BLIND ESTIMATION OF UNKNOWN TIME DELAY IN PERIODIC NON-UNIFORM SAMPLING: APPLICATION TO DESYNCHRONIZED TIME INTERLEAVED-ADCS

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ABSTRACT

Increasing the sampling rate of Analog-to-Digital Converters (ADC) is a main challenge in many fields and especially in telecommunications. Time-Interleaved ADCs (TI-ADC) were introduced as a technical solution to reach high sampling rates by time interleaving and multiplexing several low-rate ADCs at the price of a perfect synchronization between them. Indeed, as the signal reconstruction formulas are derived under the assumption of uniform sampling, a desynchronization between the elementary ADCs must be compensated upstream with an online calibration and expensive hardware corrections of the sampling device. Based on the observation that desynchronized TI-ADCs can be effectively modeled using a Periodic Non-uniform Sampling (PNS) scheme, we develop a general method to blindly estimate the time delays involved in PNS. The proposed strategy exploits the signal stationarity properties and thus is simple and quite generalizable to other applications. Moreover, contrarily to state-of-the-art methods, it applies to bandpass signals which is the more judicious application framework of the PNS scheme.

Index Terms— Nonuniform sampling, Estimation, Stationary random process, Analog-to-Digital Converters

1. INTRODUCTION

The evolution of communication systems implies the transmission of signals with increasing frequencies, requiring subsequent adaptations of the sampling devices. In this context, the design of ADCs performing at very high frequency is a huge economical and technological challenge. Thus, an alternative solution has been developed by time-interleaving and multiplexing several low-frequency and thus low-cost ADCs to form a TI-ADC [1, 2]. The sampling operation is shared between elementary ADCs to reach a high global sampling frequency. However, since a TI-ADC is expected to perform a global uniform sampling operation, the elementary ADCs must have similar intrinsic properties and, especially, they must be perfectly synchronized [3, 4]. Online solutions have been previously considered [5–7] for the estimation and correction since the desynchronization can appear and vary during functioning. Nevertheless, these methods consider baseband signals in their theoretical developments which is not realistic for telecommunications. They require hardware calibrations and corrections to impose uniform sampling, unfortunately requiring a system disconnection and increasing complexity and power consumption. An alternative and more flexible sampling model can be considered for TI-ADCs in order to avoid hardware operations: the PNS scheme [8–12]. This model allows to take into account the desynchronization, once estimated, through generalized reconstruction formulas. Based on this observation, this paper develops a blind strategy for the desynchronization estimation and more generally for the blind estimation of time delays in a PNS scheme. This strategy operates directly on the transmitted bandpass signal, with no need for a training sequence. For telecommunication purpose and for more generality, we consider a bandpass random stationary signal model and we exploit this stationarity property. The estimation is performed from the reconstructed bandpass signal. Afterwards, the estimated delay is used to adapt the PNS reconstruction formulas. The paper is organized as follows. Section 2 formulates the problem and presents the signal and sampling models. Section 3 details the proposed method. The performance analysis is conducted in section 4. Section 5 contains concluding remarks and future work discussion.

2. PROBLEM FORMULATION

The method proposed in this paper is very general and could be envisioned in a theoretical way only, in relation with non uniform sampling. However, TI-ADCs provide a natural and illustrative application framework for this method because it is nontrivial and requires general methods applicable to ran-

dom bandpass signals.

2.1. Application: Desynchronization in TI-ADCs

TI-ADCs are composed of L elementary time-interleaved and multiplexed ADCs, each operating at a frequency f_s. If the delay between two consecutive ADCs is constant and equal to $\frac{1}{Lf_s}$, the TI-ADC performs a global uniform sampling operation at frequency Lf_s. However, in practice, disparities and design imperfections lead to the so-called mismatch errors: gain, offset and time-skew errors. These errors lead to non-linear distortions upon the reconstructed signal and many studies are devoted to their characterization and correction [4–6]. Moreover, under adverse (mainly thermic) operating conditions, this delay may vary during system functioning and perfect synchronization can be hardly guaranteed and maintained. Fig. 1 shows the modified architecture that we will consider in the following, taking into account the desynchronization (or time-skew error) which is considered as the most damaging error [3]. The classical multiplexing stage after the ADCs must be replaced by a reconstruction stage (REC) that uses the desynchronization between elementary ADCs denoted as $\delta_i, i = 0, ..., L - 1$ as an input. This paper proposes a method for the blind estimation of the desynchronization during system functioning. This estimation is then put back into the sampling model for an accurate reconstruction without hardware delay correction.
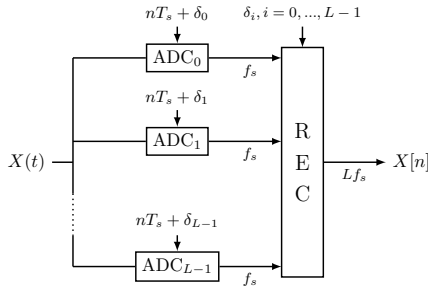

Fig. 1: General architecture of a TI-ADC

2.2. Signal model

We model the analog signal as a real wide-sense stationary random process $\mathbf{X} = \{X(t),\ t \in \mathbb{R}\}$ with zero mean, finite variance and power spectral density $s_X(f)$ defined by:

$$s_X(f) = \int_{-\infty}^{\infty} e^{-2i\pi f\tau} R_X(\tau)\, d\tau \qquad (1)$$

where $R_X(\tau) = \mathrm{E}[X(t)X^*(t - \tau)]$ is the auto-correlation function of $\mathbf{X}$ ($\mathrm{E}[\cdot]$ stands for the mathematical expectation and the superscript $*$ for the complex conjugate). This model fits the targeted applications in the field of telecommunications where the multiple carriers can be approximated by a random process model. Moreover, this telecommunications-modeling signal is assumed to be bandpass: its power spec-

trum $s_X(f)$ is included in the so-called k^{th} Nyquist band defined in normalized frequency by:

$$\mathcal{B}_{\mathcal{N}}(k) = \left(-(k + \frac{1}{2}), -(k - \frac{1}{2})\right) \cup \left(k - \frac{1}{2}, k + \frac{1}{2}\right). \quad (2)$$

In [13], the authors emphasized the challenge of using high frequencies (high k values) in the transmission because of the resulting sensitivity towards synchronization.

2.3. PNS2 sampling scheme

The PNSL (Periodic Non-uniform Sampling (PNS) of order L) is a sampling scheme composed of L uniform sampling sequences $\mathbf{X}_{\delta_i} = \{X(nT_s + \delta_i), n \in \mathbb{Z}\}$, $i = 0, ..., L - 1$ that are non-uniformly time-interleaved. $T_s = \frac{1}{f_s}$ denotes the common sampling period and δ_i denotes the delay of each sequence. We set that $\delta_i \neq \frac{iT_s}{L}$ (except $\delta_0 = 0$ by convention) and $\delta_i - \delta_{i-1} \neq \frac{T_s}{L}$, modeling the desynchronization. The sampling times are thus non uniformly distributed in a given sampling period. In the following we will consider a normalized sampling frequency for simplification, giving $T_s = 1$. PNSL has been investigated for its anti-aliasing properties [8, 10, 11] and its lower sampling frequency requirement in the case of bandpass signals [9, 12].

For simplicity and without loss of generality, this paper focuses on the PNS2 scheme [8]. Results can be extended to the case of L ADCs leading to more complex expressions for the reconstruction formulas [14]. Concerning the delay estimation step, a TI-ADC with L ADCs denoted $\text{ADC}_{0,...,L-1}$ can be calibrated by choosing ADC_0 as a reference and by estimating successively the delay between each $\text{ADC}_{1,...,L-1}$ and ADC_0 according to a simple PNS2 scheme.

In the case of PNS2, the samples are distributed according to two uniform sequences defined as $\mathbf{X}_0 = \{X(n), n \in \mathbb{Z}\}$ and $\mathbf{X}_{\delta} = \{X(n + \delta), n \in \mathbb{Z}\}$ with $\delta \in\ ]0, 1[$. The resulting mean sampling rate equals 2 and thus fits the signal effective bandwidth, for a real bandpass signal whose band is composed of two symmetric intervals of unit length (2). Under the condition that $2k\delta \notin \mathbb{Z}$, the exact reconstruction from an infinite number of samples is derived using the formula [15]:

$$\begin{aligned} X(t) &= \frac{A_0(t)\sin\left[2\pi k(\delta - t)\right] + A_\delta(t)\sin\left[2\pi kt\right]}{\sin\left[2\pi k\delta\right]}, \\ A_\lambda(t) &= \sum_{n \in \mathbb{Z}} \frac{\sin\left[\pi(t - n - \lambda)\right]}{\pi(t - n - \lambda)} X(n + \lambda). \end{aligned} \qquad (3)$$

with $\lambda \in \{0, \delta\}$. Note that, in previous works, the authors have proposed errorless reconstruction formulas (in term of mean-squared error) [16, 17], whose convergence rate can be increased by introducing appropriate filters [15] and which can perform joint reconstruction and interference cancelation or direct analytic signal construction [18]. These additional functionalities are of great interest in telecommunications.

The reconstruction formulas associated to PNS2 assume that the time delay δ is *a priori* known. For TI-ADC modeling

purpose, the time delay must be considered as unknown and possibly time-varying. However, the variations of δ, mainly resulting from thermal constraint evolution, can be assumed very slow with respect to its estimation computational time. Consequently, the proposed method considers a fixed value of δ during the observation time. Next section presents a strategy for the estimation of δ from the observation of the sequences $\mathbf{X}_0$ and $\mathbf{X}_\delta$ using stationarity properties.

3. PROPOSED METHOD

Now, we consider that the true delay δ is unknown. In this case, the reconstruction is performed using a wrong value of the delay denoted as $\tilde{\delta}$, whereas the sampling times are actually $\{n + \delta, n \in \mathbb{Z}\}$. Let $\mathbf{X}^{(\tilde{\delta})} = \{X^{(\tilde{\delta})}(t),\ t \in \mathbb{R}\}$ denote the reconstructed signal using $\tilde{\delta}$ instead of δ in (3):

$$X^{(\tilde{\delta})}(t) = \frac{A_0(t)\sin[2\pi k(\tilde{\delta}-t)] + A_\delta(t+\delta-\tilde{\delta})\sin[2\pi kt]}{\sin[2\pi k\tilde{\delta}]} \tag{4}$$

The strategy we propose for the estimation of δ is based on the observation that the reconstructed signal $\mathbf{X}^{(\tilde{\delta})}$ is not stationary in the general case of desynchronization (i.e. $\tilde{\delta} \neq \delta$). This property is demonstrated in the Appendix.

Our method exploits this non stationarity property to estimate δ. The reconstructed signal mean power is estimated at different times. Comparison of the estimates allows to detect whether the reconstructed signal is wide-sense stationary (time-independent mean power) or not.

First, the formula (4) is used to reconstruct the signal at uniform discrete times expressed as $t_m = n + \frac{m}{M+1}$, $m = 1,...,M$ and $n \in \mathbb{Z}$. Let $P_m^{(\tilde{\delta})}$ denote the expected power of the reconstructed signal at times t_m as:

$$P_m^{(\tilde{\delta})} = \mathrm{E}\left[\left|X^{(\tilde{\delta})}(t_m)\right|^2\right] \tag{5}$$

Let P_{ref} denote the reference power defined from the two available sampling sequences $\mathbf{X}_0$ or $\mathbf{X}_\delta$:

$$P_{\mathrm{ref}} = \mathrm{E}\left[|X(n)|^2\right] = \mathrm{E}\left[|X(n+\delta)|^2\right] \tag{6}$$

If $\tilde{\delta} = \delta$, the stationarity property implies that:

$$P_{m_1}^{(\tilde{\delta})} = P_{m_2}^{(\tilde{\delta})} (= P_{\mathrm{ref}}) \ \forall (m_1, m_2) \in [1, M] \tag{7}$$

On the contrary, if $\tilde{\delta} \neq \delta$, the equality (7) does not hold anymore. The principle of our method is to identify the value $\hat{\delta}$ of $\tilde{\delta}$ that respect (7). Next section studies the performance analysis of this estimation method.

4. PERFORMANCE ANALYSIS

Simulations are performed for a random stationary bandpass process in Nyquist band $\mathcal{B}_\mathcal{N}(7)$ ($k = 7$ in (2)). In the context of PNS2 sampling, a classical scheme has been identified

in telecommunications as *quadrature sampling*. This special sampling scheme has the property to give direct access to the in-phase and quadrature components by sampling using PNS2 with the use of $\delta = \frac{1}{4f_c}$ where f_c denotes the signal central frequency [13]. Here, $f_c = k = 7$ according to (2).

The desynchronization is modeled as an additive uniform random variable A and we have $\delta = \frac{1}{4k} + A$, $A \sim \mathrm{U}(-\varepsilon, \varepsilon)$ where ε is chosen according to the signal band properties. Indeed, following PNS2 definition, δ must respect the condition $2k\delta \notin \mathbb{Z}$ in order to perform the reconstruction. Applying that condition here, we have : $\delta \in]0, \frac{1}{2k}[$ resulting in $\varepsilon = \frac{1}{4k}$.

Assuming a delay $\tilde{\delta}$, the signal is reconstructed at times t_m using a window of N samples for each sampling sequence and a truncated version of (4):

$$\begin{cases} A_0(t) = \sum_{n=-\frac{N}{2}}^{\frac{N}{2}} \frac{\sin[\pi(t-n)]}{\pi(t-n)} X(n) \\ A_\delta(t+\delta-\tilde{\delta}) = \sum_{n=-\frac{N}{2}}^{\frac{N}{2}} \frac{\sin[\pi(t-n-\tilde{\delta})]}{\pi(t-n-\tilde{\delta})} X(n+\delta) \end{cases} \tag{8}$$

Mean powers are estimated for N reconstruction times using classical expectation estimators:

$$\widehat{P}_m^{(\tilde{\delta})} = \frac{1}{N} \sum_{n=-\frac{N}{2}}^{\frac{N}{2}} \left| X^{(\tilde{\delta})}\left(n + \frac{m}{M+1}\right)\right|^2 ,\ m = 1,...,M$$

We consider $M = 14$ and $N = 500$ in the following. Fig. 2 displays the estimated power curves, for a randomly chosen $\delta = 0.011$ and $\tilde{\delta} \in [0.001, 0.05]$.
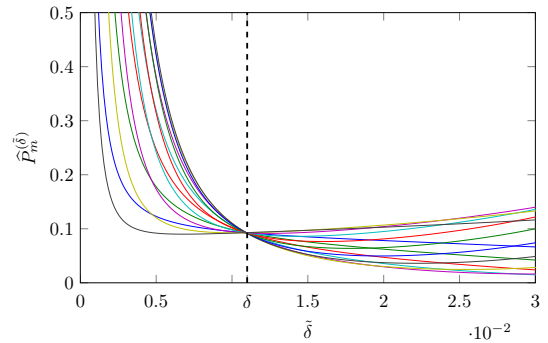

Fig. 2: Estimated power curves for $M = 14$.

According to Fig. 2, the curves all cross around the same point with coordinate δ. It validates the stationarity behavior: when the curves cross, all the estimated powers are equal and then the signal power is time-independent. The problem is that the equality in (7) does not strictly hold. Then, to find the point where the curves are the closer from each other, we estimate the variance of $\widehat{P}_m^{(\tilde{\delta})}$ for each value of $\tilde{\delta}$. This variance is displayed on Fig. 3 as a function of $\tilde{\delta}$. The minimum gives the best estimation $\hat{\delta}$ of δ defined as:

$$\hat{\delta} = \min_{\tilde{\delta} \in \left[\frac{1}{4k}-\varepsilon, \frac{1}{4k}+\varepsilon\right]} \left[\mathrm{var}\left(\widehat{P}_{m,\ m=1..M}^{(\tilde{\delta})}\right)\right] \tag{9}$$

We performed the estimation with an increasing number N of samples and we tested the results by computing the Signal-to-Error Ratio (SER) of the reconstructed signal using the estimation $\hat{\delta}$. Initially, without estimation, we have a SER of 2.4dB as the reconstruction is seriously damaged by the desynchronization. Fig. 4 plots the mean SER in dB as a function of N for $N_{\text{ite}} = 1000$ iterations and it shows that the estimation step helps compensating desynchronization, considering that, above 40dB, the reconstruction is satisfactory.

$$\text{SER}_{\text{dB}} = 10\log_{10}\left[\frac{P_{\text{signal}}}{P_{\text{error}}}\right] = 10\log_{10}\left[\frac{\text{E}[|X(t)|^2]}{\text{E}[|X(t)-X^{(\delta)}(t)|^2]}\right]$$
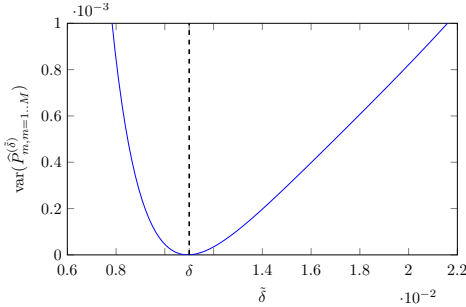

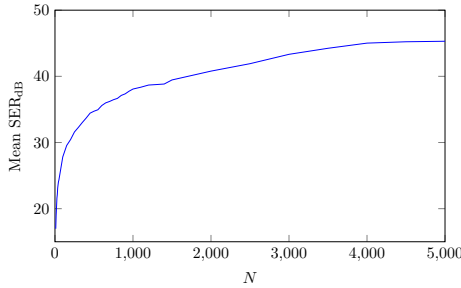**Fig. 3**: Variance of the estimated power curves for $M = 14$.

Fig. 4: Mean reconstruction SER in dB as a function of the number of samples, estimated for $N_{\text{ite}} = 1000$.

5. CONCLUSION AND FUTURE WORKS

This paper proposes a blind method to estimate and compensate desynchronization in a PNS2 sampling scheme. Exploiting stationarity properties of the reconstructed signal, it helps to build a flexible model for TI-ADCs in the case of bandpass signals. This method requires few a priori information. It performs on the signal samples and does not require learning sequences for system calibration. Moreover, for application to telecommunication purpose, the method applies on a realistic random wide-sense stationary bandpass signal model contrarily to the state-of-the-art methods that often deal with simple baseband signals such as sine waves. The simulations show that the estimation/compensation helps to retrieve satisfying reconstruction performance. Note that the desynchronization is assumed constant during the estimation. Consequently, the method must be performed periodically when the

desynchronization varies. The next step should be to develop an adaptive algorithm for online estimation of desynchronization variations. This will be part of a future work.

Appendix: Proof of the stationarity property

As the signal of interest is real and with spectrum relying into two separate bands (2), it can be decomposed as $\mathbf{X} = \mathbf{X}_+ + \mathbf{X}_-$, where $\mathbf{X}_+$ (respectively $\mathbf{X}_-$) stands for the signal component whose spectrum relies in the positive frequencies (respectively negative frequencies). Following the developments of [19], the principle of our derivations is to consider the isometry involving $\mathbf{X}_+$ and $\mathbf{X}_-$ as:

$$X_+(t) \xleftrightarrow{\mathcal{I}_{\mathbf{X}}} e^{2i\pi ft}\mid_{f>0} \text{ and } X_-(t) \xleftrightarrow{\mathcal{I}_{\mathbf{X}}} e^{2i\pi ft}\mid_{f<0}$$

From the equation above applied to $\mathbf{X}_+$ and $\mathbf{X}_-$, the following system can be obtained then leading to (3).

$$\begin{cases} A_0(t) = X_+(t)e^{-2i\pi kt} + X_-(t)e^{2i\pi kt} \\ A_\delta(t) = X_+(t)e^{-2i\pi k(t-\delta)} + X_-(t)e^{2i\pi k(t-\delta)} \end{cases} \quad (10)$$

Considering (5) and similar derivations as those leading to (10), the isometry relates $X_-^{(\tilde{\delta})}(t)$ (respectively $X_+^{(\tilde{\delta})}(t)$) to:

$$e^{2i\pi t(f-k)}\frac{\sin[2\pi k(\tilde{\delta}-t)]}{\sin[2\pi k\tilde{\delta}]} + e^{2i\pi f(t+\delta-\tilde{\delta})}e^{-2i\pi k(t-\tilde{\delta})}\frac{\sin[2\pi kt]}{\sin[2\pi k\tilde{\delta}]},$$

for $f < 0$ (respectively $f > 0$). Then, we obtain that the isometry relates $X(t) - X^{(\tilde{\delta})}(t)$ to:

$$e^{2i\pi ft}\frac{\sin[2\pi kt]}{\sin[2\pi k\tilde{\delta}]}\left[1 - e^{2i\pi f(\delta-\tilde{\delta})}\right]e^{2i\pi k(\tilde{\delta}-t)\text{sign}(f)} \quad (11)$$

where $\text{sign}(f) = 1$ if $f > 0$ and $\text{sign}(f) = -1$ if $f < 0$.

Now, let us consider the mean square reconstruction error defined by $\varepsilon_{\tilde{\delta}}^2(t) = \text{E}[|X(t) - X^{(\tilde{\delta})}(t)|^2]$. From (11), $\varepsilon_{\tilde{\delta}}^2(t)$ can be expressed as a function of $\tilde{\delta} - \delta$:

$$\begin{aligned} \varepsilon_{\tilde{\delta}}^2(t) &= \int_{-\infty}^{\infty}\left|\frac{\sin[2\pi kt]}{\sin[2\pi k\tilde{\delta}]}\left[1 - e^{2i\pi f(\delta-\tilde{\delta})}\right]\right|^2 s_X(f)df \\ &= 8\left(\frac{\sin[2\pi kt]}{\sin[2\pi k\tilde{\delta}]}\right)^2\int_0^{\infty}\sin^2\left[\pi f(\delta-\tilde{\delta})\right]s_X(f)df \end{aligned}$$

The reconstruction is not errorless except when $\tilde{\delta} = \delta$. Similarly we can develop the expression of the power of $X^{(\tilde{\delta})}(t)$:

$$\begin{aligned} \text{E}\left[\left|X^{(\tilde{\delta})}(t)\right|^2\right] &= \frac{8}{\sin^2[2\pi k\tilde{\delta}]}\int_0^{\infty}(\sin^2[\pi k\tilde{\delta}]\cos^2[\pi k(2t-\tilde{\delta})] \\ &+ \sin[2\pi kt]\sin[2\pi k(t-\tilde{\delta})]\sin^2[\pi f(\tilde{\delta}-\delta)+\pi k\tilde{\delta}])s_X(f)df \end{aligned} \quad (12)$$

showing that the power of $X^{(\tilde{\delta})}(t)$ depends on t so the result is not stationary, except when $\tilde{\delta} = \delta$. In this case, the expression in (12) simplifies using trigonometry formulas, to give $\text{E}\left[\left|X^{(\delta)}(t)\right|^2\right] = \text{E}\left[\left|X(t)\right|^2\right] = \int_{-\infty}^{\infty}s_X(f)df$ (which does not depend on t) as expected by definition of (1).

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