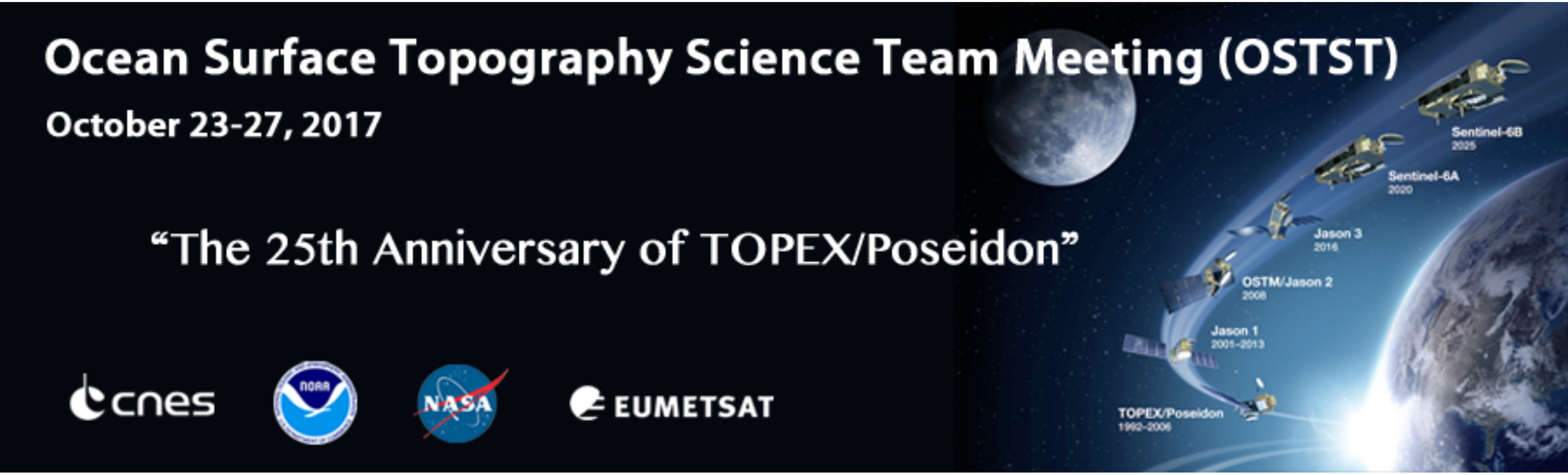

A promising parametric spectral analysis method applied to sea level anomaly signals

C. Mailhes[1], D. Bonacci[1],
A. Guillot[2], S. Le Gac[2], N. Steunou[2], C. Cheymol[2], N. Picot[2], G. Dibarboure[2]

1. Telecommunications for Space and Aeronautics Lab. (TeSA), Toulouse, France
2. Centre National d’Etudes Spatiales (CNES), Toulouse, France

TéSA
Telecommunications for Space and Aeronautics

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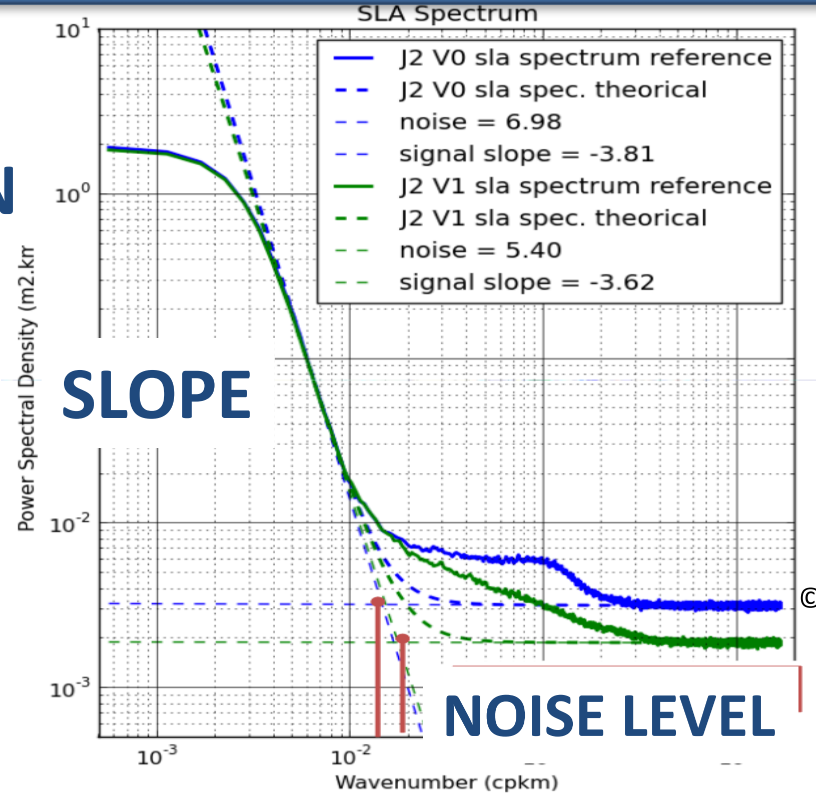
CENTRE NATIONAL D’ÉTUDES SPATIALES

Context of the presentation

SPECTRAL ANALYSIS of sea level anomalies (SLA)
widely used in the altimetry community:

To understand the geophysical content of measured signals,
To assess and compare the performance of missions

SPECTRUM ESTIMATION



SPECTRAL ANALYSIS = usually based on Fourier transform

Outline of the talk

Study funded by cnes
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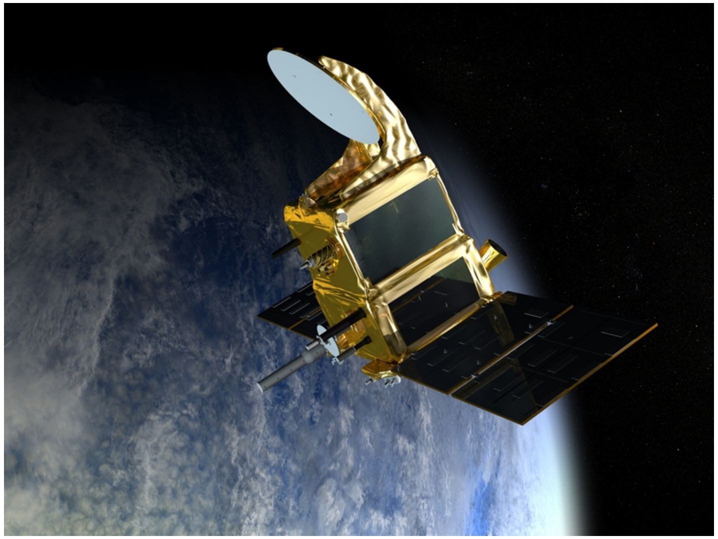
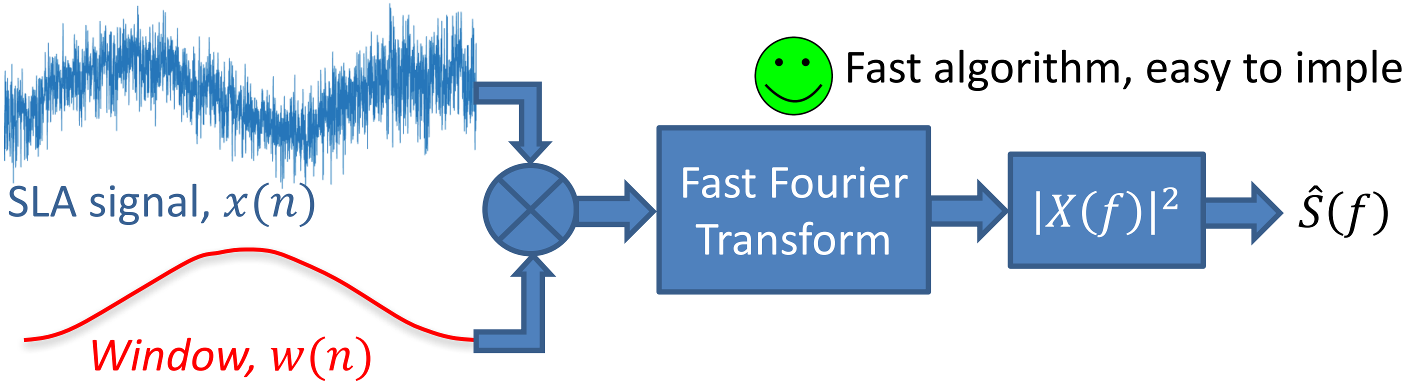
1. Spectral analysis based on Fourier transform
2. Spectral analysis based on parametric modeling
3. A parametric spectral analysis for SLA: ARWARP
4. Validation on simulated signals
5. Results on real signals
6. Conclusions and perspectives

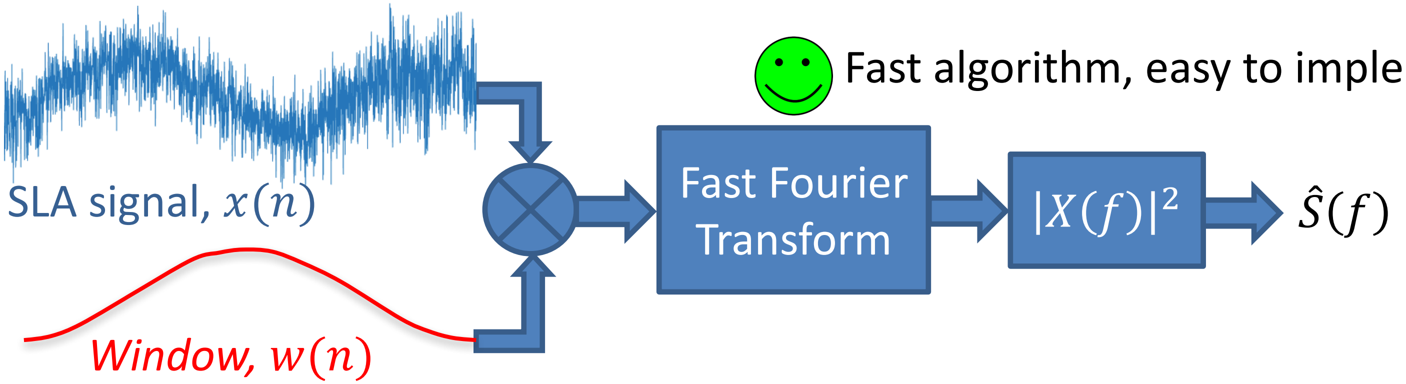


Comparisons made in this presentation
on simulated Sea Level Anomalies (SLA)
and
on real signals from SARAL/AltiKa, Agulhas current area

1. Spectral Analysis based on Fourier Transform

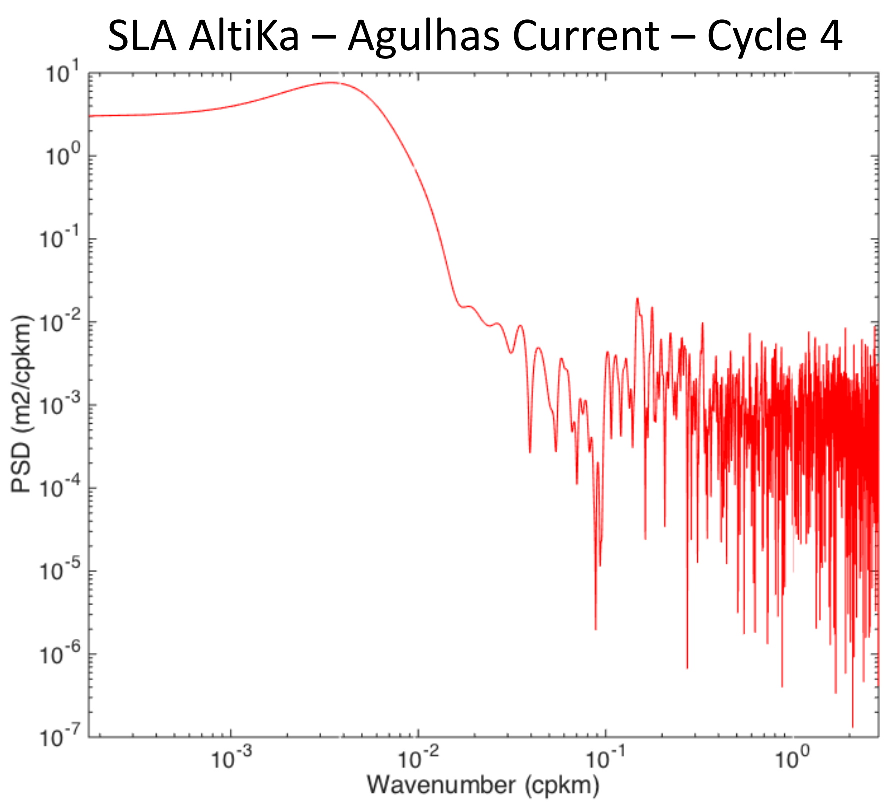


1. Spectral Analysis based on Fourier Transform



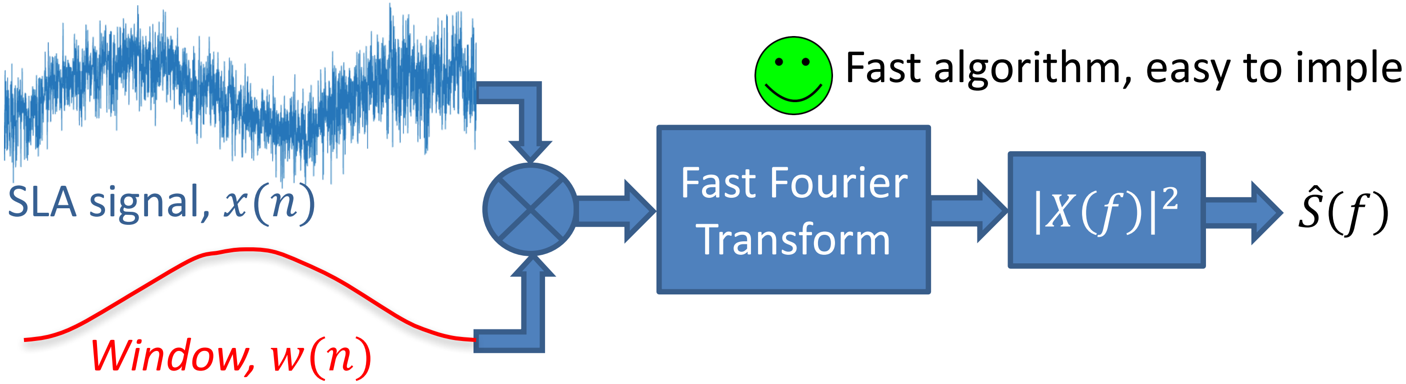
Fast algorithm, easy to implement

Spectral estimation on 1 segment (3000 samples)



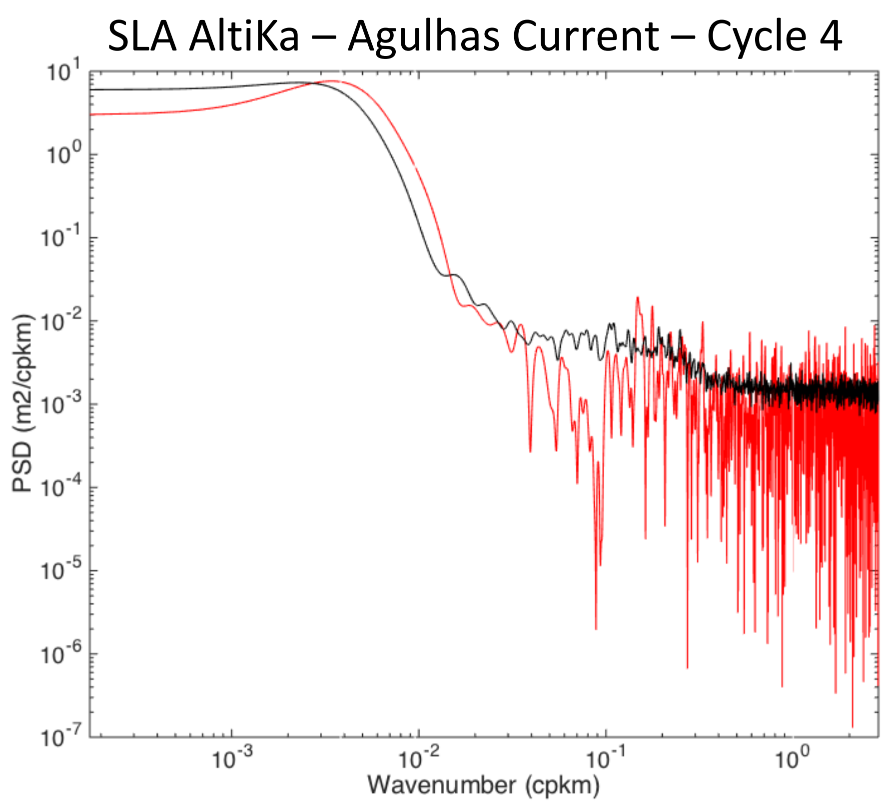
Variance

1. Spectral Analysis based on Fourier Transform



Spectral estimation on 1 segment (3000 samples)

Averaging several segments (25) (Welch estimation)



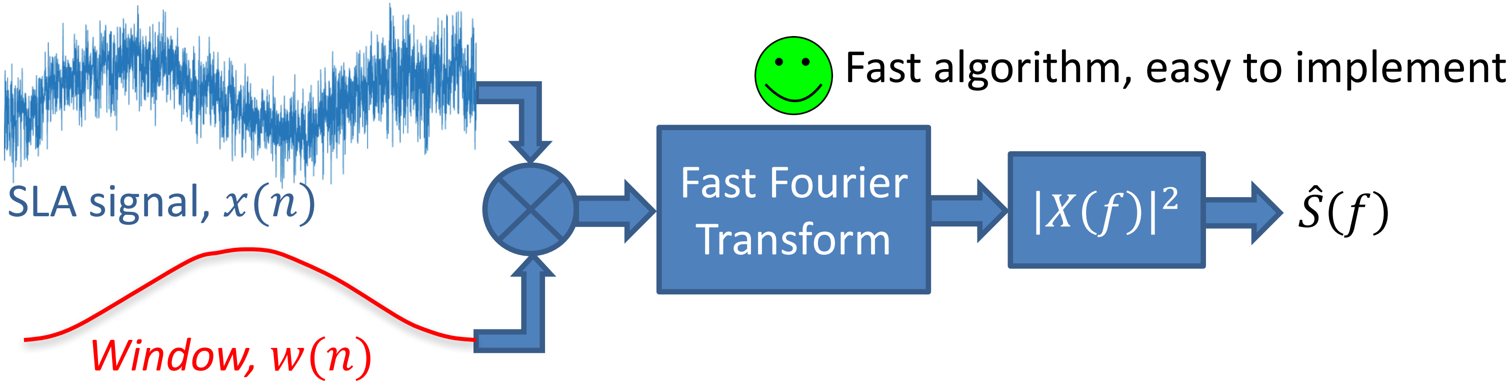
Variance

Average spectra

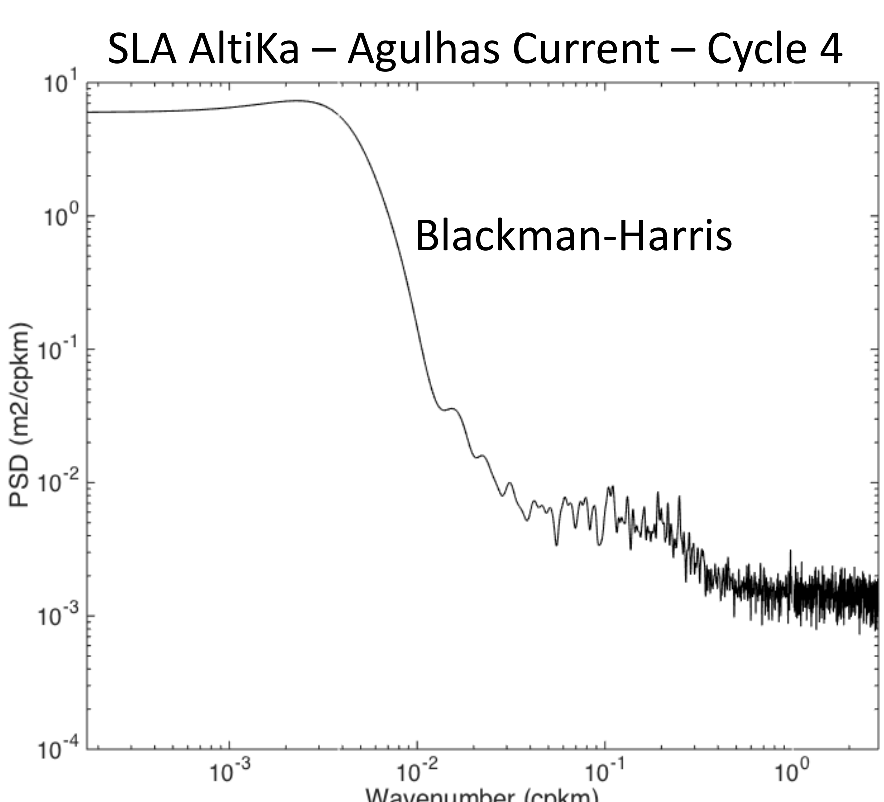
Variance reduced

Stationarity?

1. Spectral Analysis based on Fourier Transform

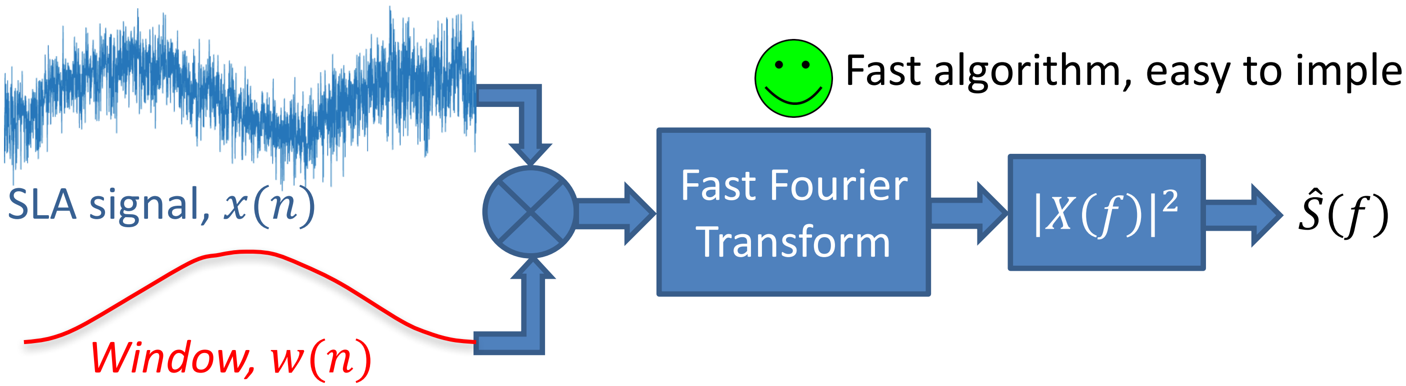


Averaging several segments (25) (Welch estimation)
Blackman-Harris window



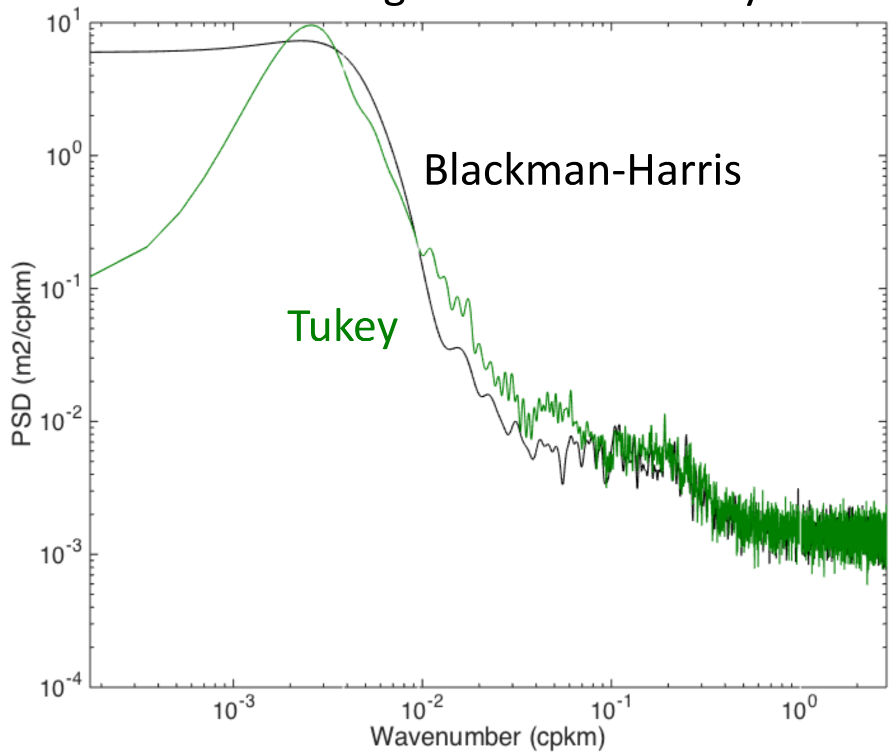
Variance
Average spectra
Variance reduced
Stationarity?
Which window?

1. Spectral Analysis based on Fourier Transform



Fast algorithm, easy to implement

SLA AltiKa – Agulhas Current – Cycle 4



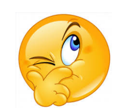
Averaging several segments (25) (Welch estimation)
Blackman-Harris window
Tukey window

 Variance

 Average spectra

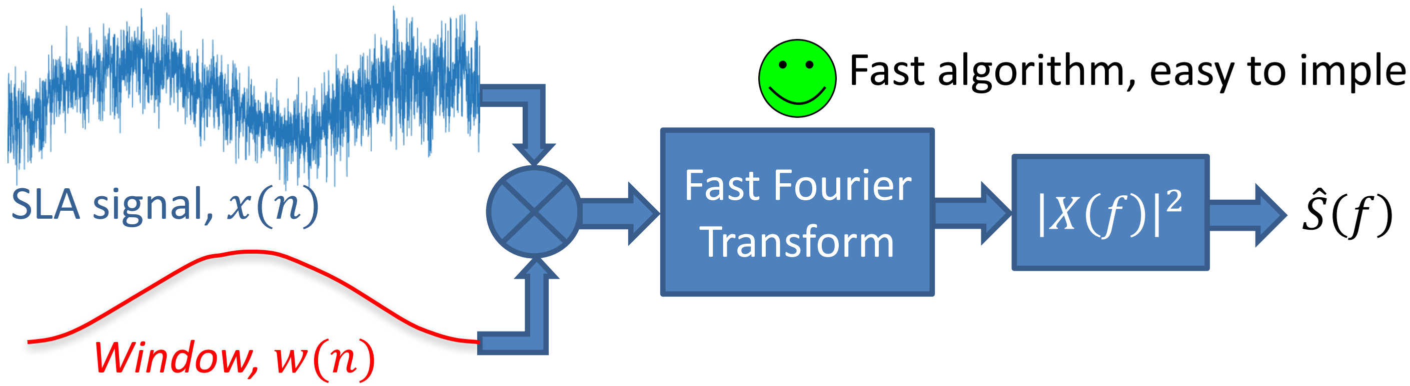
 Variance reduced

 Stationarity?

 Which window?

1. Spectral Analysis based on Fourier Transform

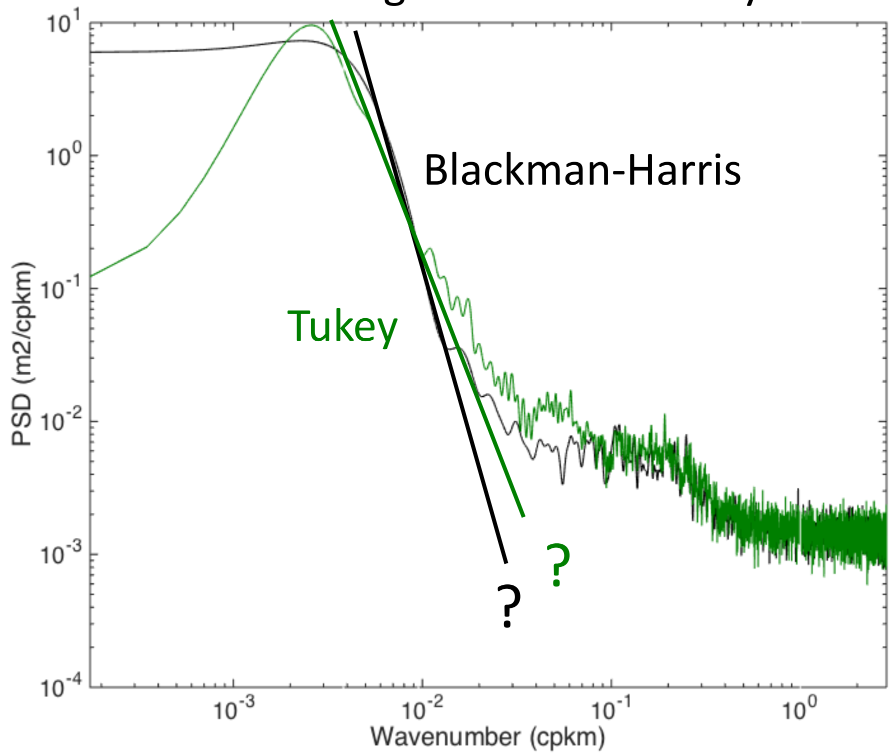
Fast algorithm, easy to implement



Averaging several segments (25) (Welch estimation)

Blackman-Harris window

Tukey window



Variance

Average spectra

Variance reduced

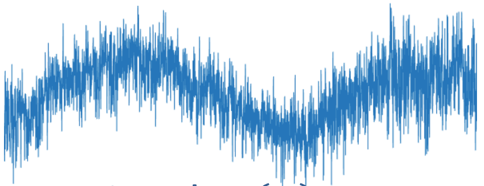
Stationarity?

Which window?

Different slope estimations

Bias

2. Spectral Analysis based on **parametric modeling**



SLA signal, $x(n)$

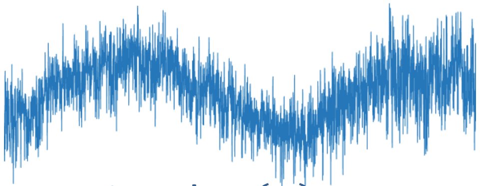
AutoRegressive (AR) modeling of a SLA signal

$$x(n) = \sum_{k=1}^{p} a_k x(n-k) + e(n)$$

Actual sample = Linear combination of past samples + unpredictable part (model error)

[1] S. M. Kay and S. L. Marple, "Spectrum analysis—A modern perspective," in *Proceedings of the IEEE*, vol. 69, no. 11, pp. 1380-1419, Nov. 1981.
[2] L. Marple, « Digital Spectral Analysis: with Applications », Prentice Hall Ed., 1987.
[3] S.M.Kay, « Modern spectral estimation: Theory and applications », Prentice Hall Ed., 1988.
[4] P. Soica, R. Moses, « Spectral analysis of signals », Prentice Hall Ed., 2005 (expanded version of the book of 1997).
[5] R.E..Thomson, W.J.Emery, « Data analysis methods in physical oceanography », Elsevier Ed., 3rd edition, 2014.

2. Spectral Analysis based on **parametric modeling**



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AR modeling — Find the AR coefficients $\{a_1, a_2, \cdots, a_p\}$ such that error is minimized

Several algorithms: e.g., in Matlab *lpc.m, levinson.m, aryule.m*

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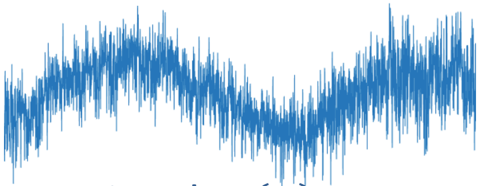
2. Spectral Analysis based on **parametric modeling**



SLA signal, $x(n)$

AutoRegressive (AR) modeling of a SLA signal

$$x(n) = \sum_{k=1}^{p} a_k x(n-k) + e(n)$$

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AR modeling

Find the AR coefficients $\{a_1, a_2, \cdots, a_p\}$ such that error is minimized

Several algorithms: e.g., in Matlab *lpc.m, levinson.m, aryule.m*

AR spectral estimation

$$\boxed{S_{AR}(f) = \frac{\sigma_e^2}{\left|1 - \sum_{k=1}^{p} a_k e^{i2\pi f k}\right|^2}}$$

[1] S. M. Kay and S. L. Marple, "Spectrum analysis—A modern perspective," in *Proceedings of the IEEE*, vol. 69, no. 11, pp. 1380-1419, Nov. 1981.

[2] L. Marple, « Digital Spectral Analysis: with Applications », Prentice Hall Ed., 1987.

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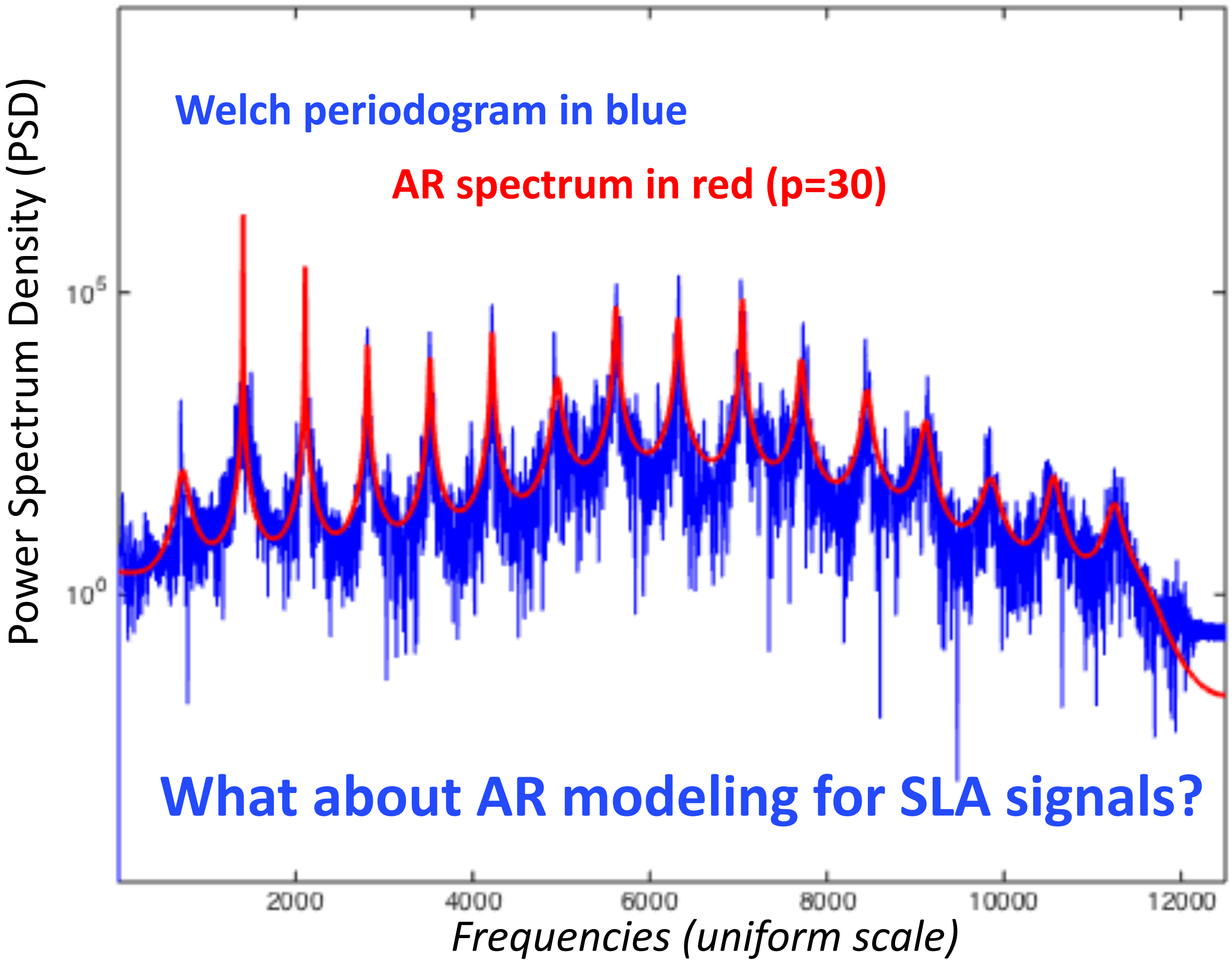
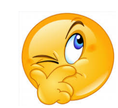
2. Spectral Analysis based on **parametric modeling**

PSD of an audio signal (industrial system)



- ✓ No window choice
- ✓ Variance decreased
- ✓ Better resolution for short signal segments

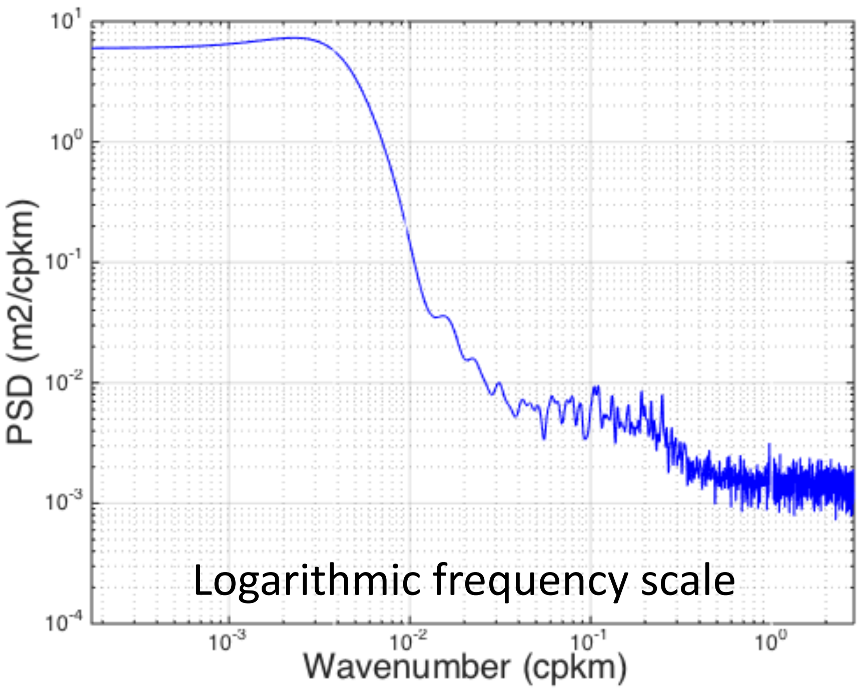


- ➢ More complex to implement
- ➢ Choice of order *p*

2. Spectral Analysis based on **parametric modeling**

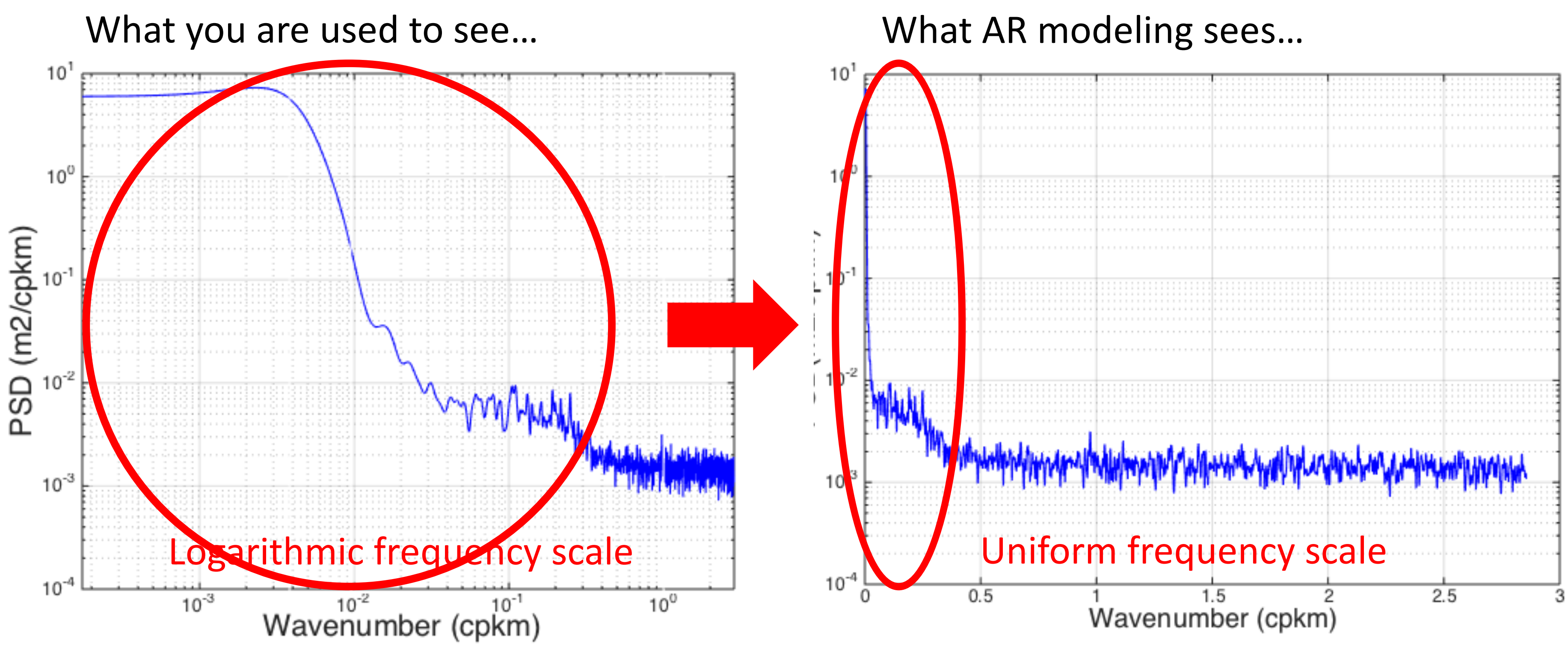
SLA AltiKa – Agulhas Current – Cycle 4

What you are used to see…



2. Spectral Analysis based on **parametric modeling**

SLA AltiKa – Agulhas Current – Cycle 4



AR spectral analysis will fit a model on the whole spectrum,
on a uniform frequency scale basis.

Necessary to adapt to fit well the interesting part of the PSD

3. A parametric spectral analysis for SLA: ARWARP

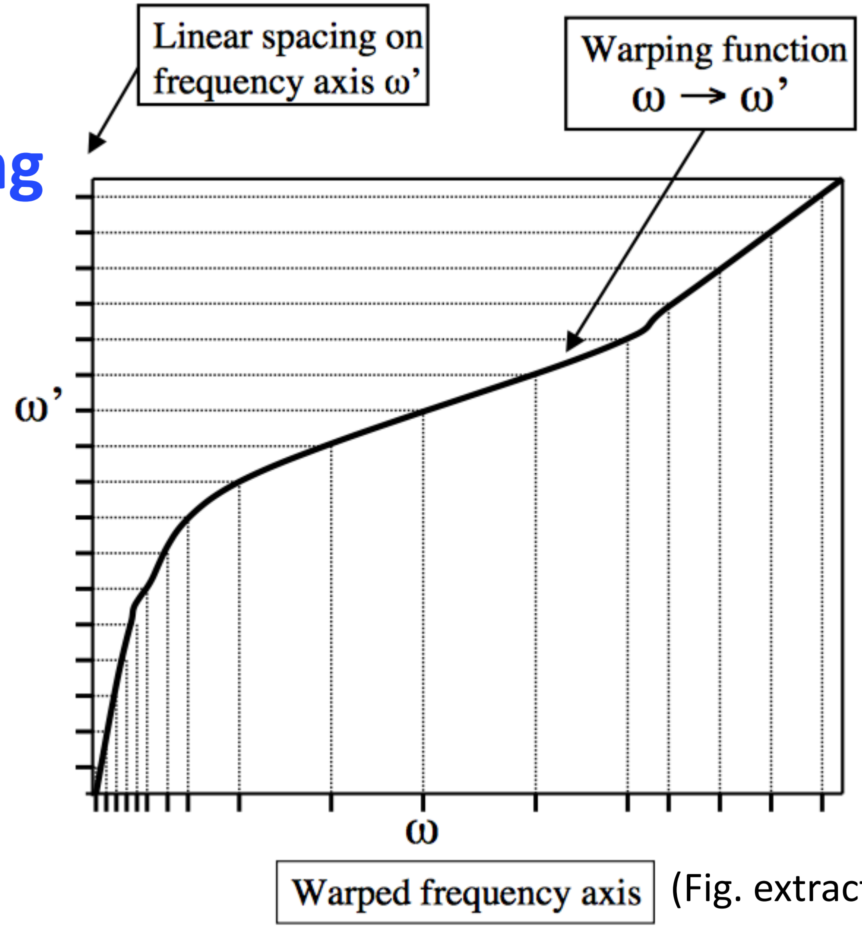
Introduce pre-processing to adapt AR analysis to SLA signals

Frequency warping

Dilatation of the spectrum

Reversible operation

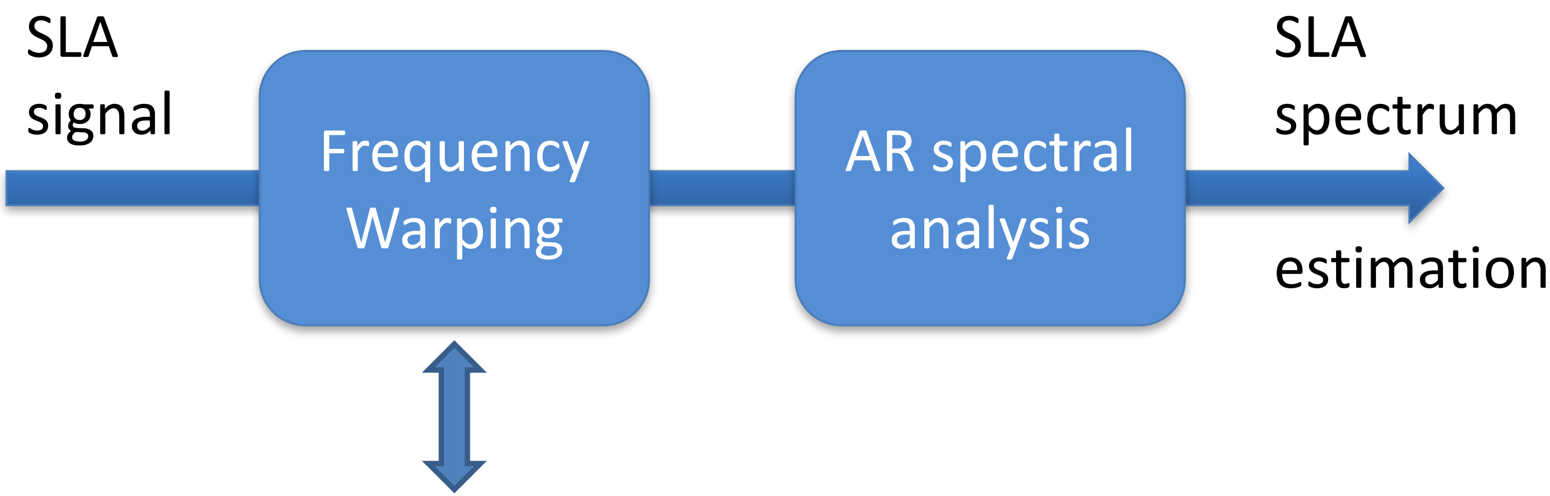
Already used in speech processing



(Fig. extract from [1])

[1] T. Kinnunen, *Spectral features for automatic text-independent speaker recognition*, PhD Thesis, 2003, available on ftp://ftp.cs.uef.fi/pub/PhLic/2004_PhLic_Kinnunen_Tomi.pdf.

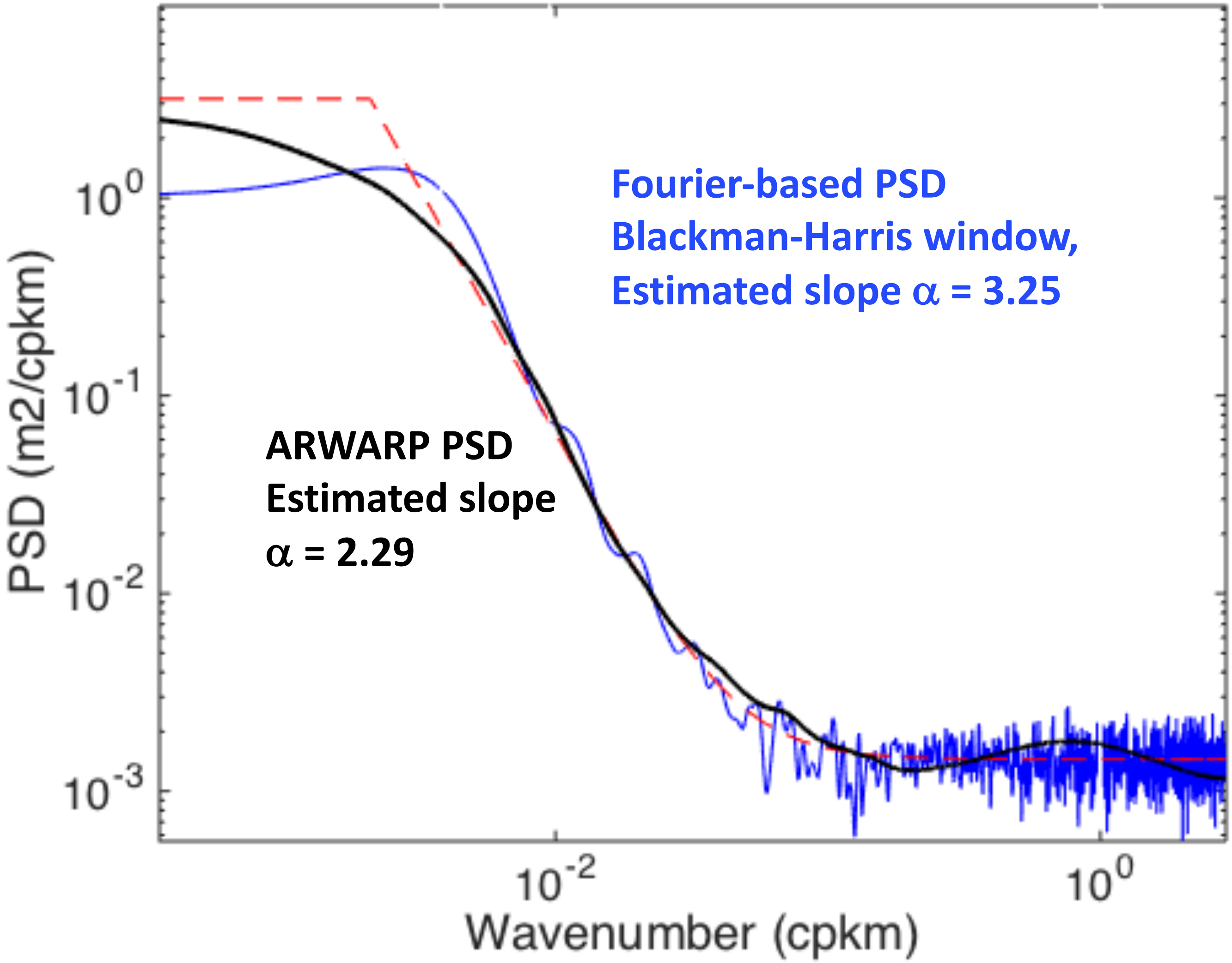
3. A parametric spectral analysis for SLA: ARWARP



$$f' = f + \frac{1}{\pi}\tan^{-1}\left(\frac{b\sin(2\pi f)}{1 - b\cos(2\pi f)}\right)$$

4. Validation on simulated SLA signals

Simulated SLA, Slope α = 2.5, 20 segments of 3000 samples

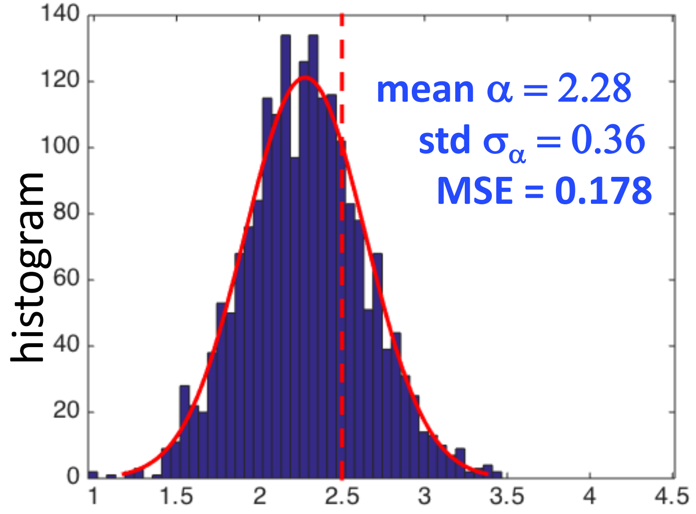


ARWARP PSD « smoother » PSD

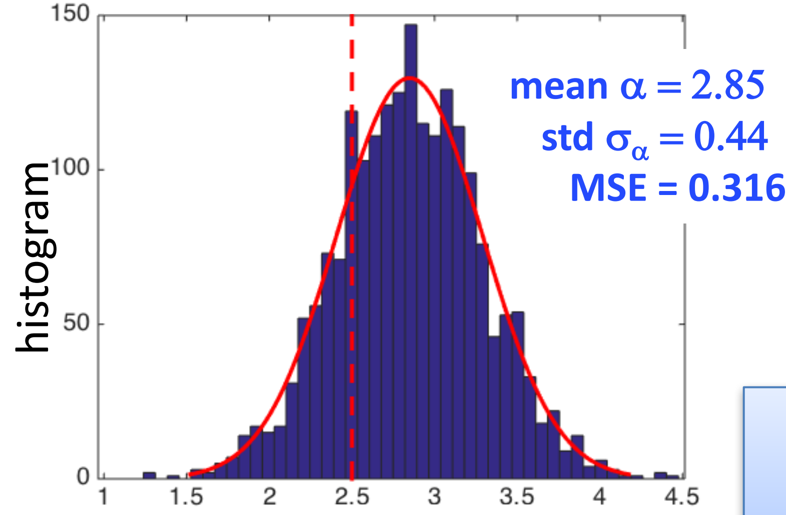
4. Validation on simulated SLA signals

Simulated SLA, Slope α = 2.5, 20 segments of 3000 samples

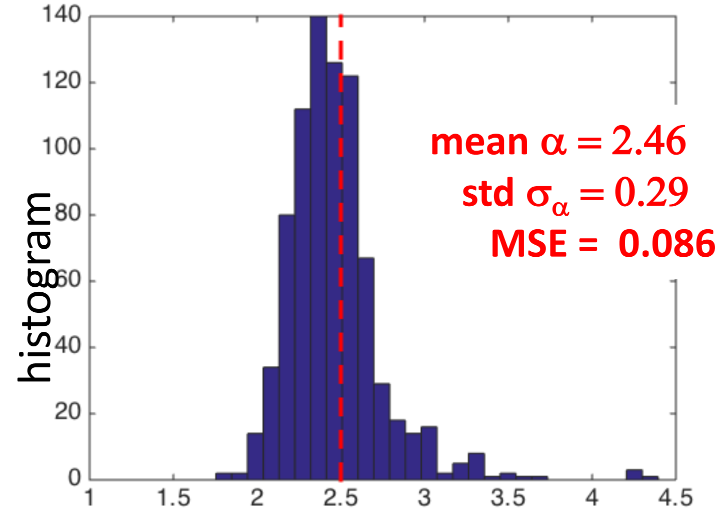
Fourier-based PSD, Rect. window



Fourier-based PSD, B-H. window



ARWARP PSD

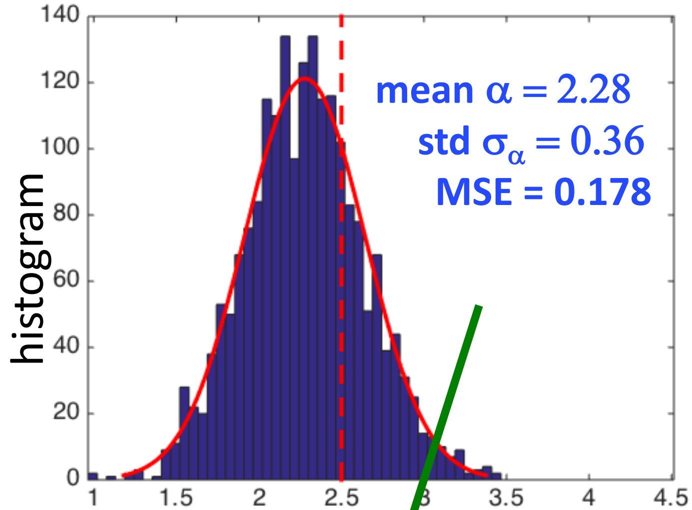


2000 slope estimations

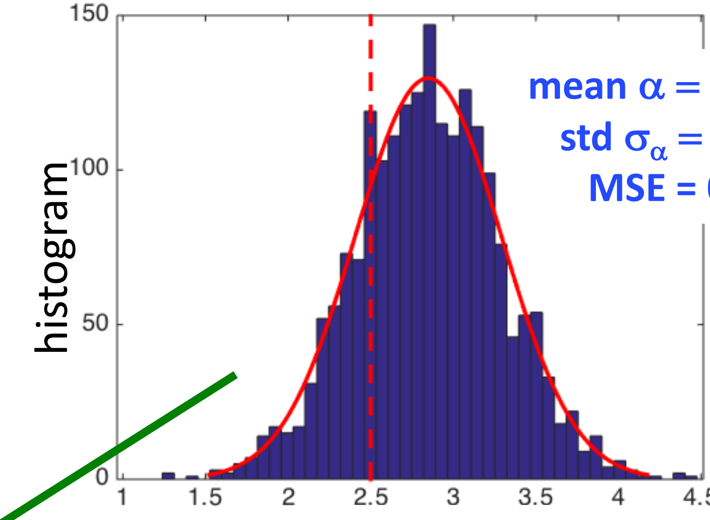
4. Validation on simulated SLA signals

Simulated SLA, Slope α = 2.5, 20 segments of 3000 samples

Fourier-based PSD, Rect. window



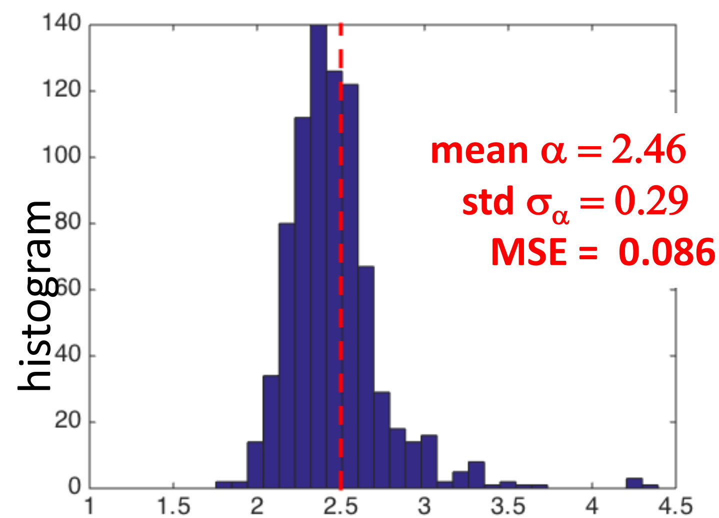
Fourier-based PSD, B-H. window



2000 slope estimations

Average both slope estimations ?

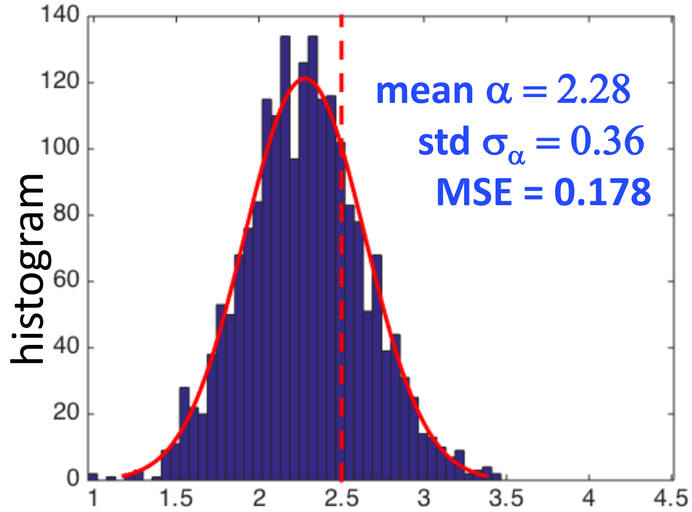
ARWARP PSD



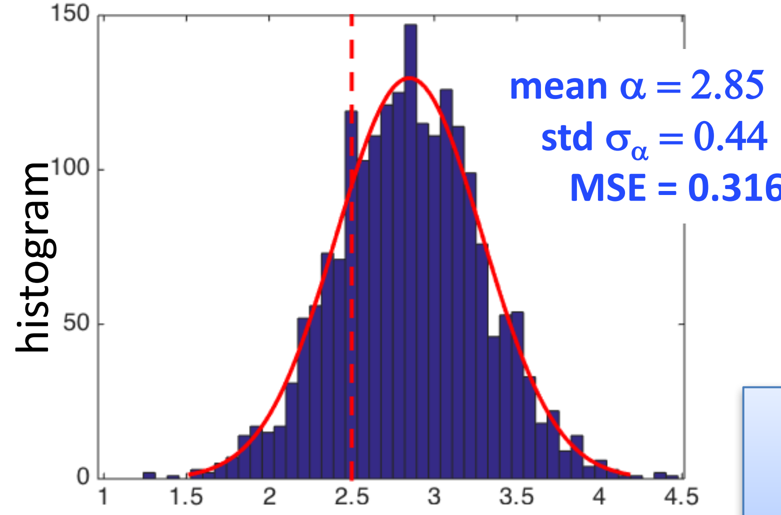
4. Validation on simulated SLA signals

Simulated SLA, Slope α = 2.5, 20 segments of 3000 samples

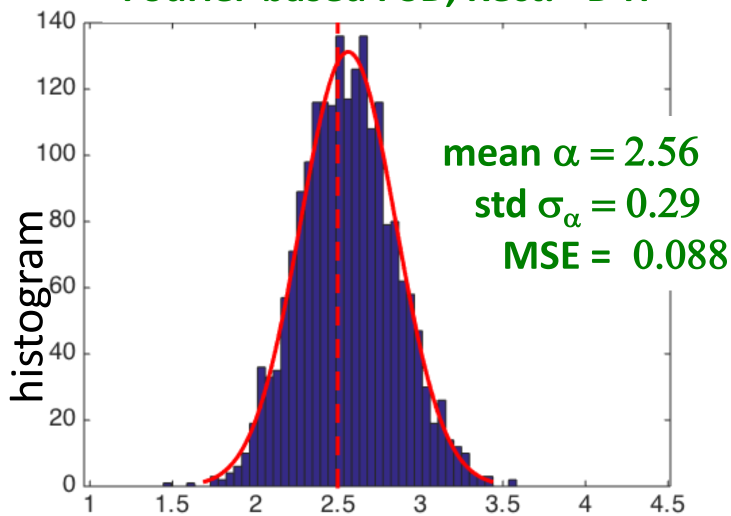
Fourier-based PSD, Rect. window



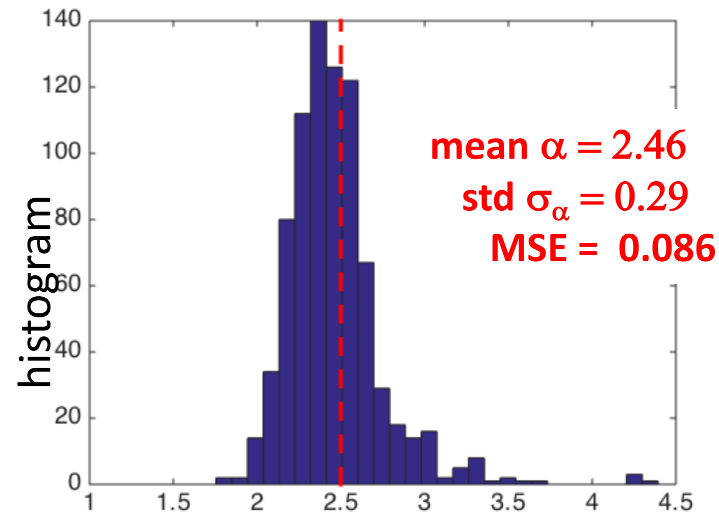
Fourier-based PSD, B-H. window



Fourier-based PSD, Rect.+ B-H



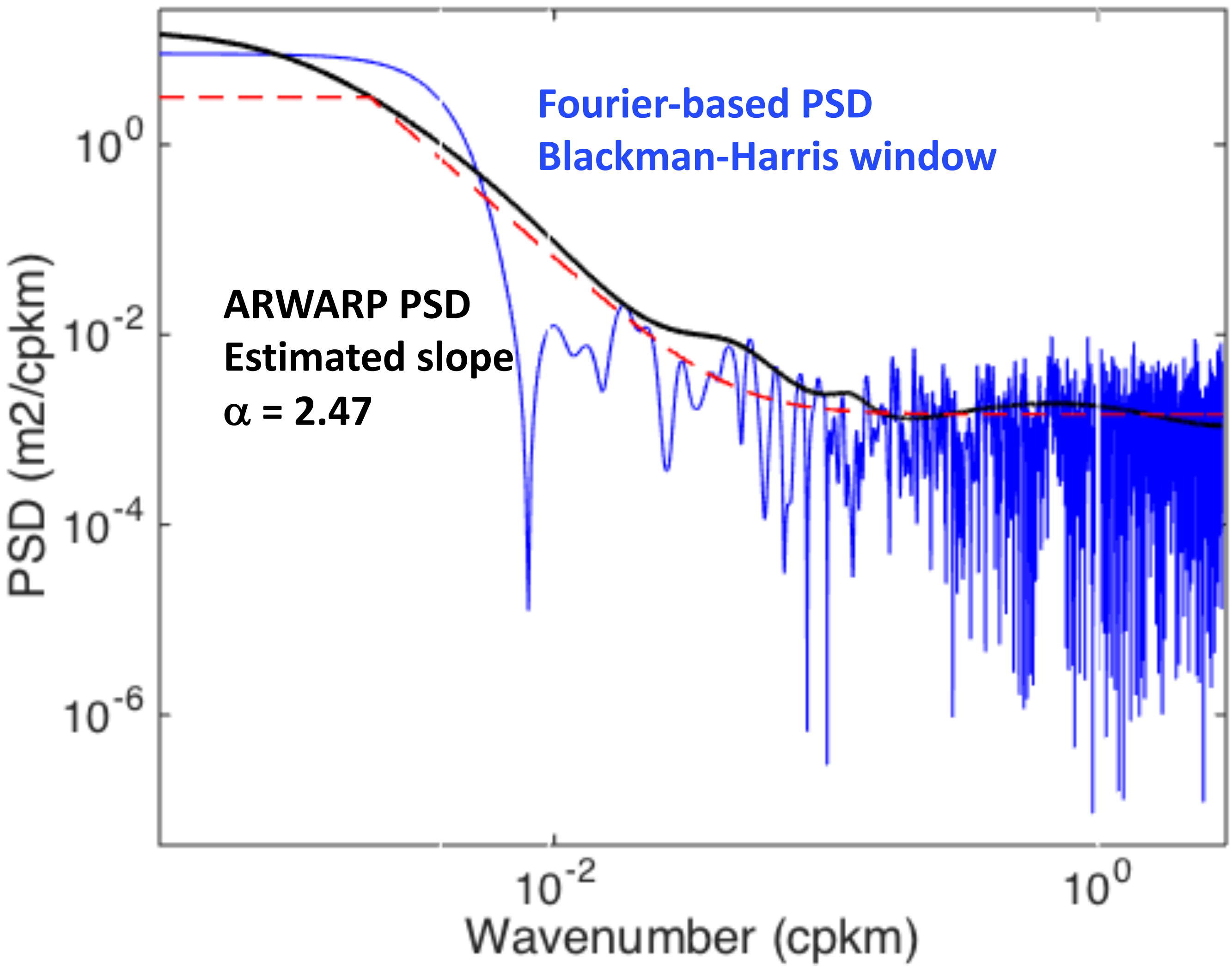
ARWARP PSD



2000 slope estimations

4. Validation on simulated SLA signals

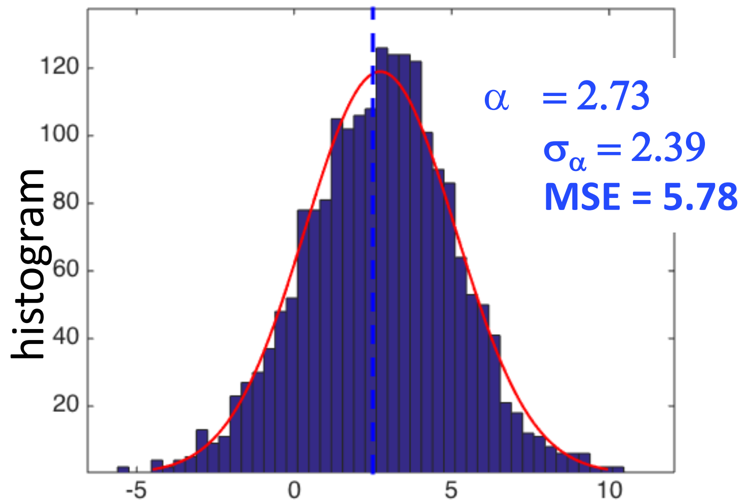
Simulated SLA, Slope α = 2.5, 1 segment of 3000 samples



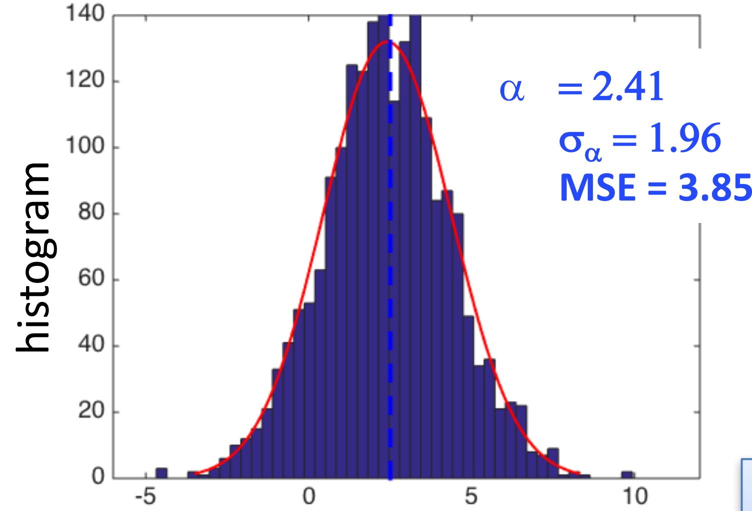
4. Validation on simulated SLA signals

Simulated SLA, Slope α = 2.5, 1 segment of 3000 samples

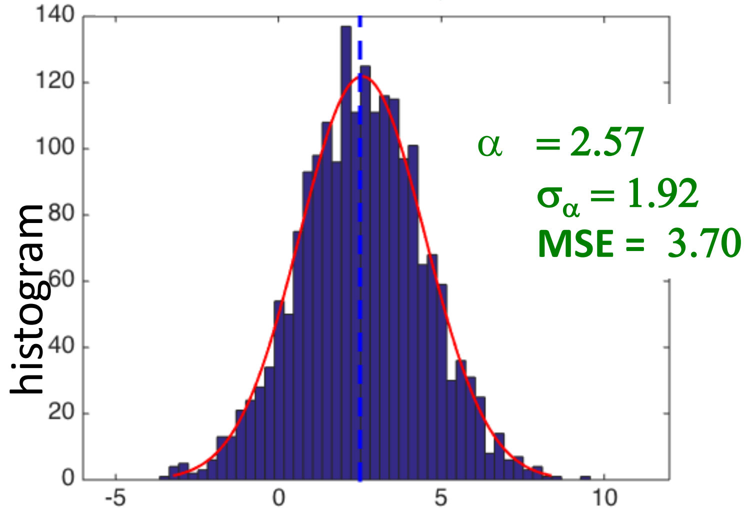
Fourier-based PSD, Rect. window



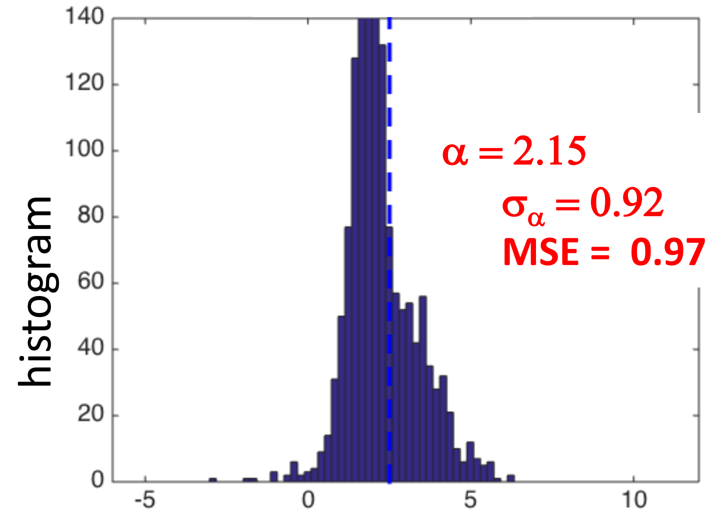
Fourier-based PSD, B-H. window



Fourier-based PSD, Rect.+ B-H



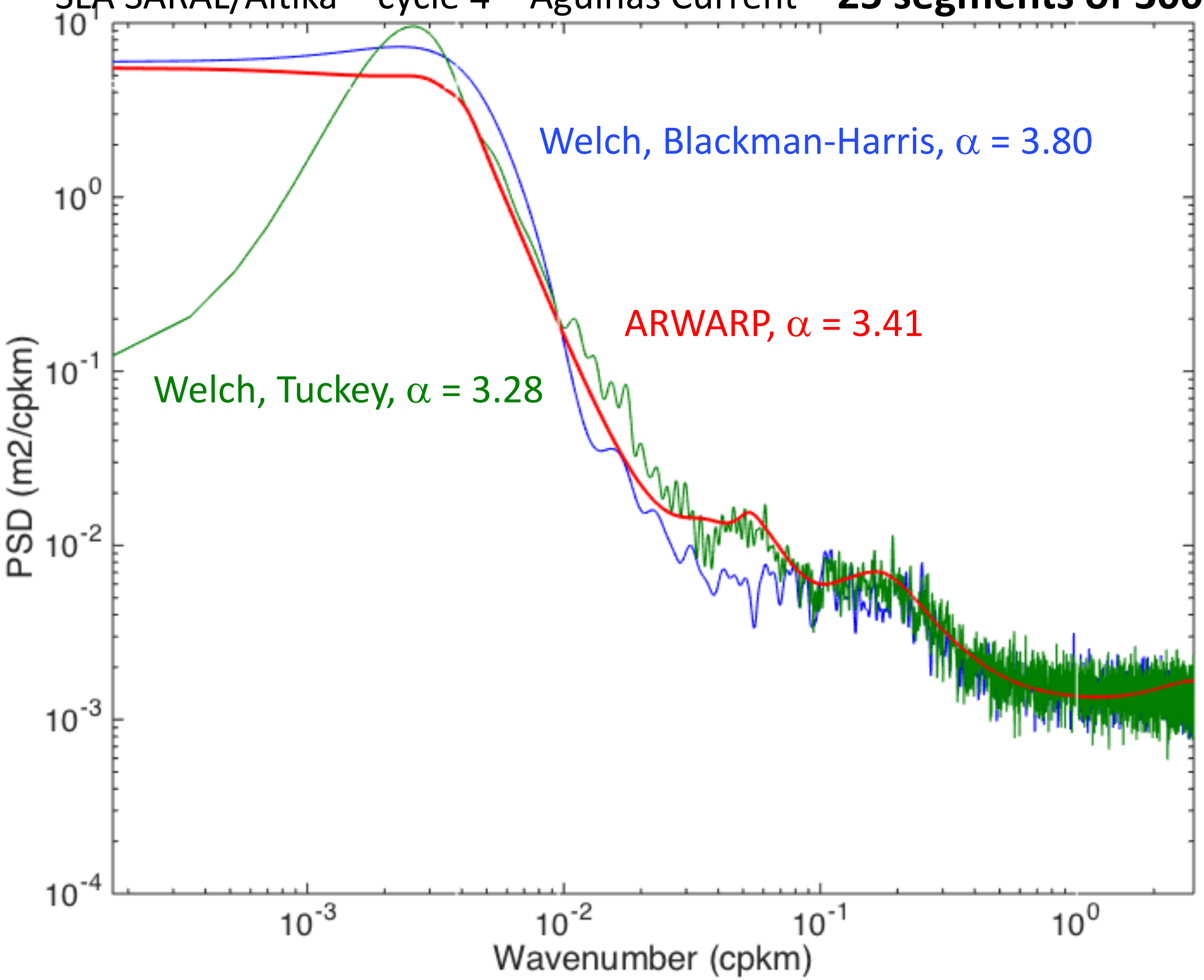
ARWARP PSD



2000 slope estimations

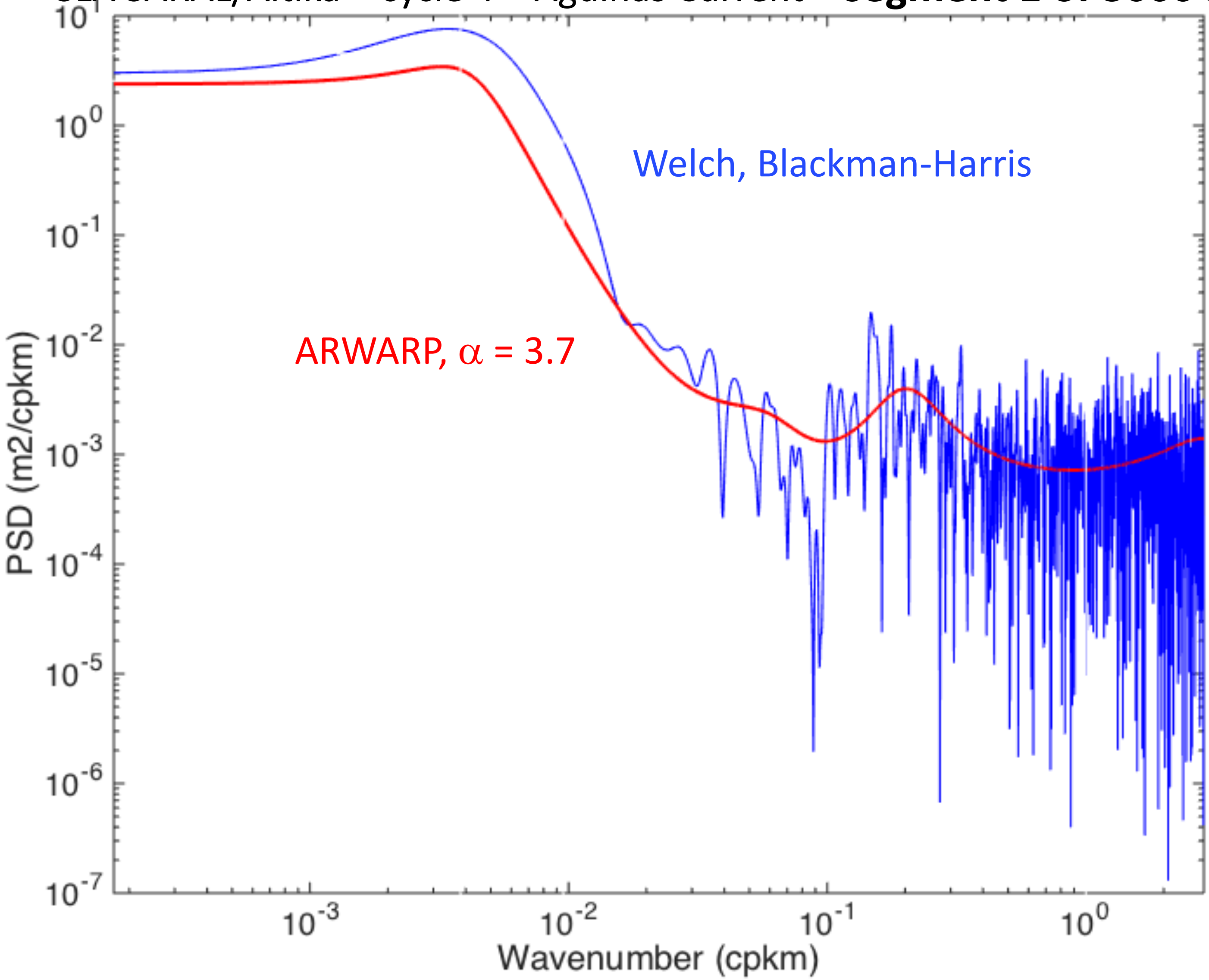
5. Results on real signals

SLA SARAL/Altika – cycle 4 – Agulhas Current – **25 segments of 3000 samples**



5. Results on real signals

SLA SARAL/Altika – cycle 4 – Agulhas Current – **segment 1 of 3000 samples**



6. Conclusions

Study funded by cnes

Spectral analysis of sea level anomaly signals

Fourier-based PSD	ARWARP PSD
For small signal sample size, not interesting	**Can be used on small sample size**, no need to average
Necessary to average PSD (Welch)	Averaging PSD possible
PSD variance	Smooth **PSD**
Estimation of the **slope:** biased (window), large variance	Estimation of the **slope**: small bias, small variance
When averaging PSDs, **slope** estimation combining rectangular and BH windows	≈ ARWARP **slope** estimation (equivalent MSEs)
Estimation of the **noise level**: good estimator, whatever the window (except rect.)	Estimation of the **noise level**: biased

6. Conclusions

Study funded by cnes
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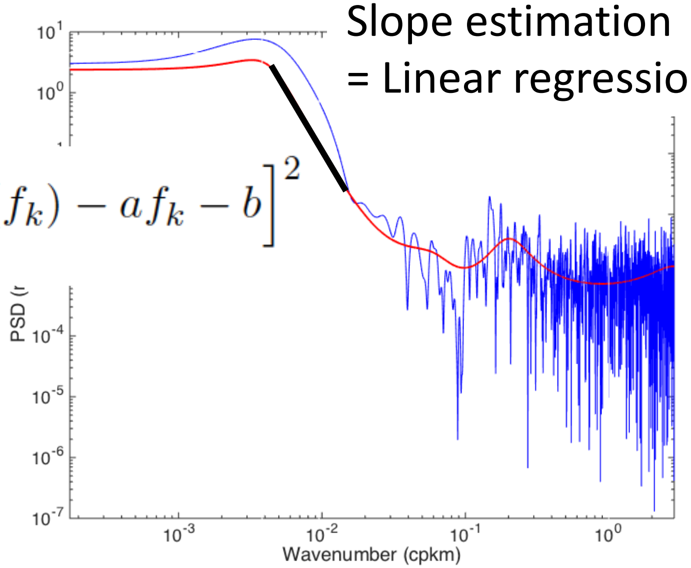
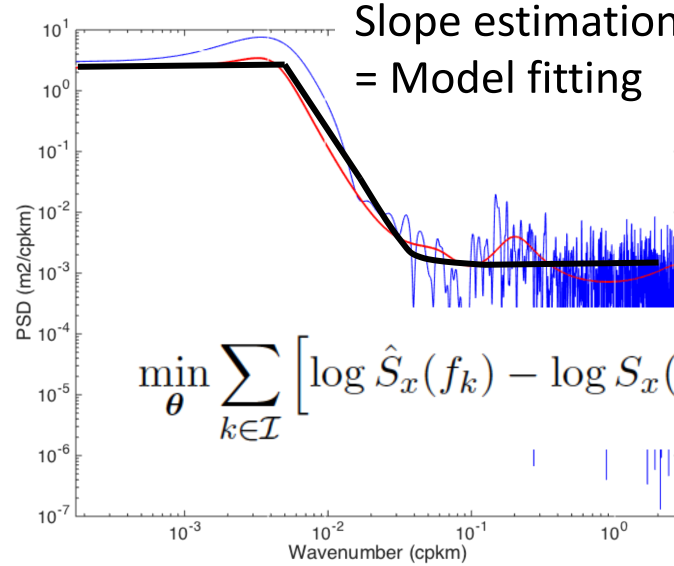
Spectral analysis of sea level anomaly signals using ARWARP or TF-based methods

- ❑ Extended paper in preparation: more details, more results
- ❑ To be used on other kinds of signals
  (SLA 1 Hz, wet troposheric correction, …)
- ❑ Next: study on error bounds
  Cramer–Rao bound on slope estimation = bound on estimation variance
- ❑ Nexte: study to reduce the slope estimation bias



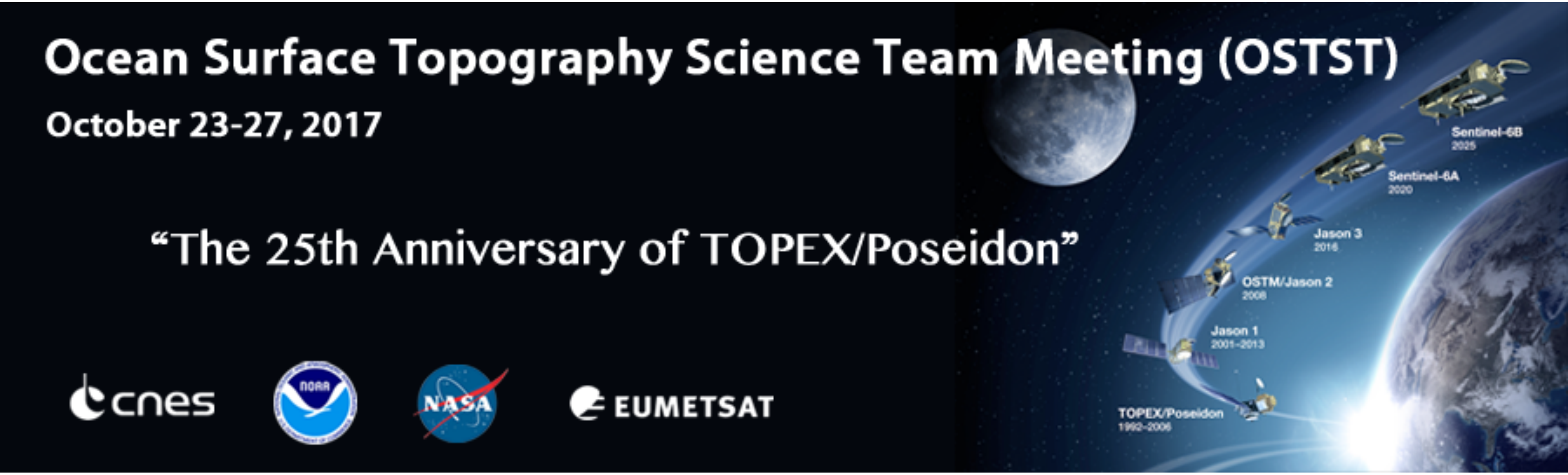
Slope estimation
= Linear regression

$$\min_{a,b} \sum_{k \in \mathcal{J}} \left[\log \hat{S}_x(f_k) - a f_k - b \right]^2$$



Slope estimation
= Model fitting

$$\min_{\boldsymbol{\theta}} \sum_{k \in \mathcal{I}} \left[\log \hat{S}_x(f_k) - \log S_x(f_k; \boldsymbol{\theta}) \right]^2 .$$



Thank you for your attention