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CDMA satellite capacity dynamics with imperfect power control for simultaneous QoS classes

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ABSTRACT

In this paper, we focus on a non-GEO scenario and investigate the effect of actual power control performance on the return capacity dynamics of a DS-CDMA system. We consider mobile users at different speeds and transmission rates yielding several classes of services. We compute analytically the ideal maximum return capacity of the system which is shown not to be fixed but limited by an hyperplane in the space of the considered Quality of service (QoS) so that different resource assignment criteria can be applied. At maximum ideal capacity it is shown the different percentages of users that can be served, the more restraining QoS the more resources that must be assigned. We then explore the appropriate efficiency of the system, i.e. the actual used resources as a function of the distribution of QoS. We finally look into the inter-beam effect by applying different degrees of diversity gains and we show the trade-off between improving the capacity efficiency which is poor due to power control error and the net capacity achieved by the overall system.

I. Introduction

Unlike TDMA-based second generation planning tools which concentrated on coverage for a fixed given load, CDMA-based planning tools need to consider radio resource algorithms dynamic performance to achieve optimum dimensioning. Following we firstly describe the general assumptions about the channel and the non-GEO scenario under study which is based on [1]. We then compute the maximum capacity of the system by assuming ideal power control according to the algorithm introduced in [2] and then we apply power control error from [3].

We define QoS groups of users according to their speed and transmission rate. Since this power control error strongly depends on the mobile speed and environment yielding outage percentages QoS-dependent. We finally look into the inter-beam effect by applying different degrees of diversity gains and we show the trade-off between improving the capacity efficiency which is poor due to power control error and the net capacity achieved by the overall system.

II. Channel and power control models

The physical layer has been modeled with the wideband LMS (Land Mobile Satellite) channel model described in [1] which is based on a 3-state Markov channel model valid for any kind of satellite constellation and for L-, S- and Ka-bands. Fig. 1 shows a block diagram of the model.

Data from measurement campaigns was used to extract the parameters for the statistical distributions involved in the model. A basic feature of the model is its capability to generate timeseries of a number of channel parameters whose study is required: signal envelope, phase, instantaneous power delay profiles, Doppler spectra, etc. All these parameters can be generated for any elevation and environment for which empirical data is available. Sampling time can be selected allowing the generation/tracking of very slow (Markov states whose average duration depends on the environment), slow (long-term variations) or fast (short-term variations) fading as desired.

In [3] the timeseries generated by the model are used to parameterize power control imperfection as a function of the environment and user speed by acting upon the timeseries through a 7-level power control algorithm. The power correction step sizes defining the change introduced in the transmission power are 0 dB, ± 0.75 dB, ± 1.10 dB and ± 1.9 dB as in [2]. A simple up/down adjustment or several power adjustment levels are possible. Power control adjustment is always relative to the previous power setting, since absolute power setting would not be realistic due to the expensive circuitry needed.
Simulations were performed for constellation snapshots with satellite elevations within the following ranges: 15º-45º, 45º-65º, 65º-85º and then we computed average PC errors by applying PC algorithm with the satellite constellation in movement. These errors are totally defined by the standard deviation of the log-normal statistical behavior shown by the power control imperfections. Table I lists the power control error standard deviation for different user speeds, environments and satellite elevations considering an update time of 50 ms. It is apparent the worsening effect of the environment which decreases with elevation. The average values were computed by generating timeseries while the constellation was in movement. Iridium and Globalstar produced similar averages, which means that average values only depend on the physical environment.

<table>
<thead>
<tr>
<th>User speed</th>
<th>Elevation</th>
<th>Urban</th>
<th>Suburban</th>
<th>Open</th>
<th>Heavy Tree</th>
<th>Interm tree</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 m/s</td>
<td>15º - 45º</td>
<td>1.75</td>
<td>0.75</td>
<td>0.5</td>
<td>2</td>
<td>1.75</td>
</tr>
<tr>
<td></td>
<td>45º - 65º</td>
<td>1.2</td>
<td>1</td>
<td>0.5</td>
<td>1.75</td>
<td>1.75</td>
</tr>
<tr>
<td></td>
<td>65º - 85º</td>
<td>1</td>
<td>0.75</td>
<td>0.5</td>
<td>1.5</td>
<td>1</td>
</tr>
<tr>
<td>15 m/s</td>
<td>15º - 45º</td>
<td>2.25</td>
<td>2</td>
<td>0.75</td>
<td>2.5</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>45º - 65º</td>
<td>2</td>
<td>1.5</td>
<td>0.75</td>
<td>2</td>
<td>1.5</td>
</tr>
<tr>
<td></td>
<td>65º - 85º</td>
<td>1</td>
<td>1.5</td>
<td>0.75</td>
<td>1.5</td>
<td>1</td>
</tr>
<tr>
<td>30 m/s</td>
<td>15º - 45º</td>
<td>3</td>
<td>2.3</td>
<td>0.75</td>
<td>3.5</td>
<td>1.75</td>
</tr>
<tr>
<td></td>
<td>45º - 65º</td>
<td>2</td>
<td>1.75</td>
<td>0.75</td>
<td>3</td>
<td>1.5</td>
</tr>
<tr>
<td></td>
<td>65º - 85º</td>
<td>2</td>
<td>1.75</td>
<td>0.75</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

Table I. Power control error standard deviation for different user speeds, environments and satellite elevations. Power control update time is 50 ms.

In the following sections we select a number of speed and environment combinations defining different classes of services for a given transmission rate.

### III. DS-CDMA Up-Link Capacity

#### A. Ideal Power Control and no diversity
As a first approach to the problem of uplink capacity calculation, we start by studying the case where power control operates ideally and diversity capability is disabled.

Let us assume a beam serving \( N \) users that can be classified in one out of \( K \) groups with \( N_j, j = 1, \ldots, K \) users in each group. We consider that the attributes characterizing each group are the speed \( v_j \) and the binary rate \( R_j \). In the uplink reception, each one of the \( N \) received signals has to stay above a given threshold of bit energy-to-noise plus interference density, \( \eta = E_b/(N_o + I_0) \), to guarantee the required Quality of Service, QoS. Note that there are as many thresholds as groups: \( \eta_j, j = 1, \ldots, K \). The bit energy-to-noise plus interference density at the satellite receiver for each user is:

\[
\frac{E_b}{N_o + I_0} = \frac{B}{R_u} \frac{P_u}{N_t} \sum_{j=1}^{N} P_j
\]  

(1)
where $E_b$ is the bit energy, $B$ is the channel bandwidth, $R_u$ is the binary rate of the user of interest, $u$, $N_b$ is the AWGN density power, $N_j$ is the AWGN power and $P_j$, $j = 1,...,K$, is the received power from the rest of the mobile users in the antenna. Let us make the following change of notation

$$C_u = \frac{E_b}{N_b + I_{ru}} \frac{R_u}{B} \Rightarrow C_u = \frac{\text{SNR}_u}{1 + \sum_{j=1}^{N_j} \text{SNR}_j}$$

(2)

where $C_u$ is the power-to-noise plus interference power ratio threshold for the required QoS of user $u$ and $\text{SNR}_{j,u}$ is the power-to-noise ratio of user $j$ or $u$. Assuming without lose of generality that $u$ belongs to the first group, we can write

$$C_1 = \frac{\text{SNR}_1}{1 + (N_1 - 1) \text{SNR}_1 + \sum_{j \neq 1}^{K} N_j \text{SNR}_j}$$

(3)

Now we can combine the equations for the $K$ groups to obtain an expression relating the capacity of the up-link in terms of number of users of each group with the constants $C_j$, representing the quality required by a member of each group to establish a communication:

$$\sum_{j=1}^{K} C_j N_j < 1$$

(4)

Expression (4) is an hyperplane whose associated k-dimension vector is simply defined by the constants $C_j$. (4) can be easily proved by induction (see Appendix A) and allows computing permitted maximum number of users of each QoS. In this sense, expression (4) imposes a bound on the total capacity of the uplink. Since $C_j > 0$ and $N_j \geq 0$, the capacity reaches its maximum value when the equality holds as it was also obtained through different considerations in [4]. On the other hand, expression (4) serves as a criterion to determine whether the up-link has enough resources to hold a certain number of users of each group. This way, algorithms for selective admittance of communications in the beam can be developed from this criterion: a new user demanding a certain QoS is admitted in the beam if the new configuration of users served by the beam satisfies (4).

As a particular example of application, let us consider a situation where all the users belong to the same group $u$. Then the capacity of the beam ranges between 0 and $\lceil 1/C_u \rceil$, with $\lceil x \rceil$ being the integer closest to $x$ and $x \geq 0$. Furthermore, if a beam is serving two types of users (1 and 2), then for a fixed $N_j$, $N_2$ ranges from 0 to $\lceil (1-C_j/N_j/(C_j+1))/C_j \rceil$. According to this, when 3 groups are considered, the capacity of the up-link can be geometrically represented in terms of a vector of users of each group $(N_1, N_2, N_3)$. This vector of number of users is actually any point within the pyramid obtained as the intersection of planes $N_1 = 0$, $N_2 = 0$, $N_3 = 0$ and $C_1 N_1/(C_1+1)+C_2 N_2/(C_2+1)+C_3 N_3/(C_3+1)=1$. This way, the combinations of users $(N_1, N_2, N_3)$ that lie inside the volume of the pyramid are allowed, and those on the upper face maximize the up-link capacity.

The example showed in Fig. 2 compares two scenarios. Scenario 1 involves three groups of users transmitting at 2.4 kb/s characterized by speeds of 0 km/h (class 1), 3 km/h (class 2) and 70 km/h (class 3). The maximum values of capacity when only one of the three groups is present are $N_{1,max}=349$, $N_{2,max}=86$, $N_{3,max}=192$ respectively when considering $C_j=8.3-F$ dB, $C_j=3.8-F$ dB and $C_j=3.8-F$ dB [5] where $F$ is the spreading factor which is the ratio between chip rate (2048 Mc/s in UMTS) and bit rate. It can be observed that Class 2 is the most restrictive in terms of QoS, since each user demands a larger amount of uplink resources. The second scenario showed in Fig. 2 involves three groups of users transmitting at 64 kb/s also characterized by speeds of 0 km/h (class 1), 3 km/h (class 2) and 70 km/h (class 3). In this case the maximum values of capacity when only one of the three groups is present are $N_{1,max}=17$, $N_{2,max}=14$, $N_{3,max}=5$ respectively when considering $C_j=3-F$ dB, $C_j=3.8-F$ dB and $C_j=8.7-F$ dB [5]. It can be observed the dramatic decrease in the number of users that can be served when QoS are more demanding.

B. Resources assignment

By resource assignment we refer to the procedure followed to assign the up-link resources among the different users served by the beam. The procedure should maintain the capacity of the up-link at its maximum possible value, i.e., to serve as many users as possible. This means that even though all combinations of users lying inside the volume of the pyramid described in the previous section are allowed, we are only interested in those distributions of users that geometrically speaking belong to the plane of maximum capacity (see Fig. 2). It is clear that given a pyramid of a set of bit rates and speeds, the selection of a working point on the surface requires that a distribution of $(\alpha_i N_{i,max}, \alpha_2 N_{2,max}, \alpha_3 N_{3,max})$ must meet $\alpha_1 + \alpha_2 + \alpha_3 = 1$. The most
restrictive classes will consume most of the resources of the system and therefore a larger percentage must be assigned to them. Figure 3 shows different assignments.

C. From ideal case to real systems: Power Control Errors and Diversity Gain

Now we introduce in our model the influence of Power Control Error (PCE). PCE affects the system in two ways. In the one hand, a user suffering a power control error may transmit a weaker signal that will not reach the $E_b/N_0 + I_o$ needed in the up-link to guarantee the required BER. On the other hand, an incorrect power control may force a user to transmit with an extremely high power that increases the interference power experimented by the rest of the users sharing the up-link.

In the model adopted in this work, PCE follows a log-normal statistical distribution as it was explained in section II, which becomes a zero mean Gaussian distribution in the logarithmic units domain. The contribution of the PCE modifies the expression that calculates the $E_b/N_0$ for each type of traffic in the following way:

$$C_1 = \frac{e^{\gamma \text{SNR}_i}}{1 + (N_1 - 1)e^{\gamma \text{SNR}_i} + \sum_{j=2}^{K} N_je^{\gamma \text{SNR}_j}}$$

where $\gamma = \ln(10)/10$ and $\text{SNR}_j$ is a Normal random variable with mean the nominal signal-to-noise ratio in dB of the $j$-th user of a given class assuming ideal power control. In Fig. 4 the working point is shown in the pyramid while the reduction of capacity is plotted after taking into account the PCE.

IV. Simulations

Simulations were carried out taking into account significant scenarios to evaluate the impact of PCE and diversity on the real capacity of the up-link. For real capacity we mean the average number of users of each class whose QoS is above the required threshold.

Input data is shown in Table II. Figure 4 shows the contour of maximum uplink capacity in ideal conditions (left) and with error of power control (right). It can be observed the decrease of capacity due to the interference of those users transmitting above the required. As a consequence, those affected users actually have the up-link closed only during a certain percentage of the time. This percentage of the time is what we call availability of the up-link. To calculate this availability, a beam population that occupies the full capacity of the up-link (working point on the maximum capacity face of the pyramid) is created and real Power Control is simulated by means of Montecarlo. The second study concerns to the calculation of the amount of up-link resources that have to be left unassigned to any of the groups in order to guarantee 99% of the time with service to all the users of the different groups that coexist in the beam.
Fig. 4. Left, contour of the maximum up-link capacity pyramid under ideal conditions. Right, same scenario under log-normal PCE with standard deviations $\sigma_1=2.3$dB, $\sigma_2=1.4$dB and $\sigma_3=0$dB.

<table>
<thead>
<tr>
<th>Number of users</th>
<th>Up-link availability</th>
<th>Up-link capacity for availability of 99%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type C</td>
<td>63</td>
<td>Resources type C</td>
</tr>
<tr>
<td>Type A</td>
<td>117</td>
<td>Resources type A</td>
</tr>
<tr>
<td>Type B</td>
<td>29</td>
<td>Unused</td>
</tr>
<tr>
<td>Type C</td>
<td>63</td>
<td>Resources type C</td>
</tr>
<tr>
<td>Type A</td>
<td>117</td>
<td>Resources type A</td>
</tr>
<tr>
<td>Type B</td>
<td>29</td>
<td>Unused</td>
</tr>
</tbody>
</table>

Fig. 5. Power Control Error Impact on the real capacity of the up-link. For the scenarios depicted in Fig. 2, we plot the assigned capacity (left), the availability of the up-link experimented by each class of users (center) and the capacity that should be assigned to those classes of users if we want to guarantee that all of them experiment an up-link availability of 99%. Types A and D correspond to static users, B and E to pedestrians at 3 Km/h and C and F to vehicles at 70 Km/h; A, B and C transmit at 2.4 Kbps while D, E and F do at 64 Kbps. Finally, pedestrians are supposed to move in an open environment and vehicles in an urban one.

Table I. Numerical results for the Scenarios of study

<table>
<thead>
<tr>
<th></th>
<th>Type A</th>
<th>Type B</th>
<th>Type C</th>
<th>Type D</th>
<th>Type E</th>
<th>Type F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ideal Number of Users</td>
<td>117</td>
<td>29</td>
<td>63</td>
<td>2</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>Up-link availability</td>
<td>0.88 %</td>
<td>30.89 %</td>
<td>44.78 %</td>
<td>28.68 %</td>
<td>45.55 %</td>
<td>34.41 %</td>
</tr>
<tr>
<td>Number of users served with 99 % of availability</td>
<td>18</td>
<td>5</td>
<td>10</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

The population of the beam is iteratively reduced from the initial working point (maximum capacity) preserving the proportion of users belonging to each one of the groups initially in the beam until the availability experimented by those groups fall beyond 99%. The results of this experiment on the scenarios of study are depicted in Fig. 4 (right column) and in Table II.

REFERENCES

[3]. M. Vázquez-Castro, F. Pérez-Fontán, “LMS Markov model and its use for power control error impact analysis on CDMA system capacity”, Accepted by IEEE JSAC wireless series, special issue
Appendix

We start the proof of expression (4) rewriting equation (3) in a matrix form:
\[ \mathbf{A}_n \mathbf{s} = -\mathbf{c} \]  
(5)
\[
\begin{bmatrix}
C_1(N_1-1) & C_{N_1} & \ldots & C_N \\
C_{N_1} & C_1(N_1-1) & \ldots & C_{N_1} \\
\vdots & \vdots & \ddots & \vdots \\
C_N & C_{N_1} & \ldots & C_1(N_1-1)
\end{bmatrix}
\]
\[ \mathbf{s} = [S_1, S_2 \cdots S_n]^T \] and \[ \mathbf{c} = [C_1, C_2 \cdots C_n]^T. \]
The key of the demonstration is to show that equation (5) has positive solutions \(S_k\) iff expression (4) holds. The condition for (5) to have a solution is that \(\det(\mathbf{A}_n) \neq 0\) and, if so, solutions are given by \(S_k = \det(\mathbf{A}_n^k)/\det(\mathbf{A}_n)\), where \(\mathbf{A}_n^k\) is the matrix resulting from replacing column \(k\) in \(\mathbf{A}_n\) by \(-\mathbf{c}\).

The next step is to calculate \(\det(\mathbf{A}_n)\). We are going to prove by induction that
\[
\Delta = \left| \begin{array}{cccc}
C_{N_1}-C_{-1} & \ldots & C_N & C_{N_1} \\
C_N & \ldots & C_{N_1} & \ldots \\
\vdots & \ddots & \ddots & \vdots \\
C_{-1} & \ldots & \ldots & C_{N_1}
\end{array} \right| 
\]
(6)
To proceed this way we show that (6) is true for \(n=2\) and that if it is true for \(n\), then it will also be true for \(n+1\).

Now we need to prove that if expression (6) holds for \(n\), then it also does for \(n+1\). First we develop \(\Delta_{n+1}\) by its last column:
\[ \Delta_{n+1} = (-1)^{(n+1)-1} (C_{n+1}(N_{n+1}-1)) \Delta_n + \sum_{j=1}^{n} (-1)^{n+j} C_{n+j} N_j \Delta_i \]  
(7)
with
\[
\mathbf{A}_n = \begin{bmatrix}
C_{N_1} & C_{N_1} & \ldots & C_{N_1} \\
C_{N_1} & C_{N_1} & \ldots & C_{N_1} \\
\vdots & \vdots & \ddots & \vdots \\
C_{N_1} & C_{N_1} & \ldots & C_{N_1}
\end{bmatrix}
\]
To solve each \(\Lambda_i\), we take out \(C_{n+1}\) as a common factor of the last row and then subtract from each row \(j\) the last row multiplied by \(C_j\). After these transformations, \(\Lambda_i\) turns out to
\[ \Lambda_i = C_{n+1} \begin{bmatrix}
-C_{-1} & 0 & \ldots & 0 & 0 & 0 & \ldots & 0 \\
0 & -C_{-2} & \ldots & 0 & 0 & 0 & \ldots & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & \ldots & -C_{-1} & 0 & 0 & \ldots & 0 \\
0 & 0 & \ldots & 0 & 0 & -C_{-1} & \ldots & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & \ldots & 0 & 0 & 0 & \ldots & -C_{-1}
\end{bmatrix} 
\]
Now we develop \(\Lambda_i\) by its \(i\)th column, arriving to
\[ \Delta_i = C_{n+1} (-1)^{n+i} N_i \prod_{j=1}^{i-1} (-C_j - 1) = C_{n+1} (-1)^{n+i} N_i \prod_{j=1}^{i-1} (C_j + 1) \]
If we introduce it in (7), we get
\[ \Delta_n = (-1)^{(n+1)i} \left[ (-1)^{n+i} (C_{n+1} (N_{n+1} - 1) - 1) \Delta_n + C_{n+1} N_i \prod_{j=1}^{i} (C_j + 1) \sum_{j=1}^{n-i} \frac{C_{n+1} N_i}{C_j + 1} \right] \]
Now we substitute $\Delta_n$ by its assumed value, 6, and calculate the products inside the brackets arriving to

$$\Delta_{nt} = (-1)^{n+1} \prod_{k=1}^{n} \left[ -C_{nt} N_{nt} + (C_{nt} + 1) \sum_{m=1}^{n} \frac{CN_m}{C_m + 1} \right]$$

Finally, we take $(C_{n+1} + 1)$ as common factor and introduce the $-C_{n+1} N_{n+1} (C_{n+1} + 1)$ in the sum, resulting in

$$\Delta_{nt} = (-1)^{n+1} \prod_{k=1}^{n} \left[ 1 - \sum_{m=1}^{n} \frac{CN_m}{C_m + 1} \right]$$

what is equal to expression (6) evaluated for $n=n+1$, and therefore (6) is proved. Then, equation (5) has solutions when

$$1 - \sum_{j=1}^{\infty} \frac{C_j N_j}{(C_j + 1)} > 0$$

Next step is to calculate the numerator of $S_k$, $\det(A_n^k)$.

$$\det(A_n^k) = \begin{vmatrix} C_j N_j - C_j - 1 & C_j N_{j+1} - C_j & C_j N_{j+1} & \cdots & C_j N_n \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ C_j N_1 & \cdots & C_j N_{j+1} - C_j & C_j N_{j+1} & \cdots & C_j N_n - C_j - 1 \end{vmatrix}$$

Now we sum column $j$ multiplied by $N_i$ to each column $i$ of the determinant, and then develop it by column $j$, arriving at

$$\det(A_n^k) = (-1)^k C_k \prod_{j=k}^{n} (C_j + 1) \tag{8}$$

Combining (6) and (8), we can obtain that the sign of each $S_k$ is given by

$$\text{sign}(S_k) = \text{sign} \left( \frac{\det(A_n^k)}{\det(A_n)} \right) = \text{sign} \left( \frac{1}{1 - \sum_{j=1}^{\infty} \frac{C_j N_j}{(C_j + 1)}} \right) = \text{sign} \left( 1 - \sum_{j=1}^{\infty} \frac{C_j N_j}{(C_j + 1)} \right)$$

So, the condition for (5) to have positive solutions is,

$$1 - \sum_{j=1}^{\infty} \frac{C_j N_j}{(C_j + 1)} > 0 = \sum_{j=1}^{\infty} \frac{C_j N_j}{(C_j + 1)} < 1$$

that coincides with expression (4).