Robust Standalone GNSS Navigation

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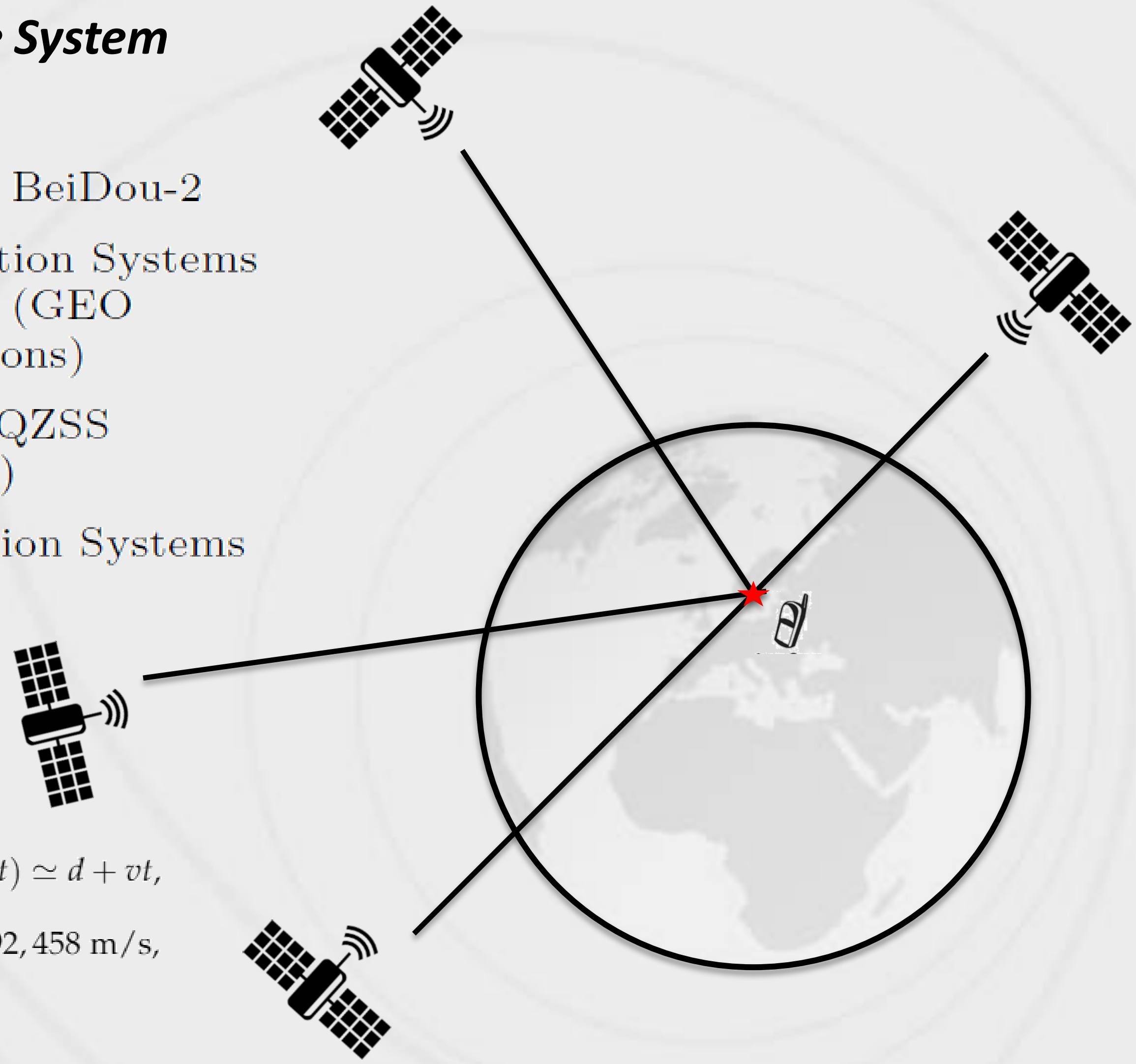
1-Introduction to the GNSS

Global Navigation Satellite System

- Galileo, GPS, GLONASS, BeiDou-2
- Satellite Based Augmentation Systems (SBAS) : WAAS, EGNOS (GEO satellites and ground stations)
- Regional NAVIC (India), QZSS (Japan), BeiDou-1 (China)
- Ground Based Augmentation Systems (GBAS)



$$\|\mathbf{p}_{TR}(t)\| \triangleq \|\mathbf{p}_R(t) - \mathbf{p}_T(t - \tau(t))\| = c\tau(t) \simeq d + vt,$$

$$\tau(t) \simeq \tau + bt, \quad \tau = \frac{d}{c}, \; b = \frac{v}{c}, \quad c = 299{,}792{,}458 \text{ m/s},$$

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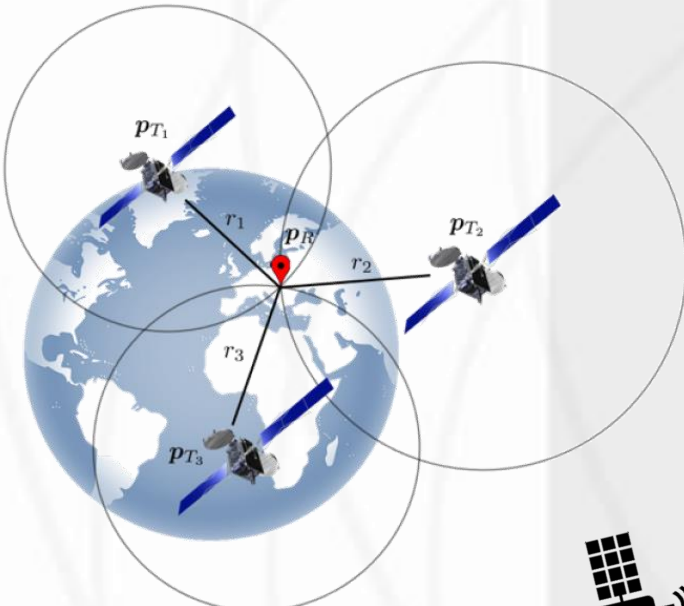

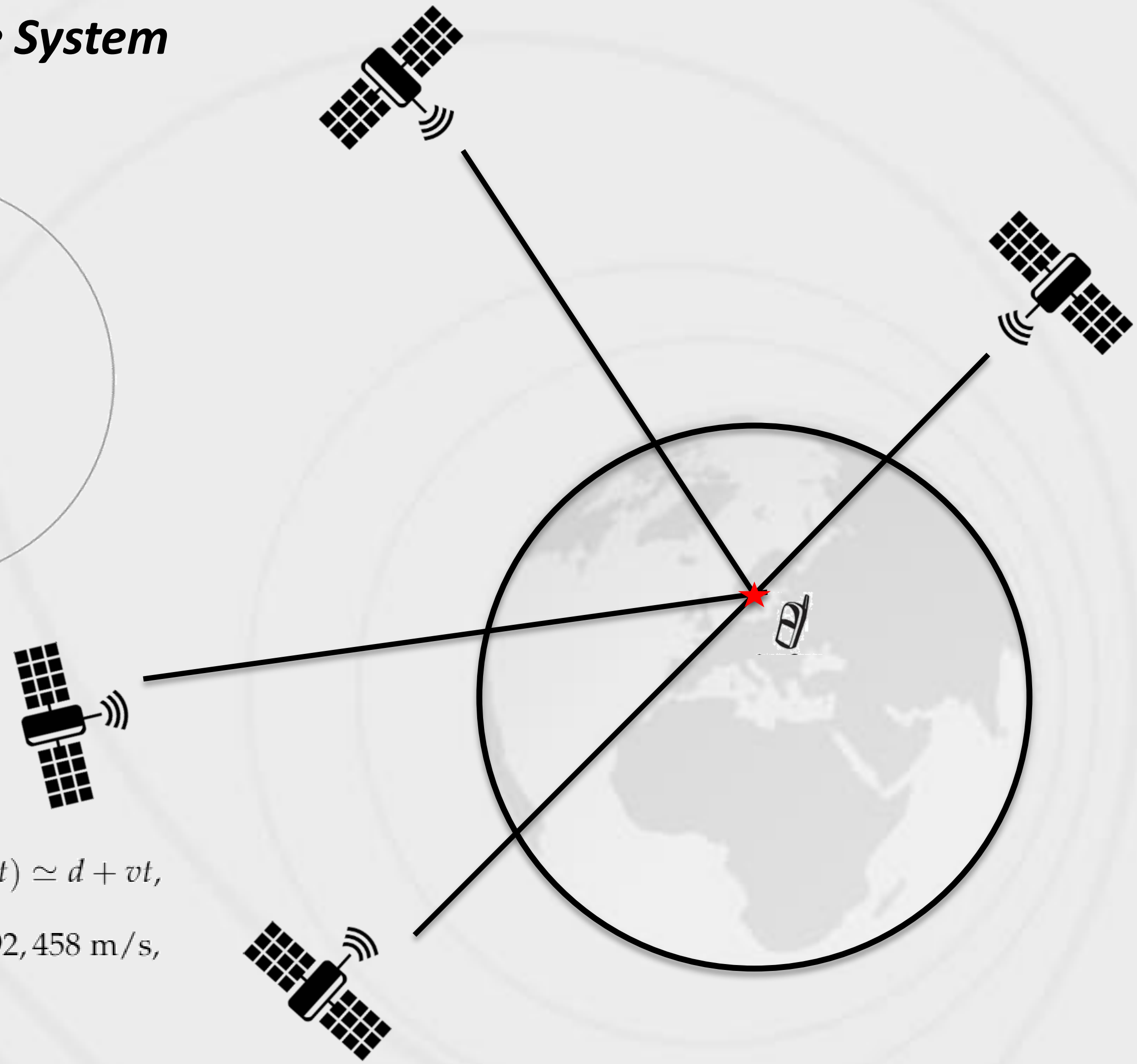

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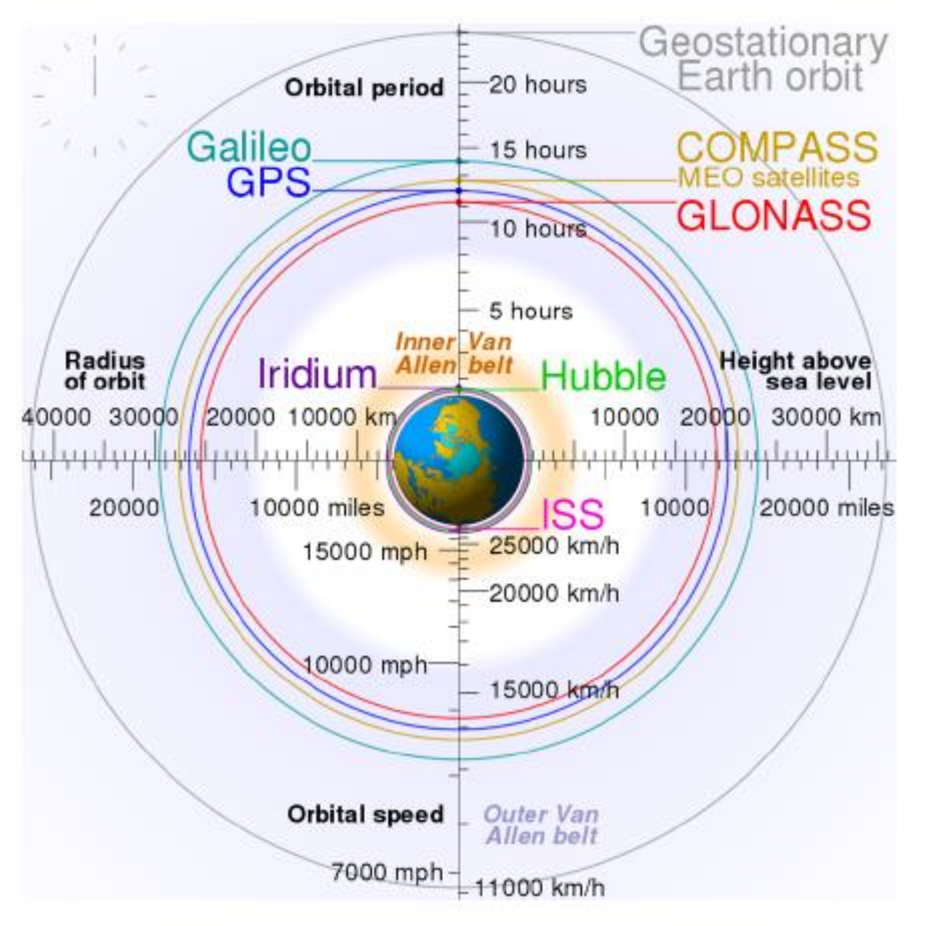

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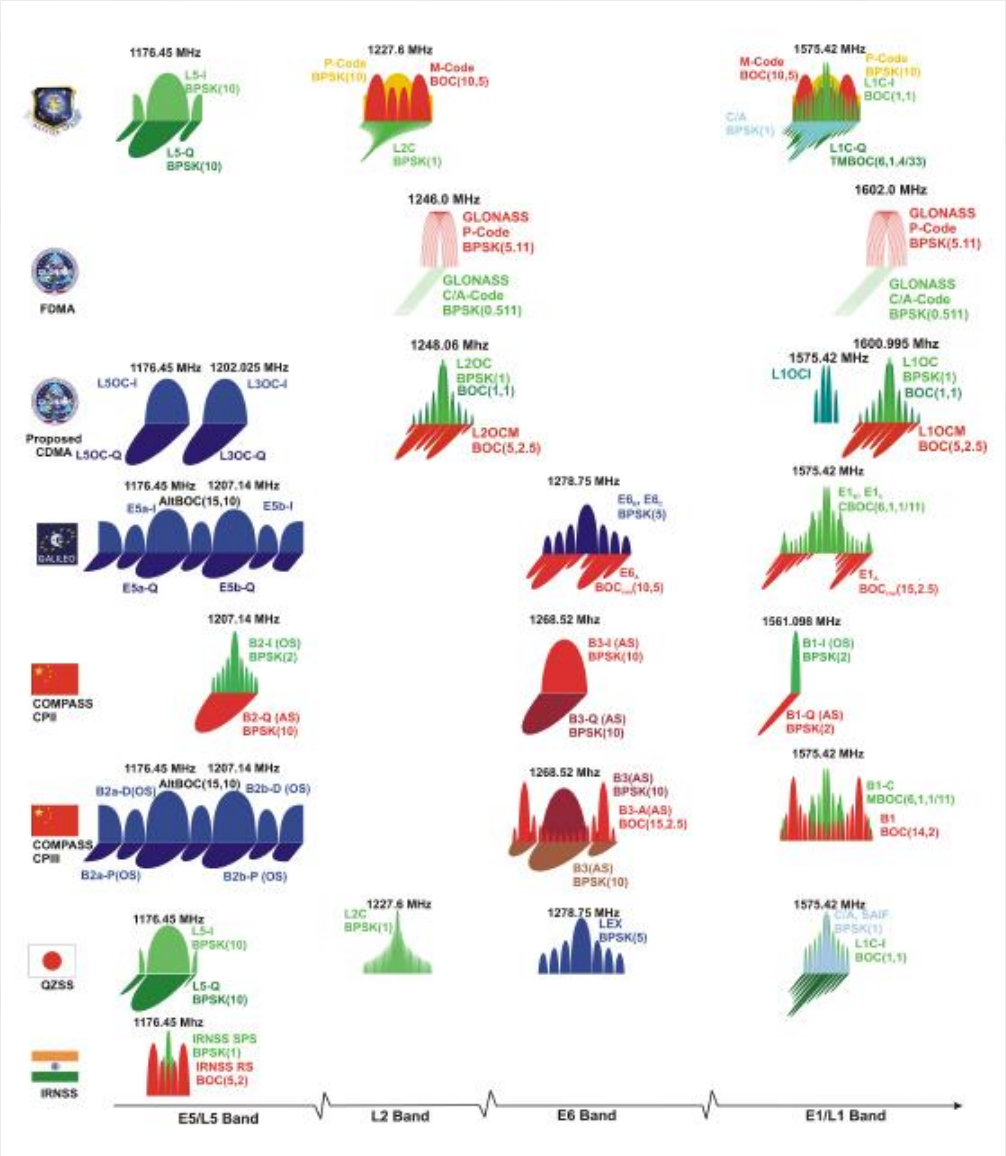

http ://www.emfrf.com/wp-content/uploads/2014/03/spectrum.jpg & Navipedia

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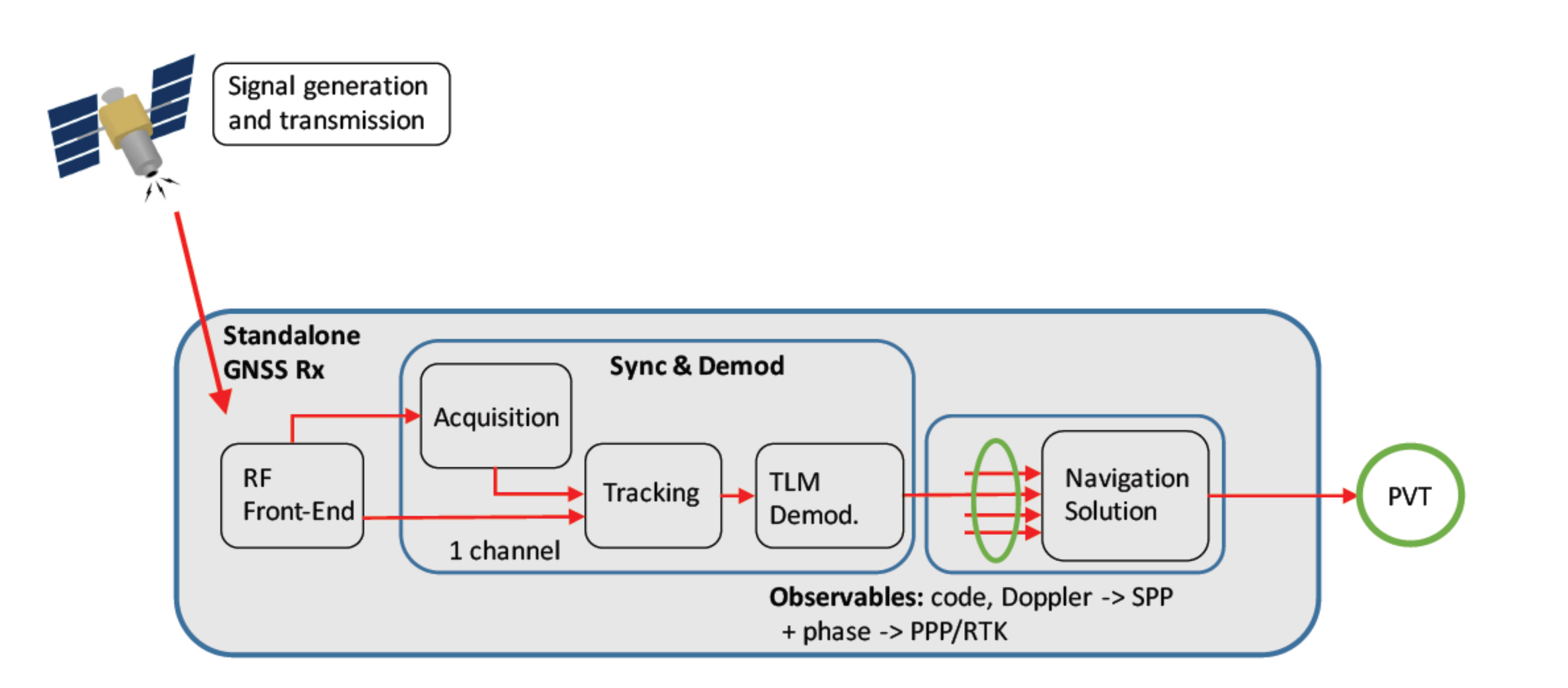

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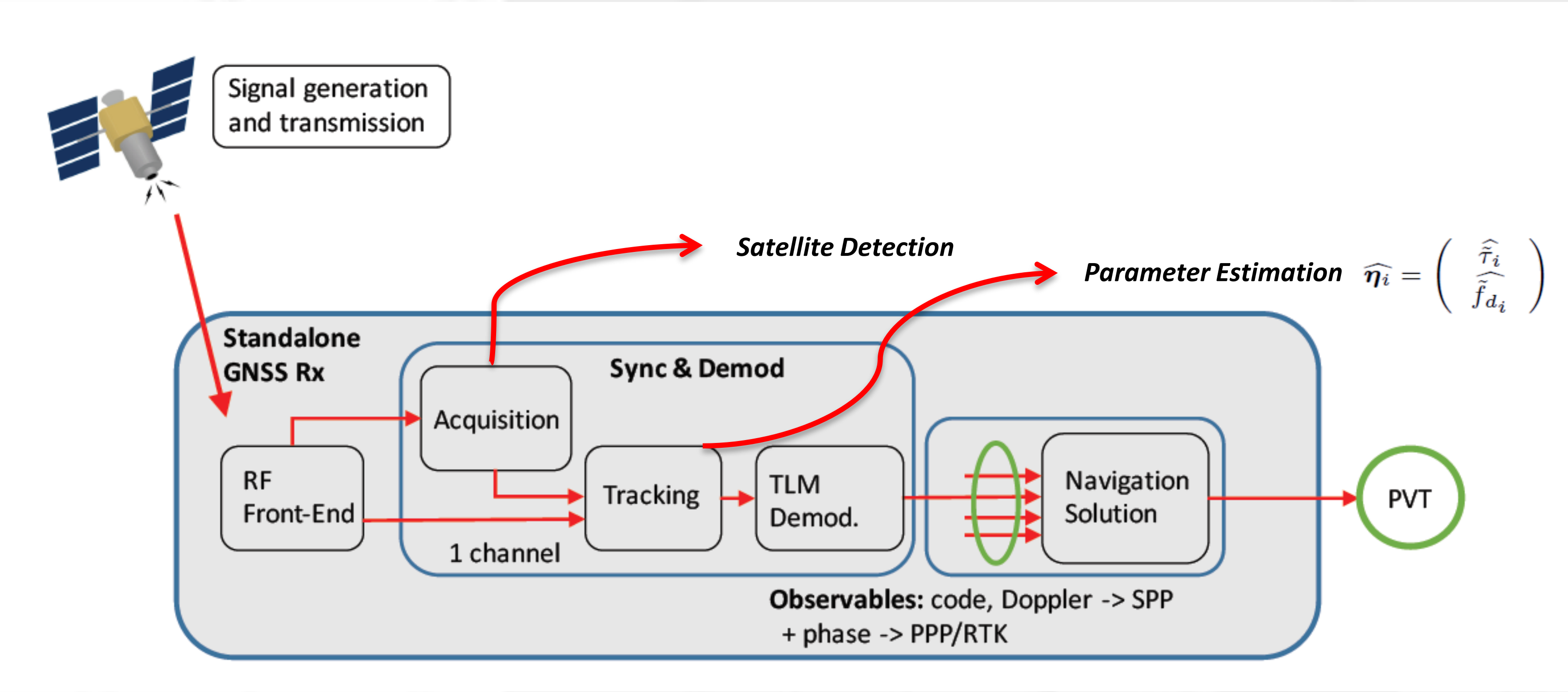

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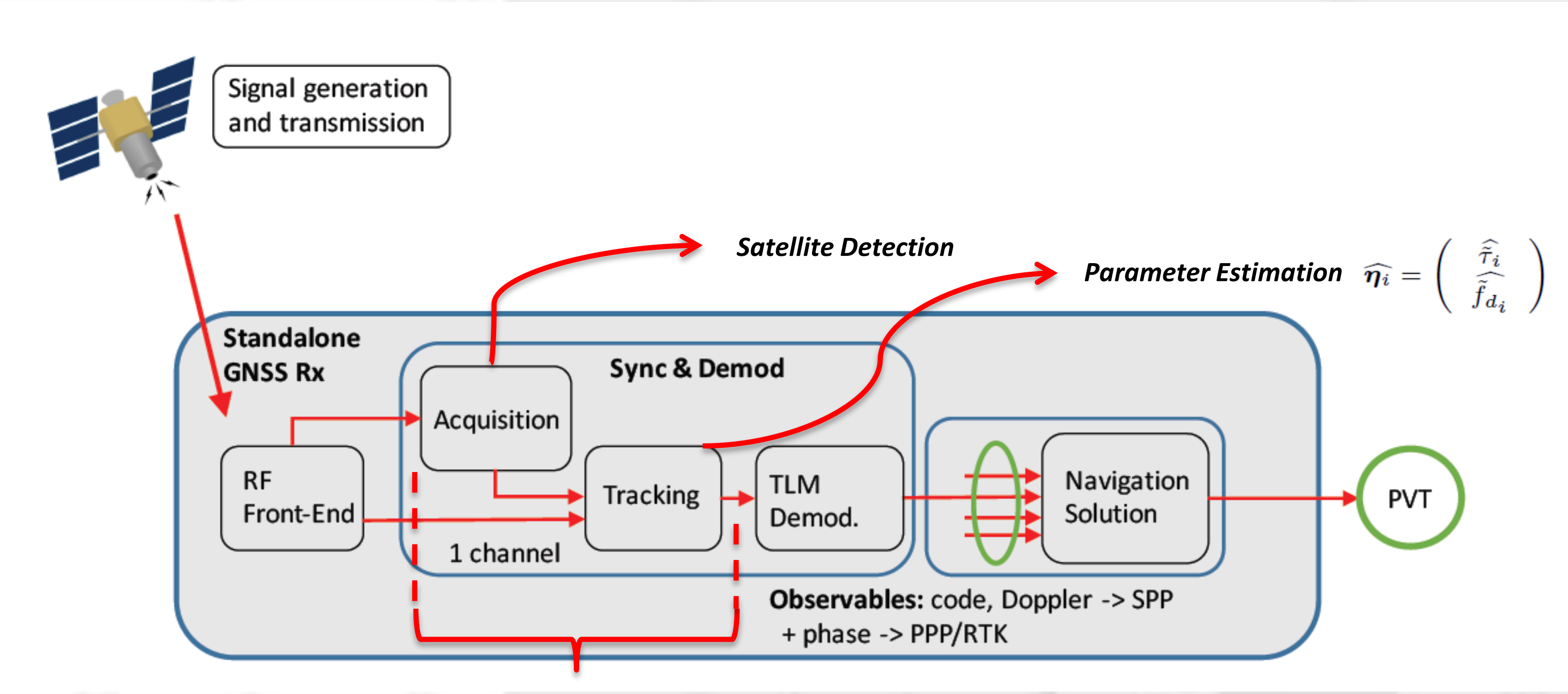

Two Steps Estimation Algorithm
to maximize the Autocorrelation function → Maximum Likelihood Approximation

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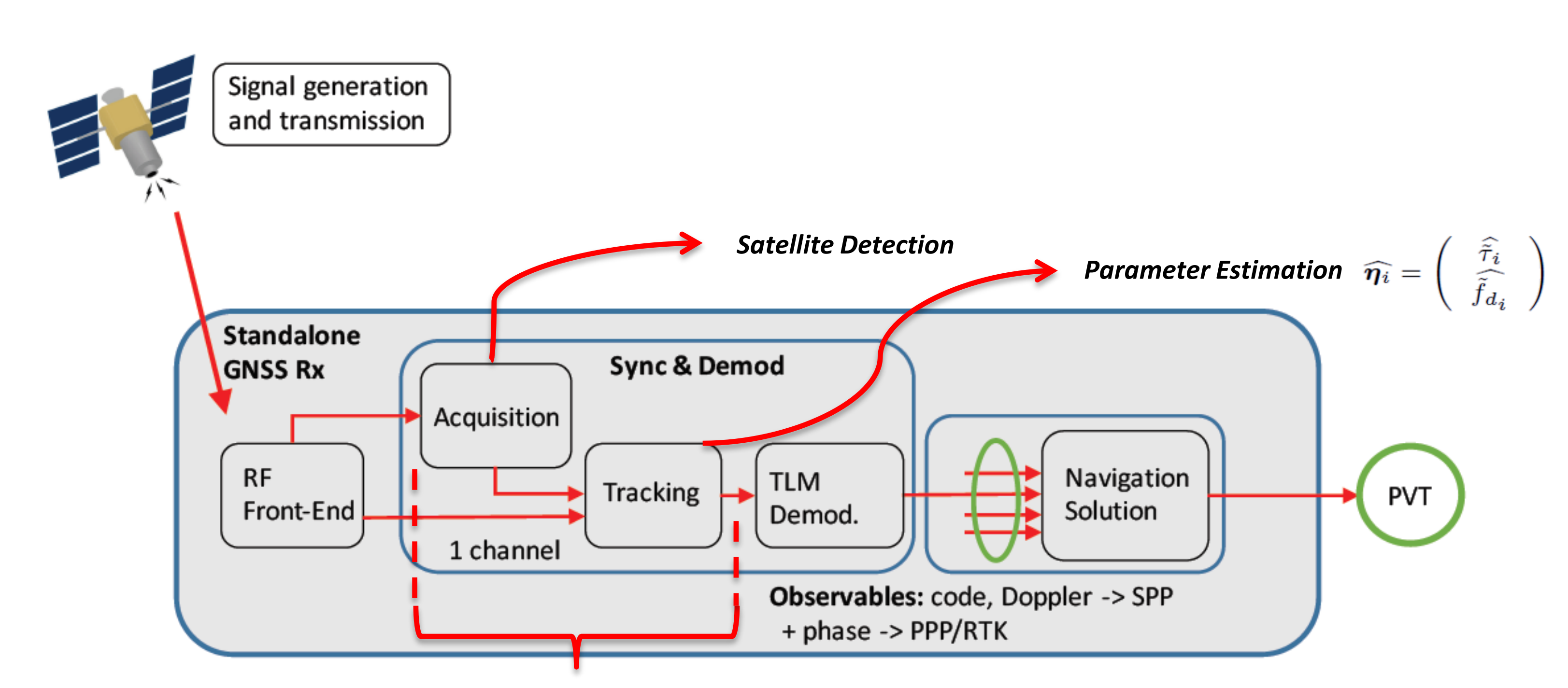

Two Steps Estimation Algorithm → Maximum Likelihood Approximation

Theoretical Estimation Limits → CRB

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Theoretical Estimation Limits → CRB

Why??

Some GNSS applications :

- Air navigation (aircrafts & drones), spacecrafts, autonomous cars, boats and ships
- Mining, precise agriculture
- Leisure (sailing, cycling, hiking, climbing), eHealth, search & rescue (SAR)
- Surveying, mapping, geophysics (ground movement, earthquake prediction, tsunami prediction)
- Archaeology, Earth observation (remote sensing - GNSS-R and GNSS-RO)
- Precise timing (synchronisation in power grids, seismology, communications nets)
- IoT, Big Data, augmented reality, smart cities

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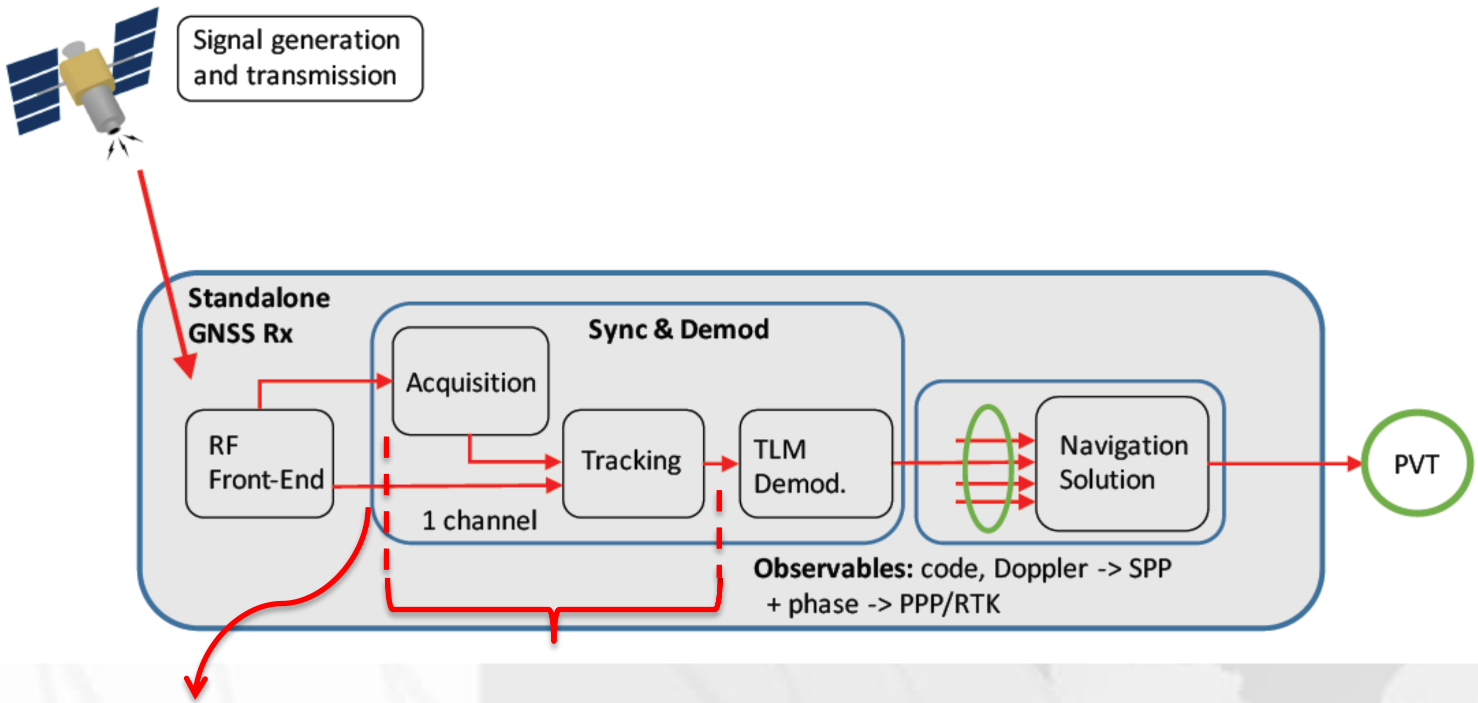

How is this signal?

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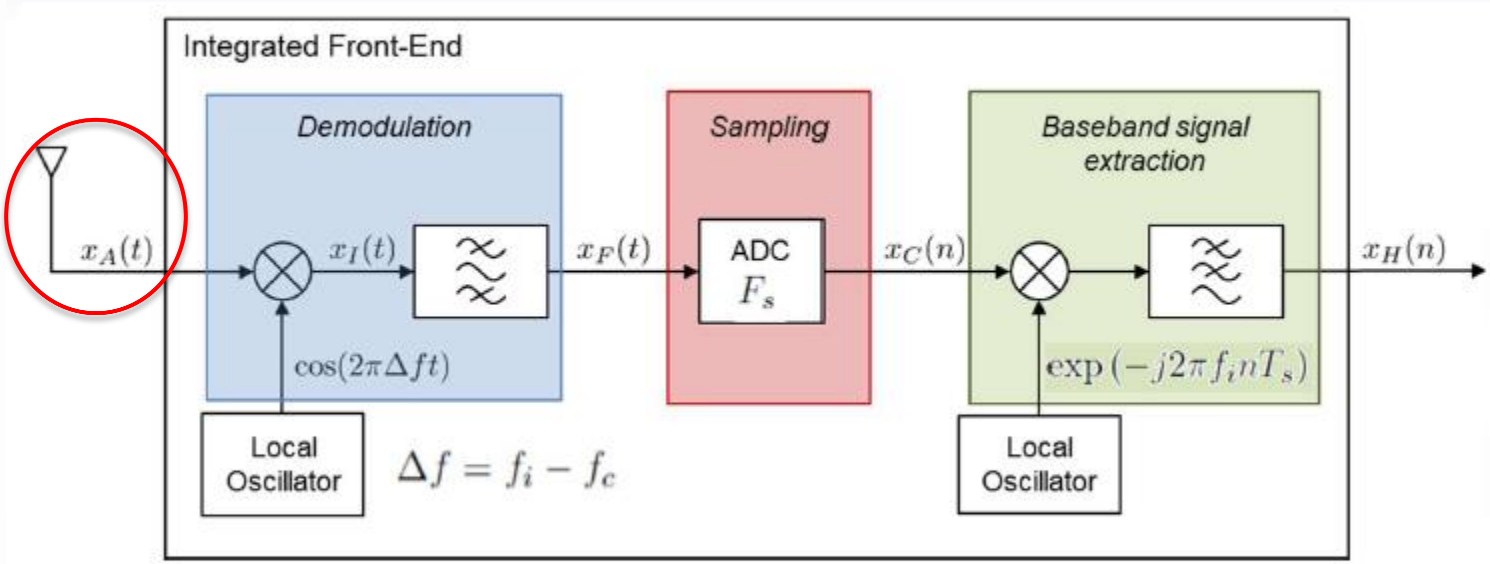
Global Navigation Satellite System



$$s_A(t) = s((1-b)(t-\tau))\, e^{j2\pi f_c (1-b) t} e^{-j2\pi f_c \tau}$$

Narrowband assumption

$$s_A(t) = s((1-b)(t-\tau))\, e^{j2\pi f_c (1-b) t} e^{-j2\pi f_c \tau}$$

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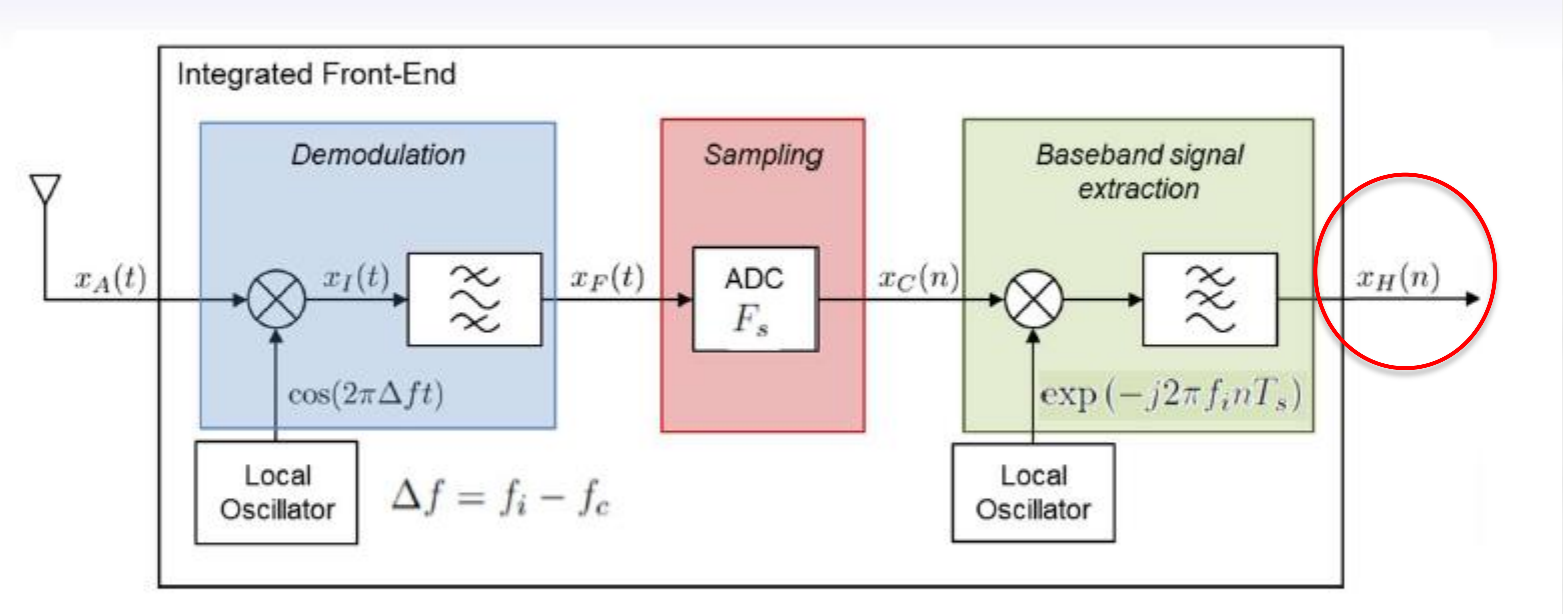

$$x(t) = \alpha s(t;\boldsymbol{\eta})\, e^{-j\omega_c b(t-\tau)} + n(t)$$

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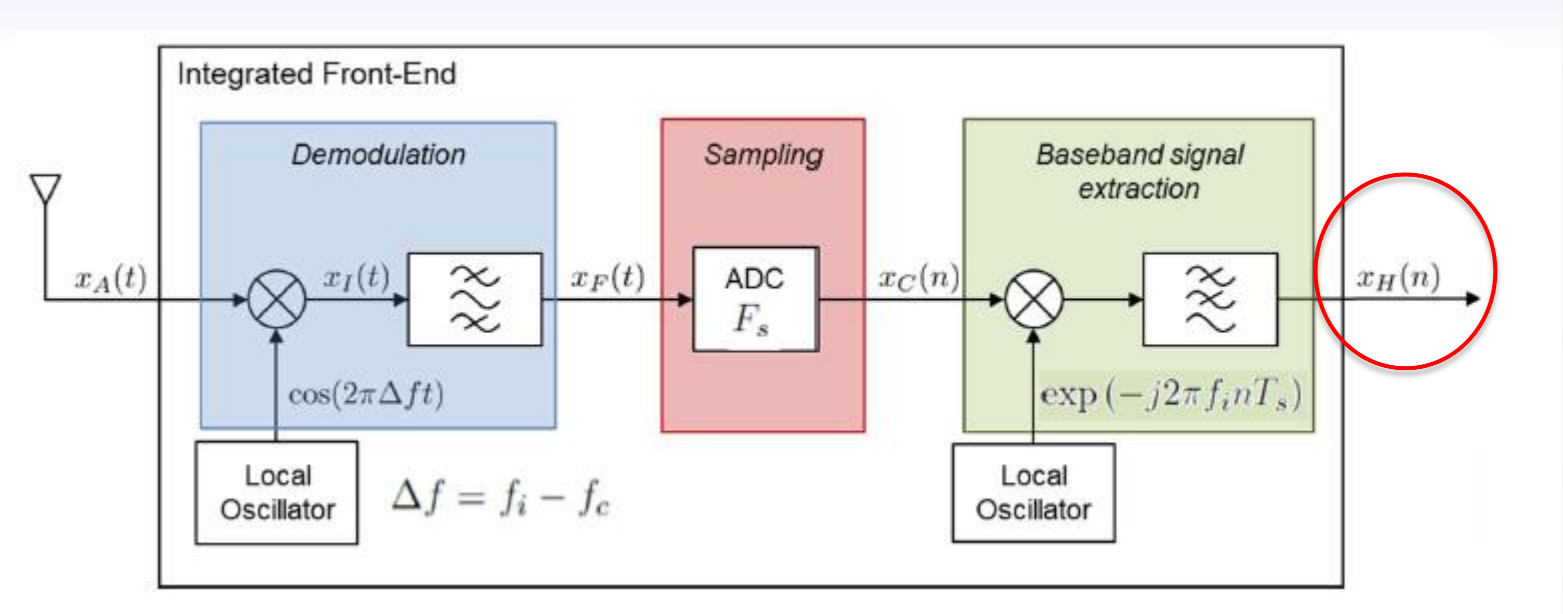
Global Navigation Satellite System



$$\mathbf{x} = \alpha \mathbf{a}(\boldsymbol{\eta}) + \mathbf{n},$$

$$\mathbf{x} = \left(x\left(N_1' T_s\right), \ldots, x\left(N_2' T_s\right)\right)^\top,$$

$$\mathbf{n} = \left(n\left(N_1' T_s\right), \ldots, n\left(N_2' T_s\right)\right)^\top,$$

$$\mathbf{s}(\boldsymbol{\eta}) = \left(s\left(N_1' T_s; \boldsymbol{\eta}\right), \ldots, s\left(N_2' T_s; \boldsymbol{\eta}\right)\right)^\top,$$

$$\mathbf{a}(\boldsymbol{\eta}) = \left((\mathbf{s}(\boldsymbol{\eta}))_1 e^{-j\omega_c b\left(N_1' T_s - \tau\right)}, \ldots, (\mathbf{s}(\boldsymbol{\eta}))_{N'} e^{-j\omega_c b\left(N_2' T_s - \tau\right)}\right)^\top$$

2-Theoretical Limits: Cramér-Rao Bound

$$\mathbf{x} = \mathbf{A}(\eta)\,\alpha + \mathbf{n},\ \ \mathbf{x}, \mathbf{n} \in \mathbb{C}^N,\ \ \mathbf{A}(\eta) \in \mathbb{C}^{N \times Q},\ \ \alpha \in \mathbb{C}^Q$$

Unknown deterministic parameter vector: $\eta \in \mathbb{R}^P.$

Signal Models:

- ***Conditional signal model (CSM)***
- ***Unconditional signal model (USM)***

Single Source CSM

$$\mathbf{x} = \mathbf{a}(\eta)\,\alpha + \mathbf{n},\ \ \mathbf{x}, \mathbf{n} \in \mathbb{C}^N,\ \ \mathbf{a}(\eta) \in \mathbb{C}^N,\ \ \alpha \in \mathbb{C}.$$

2-Theoretical Limits: Cramér-Rao Bound

Single Source CSM

$$\mathbf{x} = \mathbf{a}(\boldsymbol{\eta})\,\alpha + \mathbf{n},\ \mathbf{x},\mathbf{n} \in \mathbb{C}^N,\ \mathbf{a}(\boldsymbol{\eta}) \in \mathbb{C}^N,\ \alpha \in \mathbb{C}.$$

Re-parametrization

$$\mathbf{x} = \mathbf{a}(\boldsymbol{\eta})\,\rho e^{j\varphi} + \mathbf{w},\ \mathbf{x},\mathbf{w} \in \mathbb{C}^N,\ \mathbf{a}(\boldsymbol{\eta}) \in \mathbb{C}^N,\ \rho \in \mathbb{R}^+$$

Goal: compact CRB formula for the joint estimation

$$\boldsymbol{\epsilon}^\top = (\sigma_w^2, \rho, \varphi, \boldsymbol{\eta}^\top)$$

2-Theoretical Limits: Cramér-Rao Bound

$$\mathbf{x} = \mathbf{a}'(\boldsymbol{\theta})\,\rho + \mathbf{n},\ \ \mathbf{a}'(\boldsymbol{\theta}) = \mathbf{a}(\boldsymbol{\eta})\,e^{j\varphi},\ \ \boldsymbol{\theta}^T = \left(\varphi, \boldsymbol{\eta}^T\right),$$

$$CRB_\rho = \frac{\sigma_n^2}{2\left\|\mathbf{a}(\boldsymbol{\eta})\right\|^2} + \rho^2 \frac{\mathrm{Re}\left\{\mathbf{a}^H(\boldsymbol{\eta})\frac{\partial \mathbf{a}(\boldsymbol{\eta})}{\partial \boldsymbol{\eta}^\top}\right\}\mathbf{CRB}_{\boldsymbol{\eta}}\,\mathrm{Re}\left\{\mathbf{a}^H(\boldsymbol{\eta})\frac{\partial \mathbf{a}(\boldsymbol{\eta})}{\partial \boldsymbol{\eta}^\top}\right\}^\top}{\left\|\mathbf{a}(\boldsymbol{\eta})\right\|^4},$$

$$\mathbf{CRB}_{\boldsymbol{\theta}} = \begin{bmatrix} CRB_\varphi & \mathbf{CRB}_{\boldsymbol{\eta},\varphi}^\top \\ \mathbf{CRB}_{\boldsymbol{\eta},\varphi} & \mathbf{CRB}_{\boldsymbol{\eta}} \end{bmatrix},\ \boldsymbol{\theta}^\top = \left(\varphi, \boldsymbol{\eta}^\top\right),$$

$$CRB_\varphi = \frac{\sigma_n^2}{2\rho^2}\frac{1}{\left\|\mathbf{a}(\boldsymbol{\eta})\right\|^2} + \frac{\mathrm{Im}\left\{\mathbf{a}^H(\boldsymbol{\eta})\frac{\partial \mathbf{a}(\boldsymbol{\eta})}{\partial \boldsymbol{\eta}^\top}\right\}\mathbf{CRB}_{\boldsymbol{\eta}}\,\mathrm{Im}\left\{\mathbf{a}^H(\boldsymbol{\eta})\frac{\partial \mathbf{a}(\boldsymbol{\eta})}{\partial \boldsymbol{\eta}^\top}\right\}^\top}{\left\|\mathbf{a}(\boldsymbol{\eta})\right\|^4},$$

$$\mathbf{CRB}_{\boldsymbol{\eta},\varphi} = -\mathbf{CRB}_{\boldsymbol{\eta}}\frac{\mathrm{Im}\left\{\mathbf{a}^H(\boldsymbol{\eta})\frac{\partial \mathbf{a}(\boldsymbol{\eta})}{\partial \boldsymbol{\eta}^\top}\right\}^\top}{\left\|\mathbf{a}(\boldsymbol{\eta})\right\|^2}.$$

$$\mathbf{CRB}_{\boldsymbol{\eta}} = \frac{\sigma_n^2}{2\rho^2}\,\mathrm{Re}\left\{\left(\frac{\partial \mathbf{a}(\boldsymbol{\eta})}{\partial \boldsymbol{\eta}^T}\right)^H \boldsymbol{\Pi}_{\mathbf{a}(\boldsymbol{\eta})}^{\perp}\frac{\partial \mathbf{a}(\boldsymbol{\eta})}{\partial \boldsymbol{\eta}^T}\right\}^{-1},$$

“Compact CRB for Delay, Doppler and Phase Estimation – Application to GNSS SPP & RTK Performance Characterization”, **IET Radar, Sonar & Navigation**, vol. 14, no. 10, pp. 1537-1549, September 2020.

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2-Theoretical Limits: Cramér-Rao Bound

Cramér-Rao Bound for Band-limited Signals/ Narrowband approx

$$s(t) = c(t)$$

$$c(t) = \sum_{n=N_1}^{N_2} c(nT_s)\,\mathrm{sinc}\left(\pi F_s (t - nT_s)\right) \rightleftharpoons$$

$$c(f) = \left(T_s \sum_{n=N_1}^{N_2} c(nT_s)\, e^{-j2\pi n T_s}\right) 1_{\left[-\frac{B}{2}, \frac{B}{2}\right]}(f),$$

$$\mathrm{CRB}_{\boldsymbol{\eta}} = \frac{1}{2\mathrm{SNR}_{\mathrm{out}}} \boldsymbol{\Delta}_{\boldsymbol{\eta}}^{-1},$$

$$[\boldsymbol{\Delta}_{\boldsymbol{\eta}}]_{1,1} = F_s^2 \left(\frac{\mathbf{c}^H \mathbf{V} \mathbf{c}}{\mathbf{c}^H \mathbf{c}} - \left| \frac{\mathbf{c}^H \boldsymbol{\Lambda} \mathbf{c}}{\mathbf{c}^H \mathbf{c}} \right|^2 \right)$$

$$[\boldsymbol{\Delta}_{\boldsymbol{\eta}}]_{2,2} = \frac{\omega_c^2}{F_s^2} \left(\frac{\mathbf{c}^H \mathbf{D}^2 \mathbf{c}}{\mathbf{c}^H \mathbf{c}} - \left(\frac{\mathbf{c}^H \mathbf{D} \mathbf{c}}{\mathbf{c}^H \mathbf{c}} \right)^2 \right)$$

$$[\boldsymbol{\Delta}_{\boldsymbol{\eta}}]_{1,2} = [\boldsymbol{\Delta}_{\boldsymbol{\eta}}]_{2,1} = \omega_c \,\mathrm{Im} \left\{ \frac{\mathbf{c}^H \mathbf{D} \boldsymbol{\Lambda} \mathbf{c}}{\mathbf{c}^H \mathbf{c}} - \frac{\mathbf{c}^H \mathbf{D} \mathbf{c}}{\mathbf{c}^H \mathbf{c}} \frac{\mathbf{c}^H \boldsymbol{\Lambda} \mathbf{c}}{\mathbf{c}^H \mathbf{c}} \right\}$$

$$CRB_{\varphi} = \frac{1}{2\mathrm{SNR}_{\mathrm{out}}} + \begin{pmatrix} F_s \,\mathrm{Im}\left\{ \frac{\mathbf{c}^H \boldsymbol{\Lambda} \mathbf{c}}{\mathbf{c}^H \mathbf{c}} \right\} - b\omega_c \\ \frac{\omega_c}{F_s} \frac{\mathbf{c}^H \mathbf{D} \mathbf{c}}{\mathbf{c}^H \mathbf{c}} \end{pmatrix}^{\top} \times \mathrm{CRB}_{\boldsymbol{\eta}} \begin{pmatrix} F_s \,\mathrm{Im}\left\{ \frac{\mathbf{c}^H \boldsymbol{\Lambda} \mathbf{c}}{\mathbf{c}^H \mathbf{c}} \right\} - b\omega_c \\ \frac{\omega_c}{F_s} \frac{\mathbf{c}^H \mathbf{D} \mathbf{c}}{\mathbf{c}^H \mathbf{c}} \end{pmatrix},$$

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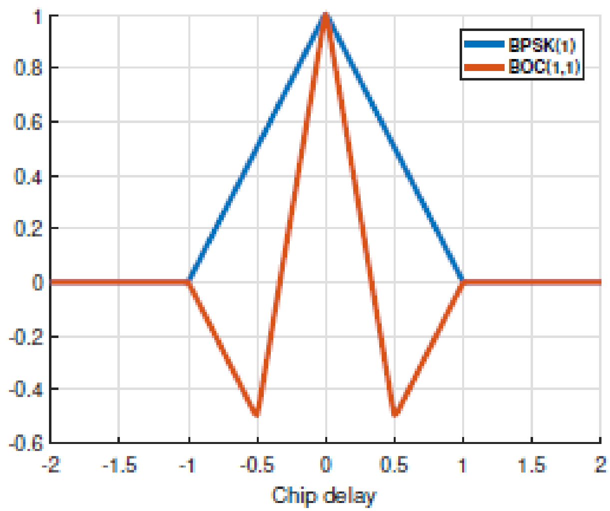
2-Theoretical Limits: GPS C/A signal

Time-Delay Estimation of the GPS C/A signal

$$ACF(t) = \int_{-\frac{B_r}{2}}^{\frac{B_r}{2}} G_s(f) e^{-j2\pi f t} df.$$

$$G_{BPSK}(f_c) = f_c \frac{\sin^2\left(\frac{\pi f}{f_c}\right)}{(\pi f)^2}.$$



The PRN correlation is assume ideal !!

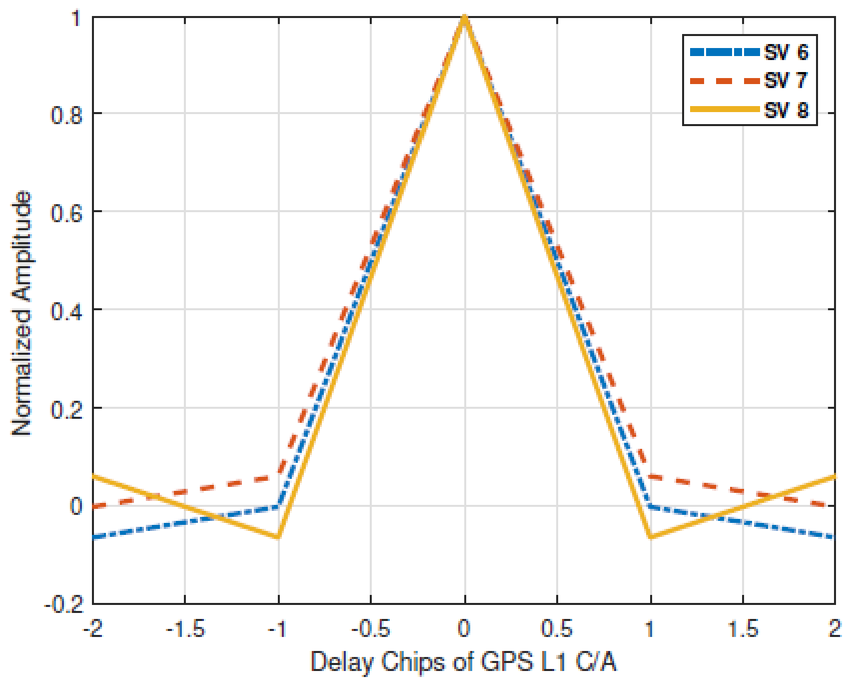
$$R_{c_i} \approx \delta(m)$$

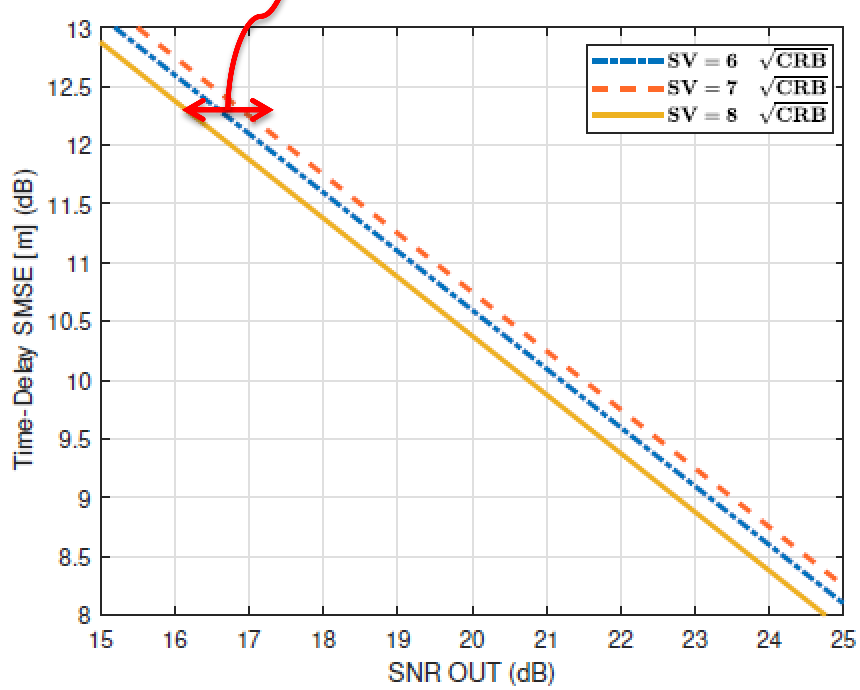
$$B_{Gabor} = \sqrt{\int_{-\frac{B_r}{2}}^{\frac{B_r}{2}} f^2 G_s(f) df}.$$

2-Theoretical Limits: *GPS C/A signal*

Time-Delay Estimation of the GPS C/A signal

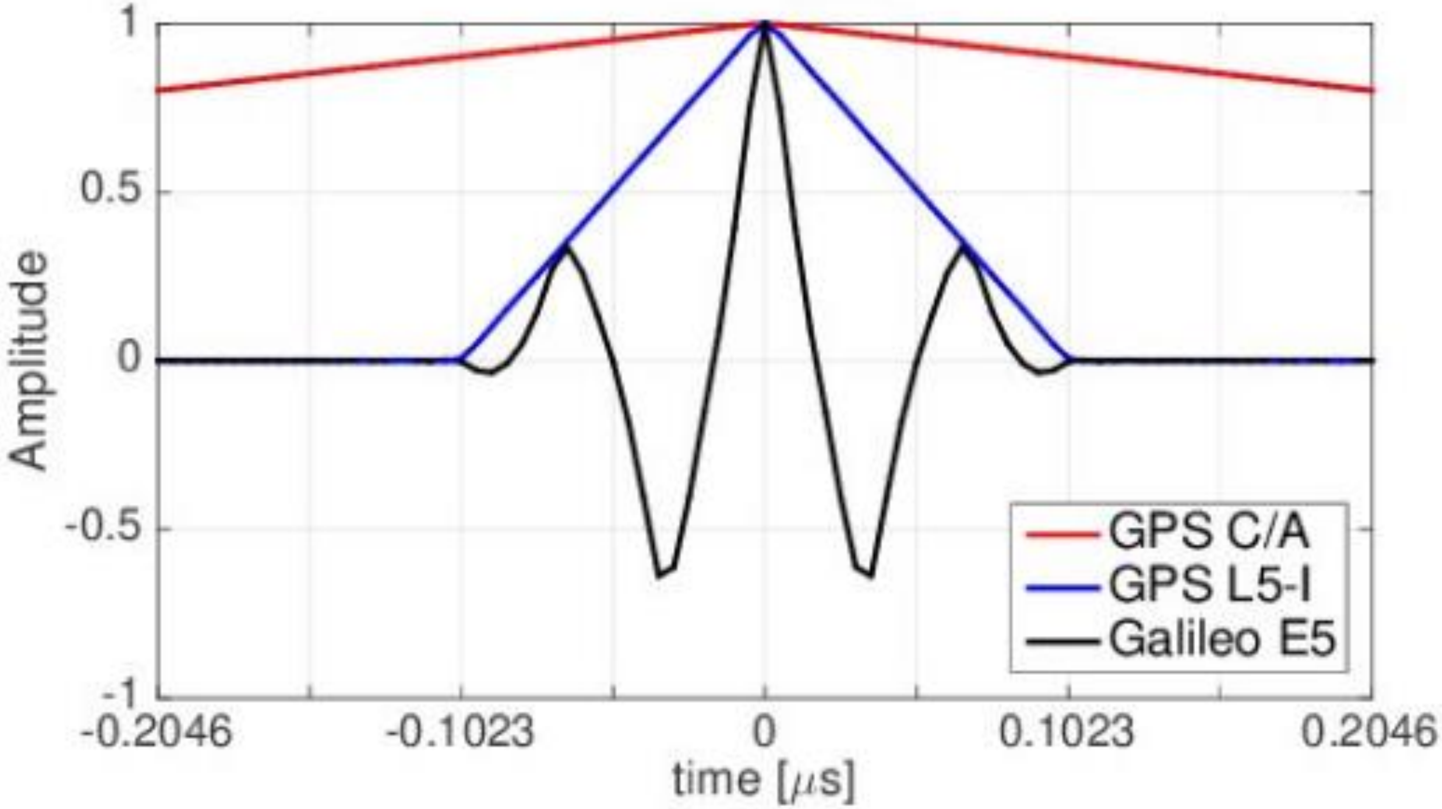
Extra of energy around 15%





"On the Time-Delay Estimation Performance Limit of New GNSS Acquisition Codes", in the Proceedings of the International Conference on Localization and GNSS (ICL-GNSS '20), 2-4 June 2020, Tampere, Finland

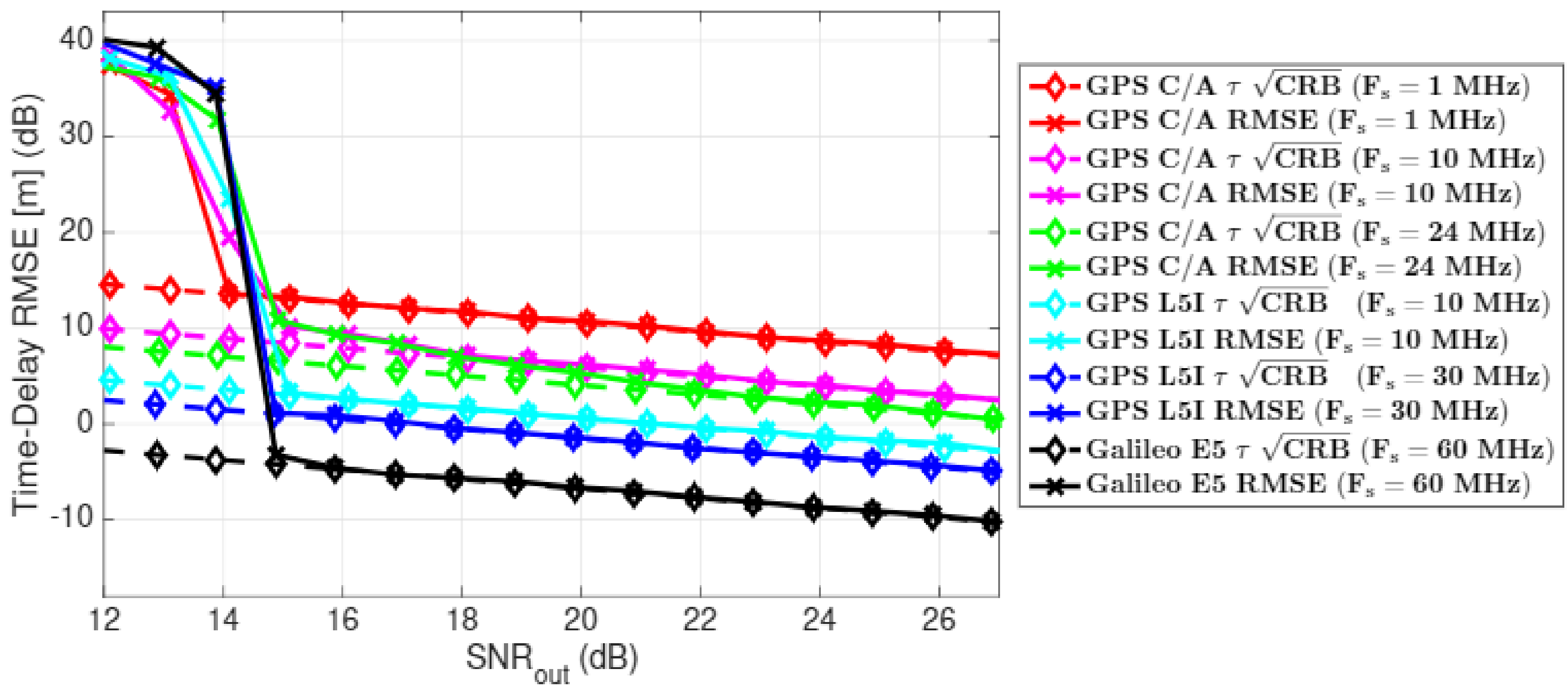
2-Theoretical Limits: GNSS Signals



2-Theoretical Limits: GNSS Signals

Maximum Likelihood Estimation

$$\hat{\tau} = \arg\max_{\tau} \left\{ \left| \left(\mathbf{c}(\tau)^H \mathbf{c}(\tau) \right)^{-1} \mathbf{c}(\tau)^H \mathbf{x} \right|^2 \right\},$$

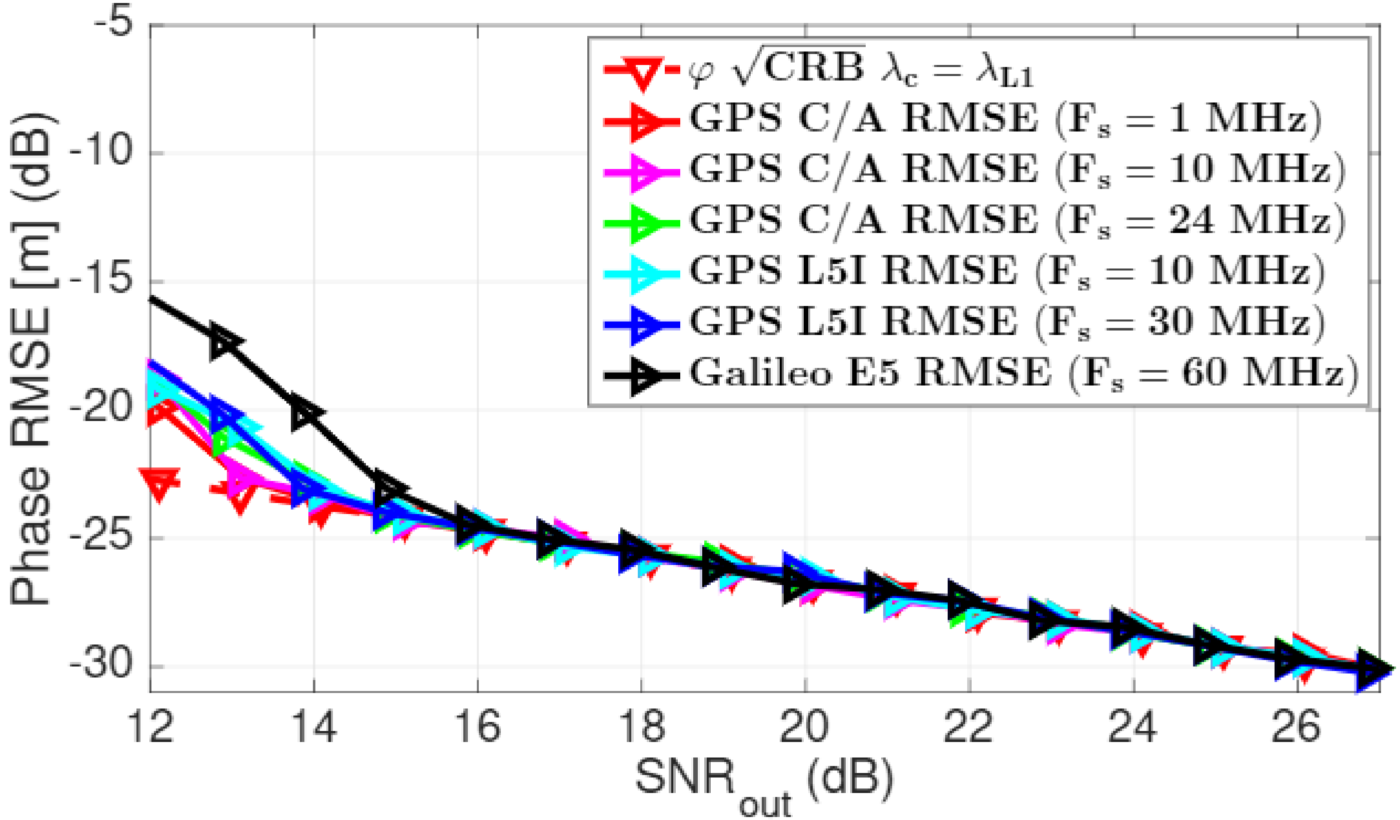


"Performance Limits of GNSS Code-based Precise Positioning : GPS, Galileo & Meta-Signals", **Sensors**, 20 (8), 2196, April 2020.

2-Theoretical Limits: GNSS Signals

Maximum Likelihood Estimation

$$\widehat{\varphi}\left(\widehat{\tau}\right) = \arg\left\{\left(\mathbf{c}\left(\widehat{\tau}\right)^H \mathbf{c}\left(\widehat{\tau}\right)\right)^{-1} \mathbf{c}\left(\widehat{\tau}\right)^H \mathbf{x}\right\},$$



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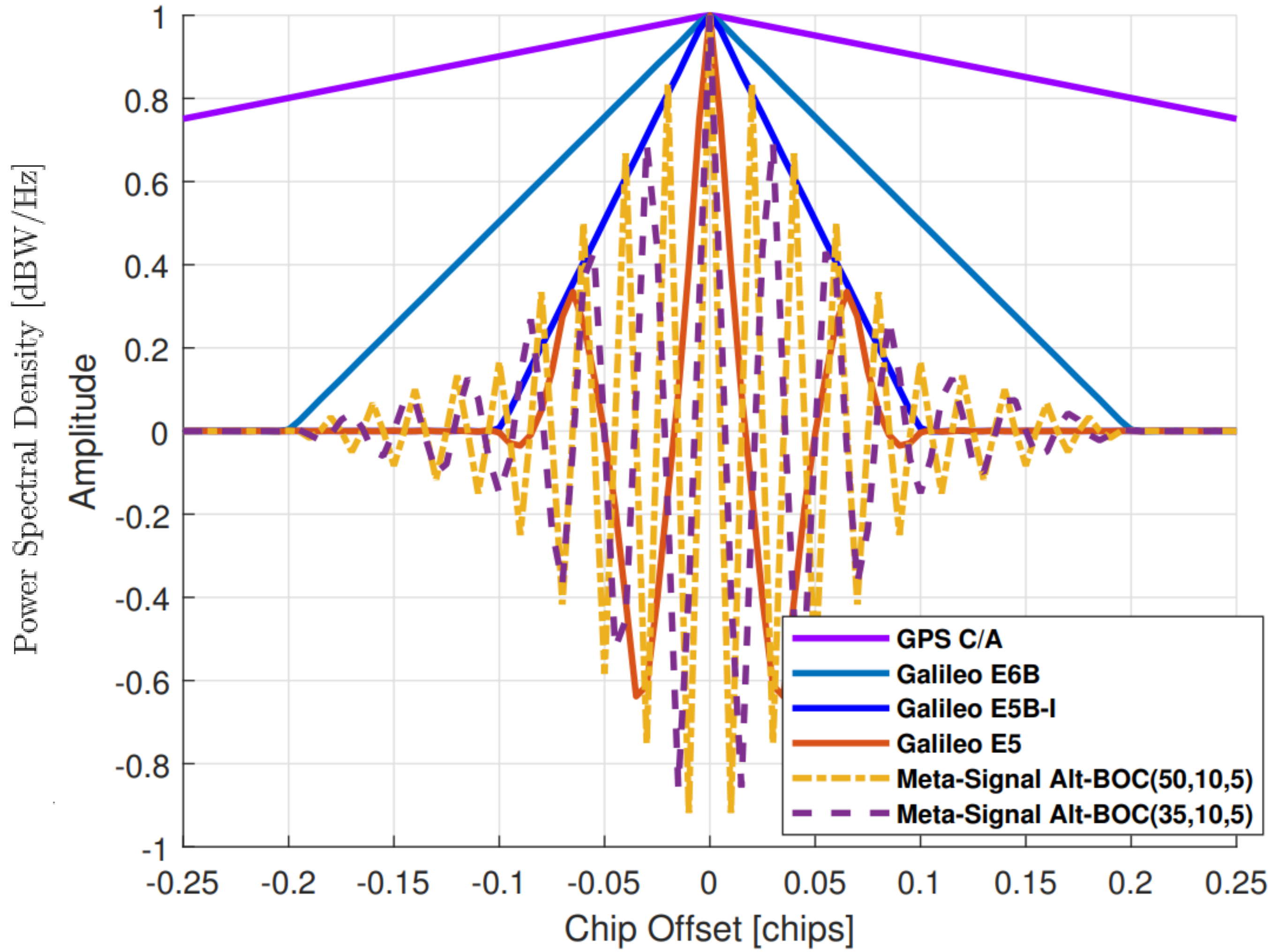
2-Theoretical Limits: GNSS to GNSS Meta-Signals



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2-Theoretical Limits: GNSS Meta-Signals

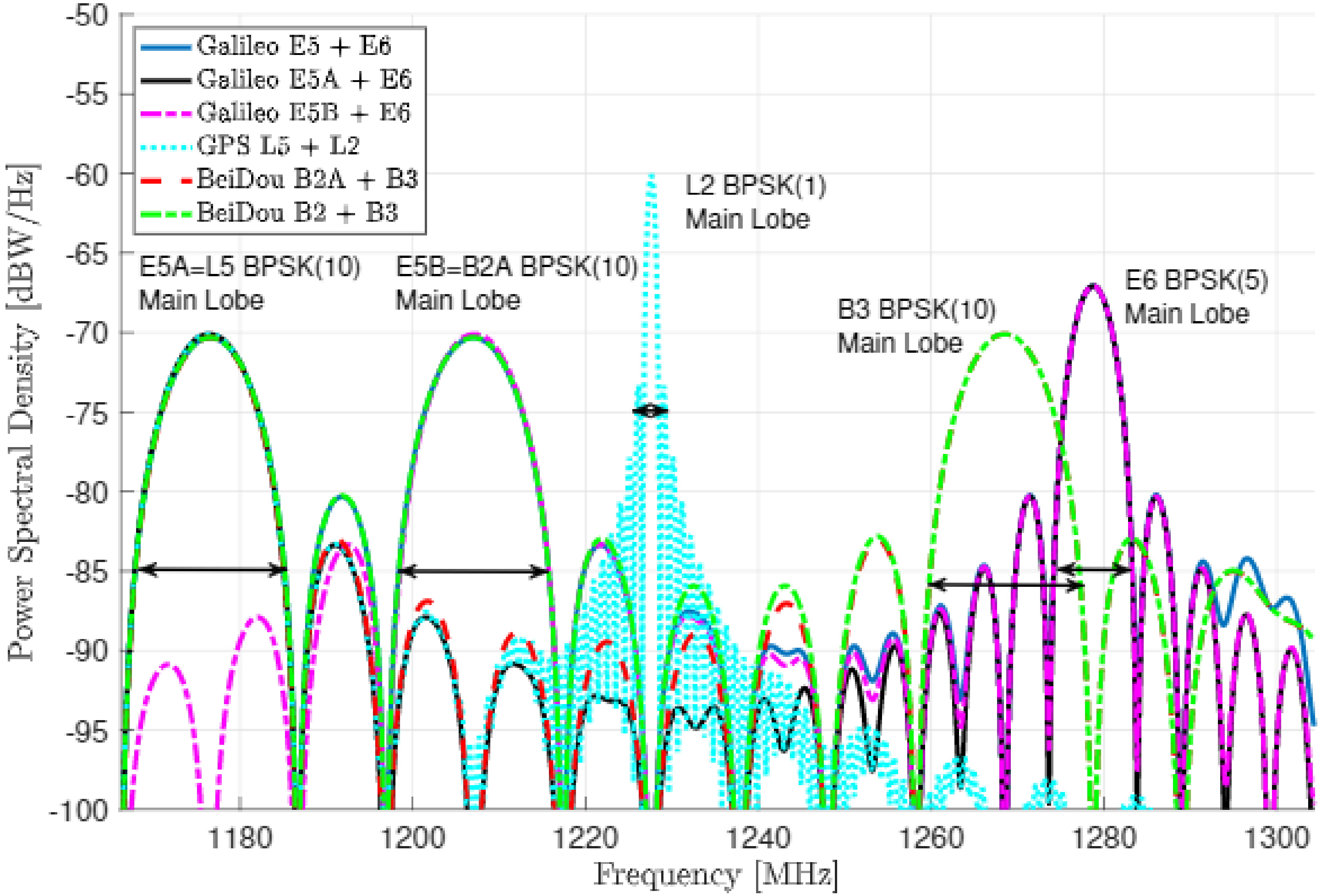
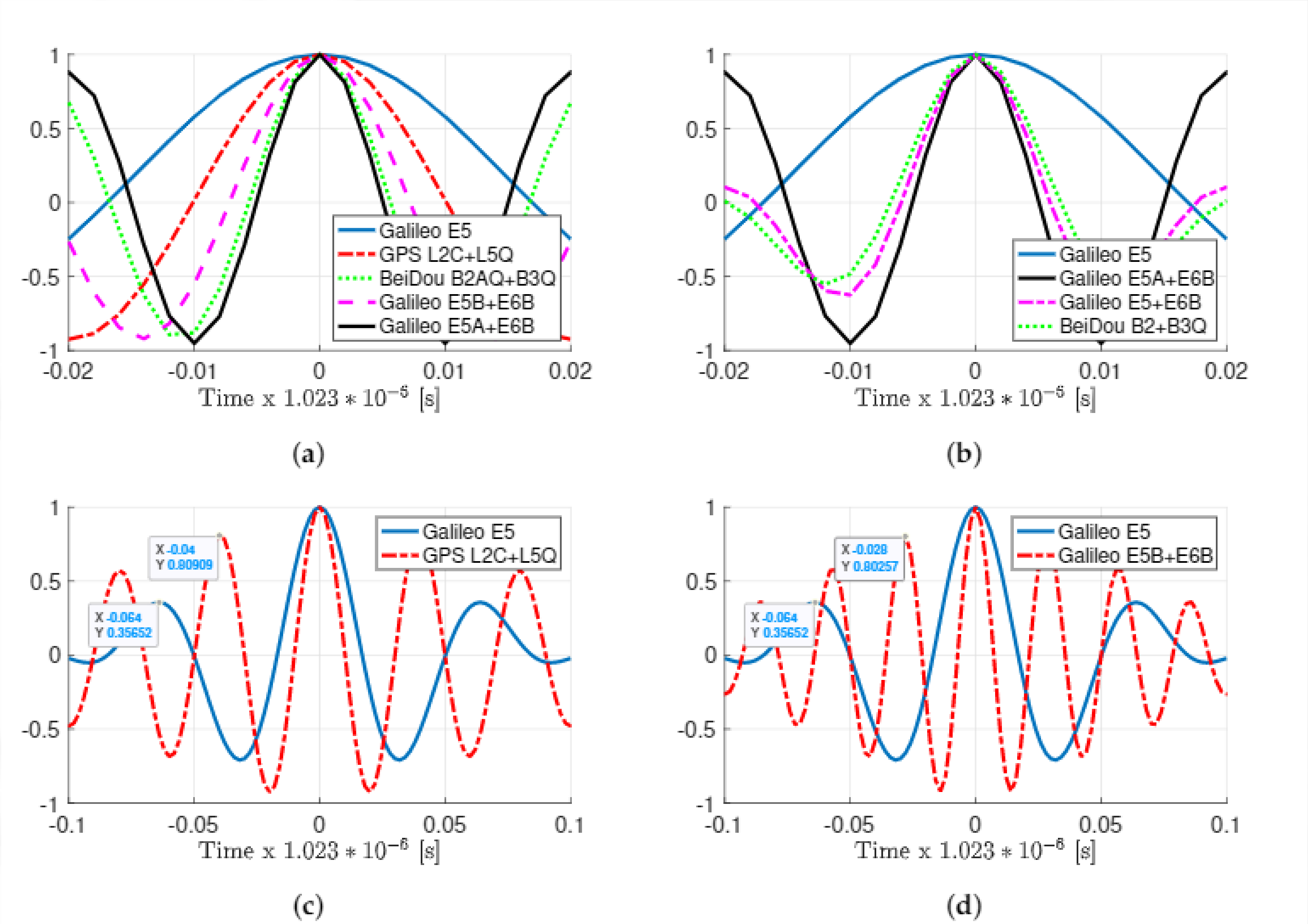


Figure 1. PSD for the different GNSS meta-signals.

“Positioning Performance Limits of GNSS Meta-Signals and HO-BOC Signals”, **Sensors**, 20 (12), 3586, June 2020.

2-Theoretical Limits: GNSS Meta-Signals



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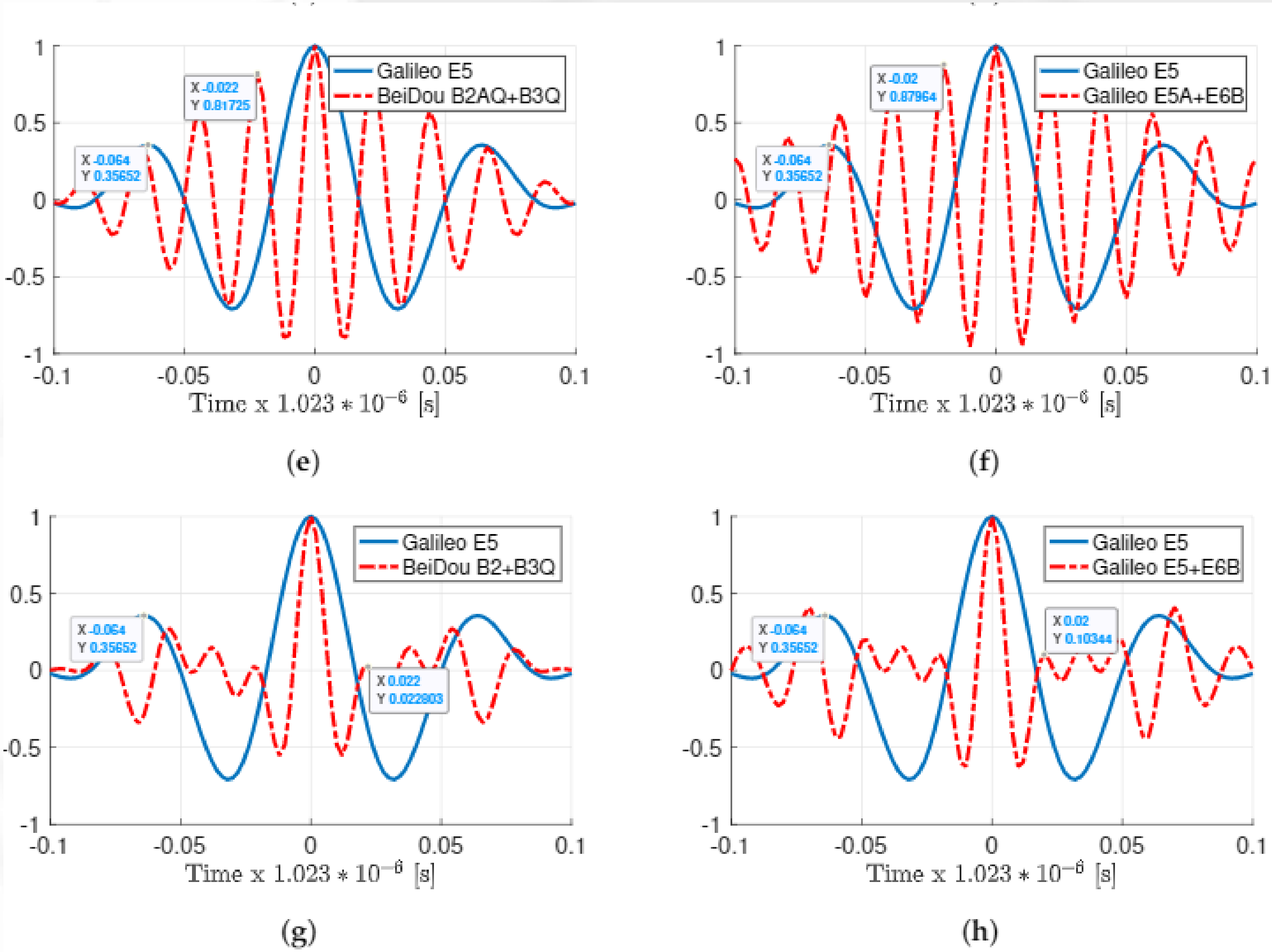

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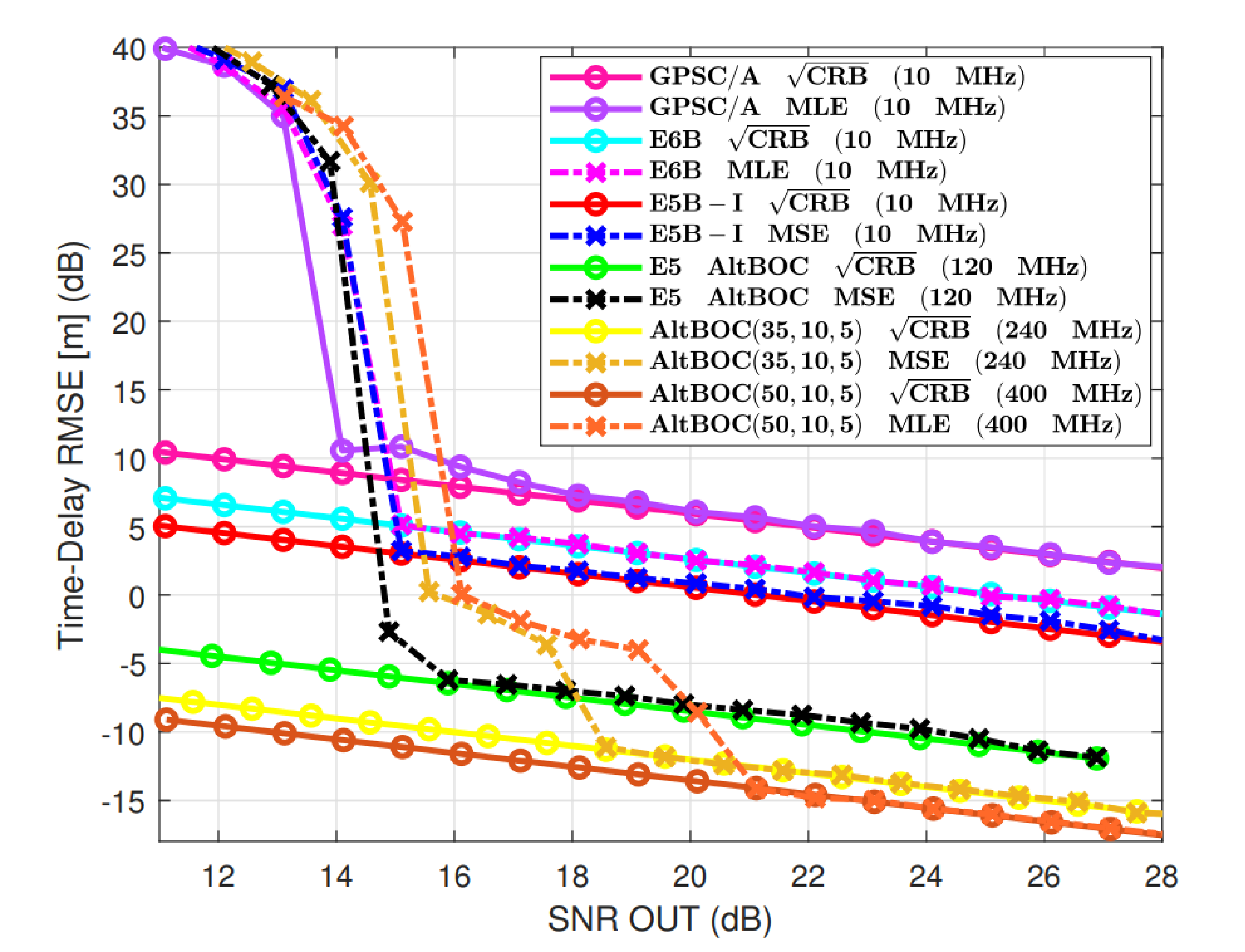
2-Theoretical Limits: GNSS Meta-Signals



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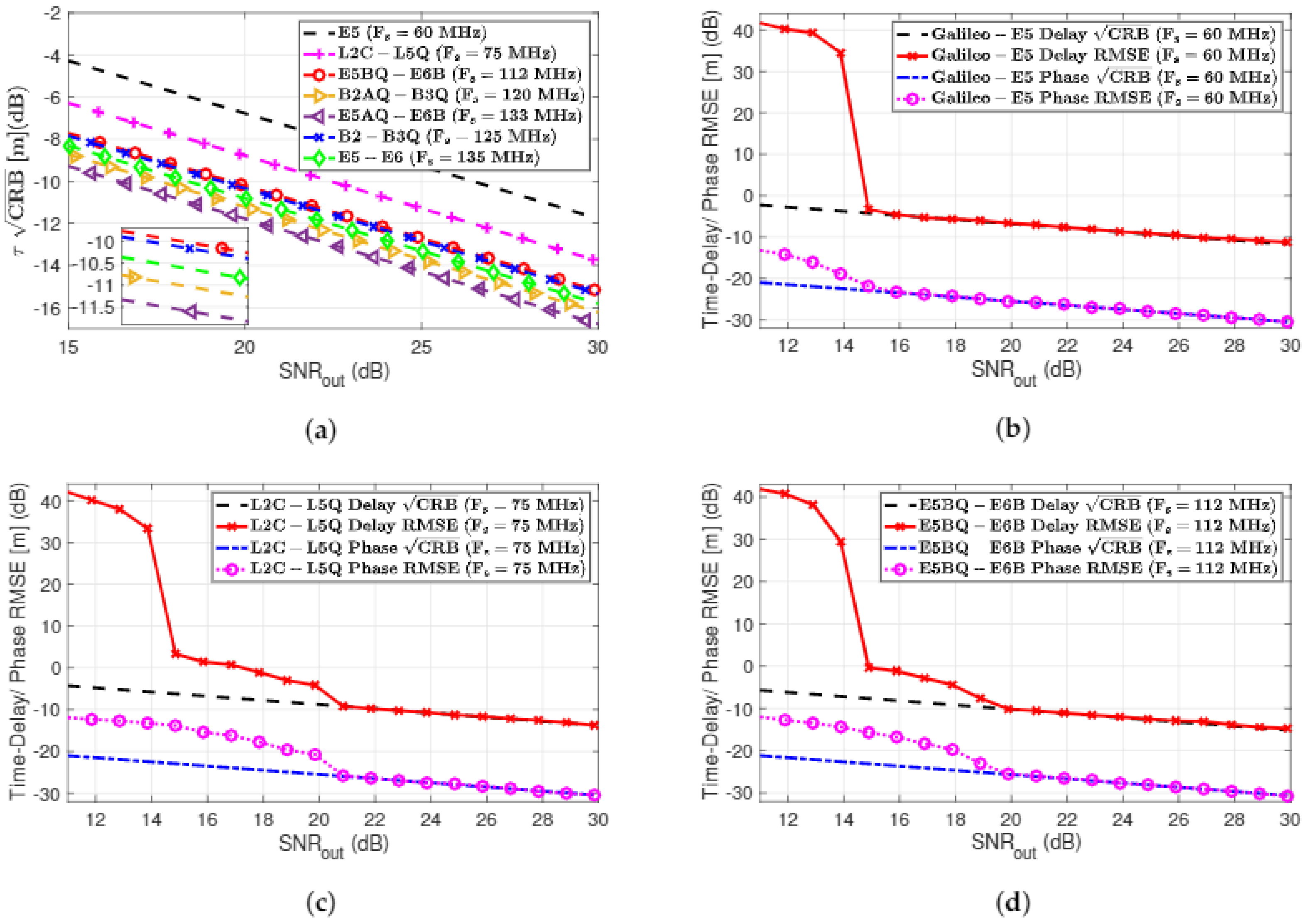
2-Theoretical Limits: GNSS Meta-Signals



(a) (b)

(c) (d)

“Positioning Performance Limits of GNSS Meta-Signals and HO-BOC Signals”, **Sensors**, 20 (12), 3586, June 2020.

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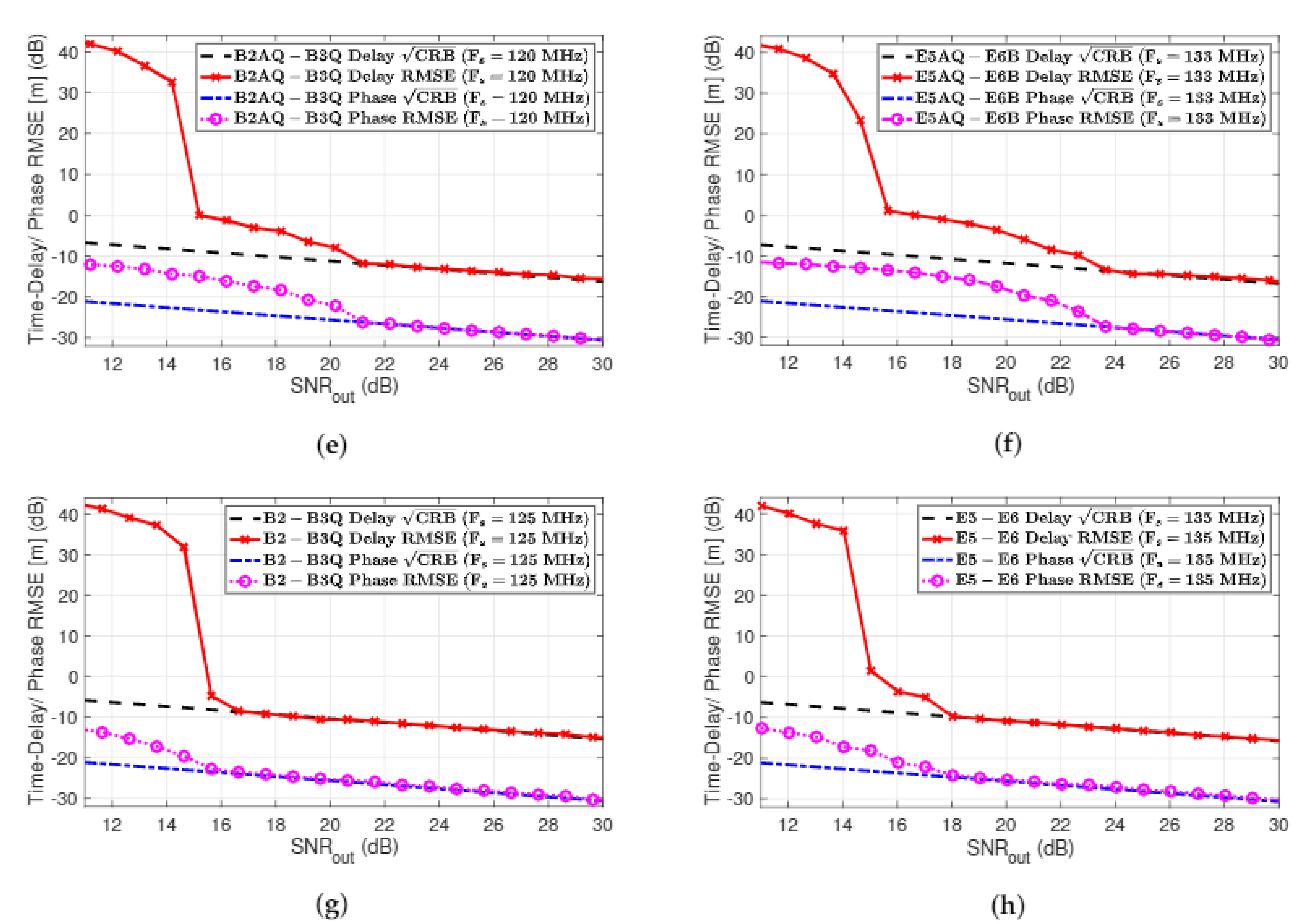
2-Theoretical Limits: GNSS Meta-Signals



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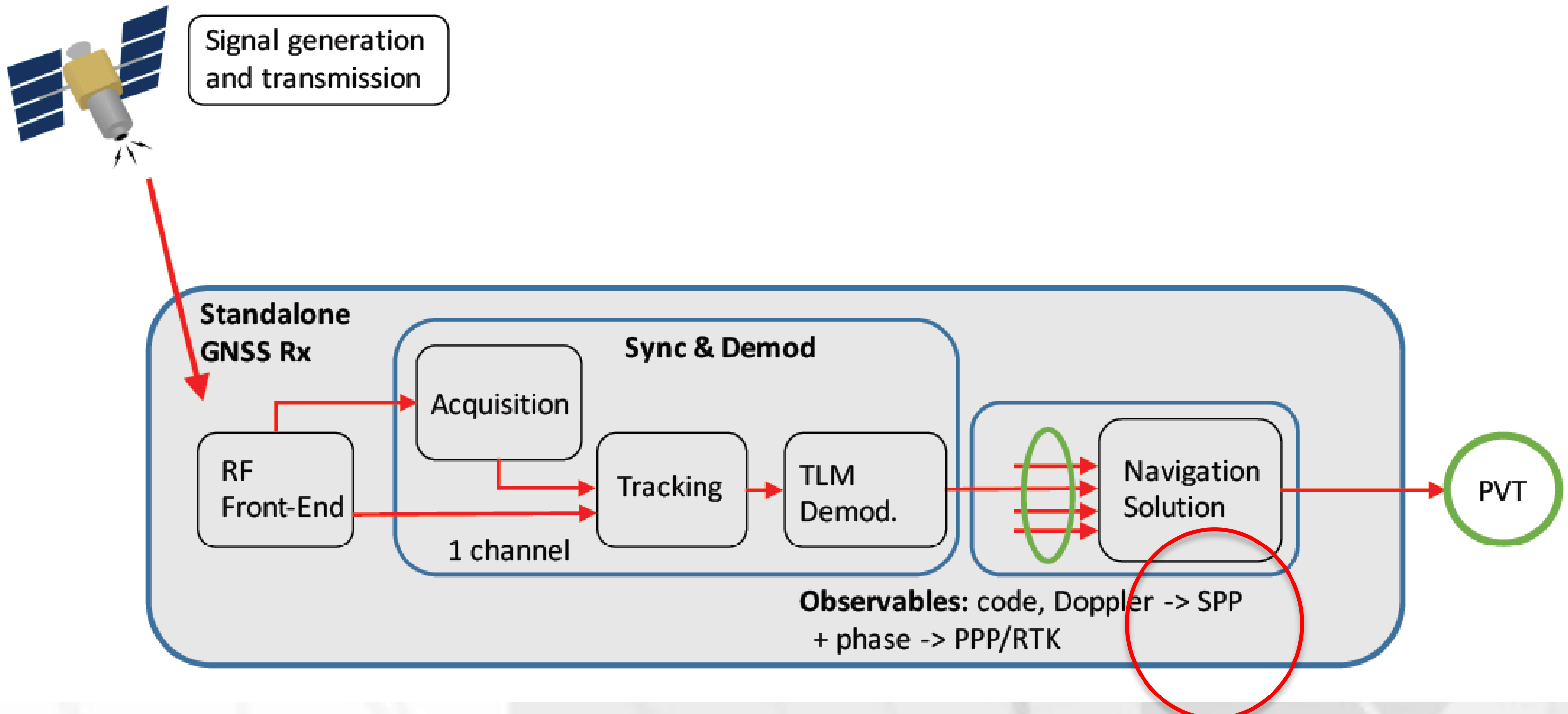
"Positioning Performance Limits of GNSS Meta-Signals and HO-BOC Signals", **Sensors**, 20 (12), 3586, June 2020.

2-Theoretical Limits: GNSS SPP Solution



$$\hat{\rho}_i = c\hat{\tau}_i = \rho_i(\mathbf{p}_R) + c\,(\delta t_r - \delta t_i) + \epsilon_i$$

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2-Theoretical Limits: GNSS SPP Solution

$$\hat{\rho}_i = c\hat{\tau}_i = \rho_i(\mathbf{p}_R) + c\,(\delta t_r - \delta t_i) + \epsilon_i$$

$$y_i = \hat{\rho}_i + c\delta t_i - \epsilon_i^{iono} - \epsilon_i^{tropo} = ||\mathbf{p}_{T_i} - \mathbf{p}_R|| + c\delta t_r$$

$$\hat{\rho}_i + c\delta t_i - \epsilon_i^{iono} - \epsilon_i^{tropo} \approx \rho_i(\mathbf{p}^0) - \mathbf{u}_i(\mathbf{p}^0)\boldsymbol{\delta}_p + \varepsilon_i$$

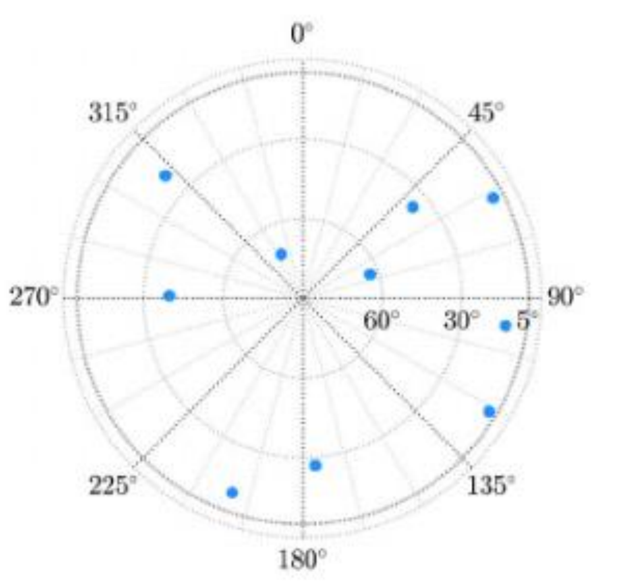
$$\mathbf{y} = \mathbf{H}\boldsymbol{\delta} + \boldsymbol{\varepsilon}$$

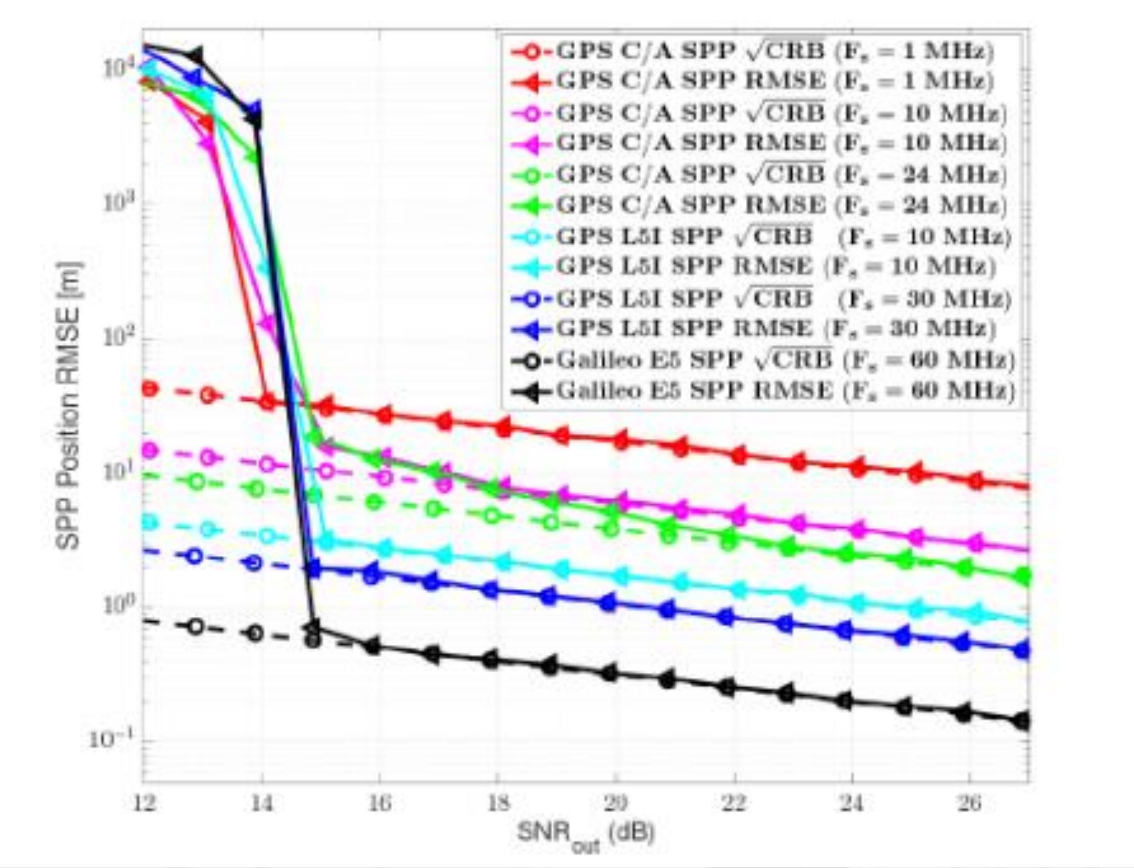
Weighted LS (WLS) problem :

$$\hat{\boldsymbol{\delta}}_{WLS} = \arg\min_{\boldsymbol{\delta}}\{||\mathbf{y} - \mathbf{H}\boldsymbol{\delta}||^2_{\mathbf{W}}\} = \arg\min_{\boldsymbol{\delta}}\{(\mathbf{y} - \mathbf{H}\boldsymbol{\delta})^T\mathbf{W}(\mathbf{y} - \mathbf{H}\boldsymbol{\delta})\}$$

$$\begin{pmatrix} \hat{\mathbf{p}}_R \\ \widehat{c\delta t_r} \end{pmatrix} = \begin{pmatrix} \mathbf{p}^j \\ 0 \end{pmatrix} + \left(\mathbf{H}^T\mathbf{W}\mathbf{H}\right)^{-1}\mathbf{H}^T\mathbf{W}\mathbf{y}$$

2-Theoretical Limits: GNSS SPP Solution





"Compact CRB for Delay, Doppler and Phase Estimation – Application to GNSS SPP & RTK Performance Characterization", **IET Radar, Sonar & Navigation**, vol. 14, no. 10, pp. 1537-1549, September 2020.

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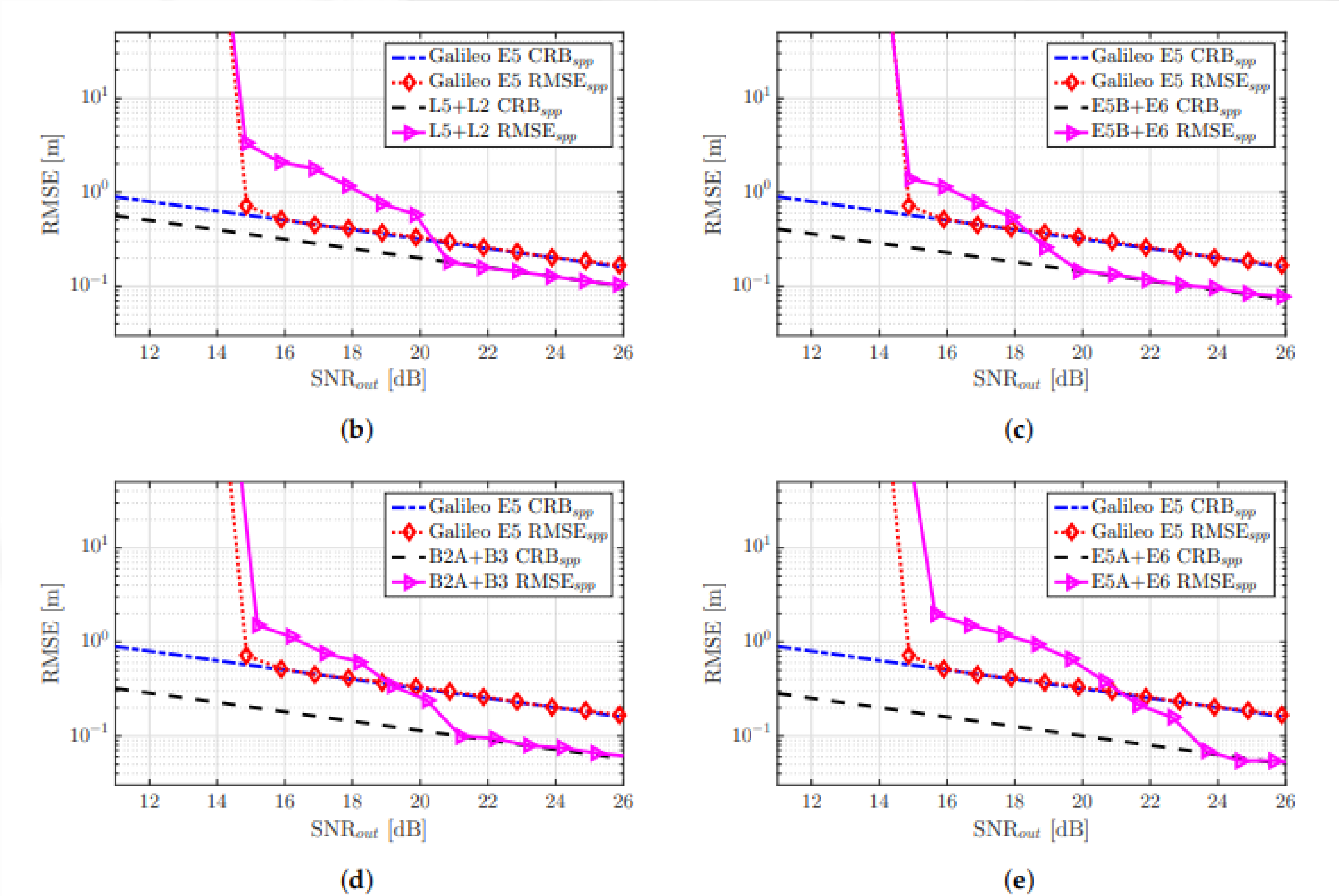

"Positioning Performance Limits of GNSS Meta-Signals and HO-BOC Signals", **Sensors**, 20 (12), 3586, June 2020.

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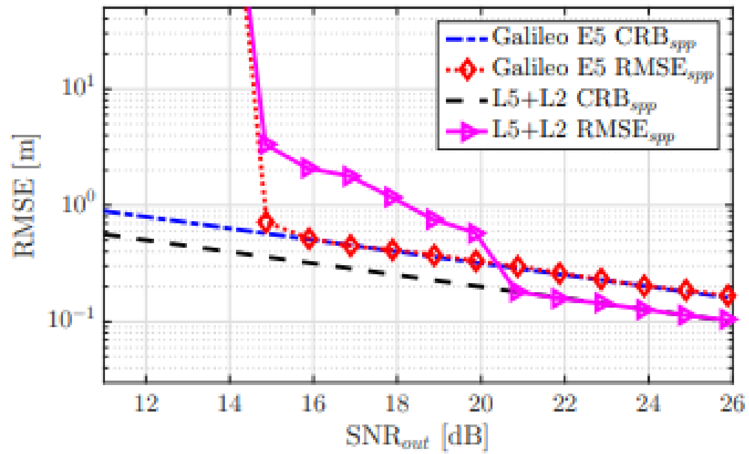
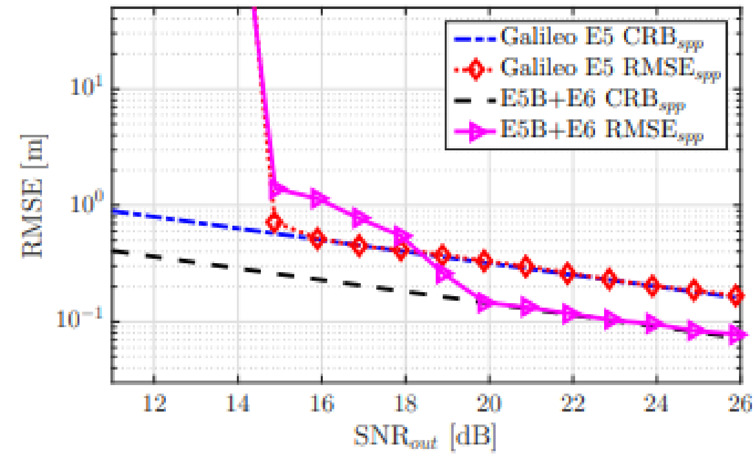
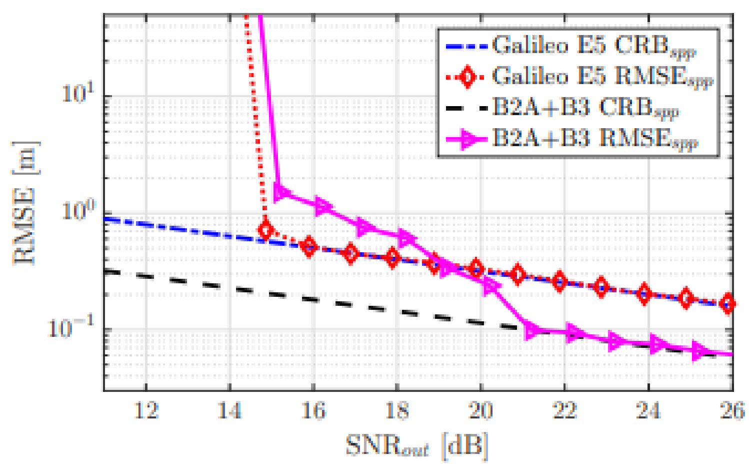


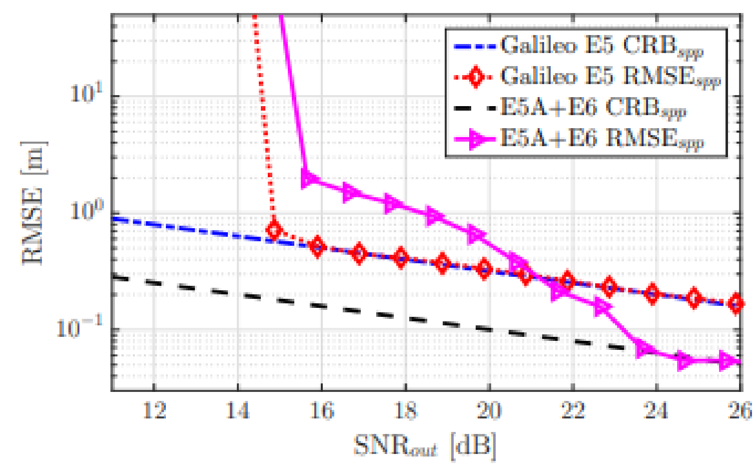
(b)



(c)



(d)



(e)

"Positioning Performance Limits of GNSS Meta-Signals and HO-BOC Signals", **Sensors**, 20 (12), 3586, June 2020.

Conclusions of Section 2.

- *Derivation of CRB for Band limited signals. The latter CRB is particularly useful because it is expressed only from the signal samples.*

- *Evaluation of the time-delay and phases estimation of the GNSS Signals and GNSS Meta-signals.*

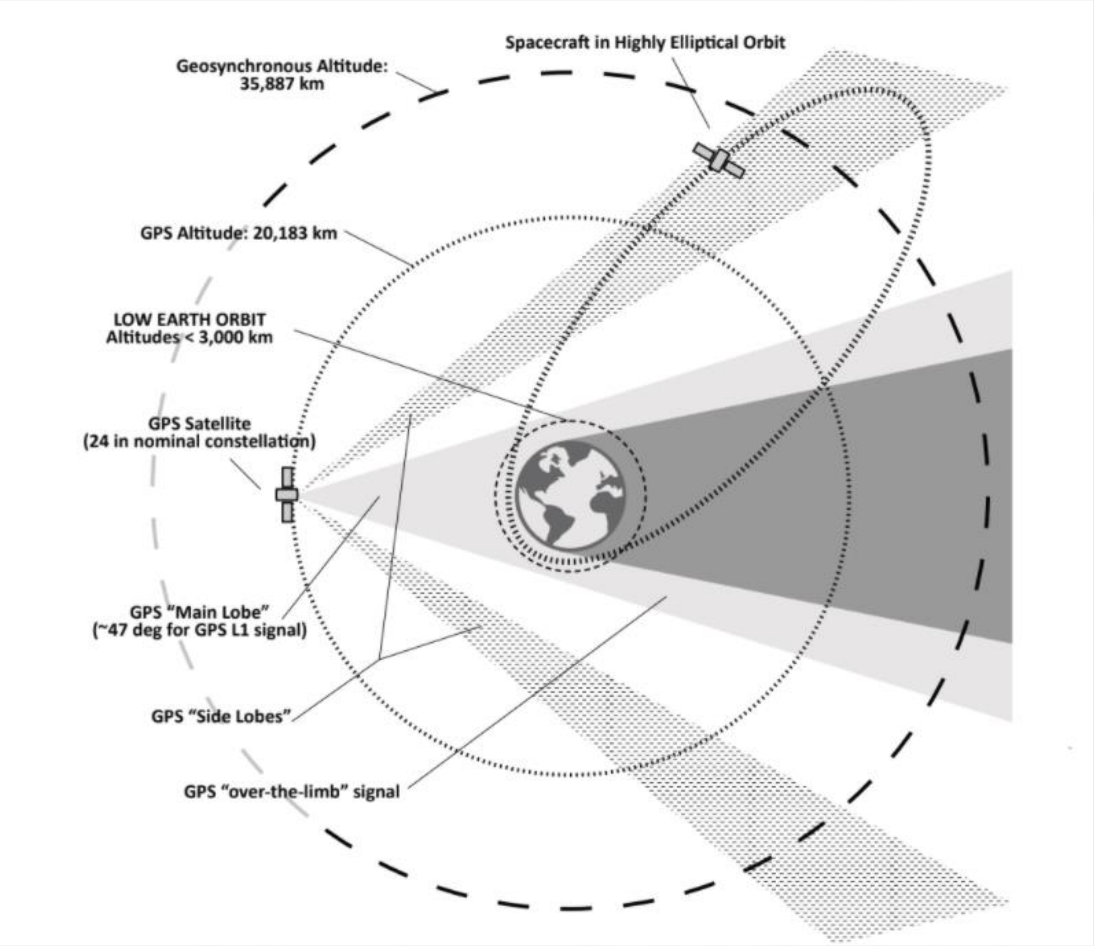
- *Evaluation of the SPP solution of the GNSS Signals and GNSS Meta-signals.*

- *Large Bandwidth GNSS Meta-Signals can have possible false locks due to high secondary correlation peaks. This issue can degraded the time-delay estimation performance. Evaluate the threshold through the MLE is required.*

- *RTK Theoretical Limits – Collaboration with the DLR (Daniel Medina)*
  - *“Compact CRB for Delay, Doppler and Phase Estimation – Application to GNSS SPP & RTK Performance Characterization”, IET Radar, Sonar & Navigation, vol. 14, no. 10, pp. 1537-1549, September 2020.*
  - *“Positioning Performance Limits of GNSS Meta-Signals and HO-BOC Signals”, Sensors, 20 (12), 3586, June 2020.*

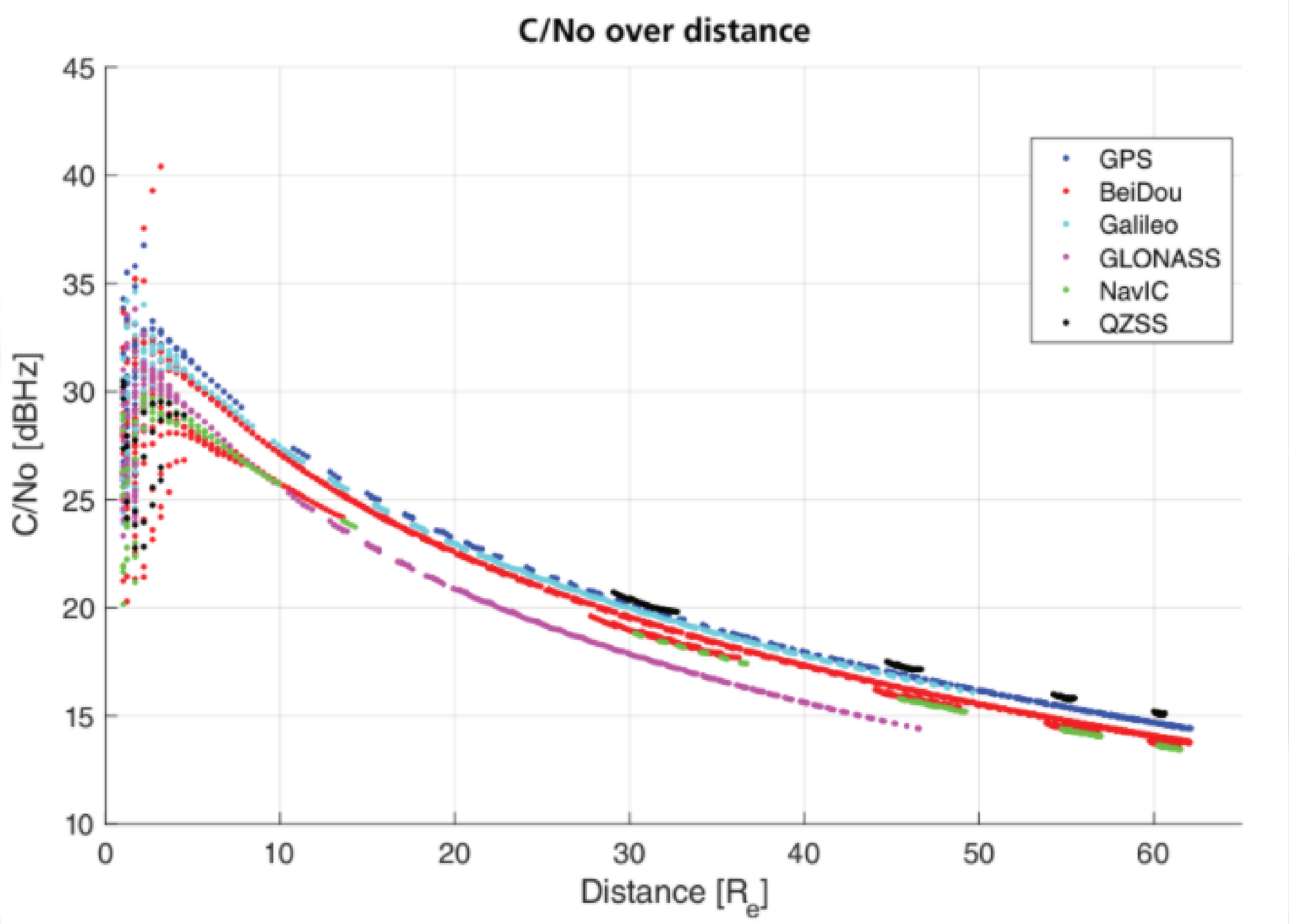
3-Theoretical Limits: Space Applications

- **Space missions: Launch, orbits**
- **Orbits: LEO, GEO, HEO, LTO**
- **Orbital parameters: Acceleration!**
- **Receivers require a minimum Carrier-noise density ratio C/N_0**



W. Enderle et al. Space Service Volume Booklet: The Interoperable Global Navigation Satellite Systems Space Service Volume. 2018

3-Theoretical Limits: Space Applications



Distance of 60 R_E is approximately in Lunar orbit / This gives most restricting case: 15 dB-Hz

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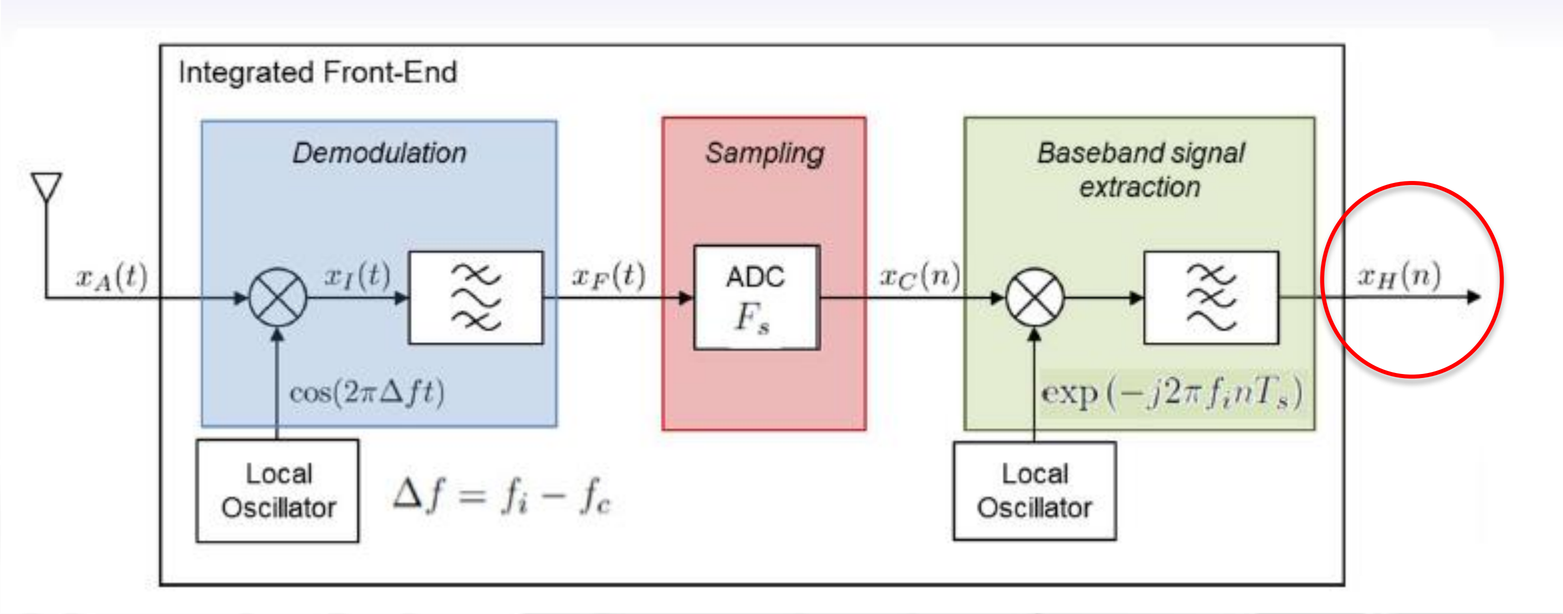
3-Theoretical Limits: Space Applications

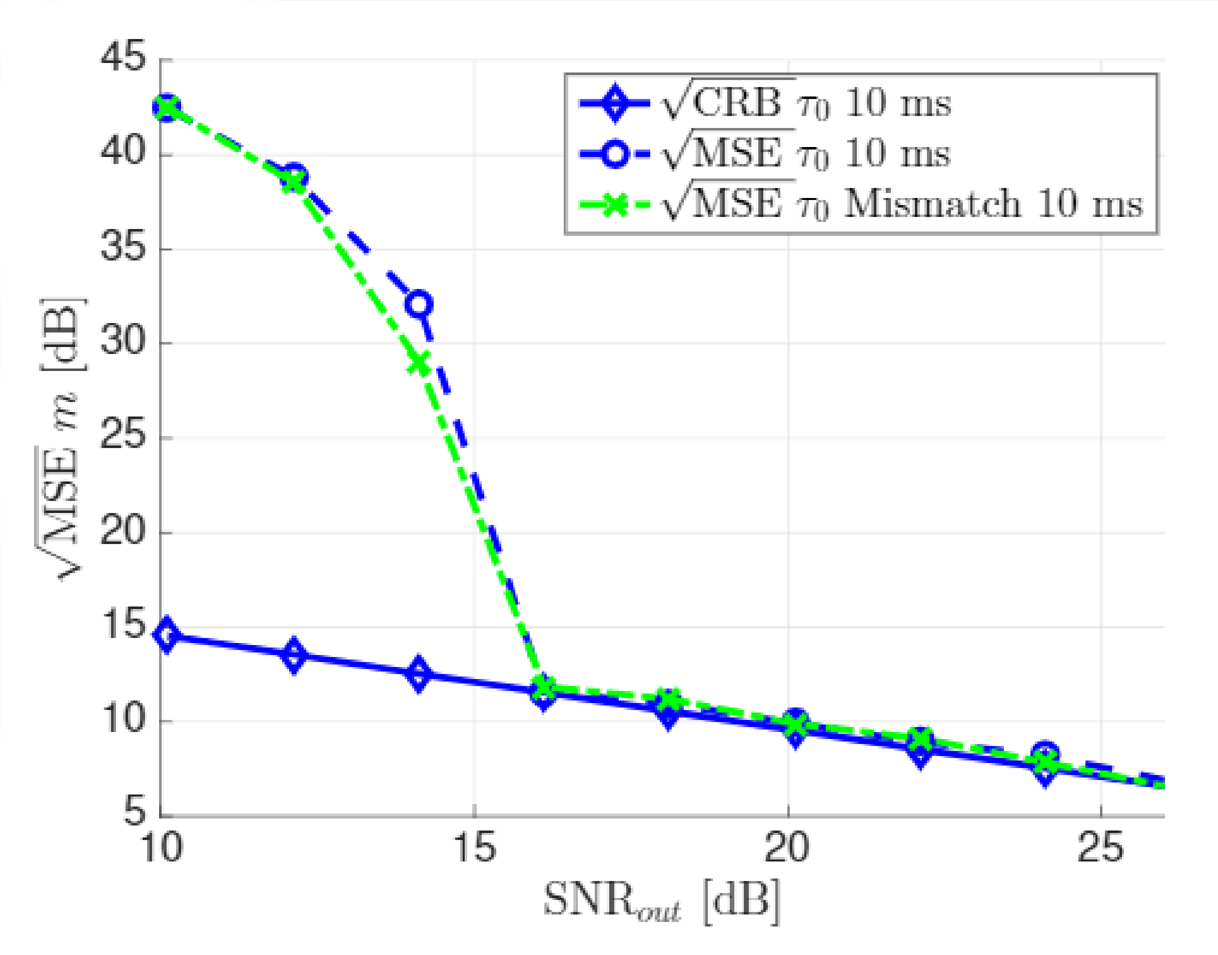


$$x(t) = x_A(t)\, e^{-j2\pi f_c t} = \alpha a(t;\eta) + n(t),$$
$$a(t;\eta) = e^{-j2\pi f_c (b(t-\tau)+d(t-\tau)^2)} s((1-b)(t-\tau))$$

$$\eta = [\tau, b, d]$$
$$d = a/2c$$

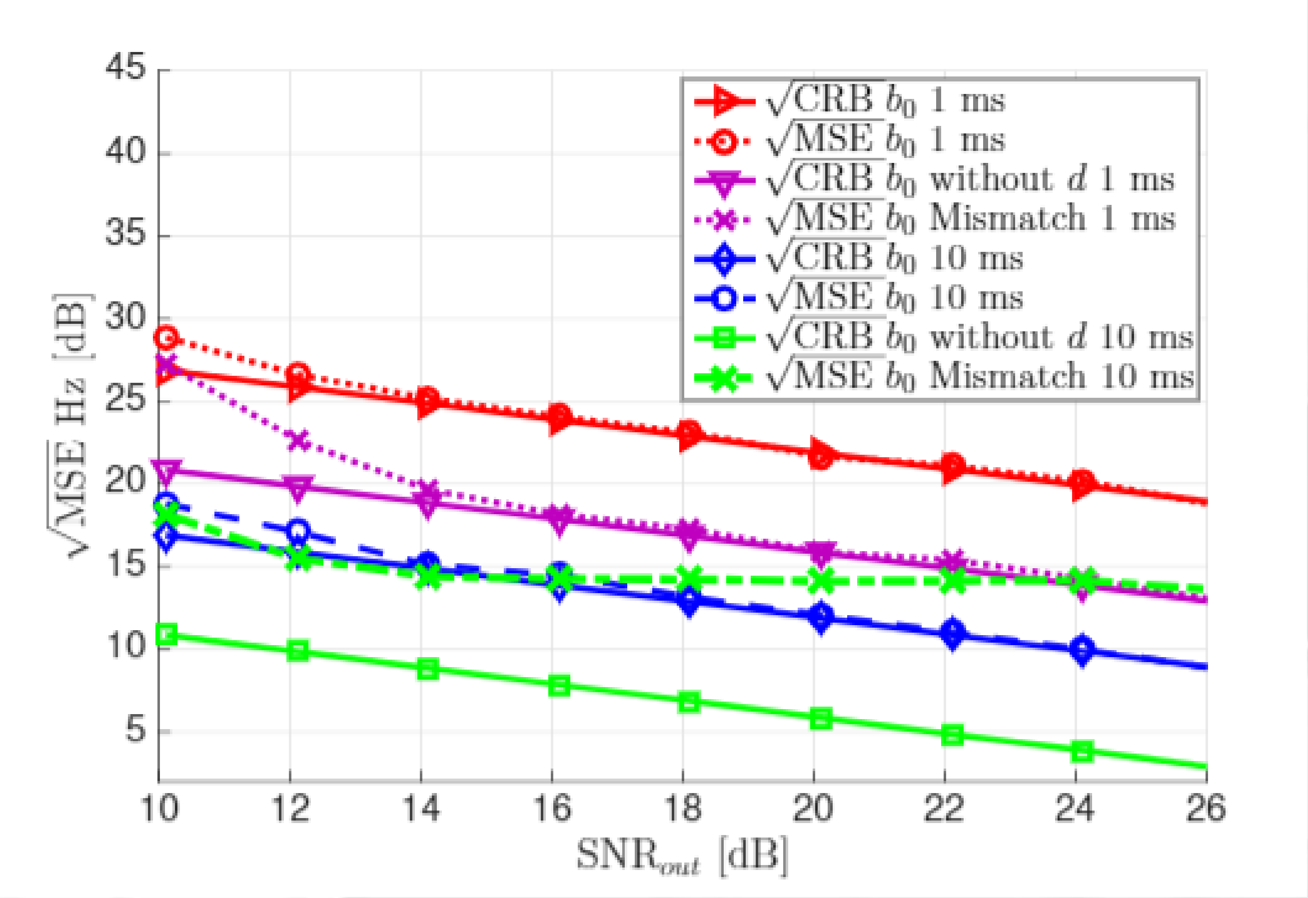
3-Theoretical Limits: Space Applications

Delay Estimation



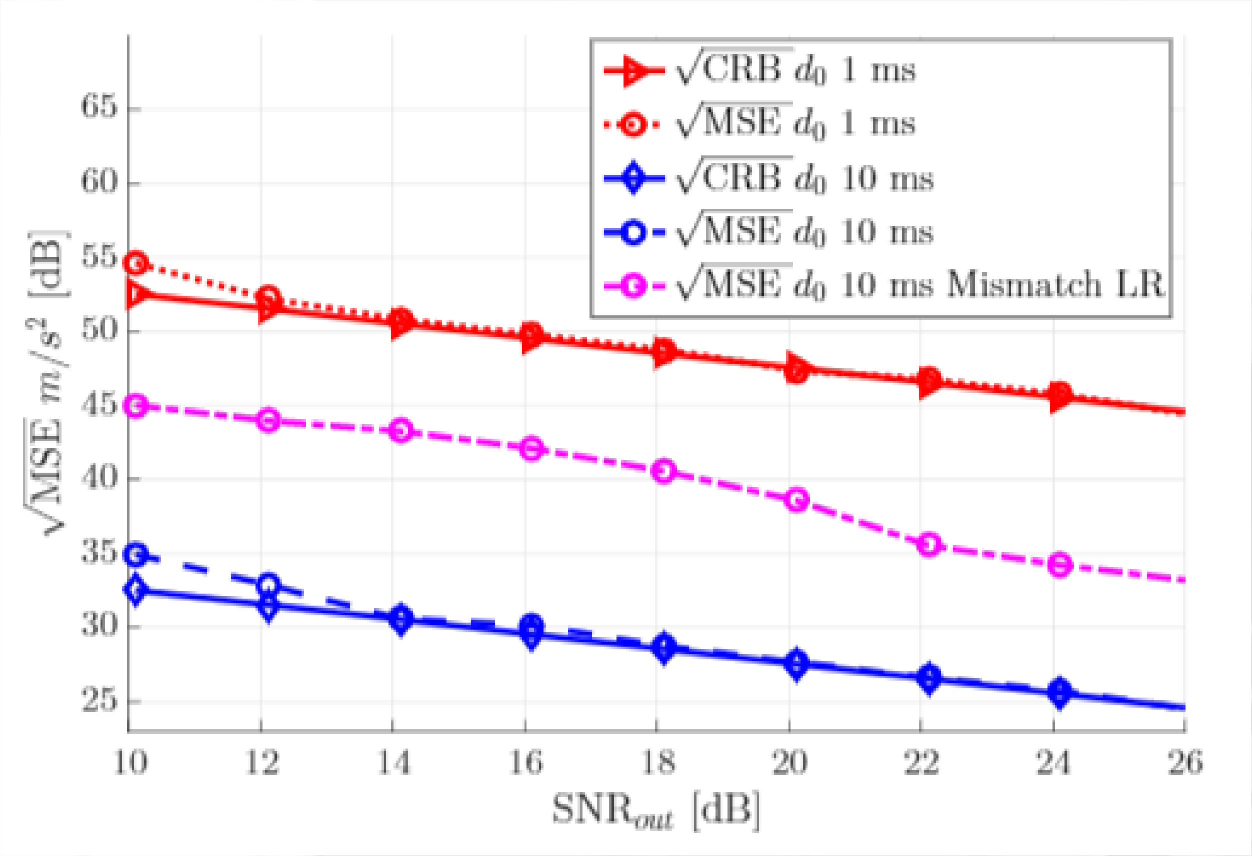
3-Theoretical Limits: Space Applications

Doppler Estimation

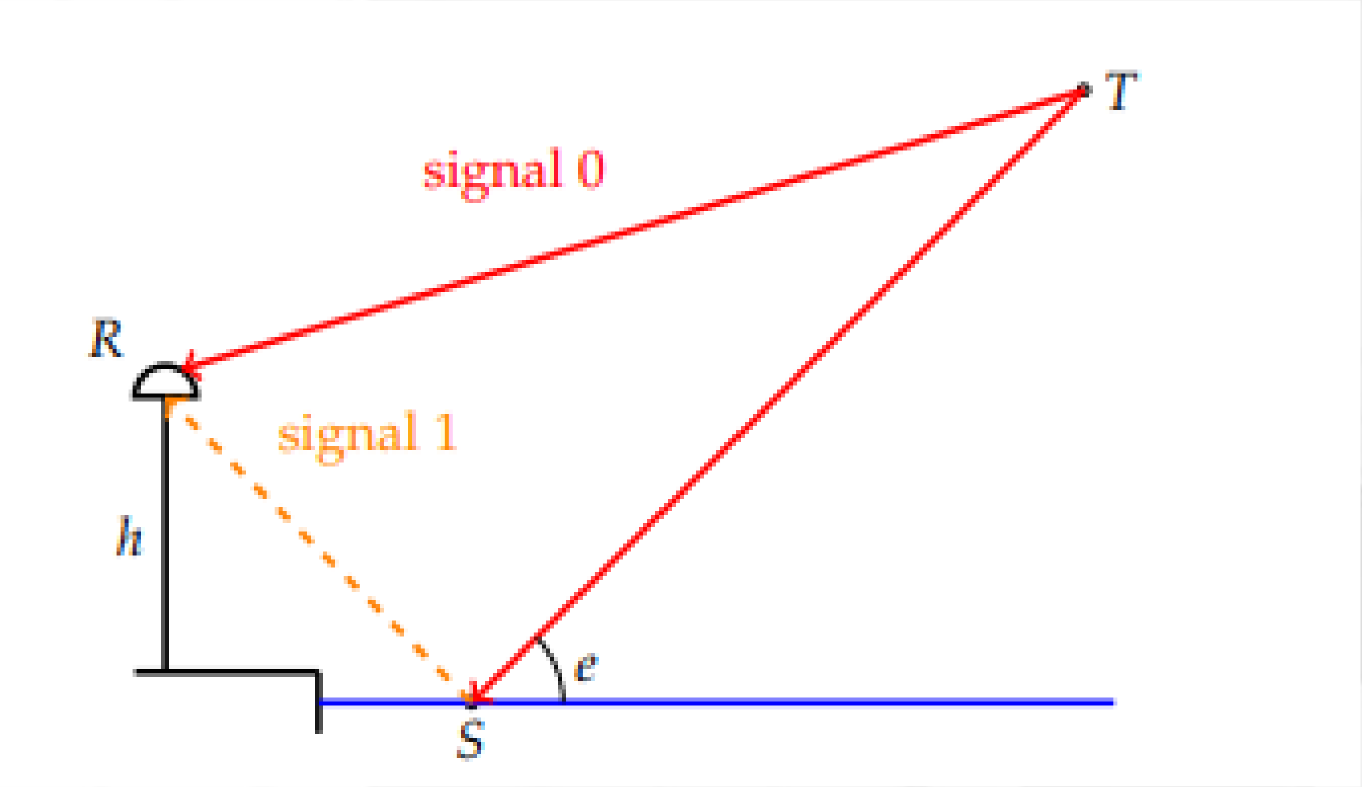


3-Theoretical Limits: Space Applications

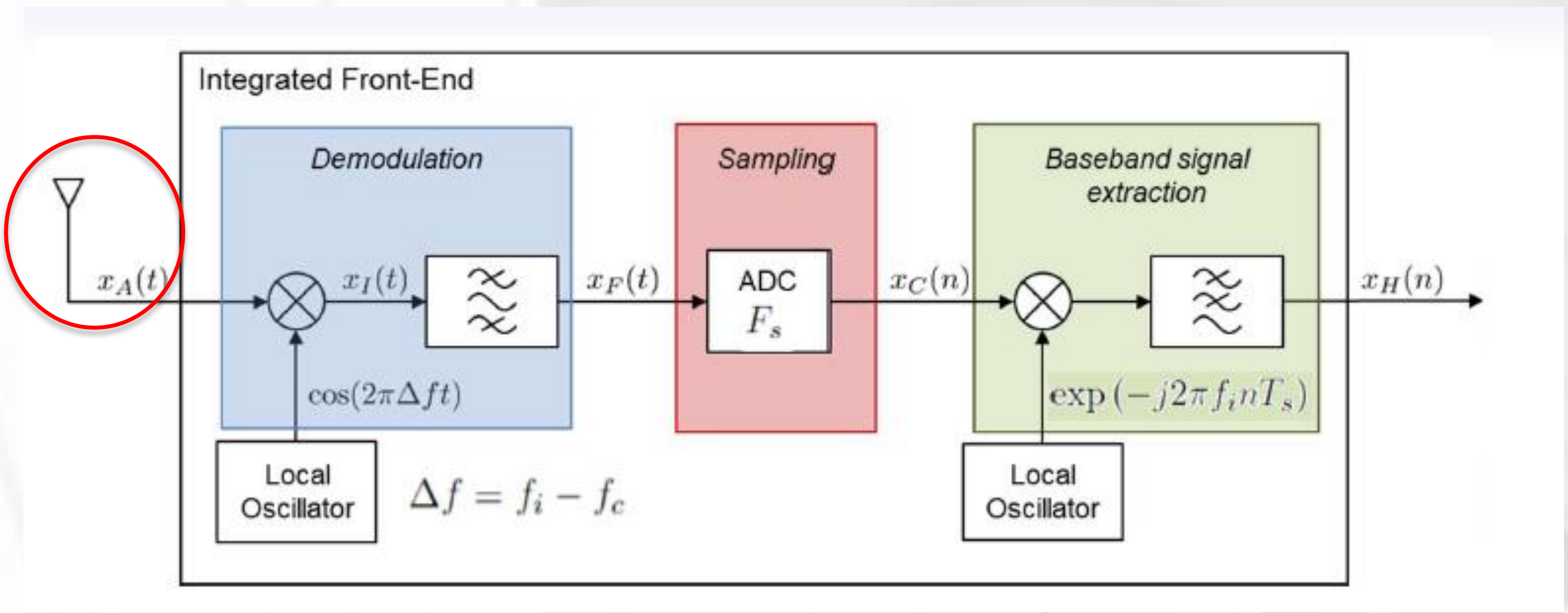
Acceleration Estimation



4-Theoretical Limits: Multipath



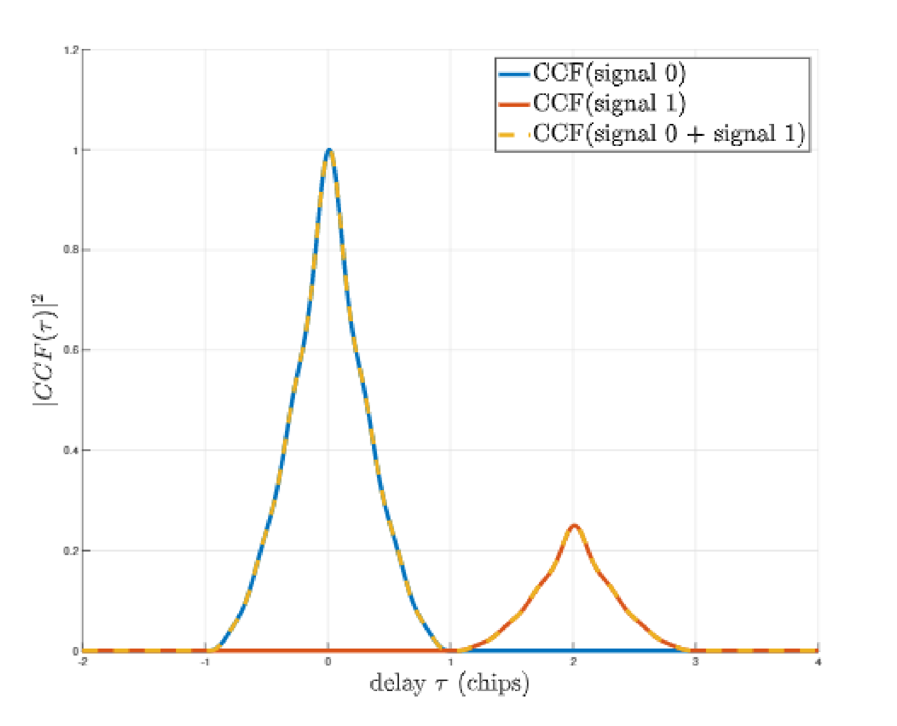
4-Theoretical Limits: Multipath

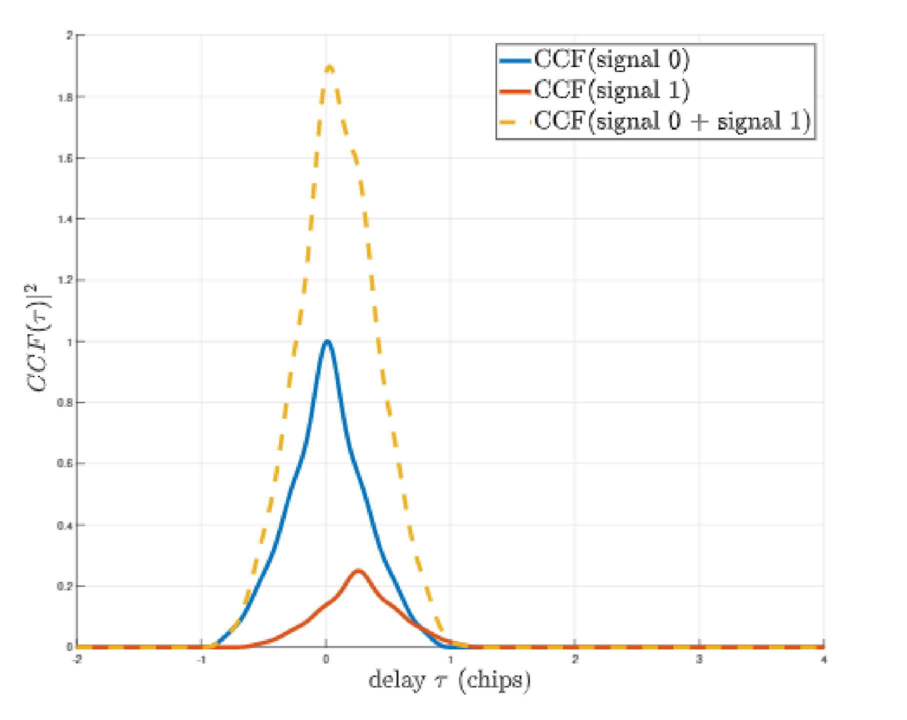


$$x_R(t) = d_R(t;\eta_0,\rho_0,\phi_{R,0}) + d_R(t;\eta_1,\rho_1,\phi_{R,1}) + w_R(t),$$
$$d_R(t;\eta_i,\rho_i,\phi_{R,i}) = \rho_i e^{j\phi_{R,i}} s((1-b_i)(t-\tau_i)) e^{j\omega_c(1-b_i)t} e^{-j\omega_c\tau_i},$$

$$\mathbf{x} = \mathbf{A}(\eta_0,\eta_1)\boldsymbol{\alpha} + \mathbf{w},\ \mathbf{w} \sim \mathcal{CN}(0,\sigma_n^2\mathbf{I}_N)\ ,$$

4-Theoretical Limits: Multipath





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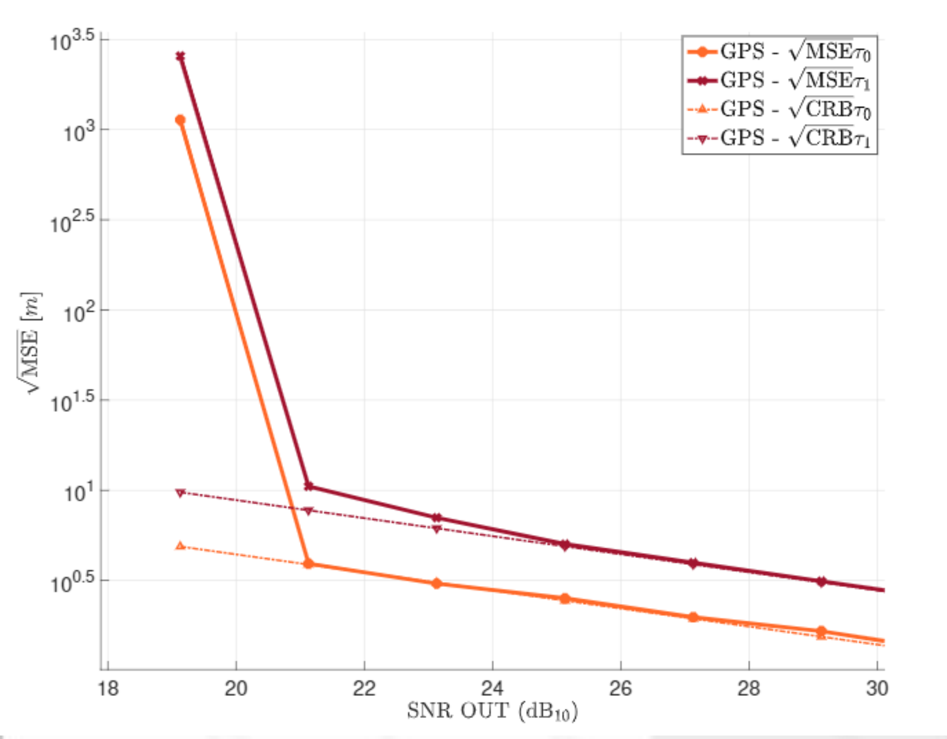

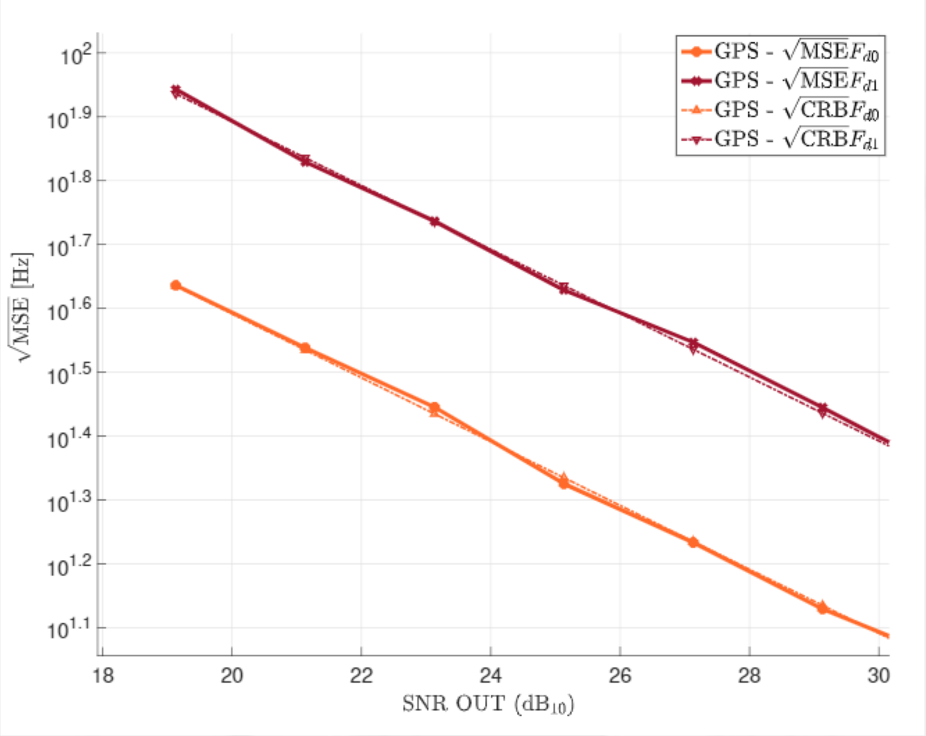

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