Equivalent random propagation time for coaxial cables

B. Lacaze
Tesa, 14/16 Port St-Etienne 31000 Toulouse France

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Abstract

Propagation of monochromatic electromagnetic waves in free space results in a widening of the spectral line. On the contrary, propagation preserves monochromaticity in the case of acoustic waves. In this case, the propagation can be modelled by a linear invariant filter leading to attenuations and phase changes. Due to the Beer-Lambert law, the associated transfer function is an exponential of power functions with frequency-dependent parameters.

In recent papers, we have proved that the acoustic propagation time can be modelled as a random variable following a stable probability distribution. In this paper, we show that the same model can be applied to the propagation in coaxial cables.

keywords: random propagation time, stationary process, stable probability distributions, coaxial cables.

1 Introduction

1) Coaxial cables are characterized by four parameters R, C, L, G and a length l with respective units metre (m), ohm (Ω), farad (F), henry by metre (H/m). The Beer-Lambert law and elementary properties of circuits lead to the following tranfer function

$$H_l(\omega) = e^{-l\sqrt{(R+i\omega L)(G+i\omega C)}}. \qquad (1)$$

In practice, the dielectric loss is generally negligible compared to ωC and R depends on the frequency $\omega/2\pi$ [4]. Consequently, the transfer function simplifies as follows:

$$H_l(\omega) = e^{-l\left(im\omega+a\sqrt{\omega}(1+i)\right)}, \text{ for } \omega > 0$$

where the parameters m and a characterize the coaxial cable in the considered frequency band. This formula is generalized to $\omega \in \mathbb{R}$:

$$H_l(\omega) = e^{-l\left(im\omega+a\sqrt{|\omega|}(1+i\,\mathrm{sgn}\,\omega)\right)} \qquad (2)$$

using the relation $H_l(\omega) = H_l^*(-\omega)$ since a real input must yield a real output. The exponential expression $H_l(\omega) = e^{-l\gamma(\omega)}$ is in accordance with the Beer-Lambert law and thus to the relation relating filters in series:

$$H_{l+l'}(\omega) = H_l(\omega)\, H_{l'}(\omega).$$

The relation (2) means that the monochromatic wave $e^{i\omega t}$ at the cable input is transformed in the monochromatic wave $H_l(\omega)\, e^{i\omega t}$ at the distance l, assuming that the cable is semi-infinite or well-matched.

2) This result can be compared with propagation in other media. Propagation of electromagnetic waves in free (i.e. not guided) medium often leads to a widening of the spectral line. Backscattering of monochromatic waves on trees in the X-band (8 GHz) results in a power spectrum with a bandwidth around few tens Hz [5], [16]. The same behavior has been observed for laser propagation in the atmosphere [7], [17]. In sea water, radar backscattering leads to a mixing of Gaussian spectra depending on the polarization and with Doppler shift [12], [20]. The same phenomena are found in wind profilers [7], [1]. We have known also for a long time that emissions or absorptions of sky light lead to line broadening and Doppler shift mainly explained by star composers and movements [9]. In all these practical situations, the observed spectral widening changes a line spectrum into a continuous one. To my knowledge and for small powers, spectral widening has not been noticed in coaxial cables. The cable behaves like a linear invariant filter subject to the Beer-Lambert law which results to an exponential expression of its transfer function. Consequently, for a coaxial cable, a pure monochromatic wave remains a pure monochromatic wave. The amplitude and the phase are two functions of the frequency $\omega/2\pi$ defined by formula (2).

3) The same behavior is observed for acoustic propagation. The received wave is monochromatic like the transmitted wave. In the atmosphere or in the sea water, the wave attenuation expresses as $\exp[-al\omega^2]$, where the parameter a is medium-dependent and l is the distance [8]. The celerity c of the wave is constant with respect to the frequency. Actually, tables giving the influence of the temperature, the salinity and other physical parameters are very detailed. To our knowledge there exists no table giving the dependence between c and ω since they are usually assumed independent. However, ultrasonic waves have a different behavior. The crossing of media like biological tissues, liquids like castor oil (mimicking tissues), egg yolk or many other substances on small distances (of the order of the centimetre for instance) are weakened like $\exp\left[-al\omega^b\right]$ with $0 < b < 2$ [10], [11], [19]. Moreover, for $b \neq 2$, the celerity is function of ω. For $b \neq 1$, the complex gain $H_l(\omega)$ of the equivalent filter verifies:

$$H_l(\omega) = \exp\left[-ilm\omega - al\left|\omega\right|^b\left(1 + i\tfrac{\omega}{|\omega|}\tan\tfrac{\pi b}{2}\right)\right] \tag{3}$$

and for $b = 1$:

$$H_l(\omega) = \exp\left[-ilm\omega - al\left|\omega\right|\left(1 - i\tfrac{\omega}{|\omega|}\tfrac{2\ln|\omega|}{\pi}\right)\right]$$

For instance, the value $b = 1$ often characterizes biological tissues or evaporated milk. $b = 1.66$ is used for castor oil up to 300MHz with a very good accuracy, b is around 1.5 for egg yolk and $b = 0.5$ is for brass tubes in low frequency (400-2400Hz) [14]... Equations (3) are in accordance with the "near local Kramers-Kronig theory" of Szabo

[19]. We note that, in the particular case where $b=2$, the celerity of the wave is the constant m ($\tan\frac{\pi b}{2}=0$) according to Eq.(3) .

Propagation in coaxial cables verifies (2) , i.e. (3) with $b=1/2$. This result appears as a paradox: this behavior is similar to the case of acoustic waves (preserved monochromaticity) and not to electromagnetic wave propagation in free space (widened power spectrum).

4) After these considerations about physical modelling, some probabilistic models can be discussed. The probability distribution of the random variable (r.v.) X belongs to the set of stable distributions when its characteristic function (c.f.) is defined by:

$$\mathrm{E}\left[e^{-i\omega X}\right]= \exp\left[-i\alpha\omega-al\left|\omega\right|^{b}\left(1-ic\frac{\omega}{|\omega|}\tan\frac{\pi b}{2}\right)\right] \quad (4)$$

when $b\neq 1$ and, when $b=1$

$$\mathrm{E}\left[e^{-i\omega X}\right]= \exp\left[-i\alpha\omega-al\left|\omega\right|\left(1+ic\frac{\omega}{|\omega|}\frac{2\ln|\omega|}{\pi}\right)\right].$$

The set of real parameters (α,a,b,c) must verify the conditions $a>0, 0<b\leq 2, |c|\leq 1$ [15], [18]. Stable distributions generalize the Central-Limit theorem to r.v. with infinite variance. The complex gains $H_l(\omega)$ defined by (3) can be identified with the subset of functions in (4) under the condition $c=-1$. Moreover a random process $\mathbf{A}=\{A(t), t\in\mathbb{R}\}$, such that the r.v $A(t)$ and $A(t)-A(t-\tau)$ follow stable distributions with $b=\frac{1}{2}, c=-1$ and $b=\frac{1}{2}, c=0$ respectively, can be derived.

As shown in the next section, the attenuations and phase changes in coaxial cables can be explained by random propagation times. Numerical values come from the data sheet of Belden 8281. This class of cables has been studied in several theses on equalization (for instance see [2], [3]).

2 Random propagation times

1) Let $\mathbf{A}_l=\{A_l(t), t\in\mathbb{R}\}$ be a random process with the following one-dimensional characteristic function

$$\mathrm{E}\left[e^{-i\omega A_l(t)}\right]=e^{-l\left(im\omega+a\sqrt{|\omega|}(1+i\mathrm{sgn}\omega)\right)}. \quad (5)$$

This formula corresponds to a stable probability distribution (4) with parameters $\left(lm, la, \frac{1}{2}, -1\right)$ along with the transfer function (2) which defines a coaxial cable. Now, let $\mathbf{Z}_l=\{Z_l(t), t\in\mathbb{R}\}$ denote the random process defined by:

$$Z_l(t)=e^{i\omega_0(t-A_l(t))}. \quad (6)$$

$Z_l(t)$ is the output of a device with input $e^{i\omega_0 t}$ ($\omega_0>0$) and subjected to a random propagation time $A_l(t)$. Let assume that $\mathbf{A}_l$ is stationary in the sense where

$$\phi_l(\omega,\tau)=\mathrm{E}\left[e^{-i\omega(A_l(t)-A_l(t-\tau))}\right]$$

is independent of t and sufficiently regular. The process $\mathbf{Z}_l$ can be split in two additive terms [6]:

$$\mathbf{Z}_l=\mathbf{G}_l+\mathbf{V}_l \quad (7)$$

where $\mathbf{G}_l=\{G_l(t), t\in\mathbb{R}\}$ is defined by

$$G_l(t)=e^{i\omega_0(t-lm)-al\sqrt{\omega_0}(1+i)}. \quad (8)$$

The process $\mathbf{V}_l$ defined by (7) and (8) is zero-mean and stationary with autocorrelation function

$$\begin{gathered}\mathrm{E}\left[V_l\left(t\right)V_l^*\left(t-\tau\right)\right]= \\ e^{i\omega_0\tau}\left[\phi_l\left(\omega_0,\tau\right)-e^{-2al\sqrt{\omega_0}}\right].\end{gathered} \tag{9}$$

2) If we identify the wave with the model (6) ,then $\mathbf{G}_l$ is the wave measured at distance l. Because the power of $\mathbf{Z}_l$ is constant, $\mathbf{V}_l$ represents the losses in the cable and in the medium up to the distance l. This quantity is not measured by practical devices and is probably outside the observed frequency band. From (9) the $\mathbf{V}_l-$power spectrum depends on the probability distribution of $A_l\left(t\right)-A_l\left(t-\tau\right)$. In the appendix, we prove that it is possible to construct processes $\mathbf{A}_l$ which fulfill the following conditions:

a) the r.v. $A_l\left(t\right)$ possesses the stable distribution defined by (5) with arbitrary parameters m, a

b) the r.v. $A_l\left(t\right)-A_l\left(t-\tau\right)$ possesses the stable distribution defined by (4) with $\alpha=c=0, b=\frac{1}{2}$

c) the construction can be made so that the $\mathbf{V}_l-$power in any frequency band $\left(\omega_0-b,\omega_0+b\right)$ is arbitrarily small.

The last property explains why the model of propagation $\mathbf{Z}_l$ defined by (6) fulfills the theorem of the energy balance though the measured wave is an attenuated (and delayed) replica of the transmitted wave. The process $\mathbf{V}_l$ is the quantity lost and dissipated by the medium at frequencies far from the transmitted wave frequency.

3) In the coaxial cable framework $\mathbf{G}_l$ is the received wave at the distance l. For the Belden 8281 cable in the band (1,1000MHz) we have [2]

$$m=\sqrt{LC}=52.10^{-10}\mathrm{s.m}^{-1}$$

which leads to a wave celerity equal to $2.10^8\mathrm{m.s}^{-1}$ i.e. 66% of the light celerity in vacuum. Moreover, from the same source and in the usual system

$$a=39.10^{-8}.$$

From (8) , the term $al/\sqrt{\omega_0}$ is an extra delay for the wave. We have to compare the nominal delay ml with the variable delay $al/\sqrt{\omega_0}$. At 100MHz, the extra-delay is smaller than 1% of the nominal delay.

3 Remarks

1) The theory of stable probability distributions reveals the following interesting property [15], [18]. Among the c.f. (4) , the only one-sided probability densities are defined by the parameters values $c=\pm1, b<1$. $c=1$ is for a one-sided to the left and $c=-1$ for an one-sided to the right (the probability density is 0 at the left of the origin point when $\alpha=0$). This is equivalent to the causality of the filter defined by the transfer function (2). For $b\geq1$ or $b<1, c\neq\pm1$, this property does not hold. However, among the $c-$parameter values, the value -1 is the best one, because it minimizes the probability at the left of the origin point. It is indeed the value $c=-1$ which has been taken in (4) to verify (3) . Moreover, the imaginary part in the exponential of (2) is dominated by the term $-ilm\omega_0$ in real cases. Though the term $-ila\sqrt{|\omega_0|}$ seems negligible

with respect to $-iml\omega_0$, its influence on the corresponding impulse response is strong.

Finally, the conditions of the Kramers-Kronig relations are verified by $K(z)$ defined by

$$K(z) = \exp\left[-alz^b\left(1 - i\tan\frac{\pi b}{2}\right)\right]$$
$$al > 0, 0 < b < 1$$

when $z^b = \rho^b e^{ib\theta}$ with $\rho > 0, 0 < \theta < \pi$ in the upper plane. On the real axis, $K(\omega) = H_l(-\omega)$ when $m = 0$.

2) For frequencies larger than 1000MHz, the model (6) is no longer sufficient for the Belden 8281 cable (and from some other frequencies for other cables). It has to be changed in

$$H_l(\omega) = e^{-l\left(im\omega + a\sqrt{|\omega|}(1+i\mathrm{sgn}\omega) + \gamma|\omega|\right)} \tag{10}$$

with $\gamma > 0$. The new term in $\gamma|\omega|$ takes into account dielectric losses [4]. We know that $e^{-\lambda|\omega|}, \lambda > 0$, is the c.f. of the Cauchy distribution. Consequently (10) corresponds to the convolution of the stable distribution $\left(lm, la, \frac{1}{2}, -1\right)$ with a Cauchy distribution with parameter γl. Though the Cauchy distribution is stable (with parameters $(0, l\gamma, 1, 0)$) the result no longer defines a stable distribution.

3) The transfer function $H(\omega)$ is the Fourier transform of the impulse response $h(t)$:

$$H(\omega) = \int_{-\infty}^{\infty} h(t)\, e^{-i\omega t} dt.$$

This definition is coherent with the input-output relation

$$y_{out}(t) = \int_{-\infty}^{\infty} h(u)\, y_{in}(t-u)\, du$$

Equivalently $H(\omega)\, e^{i\omega t}$ is the output when $e^{i\omega t}$ is the input and/or $h(t)$ is the output when $\delta(t)$ (the "Dirac function") is the input.

In probability calculus a c.f. $\psi(\omega)$ is the Fourier transform of a probability density $f(t)$ (if it exists) in the sense

$$\psi(\omega) = \int_{-\infty}^{\infty} f(t)\, e^{i\omega t} dt.$$

To identify a transfer function with a characteristic function it is necessary to change $e^{i\omega t}$ into $e^{-i\omega t}$ in the last equation (compare formulas (4) in this paper with formula 5.7.19 in Lukacs [15]).

4) The probability density of $A_1(t)$ (and of $A_l(t)$ whatever l) is given from (5), (11), (12). For the Belden 8281 coaxial cable with $a = 39.10^{-8}$, we have approximately

$$\Pr\left[A_1(t) - m > 8.10^{-10}\right] \simeq 0.01$$

to be compared with $m = 52.10^{-10}\mathrm{s.m}^{-1}$. The mode is close to 5.10^{-14} and it is well-known that this distribution is heavy-tailed.

4 Conclusion

In circuit theory, a coaxial cable is defined by a set (R, C, L, G) where (R, L) and (C, G) represent series and parallel components. The equivalent circuit highlights the linear functions $(R + i\omega L)$ and $(G + i\omega C)$ of the frequency $\omega/2\pi$. Actually, it is not suitable for large bandwidths where components depend on the frequency (mainly due to the "skin effect"). In this case, the transfer function (1) is changed in (2) or (10) which have a very different appearance. These formulas are very close to characteristic functions of stable probability distributions. In this paper we have proved that the wave propagation in a coaxial cable is equivalent to a random propagation time. The random process $\mathbf{A}_l$ which represents it at the distance l has particular probability laws. The one-dimensional law is stable with parameters $\left(lm, la, \frac{1}{2}, -1\right)$. lm and la are the parameters of position and amplitude, $\frac{1}{2}$ is the exponent of the law. The last parameter -1 is particularly remarkable because this value is matched to the causality property of linear filters. The randomly delayed process $\mathbf{Z}_l$ (the set of r.v. $\exp[i\omega_0 (t - A_l(t))]$) can be split into two parts. The first one (the process $\mathbf{G}_l$) is the observed process at the end of the cable (in the case no reflexion). We have proved that the probability distribution of $A_l(t) - A_l(t-\tau)$ can be chosen in the class of stable distributions so that the second part $\mathbf{V}_l$ is outside the studied frequency band. The model obeys the energy balance theorem because the power of the sum $\mathbf{G}_l + \mathbf{V}_l$ is equal to the transmitted power. To conclude, this study could be applied to a more general framework. I have proved in other papers that the proposed model applies in many situations of propagation: propagation of acoustics and ultrasonics waves and also propagation of electromatic waves, in radio, radar, laser and star light (see the bibliography).

5 Appendix

1) Let $\mathbf{X} = \{X_n, n \in \mathbb{Z}\}$ be a sequence of i.i.d. (independent and identically distributed) r.v. (random variables) with c.f. (characteristic function)

$$\ln \mathrm{E}\left[e^{-i\omega X_n}\right] = -\sqrt{|\omega|}\,(1 + i\,\mathrm{sgn}\omega)\,. \tag{11}$$

We know that it is one of three stable distributions with simple probability density $f(x)$ (with the Gauss and Cauchy distributions) [15]

$$f(x) = \begin{cases} \frac{1}{\sqrt{2\pi}} x^{-3/2} e^{-1/2x}, x > 0 \\ 0, x < 0. \end{cases} \tag{12}$$

The main problem with this distribution is the lack of moments, which prevents the use of the mean-square convergence. Now, we define the r.v. Y by

$$Y = \sum_{k=-\infty}^{\infty} a_k X_k, a_k > 0$$

where $\mathbf{a} = \{a_n, n \in \mathbb{Z}\}$, is a sequence of real positive numbers. The equality

$$\begin{gathered} \mathrm{E}\left[\exp\left(-\sum_{k=m}^{n} i\omega a_k X_k\right)\right] = \\ \exp\left[-\left(\sum_{k=m}^{n} \sqrt{a_k}\right)\sqrt{|\omega|}\,(1 + i\,\mathrm{sgn}\omega)\right] \end{gathered} \tag{13}$$

comes from (11), using the (mutual) independence of the X_n. By application of the continuity theorem of P. Levy [15], we deduce that Y is defined (in the sense of the convergence in distribution) if and only if

$$\sum_{k=-\infty}^{\infty} \sqrt{a_k} < \infty. \tag{14}$$

Moreover, this condition allows to verify the hypotheses of the "three series theorem" of A. N. Kolmogorov [13], which assures the a.s. (almost sure) existence of Y. Using (13)

$$\mathrm{E}\left[e^{-i\omega Y}\right] = e^{-\lambda\sqrt{|\omega|}(1+i\mathrm{sgn}\omega)} \quad \text{with } \lambda = \sum_{k=-\infty}^{\infty}\sqrt{a_k}. \tag{15}$$

Consequently, the probability distributions of Y and $\lambda^2 X_n$ are the same.

2) Now, we define the real random process $\mathbf{U}_h = \{U_h(t), t \in \mathbb{R}\}$ by

$$U_h(t) = h^2 \sum_{k=-\infty}^{\infty} \theta(t-kh) X_k \tag{16}$$

for some positive function $\theta(t)$ symmetric and decreasing on $\mathbb{R}^+$. From the preceeding results, $\mathbf{U}_h$ is well defined for any $h > 0$ when

$$\sum_{k=-\infty}^{\infty} \sqrt{\theta(t-kh)} < \infty$$

for any $t \in \mathbb{R}$. $U_h(t)$ follows the stable distribution (15) with parameter

$$\lambda = h \sum_{k=-\infty}^{\infty} \sqrt{\theta(t-kh)}$$

which can be taken arbitrarily close to

$$\mu = \int_{-\infty}^{\infty} \sqrt{\theta(u)}du \tag{17}$$

when this quantity exists.

3) Moreover, if $t = hp_h, \tau = hq_h > 0$ where p_h and q_h are even integers and h arbitrarily small we have

$$U_h(t) - U_h(t-\tau) = h^2 [A_h - B_h]$$

with

$$\begin{cases} A_h = \sum_{k=0}^{\infty}\left[\theta\left(-\frac{\tau}{2}+kh\right) - \theta\left(\frac{\tau}{2}+kh\right)\right] X_{k+p-\frac{q}{2}} \geq 0 \\ B_h = \sum_{k=0}^{\infty}\left[\theta\left(-\frac{\tau}{2}+kh\right) - \theta\left(\frac{\tau}{2}+kh\right)\right] X_{-k+p-\frac{q}{2}} \geq 0. \end{cases}$$

Because the X_k are (mutually) independent, $U_h(t) - U_h(t-\tau)$ follows a probability distribution with c. f. in the form

$$\mathrm{E}\left[e^{-i\omega(U_h(t)-U_h(t-\tau))}\right] = \exp\left[-h\sqrt{|\omega|}\left(\alpha_h + \beta_h + i(\alpha_h - \beta_h)\mathrm{sgn}\omega\right)\right]$$

which does not depend on t. Obviously $\alpha_h = \beta_h$. When $h \to 0$ we have

$$\lim_{h\to 0}\mathrm{E}\left[e^{-i\omega(U_h(t)-U_h(t-\tau))}\right] = \exp\left[-2\sqrt{|\omega|}\int_0^{\infty}\sqrt{\theta\left(x-\frac{\tau}{2}\right) - \theta\left(x+\frac{\tau}{2}\right)}dx\right]. \tag{18}$$

Consequently we have constructed a stationary process $\mathbf{U}_h$ with c.f. $\psi(\omega)$ and $\phi(\omega,\tau)$ arbitrarily close to

$$\psi(\omega) = \exp\left[-2\sqrt{|\omega|}(1+i\mathrm{sgn}\omega)\int_0^{\infty}\sqrt{\theta(x)}dx\right]$$

$$\phi(\omega,\tau) = \exp\left[-2\sqrt{|\omega|}\int_0^{\infty}\sqrt{\theta\left(x-\frac{\tau}{2}\right) - \theta\left(x+\frac{\tau}{2}\right)}dx\right] \tag{19}$$

where $\theta(x)$ is a regular enough symmetric function decreasing on $\mathbb{R}^+$. We remark that (when $\lim_{t\to\infty}\theta(t) = 0$ quickly enough)

$$\lim_{\tau\to\infty}\phi(\omega,\tau) = |\psi(\omega)|^2 \tag{20}$$

which shows some "independence" between $U_h(t)$ and $U_h(t-\tau)$ when τ is large.

4) Now, let assume that the process $\mathbf{Z}_l$ defined by (6) and (7) is characterized by

$$\begin{cases} \psi_{l,n}(\omega) = \exp\left[-2l\sqrt{|\omega|}(1+i\mathrm{sgn}\omega)\int_0^{\infty}\sqrt{\theta_n(x)}dx\right] \\ \phi_{l,n}(\omega,\tau) = \exp\left[-2l\sqrt{|\omega|}\int_0^{\infty}\sqrt{\theta_n\left(x-\frac{\tau}{2}\right) - \theta_n\left(x+\frac{\tau}{2}\right)}dx\right] \\ \theta_n(x) = n^2\theta_1(nx) \end{cases} \tag{21}$$

where $\theta_1(x)$ is positive, even, and decreasing on $\mathbb{R}^+$, with

$$\int_0^\infty \sqrt{\theta_1(x)}dx < \infty.$$

The spectral power density $s^n_{V,l}(\omega)$ of the "noise" $\mathbf{V}_l$ is the Fourier transform of its autocorrelation function (assuming $\lim_{t\to\infty} \theta_1(t) = 0$ quickly enough)

$$s^n_{V,l}(\omega+\omega_0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-i\omega\tau}\left[\phi_{l,n}(\omega_0,\tau) - \left|\psi_{l,n}(\omega_0)\right|^2\right] d\tau.$$

Using (9) and (21) we obtain the equalities

$$\begin{cases} s^1_{V,l}(\omega+\omega_0) = n s^n_{V,l}(n\omega+\omega_0) \\ \int_{-b}^{b} s^n_{V,l}(\omega+\omega_0)\, d\omega = \\ \int_{-b/n}^{b/n} s^1_{V,l}(\omega+\omega_0)\, d\omega \to_{n\to\infty} 0. \end{cases}$$

An increase of n induces a widening of the spectral density (remember that the total power of $\mathbf{V}_l$ does not depend on n). The power of $\mathbf{V}_l$ in any interval $(\omega_0 - b, \omega_0 + b)$ can be made smaller than any quantity increasing n. Consequently a device centered on ω_0 will only measure $\mathbf{G}_l$ the harmonic part of $\mathbf{Z}_l$ if we assume that the parameter n is large enough.

References

[1] B. B. Balsey, K. S. Gage, *The MST radar technique: potential for middle atmospheric studies,* Pure Appl. Geophys. 45 (1980) 452-493.

[2] J. S-Y. Cheng, *Adaptative Equalizations System for Data Transmission over Coaxial Cables,* Thesis, Toronto, 1998.

[3] A. Hadgi-Abdolhamid, *Partial Analog Equalization and ADC Requirements in Wired Communications,* Thesis, Toronto, 2004.

[4] Jin Liu, X. Lin, *Equalization in High-Speed Communication Systems,* IEEE Circuits and Systems Magazine, 4 (2) (2004) 4-17.

[5] B. Lacaze, *A random equivalent transit time for backscattering on trees,* Signal Processing, 58 (1997) 223-225.

[6] B. Lacaze, *Stationary clock changes on stationary processes,* Signal Processing, 55 (2) (1996) 191-206.

[7] B. Lacaze, *An unifying model for spectra of transmitted monochromatic waves,* Wave Motion, 44 (2006) 70-76.

[8] B. Lacaze, *A Stochastic Model for Acoustic Attenuation*, Waves in Random and Complex Media, 17 (3) (2007) 343-356.

[9] B. Lacaze, *Weakenings and Redshifts Induced by Random Propagation Times,* Optics Com., 281 (10) (2008) 2740-2745.

[10] B. Lacaze, *A Stochastic Model for Propagation through Tissue,* IEEE Trans. Ultrason. Ferroelec. Freq. Contr. 56 (10) (2009) 2180-2186.

[11] B. Lacaze, *Random Propagation Times in Ultrasonics*, Waves in Random and Complex Media, 20 (1) (2010) 179-190.

[12] B. Lacaze, *A stochastic model for backscattering on the sea,* Waves in Random and Complex media, 16 (1) (2006) 69-74.

[13] M. Loeve, *Probability Theory,* Van Nostrand, 1955.

[14] W. P. Mason, *The propagation characteristics of sound tubes and acoustic filters,* Phys. Rev. 31 (1928) 283-295.

[15] E. Lukacs, *Characteristic Functions,* 2^{nd}ed., Griffin, London, 1970.

[16] R. M. Naranayan, D. W. Doerr, D. C. Rundquist, *Temporal decorrelation of X-band backscatter from wind- influenced vegetation,* IEEE Trans. Aeros. Electron. Sys. 28 (2) (1992) 404-412.

[17] G. Parry, *Measurement of atmospheric turbulence induced intensity fluctuations in a laser beam,* Opt. Acta, 28 (5) (1981) 715-728.

[18] G. Samorodnitsky, M. S. Taqqu, *Stable non-Gaussian random processes,* Chapman &Hall, 1994.

[19] T. L. Szabo, *Causal theories and data for acoustic attenuation obeing a frequency power law,* J. Acoust. Soc. Amer. 97 (1995) 14-24.

[20] D. Walker, *Experimentaly motivated model for law grazing angle radar Doppler spectra of the sea surface,* IEE Proc. Rad. Son. Nav. 147 (2000) 114-120.