, Alors DH est facile

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Meilleur algo classique :

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- AES-128 sécurité d'environ 126 bits, plus de doutes sur AES-256
- Chacha20 pas d'attaque significative mais algo plus récent, et moins étudié

Outline

1 Contexte

2 Motivation *aka.* la menace quantique
- Rappels de cryptographie
- L'ordinateur quantique et son impact sur la crypto actuelle

3 Cryptographie post-quantique (si on a le temps...)

4 Conclusion

Quantum computing

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- **Entanglement**: the capability of two qubits to be *correlated*. If Alice and Bob both get one of two entangled qubits, and if Alice measures a $|0\rangle$ at some point, then necessarily Bob must measure the same, as $|00\rangle$ is the only state where Alice's qubit is a $|0\rangle$.

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Qubits can be "implemented" using the spin of an electron, or the polarization of a photon, ...

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As a consequence:

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A quantum algorithm solving a problem needs to make the correct solution (state) **exponentially** more likely than the other states (cf. quantum annealing / wave function collapsing).

Shor's algorithm [Sho97]

SIAM J. COMPUT.
Vol. 26, No. 5, pp. 1484–1509, October 1997

POLYNOMIAL-TIME ALGORITHMS FOR PRIME FACTORIZATION AND DISCRETE LOGARITHMS ON A QUANTUM COMPUTER*

PETER W. SHOR†

Abstract. A digital computer is generally believed to be an efficient universal computing device; that is, it is believed able to simulate any physical computing device with an increase in computation time by at most a polynomial factor. This may not be true when quantum mechanics is taken into consideration. This paper considers factoring integers and finding discrete logarithms, two problems which are generally thought to be hard on a classical computer and which have been used as the basis of several proposed cryptosystems. Efficient randomized algorithms are given for these two problems on a hypothetical quantum computer. These algorithms take a number of steps polynomial in the input size, e.g., the number of digits of the integer to be factored.

Shor's algorithm: how it works

Algorithm 1: ShorAlgorithm(N)

Input: N

Output: p, q such that $N = pq$

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```
Pick g ∈ Z_N at random;
if gcd(g, N) ≠ 1 then
    then return (p = gcd(g, N), q = N/p)
```

ENAC

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3 then **return** $(p = \gcd(g, N), q = N/p)$

4 Find r such that $g^r \equiv 1[N]$;

5 **if** $r \equiv 0[2]$ **then**

6 **return** $\gcd\left(g^{r/2} \pm 1, N\right)$

7 **else**

8 go to 1

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Meaning that there is a non-negligible probability that $g^{r/2} \pm 1$ shares non trivial factors with N.

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- step 1. $g \leftarrow 101$
- step 2. $r \leftarrow 4347$

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- step 3. let us denote $g_p = g^{17388/2} + 1$ and $g_q = g^{17388/2} - 1$
  we have that $\gcd(g_p, N) = 829 =: p$ and $\gcd(g_q, N) = 379 =: q$
  and indeed, $p \cdot q = 829 \times 379 = 314191 = N$

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- Quantumly $\mathcal{O}\left(\log(N)^3\right)$ (polynomial in the size of the input): That's an **exponential** speedup!

Quantum period finding

How does it work? Why is it much much faster quantumly?

Fourier Transform is THE tool to analyse frequencies. Fortunately, it has a quantum equivalent: QFT.

Quantum computing allows to provide QFT a superposition of every possible states (assuming enough qubits).

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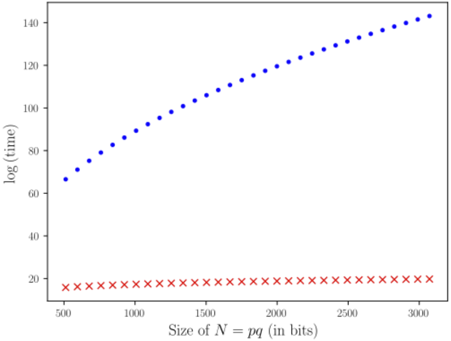

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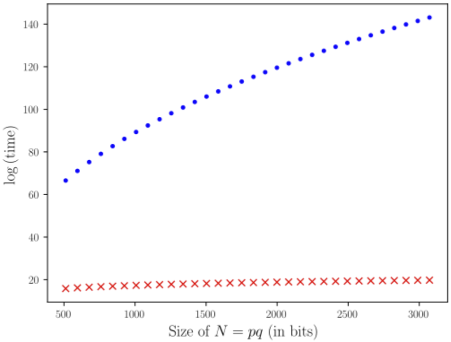

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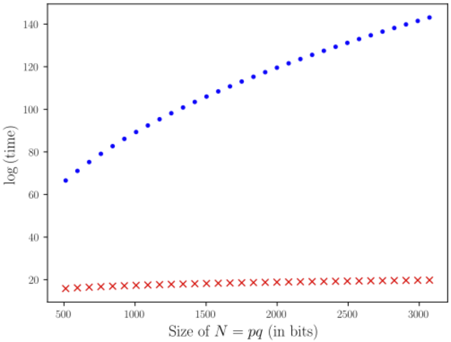

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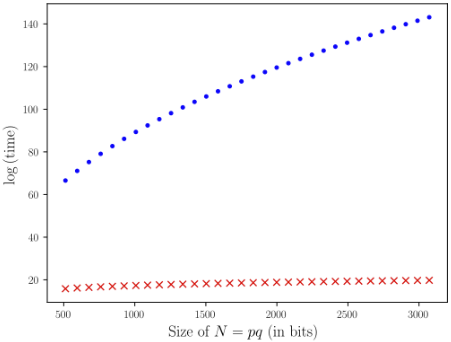

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In other words, **security as we know it collapses**...

Outline

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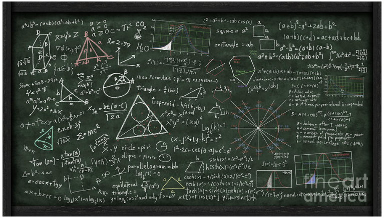
- Post-Quantum Cryptography



L'essentiel

Il existe 5 primitives populaires pour construire des problèmes *a priori* difficiles (et donc exploitables en crypto) y compris pour un ordinateur quantique :

- les réseaux euclidiens (EN: lattices)
- les codes correcteurs d'erreurs
- les fonctions de hachage
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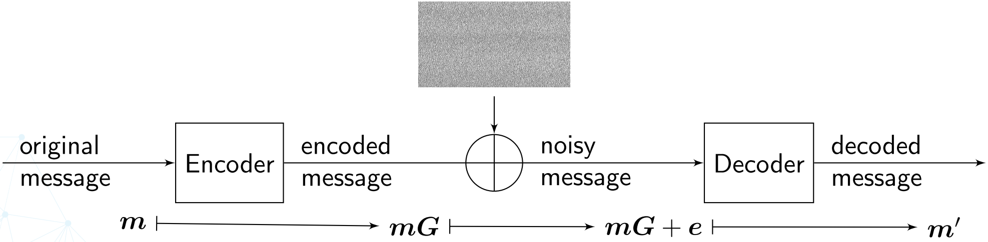

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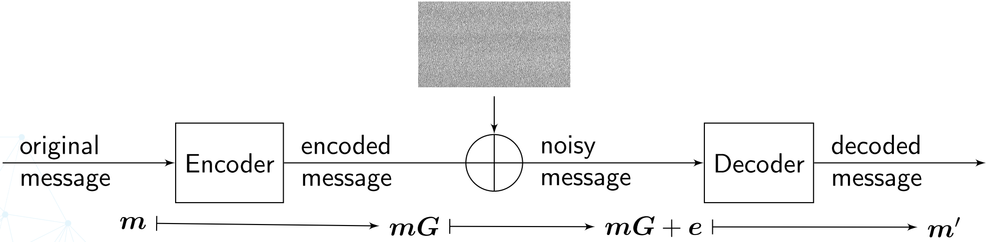

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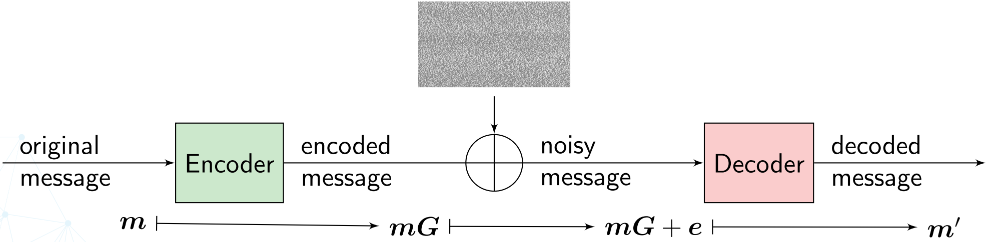

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- For code-based PKC, most of the time, public encoder / private decoder.

Definitions

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The Hamming weight of a word $\boldsymbol{u}$ is the number of its non-zero coordinates:

$$wt\left(\boldsymbol{u}\right) = \#\left\{ i \in \{0, \ldots, n-1\} \text{ such that } \boldsymbol{u}_i \neq 0 \right\}$$

$$\text{example : } wt\left((0,1,0,0,1,0,1,0)\right) = 3$$

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Exemple : le code à répétition

Message à envoyer		1			0			1	
Encodage	1	1	1	0	0	0	1	1	1
Message reçu	0	1	1	0	1	0	1	1	0
Message décodé		1			0			1	

Code-based cryptography (CBC)

Que sont les codes correcteurs ?

Des façons de rajouter de la redondance à l'information utile, afin d'être capable de détecter — voire corriger — d'éventuelles erreurs lors de la transmission.

Exemple : le code à répétition

Message à envoyer		1			0			1	
Encodage	1	1	1	0	0	0	1	1	1
Message reçu	0	1	1	0	1	0	1	1	0
Message décodé		1			0			1	

Ce code est particulièrement mauvais (bien qu'utile pédagogiquement parlant) :

- dimension : $k = 1$
- longueur : $n = 3$
- distance minimale : $d = 3$

- capacité de détection : $d - 1 = 2$ erreurs
- capacité de correction : $\left\lfloor \frac{d-1}{2} \right\rfloor = 1$ erreur
- rendement $\frac{k}{n} = \frac{1}{3}$.

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Problème du décodage de syndrome.

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Problème

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Il suffit de réaliser un pivot de Gauss sur la matrice $\mathbf{H}$. C'est purement un problème d'algèbre linéaire...

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Le problème devient *NP*-difficile [BMvT78].

(Traduction: il devient cryptographiquement intéressant)

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Soit $\mathbf{G} \in \mathbb{F}_2^{k \times n}$ la matrice génératrice d'un code (de Goppa binaire) $\mathcal{C}$ pouvant corriger jusqu'à t erreurs à l'aide de l'algorithme de décodage $\mathcal{D}_{\mathbf{G}}$.

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matrice inversible $\mathbf{S} \in \mathbb{F}_2^{k \times k}$

message $\mathbf{m} \in \mathbb{F}_2^{k}$

matrice permutation $\mathbf{P} \in \mathbb{F}_2^{n \times n}$

$$\xrightarrow{\tilde{\mathbf{G}}=\mathbf{SGP}, n, k, t}$$

$\mathbf{e} \in \mathbb{F}_2^{n}$ tel que $wt(\mathbf{e}) \leq t$

$$\xleftarrow{\mathbf{c}}$$

$$\mathbf{c} = \mathbf{m}\tilde{\mathbf{G}} + \mathbf{e}$$

$$\tilde{\mathbf{c}} = \mathcal{D}_{\mathbf{G}}\left(\mathbf{c}\mathbf{P}^{-1}\right) = \mathcal{D}_{\mathbf{G}}\left(\mathbf{mSG} + \mathbf{eP}^{-1}\right)$$

$$\mathbf{m} = \tilde{\mathbf{c}}\mathbf{S}^{-1}$$

CBC : un exemple

Soit $\mathcal{C}$ le code (de Hamming) admettant pour matrice de parité $\mathbf{H}$:

$$\mathbf{H} = \begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{pmatrix}$$

Soit $\mathbf{v} = (1, 0, 0, 0, 1, 1, 1)$ le mot reçu. Quel était le message envoyé ?

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Quasi-Cyclic Moderate Density Parity-Check Codes

KeyGen

Sample $\mathbf{h}_0, \mathbf{h}_1 \leftarrow \mathbb{F}_2^r$ of small weight w, $\mathbf{h}_0$ inversible. Compute $\mathbf{h} = \mathbf{h}_1 \mathbf{h}_0^{-1}$.

$$\mathbf{H}_{\text{secret}} = \left(\begin{array}{c|c} \mathbf{h}_0 & \mathbf{h}_1 \\ \circlearrowright & \circlearrowright \end{array} \right)$$

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Chiffrement OK. Existe-t-il un algo de signature aussi simple ?

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Claimed security	Persichetti's OTS parameters				xBF parameters		Verification	Cryptanalysis
	n	w_1	w_2	δ	τ	N	t_{verify} (ms)	t_{break} (ms)
80	4801	90	100	10	7	5	22.569	165.459
	3072	85	85	7	5	5	14.271	68.858
128	9857	150	200	12	9	10	99.492	453.680
	6272	125	125	10	7	10	42.957	288.442

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D'autres schémas de signature (plus complexes à exposer) existent, et ne souffrent pas de ce type de problème:

- WAVE [DST18]: `https://eprint.iacr.org/2018/996`
- DURANDAL [ABG+18]: `https://eprint.iacr.org/2018/1192` **(métrique rang)**

Outline

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- Most serious candidates are lattices, error correcting codes and hash functions.

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  - Hybrid encryption allows to benefit from SE efficiency while avoiding its disadvantages
- There is a real quantum threat for actual cryptography.
- Post-quantum alternatives exist and are being developed/standardized (involve classical computers, not quantum).
- Most serious candidates are lattices, error correcting codes and hash functions.
- Keys for symmetric algorithms need to be doubled.

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Thanks!

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