A New Exact Low-Complexity MMSE Equalizer for Continuous Phase Modulation

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***Abstract*— This letter introduces a new low-complexity frequency-domain equalizer for continuous phase modulations (CPM). The derivation of a fractionally spaced representation for circular block-based CPM leads, without any approximation, to a simple yet efficient frequency-domain equalization. The proposed equalizer is compared to the state-of-the-art approaches. Simulation results show the equivalence in terms of performance with a lower or similar complexity.**

***Index Terms*— Continuous phase modulation, frequency-domain equalizer, frequency-selective channel.**

I. Introduction

CONTINUOUS Phase Modulation (*CPM*) signals are commonly known for their good spectral efficiency and their constant complex envelop, which is robust to the non-linearities introduced by embedded amplifiers. They are mainly considered for applications such as deep-space, aeronautical or tactical communications and for IoT.

Compared to linear modulations, CPM transmission over frequency-selective channels is a challenging task. Optimal joint channel equalization and detection using a Maximum A Posteriori (*MAP*) trellis based detector is too prohibitive as its complexity grows exponentially with both the CPM and the channel memory. Contrary to the single carrier (SC) case using linear modulation, as the received signal is not a linear function of the transmitted data symbols, sub-optimal linear equalization and detection using the minimum mean square error (MMSE) criterion should be performed into two steps: first linear equalization is usually performed over the over-sampled complex signal envelop, then the output is fed to a non linear detector for CPM for Gaussian channels. MMSE based approaches mainly consider the generalization of SC frequency-domain equalization (FDE) for linear modulations to the case of CPMs (using cyclic prefix or unique word block transmissions). SC-FDE using linear modulations leads to a circular linear Gaussian model, linear with respect to the transmitted data symbols whose Time-Domain (*TD*) and Frequency-Domain (*FD*) auto-correlation matrices are usually both diagonal, leading to, with some abuse of terminology, the so-called 'one-tap' FD equalizer. FDE for CPM signals also ends up with a circular linear Gaussian model, but with respect to a vector of samples of the over-sampled complex signal, which is correlated by nature. Therefore, without any further hypothesis, both TD and FD auto-correlation matrices of those samples are not diagonal, and having a 'one-tap' FD equalizer highly depends on the structure of the model used. The use of SC-FDE blindly drawn from the linear case is a rough approximation leading to some heavy performance degradations. In this letter, it is shown that by carefully considering the model and the representation of the received over-sampled CPM signal, a 'one-tap' FD equalizer is achievable, by using the cyclic statistical properties of the signal in case of circular block-based transmission and a Fractionally-Spaced (*FS*) representation of the signal.

Concerning CPMs, Pancaldi and Vitetta [1] and Thillo *et al.* [2] propose minimum mean square error frequency-domain equalizers (*MMSE-FDE*). Pancaldi and Vitetta [1] use the Laurent Decomposition (*LD*) [3] of the binary CPM signals to describe the signal as a sum of P linear Pulse Amplitude Modulations (*PAM*) with complex pseudo-symbols and then equalizes those pseudo-symbols and make use of a FS representation to derive the equalizer's expression. Without using the LD, Thillo *et al.* [2] derive an equalizer based on an over-sampled version of the continuous non-linear CPM using a polyphase representation of the received signal and then outputs an equalized over-sampled version to feed a classical non linear receiver. It can be formally proved that those equalizers are equivalent up to post-treatment and thus have the same performance [4]. To further reduce the complexity of this scheme, Thillo *et al.* [2] also describe a low-complexity *approximated* MMSE-FDE. Tan and Stuber [5] propose a *symbol spaced* FD equalizer based on the LD and another one based on an orthogonal representation of the signal. Unfortunately, it can be shown that those approaches cannot cope with all types of multi-path channels [4] as they assume the delays of the paths to be multiples of the symbol duration. Those approaches have been studied for aeronautical telemetry in [6]. Brown and Vigneron [7] also use an orthogonal representation of the signal to perform a FD equalization but, unlike the Gram-Schmidt procedure of Tan and Stuber [5], it proposes to use Legendre Polynomials to generate the orthogonal basis functions. A completely different approach has been proposed in [8], using the tilted-phase representation of CPM signals and working on the over-sampled signal. Approximated low-complexity filter-based equalizers based on the MMSE criterion are proposed, in both the time and frequency domains, and the turbo principle based on soft linear filtering as proposed by Tüchler and Singer [9] is applied. Xu *et al.* [10] use a

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FS representation of the signal which is strictly similar to the one proposed in [8]. However both of them do not take into account the auto-correlation of CPM signals, resulting in a loss of performance. Darsena *et al.* [11] develop a Zero-Forcing and a MMSE linear time-varying equalizer in the Time-Domain for CPM over doubly-selective channels based on the Basis Expansion Model (*BEM*) and a widely linear counterpart of those equalizers has been introduced in [12]. However, this approach has a high computational complexity as it requires full-matrix inversion, even if only time-invariant channels are considered in case of large channel spread whereas our approach has a constant computational complexity with the channel spread.

In this letter, we derive an *exact low-complexity* MMSE-FDE based on the FS representation of the CPM waveforms, only considering time-invariant frequency-selective channels. As in [2], we perform a linear MMSE-FDE over the over-sampled complex envelope of the CPM signals, but by using the FS representation, we can fully benefit from the properties of circular block-based CPM and reduce the complexity, without making any kind of approximation in the equalizer derivation. While performing the same as the equalizer proposed by Pancaldi and Vitetta [1] and the "full complexity" polyphase domain equalizer of Thillo *et al.* [2], we will show that the proposed approach has a significant lower complexity, of the same order as the *approximated low complexity* version of Thillo *et al.* [2]. Moreover, we will also prove that the MMSE-FDE from [8] is an approximation of the proposed FS equalizer.

This letter is organized as follows. Section II presents the FS representation for circular block-based CPM signals transmitted over frequency-selective channels. In section III, the corresponding FS block MMSE-FDE is derived. In section IV, we compare the computational complexity of different equalizers of the same kind. Simulation results are given in section V, while conclusions are drawn in section VI.

II. Block-Based CPM Representation

Let $\boldsymbol{\alpha} = [\alpha_0, \cdots, \alpha_{N-1}] \in \{\pm 1, \pm 3, \ldots, \pm M-1\}^N$ be a block of N independent and identically distributed (iid) symbols drawn from a M-ary alphabet. The equivalent baseband complex envelope $s(t)$ of the transmitted CPM signal is:

$$s(t) = \sqrt{\frac{2E_s}{T_s}} \exp\left(j2\pi h \sum_{i=0}^{N-1} \alpha_i q(t - iT_s)\right) \tag{1}$$

where E_s is the symbol energy, T_s is the symbol duration, h is the modulation index, L_{cpm} is the CPM memory, $q(t) = \int_0^t g(\tau)d\tau$ if $t \leq L_{\text{cpm}}T_s$ and $q(t) = 1/2$ if $t > L_{\text{cpm}}T_s$ and $g(t)$ is the pulse response.

To perform efficient frequency domain equalization, we first need to *circularize* the channel, as for linear modulation, enabling the efficient use of a Fast Fourier Transform (FFT) at the receiver. To this end, we can use several methods such as the introduction of a Cyclic Prefix (CP) or a known Unique Word (UW) (also called Training Sequence). We assume the use of a UW despite its loss of spectral efficiency compared to a CP as it can be used to increase the performance in case of Decision Feedback Equalizer or to perform some useful estimations, such as the carrier phase and frequency or the channel parameters. However, the proposed method still applies to CP-based equalization schemes.

After appending some termination symbols and the UW [13], the CPM signal is transmitted over a frequency-selective channel with impulse response $h_c(t) = \sum_{l=0}^{L-1} a_l \delta(t - \tau_l)$ where L is the number of paths, τ_l and a_l are the delay and the complex attenuation of the l^{th} path.

As in [13], we assume that the transmitted CPM is roughly bandlimited to $B = \frac{k}{2T_s}$, $k \geq 2$. Then it can be sampled uniformly at a rate of $T_e = T_s/k$, without a significant loss of information. At the receiver, we assume ideal low-pass filtering using the front-end filter $\Psi(t)$. Denoting $h(t) = \Psi(t) * h_c(t)$, where $*$ is the convolution operator, the received signal can be written as [13]:

$$r(t) = h(t) * s(t) + w(t) = \sum_m s(m\frac{T_s}{k})h\big(t - m\frac{T_s}{k}\big) + w(t) \tag{2}$$

$w(t)$ is a complex baseband additive white Gaussian noise with power spectral density 2 N_0.

In the following, in order to derive the analytic expression of the auto-correlation of the over-sampled received signal, we will consider the LD presented in [3] for binary CPM signals with non-integer modulation indices, without loss of generality and for ease of presentation. The LD allows us to describe the CPM signal as a sum of P pulses modulated by complex pseudo-symbols. Based on this decomposition, exact expression of the time-averaged auto-correlation of the transmitted signal can be easily derived [3], [14]. The results presented in this letter can be extended as the LD has been expanded in [15] for M-ary CPM signals or for integer modulation indices in [16].

By simply reordering the samples, compared to the representation given in [2], we can derive a classical fractionally-spaced (FS) representation that does not make use of the multi-channels representation (as for the polyphase representation developed in [2]). In this FS representation, by denoting $T_e = T_s/k$, the received signal can be written as follows $\boldsymbol{r} = [r(0), r(T_e), r(2T_e), \ldots, r((kN-1)T_e)]^T$ where, from equation (2), we have

$$r[l] \triangleq r[lT_e] = \sum_m s(m\frac{T_s}{k})h\Big((l-m)\frac{T_s}{k}\Big) + w(l\frac{T_s}{k}) \tag{3}$$

$$= \sum_m s[m]h[l-m] + w[l] \tag{4}$$

This can be rewritten as $\boldsymbol{r} = \boldsymbol{hs} + \boldsymbol{w}$ where $\boldsymbol{h}$ is a circulant matrix with first column $\boldsymbol{h} = [h[0], h[1], \ldots, h[L_c - 1], 0, \ldots, 0]^T$ and L_c is the over-sampled channel impulse length. Note that $\boldsymbol{s}$ is the vector of collected samples from the over-sampled complex envelop of the transmitted CPM signal (which are correlated by nature), which is not to be confused with the transmitted data symbols vector $\boldsymbol{\alpha}$ as in SC-FDE for linear modulations. Finally, this can be stated in the frequency-domain as

$$\boldsymbol{R} = \boldsymbol{F}_{kN}\boldsymbol{r} = \boldsymbol{HS} + \boldsymbol{W} \tag{5}$$

where $\boldsymbol{F}_{kN}$ is the Fourier matrix of size $kN \times kN$, $\boldsymbol{H} = \boldsymbol{F}_{kN}\boldsymbol{h}\boldsymbol{F}_{kN}^H$, $\boldsymbol{S} = \boldsymbol{F}_{kN}\boldsymbol{s}$ and $\boldsymbol{W} = \boldsymbol{F}_{kN}\boldsymbol{w}$. By DFT properties, $\boldsymbol{H}$ is a diagonal matrix with $\boldsymbol{H} = \text{diag}(\boldsymbol{F}_{kN}\boldsymbol{h})$.

TABLE I

COMPUTATIONAL COMPLEXITY COMPARISON BETWEEN DIFFERENT EQUALIZERS FOR CPM WITH AN OVERSAMPLING FACTOR $k = 2$

Receiver Type	FFTs and IFFTs	Equalizer Calculation	Equalization and Filtering
Linear MMSE-TDE [1]	0	$+8N^3$	$+O(PN^3)$
LD-FS-MMSE-FDE [1]	$2N\log(2N) + PN\log(N)$	$+8N^3$	$+O(PN^3)$
PP-MMSE-FDE [2]	$2N\log(N) + PN\log(N)$	$+8N^3$	$+O(PN^3)$
Approximated PP-MMSE-FDE [2]	$2N\log(N) + PN\log(N)$	$+O(2N)$	$+O(PN)$
New Exact low-complexity MMSE-FDE	$2N\log(2N) + PN\log(N)$	$+O(2N)$	$+O(PN)$

III. MMSE EQUALIZATION FOR FS REPRESENTATION

The optimal equalization approach consists in performing jointly the channel equalization and the detection of the transmitted sequence. However, due to the non-linearity of CPMs, the complexity of this approach is prohibitive. An usual choice is to perform separately the channel equalization and the CPM detection. In this context, we derive in the following a linear FD equalizer based on the MMSE criterion. The output of the equalizer is then used for classical MAP detection (see for example [17]) as done in [2]. Let $\boldsymbol{J}_{\text{MMSE}}$ be the matrix of size $kN \times kN$ minimizing the following Mean Square Error (*MSE*) criterion $\text{MSE} = \mathbb{E}\Big\{(\boldsymbol{S}-\boldsymbol{J}_{\text{MMSE}}\boldsymbol{R})^H(\boldsymbol{S}-\boldsymbol{J}_{\text{MMSE}}\boldsymbol{R})\Big\}$.

A straightforward derivation [18] shows that the matrix $\boldsymbol{J}_{\text{MMSE}}$ of size $kN \times kN$ minimizing the MSE is given by:

$$\boldsymbol{J}_{\text{MMSE}} = \boldsymbol{R}_{\text{SS}}\boldsymbol{H}^H\boldsymbol{K}^{-1} \tag{6}$$

with $\boldsymbol{K} = \boldsymbol{H}\boldsymbol{R}_{\text{SS}}\boldsymbol{H}^H + N_0\boldsymbol{I}_{kN}$ and $\boldsymbol{R}_{\text{SS}}$ is the time-averaged auto-correlation matrix of $\boldsymbol{S}$. We note that the matrix $\boldsymbol{R}_{\text{SS}}$ can be precomputed using the LD [3] for binary CPM with non-integer modulation indices as shown in [2] and thus stored at the receiver side.

This equalizer is equivalent in its derivation to the one using the polyphase representation presented in [2]. The only difference is within the use of the FS representation: it leads to a different ordering of the matrices elements implied in equation (6), exhibiting nice properties. Indeed, we will show in the following that $\boldsymbol{R}_{\text{SS}}$, as $\boldsymbol{H}$, is a diagonal matrix for the FS representation (contrary to the polyphase representation), which enables a lower complexity for the equalizer. First, as for linear modulation, the use of a UW or a CP introduces an equivalence between the linear convolution and a circular convolution in Eq.(2). Thus, the received sequence corresponds to the circular convolution of the circularly extended transmitted sequence (composed by the data block and the xUW) and the channel impulse response. Therefore, over a finite-time observation window corresponding to this block, this circular convolution can be seen as a linear convolution of $\boldsymbol{h}$ and a periodic version of $\boldsymbol{s}$. Hence, by considering this periodic version of $\boldsymbol{s}$ while deriving the statistical properties, the time-averaged auto-correlation function of $\boldsymbol{s}$ is periodic of period kN over this block. As $s(t)$ is a complex signal, the auto-correlation is Hermitian, i.e. $r^*_{\text{ss}}(l) = r_{\text{ss}}(-l)$. We now obtain: $r^*_{\text{ss}}(l) = r_{\text{ss}}(-l) = r_{\text{ss}}(kN-l)$. Then, we can show that the time-domain time-averaged auto-correlation matrix is circulant:

$$\boldsymbol{r}_{\text{ss}} = \begin{bmatrix} r_{\text{ss}}(0) & r^*_{\text{ss}}(1) & r^*_{\text{ss}}(2) & \dots & r^*_{\text{ss}}(kN-1) \\ r_{\text{ss}}(1) & r_{\text{ss}}(0) & r^*_{\text{ss}}(1) & \dots & r^*_{\text{ss}}(kN-2) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ r_{\text{ss}}(kN-1) & r_{\text{ss}}(kN-2) & r_{\text{ss}}(kN-3) & \dots & r_{\text{ss}}(0) \end{bmatrix} \tag{7}$$

$$= \begin{bmatrix} r_{\text{ss}}(0) & r_{\text{ss}}(kN-1) & r_{\text{ss}}(kN-2) & \dots & r_{\text{ss}}(1) \\ r_{\text{ss}}(1) & r_{\text{ss}}(0) & r_{\text{ss}}(kN-1) & \dots & r_{\text{ss}}(2) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ r_{\text{ss}}(kN-1) & r_{\text{ss}}(kN-2) & r_{\text{ss}}(kN-3) & \dots & r_{\text{ss}}(0) \end{bmatrix} \tag{8}$$

We can see that $\boldsymbol{r}_{\text{ss}}$ is finally circulant with first column $\boldsymbol{r}_{\text{ss}}$ being $[r_{\text{ss}}(0), r_{\text{ss}}(1), \dots, r_{\text{ss}}(kN-1)]^T$. The auto-correlation matrix in the frequency domain $\boldsymbol{R}_{\text{SS}}$ is thus diagonal by DFT property. We point out that this diagonalization is not possible in case of [2] as they do not use a FS representation of the received signal resulting in a different auto-correlation matrix in the time-domain. Moreover the diagonal terms are all real. Hence, the equalizer $\boldsymbol{J}_{\text{MMSE}}$ is simply a diagonal matrix with generic term given by $J[l] = R_{\text{SS}}[l]H^*[l](R_{\text{SS}}[l]|H[l]|^2 + N_0)^{-1}$.

If we impose the coarse approximation $\boldsymbol{R}_{\text{SS}} = \boldsymbol{I}_{kN}$ (i.e. neglecting the signal correlation and assuming the signal energy normalized to 1), the equalizer becomes $J_{\text{approx}}[l] = H^*[l](|H[l]|^2 + N_0)^{-1}$ which corresponds to the highly suboptimal equalizer proposed in [8] with $k = 2$.

IV. COMPUTATIONAL COMPLEXITY ANALYSIS

In this section, we analyze the computational complexity of the proposed equalizer. The complexity will be expressed in number of floating-point operations (flops) per block as in [2], where a flop corresponds to one real multiplication plus one real addition [1].

We have shown that $\boldsymbol{R}_{\text{ss}}$ and $\boldsymbol{K}$ are diagonal matrices. Hence, the computation of $\boldsymbol{K}^{-1}$ requires $4kN$ real multiplications and $2kN$ real additions, plus kN real divisions for the inversion. The product $\boldsymbol{R}_{\text{ss}}\boldsymbol{H}^H\boldsymbol{K}^{-1}$ is done with $3kN$ real multiplications. Thus, the computation of $\boldsymbol{J}_{\text{MMSE}}$ requires $2kN$ real additions, $7kN$ real multiplications and kN real divisions. The equalization is then performed with kN complex multiplications as we only deal with a diagonal matrix of size kN. So, we can conclude that the complexity of our receiver is dominated by the DFT and IDFTs operations and is in $O\big(kN\log(kN) + PN\log(N)\big)$. This is the same for the low complexity MMSE-FDE proposed in [8] (only removing $2kN$ real multiplications), and comparing our equalizer with equalizers leading to the same performance (as it will be shown in the following section), TABLE I highlights that the computational complexity of our receiver is lower.

V. SIMULATIONS RESULTS

We first compare, by the means of simulation, the proposed equalizer (called “FS-MMSE-FDE”) with the following other exact MMSE-FDE equalizers: (a) the equalizer proposed in [2]. This equalizer uses the polyphase representation and produces an estimated signal by only considering the channel contribution. It will be called “PP-MMSE-FDE”; (b) the equalizer proposed in [1]. This equalizer both considers the

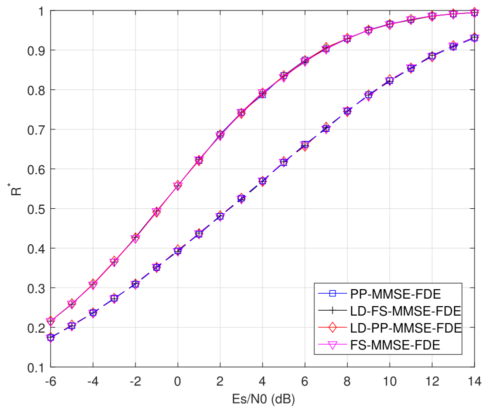

Fig. 1. Achievable coding rate over chan 1.

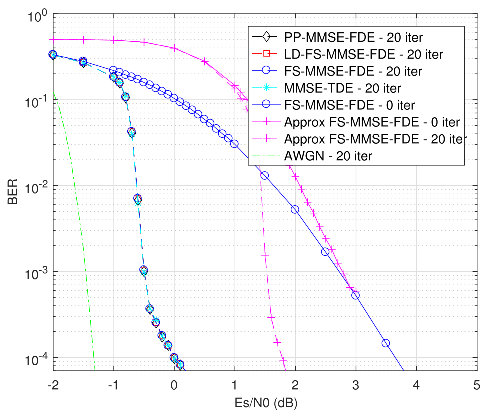

Fig. 2. BER over the aeronautical channel by satellite.

contribution of the channel and the LD to produce an estimate of the LD pseudo-symbols using the FS representation. It will be called "LD-FS-MMSE-FDE"; (c) a modified version of the equalizer proposed in [1], using a polyphase representation instead of a FS. It will be called "LD-PP-MMSE-FDE".

For the simulations, we consider two binary CPM schemes with a raised-cosine pulse shape (noted *RC*), a memory of $L_{\text{cpm}} = 3$ and a modulation index $h \in \{\frac{1}{4}, \frac{1}{2}\}$. The transmitted signal is composed of 8 blocks of 512 symbols, where each block is composed of data symbols, termination symbols and a Unique Word of size 16. The channel is the channel proposed in [5] as *chan 1*, but with modified delays: multiples of $T_s/2$ instead of T_s and the same Power Delay Profile.

Fig. 1 plots the obtained achievable rates for the 4 compared MMSE-FDE. The achievable coding rate is estimated by computing the area under the Extrinsic Information Transfer Chart (*EXIT chart*) of the detector [19]. The results show that the proposed equalizer has the same performance as the equalizers [2] and [1] (actually both having the same performance, as it has been shown in [4], up to post-treatment which is applied here).

Fig. 2 plots obtained bit error rates for the binary RC CPM scheme with $h = 1/2$, considering now a coded transmission and an iterative concatenated scheme between the CPM MAP detector and the channel MAP decoder. Used channel for simulations is an aeronautical channel, modeled as a two-paths channel with a second path delayed by $3.7T_s$ compared to the first one. A convolutional code with polynomial generators $(5, 7)_8$ has been added and 20 iterations have been done in the iterative procedure between the CPM MAP detector and the channel MAP decoder. For the same complexity, the fact that we do not approximate the auto-correlation matrix, as it is done in [8], allows our equalizer to increase the performance of 2dB at a BER of 10^{-3} and to achieve the same performance of other state-of-the-art approaches.

VI. Conclusion

A new exact MMSE-FDE for CPM waveforms is presented, having a lower complexity for the same performance compared to the MMSE-FDE proposed in the literature. Indeed, based on a FS representation for the circular block-based CPM, the proposed frequency-domain equalizer only needs the inversion of diagonal matrices as for linear modulations.

References

[1] F. Pancaldi and G. M. Vitetta, "Equalization algorithms in the frequency domain for continuous phase modulations," *IEEE Trans. Commun.*, vol. 54, no. 4, pp. 648–658, Apr. 2006.

[2] W. V. Thillo, F. Horlin, J. Nsenga, V. Ramon, A. Bourdoux, and R. Lauwereins, "Low-complexity linear frequency domain equalization for continuous phase modulation," *IEEE Trans. Wireless Commun.*, vol. 8, no. 3, pp. 1435–1445, Mar. 2009.

[3] P. Laurent, "Exact and approximate construction of digital phase modulations by superposition of amplitude modulated pulses (AMP)," *IEEE Trans. Commun.*, vol. COMM-34, no. 2, pp. 150–160, Feb. 1986.

[4] R. Chayot, N. Thomas, C. Poulliat, M.-L. Boucheret, N. Van Wambeke, and G. Lesthievent, "Channel estimation and equalization for cpm with application for aeronautical communications via a satellite link," in *Proc. IEEE MILCOM*, Oct. 2017, pp. 888–893.

[5] J. Tan and G. L. Stuber, "Frequency-domain equalization for continuous phase modulation," *IEEE Trans. Wireless Commun.*, vol. 4, no. 5, pp. 2479–2490, Sep. 2005.

[6] M. Rice and E. Perrins, "A comparison of frequency-domain equalization techniques in aeronautical telemetry," in *Proc. IEEE LATINCOM*, Nov. 2017, pp. 1–6.

[7] C. Brown and P. Vigneron, "Equalisation for continuous phase modulation using basis functions," *Proc. SPIE*, vol. 8061, p. 806109, Apr. 2011.

[8] B. Ozgul, M. Koca, and H. Delic, "Double turbo equalization of continuous phase modulation with frequency domain processing," *IEEE Trans. Commun.*, vol. 57, no. 2, pp. 423–429, Feb. 2009.

[9] M. Tüchler and A. C. Singer, "Turbo equalization: An overview," *IEEE Trans. Inf. Theory*, vol. 57, no. 2, pp. 920–952, Feb. 2011.

[10] X. Xu, H. Zhang, H. Ma, Z. Song, and Y. Cai, "Low-complexity fractional-spaced equalization approach in the frequency domain for CPM," in *Proc. ISCIT*, Sep. 2009, pp. 1301–1306.

[11] D. Darsena, G. Gelli, F. Verde, and I. Iudice, "LTV equalization of CPM signals over doubly-selective aeronautical channels," in *Proc. IEEE MetroAeroSpace*, Jun. 2016, pp. 75–80.

[12] D. Darsena, G. Gelli, I. Iudice, and F. Verdes, "Equalization techniques of control and non-payload communication links for unmanned aerial vehicles," *IEEE Access*, vol. 6, pp. 4485–4496, 2018.

[13] W. Van Thillo, V. Ramon, A. Bourdoux, F. Horlin, K. Sleurs, and R. Lauwereins, "Training sequence versus cyclic prefix for CPM with frequency domain equalization," in *Proc. IEEE GLOBECOM*, Honolulu, HI, USA, Nov. 2009, pp. 1–5.

[14] D. Darsena, G. Gelli, I. Iudice, and F. Verde, "Second-order statistics of one-sided CPM signals," *IEEE Signal Process. Lett.*, vol. 24, no. 10, pp. 1512–1516, Oct. 2017.

[15] U. Mengali and M. Morelli, "Decomposition of M-ary CPM signals into PAM waveforms," *IEEE Trans. Inf. Theory*, vol. 41, no. 5, pp. 1265–1275, Sep. 1995.

[16] X. Huang and Y. Li, "The PAM decomposition of CPM signals with integer modulation index," *IEEE Trans. Commun.*, vol. 51, no. 4, pp. 543–546, Apr. 2003.

[17] G. Colavolpe and A. Barbieri, "Simplified iterative detection of serially concatenated CPM signals," in *Proc. IEEE GLOBECOM*, vol. 3, Nov. 2005, p. 5.

[18] M. Honig, U. Madhow, and S. Verdu, "Blind adaptive multiuser detection," *IEEE Trans. Inf. Theory*, vol. 41, no. 4, pp. 944–960, Jul. 1995.

[19] J. Hagenauer, "The exit chart-introduction to extrinsic information transfer in iterative processing," in *Proc. EUSIPCO*, Vienna, Austria, 2004, pp. 1541–1548.